Fairness by design in shared-energy allocation problems

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Abstract

This paper studies how to aggregate agents with a focus on fairness, in particular in dynamic and stochastic frameworks. We suggest to use both *acceptability constraints* to ensure that each agent benefits from the aggregation, and *aggregation operators* that aim to distribute the costs and benefits fairly. Rather than using financial mechanisms to adjust for fairness issues, we focus on various objectives and constraints, within decision problems, that achieve *fairness by design*. We start from a simple single-period deterministic model and then generalize it to a dynamic and stochastic setting using *e.g.,* stochastic dominance constraints.
We illustrate our approach in the context of prosumer aggregation, where some prosumers may not be able to access the electricity market directly, although it would be beneficial to them. Therefore, new companies offer to aggregate them and promise to treat them fairly. This leads to a problem of *fair resource allocation.*

Keywords: Aggregation, Fairness, Stochastic Optimization, Prosumers

1 Introduction

Many domains, such as telecommunication networks, healthcare, disaster management, and energy-sharing systems, require fairness as a key criterion. Even if defining and enforcing fairness is difficult, mainly as it is subjective, mathematical models should not ignore this requirement. This paper investigates various methods to incorporate fairness in a multi-agent problem.

We focus on electric energy management applications, where the aggregation of prosumers is becoming more relevant due to their increasing number. Renewable energy generation capacities are becoming more affordable and effective as renewable energy investments are rising (19% in 2022, according to a report by International Renewable Energy Agency (2023) on global trends in renewable energy). This enables smaller prosumers (*i.e.,* agents that can be both producers and consumers), such as medium-sized industries, to invest in onsite energy generation and storage. However, prosumers are usually too small to access the electricity market directly, so some companies offer to aggregate them in electricity markets. For example, CPower is an American company that aggregates a total of 2.000 MW of power . We refer to Carreiro et al. (2017) for an extensive review on aggregators and their role in electricity markets.

Those aggregators can be external entities responsible for each prosumer energy transfer. In this case, it is necessary to consider how the aggregation affects the participants to ensure a fair allocation of benefits. This is highlighted in a report (EURELECTRIC, 2015) on designing fair and equitable market rules for demand response aggregation, published by the association representing the common interests of the European electricity industry, Euralectric. Indeed, in this case the aggregator needs to guarantee that each prosumer benefits from staying in the aggregation, and is not disfavored compared to others.

In the literature, one distinguishes between two main approaches in handling fairness: solve the problem efficiently and then reallocate the benefits (Yang et al., 2021; Wang et al., 2019; Yang et al., 2023); or change the objective function to get a fair solution (Xinying Chen and Hooker, 2023). In the first approach, we model a multi-agent problem with a utilitarian objective, *i.e.,* we optimize the aggregated objectives of agents. Then, the benefits are reallocated among agents according to some protocol. For example, *Shapley values* (Shapley, 1953) assess the marginal contribution of each agent in the group and determine their fair share. The second approach prioritizes fair solutions through the modeling by changing the objective function. We refer to Xinying Chen and Hooker (2023) for a comprehensive overview and guidelines on selecting an appropriate objective function to reflect fairness. The two most studied objective functions are *the minimax objective* (Rawls, 1971), which optimizes the least well-off agent's objective, and *the proportional objective* (Nash, 1950), derived from Nash's *bargaining solution*, which optimizes the logarithmic sum of agents' objectives. Note that these approaches require the well-being of different agents to be compared through a single value.

However, these approaches present some limitations. On one side, the proportional and minimax approaches focus merely on the objective function, not decisions. However, in some applications, there is more than one valuable characteristic. For example, in an energy contract, both the flexibility and the volume of energy traded are essential features. On the other hand, post-allocation distributions of benefits are not adapted to problems formulated over long periods, such as contracts in electricity markets. Indeed, those approaches require solving the whole problem before allocating costs which means that each agent have to wait until the problem's completion, which could span several months or years, to receive their fair share. Furthermore, given the inherent uncertainties linked to most problems, we also want our approach to hold in a stochastic framework. Then, fairness criteria must be redefined considering utility distributions and associated risks over time.

In this paper, we introduce various strategies for integrating fairness considerations into optimization problems. Our primary focus is what we refer to as *fairness-by-design*. Instead of relying on *ex-post* redistribution, as is usual in game theory (Shapley, 1953), we can establish a degree of fairness directly within the model. Our main contribution is to provide a framework and tools to accommodate fairness into mathematical models, particularly in prosumer aggregation. What sets our approach apart is extending this framework to dynamic and stochastic settings, allowing for risk-averse and time-consistent guarantees.

More specifically, we present two key elements for achieving fair allocation in an aggregation. First, we model fair cost allocation through an operator ordering the costs of the different prosumers. For the choice of this operator, we present three traditional approaches (*utilitarian, proportional* and *minimax*). Additionally, we propose *acceptability constraints* that ensure each agent's outcome improves in a predefined sense within the aggregation. In their simplest form, these acceptability constraints correspond to individual (or self) rationality in game theory, ensuring each agent benefits from participating to the aggregation. We then extend the problem to a dynamic framework in which decisions are made sequentially over time. In this context, the agent's costs are multidimensional, and the acceptability (or individual rationality) constraints thus need to choose a (partial) order. We discuss a few relevant partial-order choices. Similarly, in a stochastic framework, agents' costs are random variables, and we discuss relevant stochastic orders. Compared to Gutjahr et al. (2023), who propose a risk-averse stochastic bargaining game, our approach handles uncertainties through the objective function and also acceptability constraints. This enables us to consider various aspects of the impact of uncertainties on the problem. As a result, our proposed model is well-suited for addressing inherent uncertainties within multistage stochastic programs, enhancing its practical applicability. Finally, we assess these different strategies on a toy model in which we aggregate four electricity consumers to access the day-ahead market. We discuss the consequences of each modeling choice.

The remainder of the paper is organized as follows. In Section 2, we discuss the definitions of fairness and its integration into optimization models. In Section 3, we propose to model prosumers' aggregation with acceptability constraints and a fair objective function. We then illustrate the framework introduced on a toy model in Section 4. Section 5 expands the notion of acceptability into the dynamic framework, while Section 6 adapts acceptability and fairness to the stochastic framework.

Notations

To facilitate understanding, we go through some notations used in this paper. We denote for any integer n, $[n] := \{1, \dots, n\}$. Accordingly, $X_{[n]}$ denote the collection

$\{X_i\}_{i\in[n]}$. Random variables are denoted in bold characters; their realization in normal font. Finally, in this paper, the term operator always refers to a mathematical operator.

2 Fairness in the literature

In the Oxford Dictionary, fairness is defined as *the quality of treating people equally or in a way that is reasonable.* The definition is simple but subjective. Is treating people equally, regardless of any token of individuality, considered fair in society? Furthermore, what does it mean to be reasonable? Whatever take we have on fairness is necessarily subjective and context-dependent (see Konow (2003) for a philosophical analysis of fairness). In this section, we give a general overview of how fairness is defined and modeled across the scientific literature while linking it to our energy application. Bear in mind that each approach on fairness adopts a specific definition of fairness, which is not consensual.

2.1 Modeling and accommodating fairness

One of the main challenges facing fairness is the allocation of resources between agents. This naturally falls into the scope of Game Theory, where each of the N individuals (or players) is modeled with a utility function whose actual value depends on the actions of all players. For a given set of actions, we obtain a utility vector, denoted $u := (u_1, \ldots, u_N)$, representing the utility of every agent u_i. The utility vector is then said to be *fair* if it satisfies a set of properties that could vary from one specific definition of fairness to another. Among them, *individual rationality*, which ensures that every individual is better off in the aggregation and therefore accepts to be a part of it, is often required.

In a seminal contribution (Nash, 1950), John Nash introduced the bargaining problem in which two rational agents, allowed to bargain, try to maximize the sum of their utilities. If agents are rational, individual rationality must be ensured. This is modeled using a *disagreement point* which represents the outcome obtained by players if they cannot reach an agreement. For agents to cooperate, they must agree on properties a utility vector, $u_{[N]}$, should satisfy to be admissible. Nash proposed four axioms to constitute this agreement: *Pareto optimality* – we cannot improve the utility of one agent without decreasing another's utility; *Symmetry* – applying the same permutation to two utility vectors does not change their order; *Independence of irrelevant alternatives* – if a utility vector is the optimal utility vector within the feasible set, it remains so if the set is reduced; *Scale invariance* – applying affine transformations to the utility vector does not change the social ranking. Nash showed that, under some assumptions, including convexity and compactness of the set of feasible utility vectors, there exists a unique utility vector satisfying those axioms. This unique utility vector is regarded in the literature as a viable option when seeking fairness. Further, it has been shown that under convexity of the feasible set, it can be obtained by maximizing the product of utilities (Nash (1953); Muthoo (1999)), and thus by maximizing a logarithmic sum of utilities:

$$\max_{u \in \mathcal{U}} \quad \sum_{i=1}^{N} \log(u_i - d_i),$$

where $u \in \mathcal{U}$ is a feasible utility vectors among N players, and d the disagreement point. This approach is referred to as *proportional fairness*. Some papers criticized the *Independence of irrelevant alternatives* for having undesirable side effects. To overcome those issues, Kalai and Smorodinsky (1975) proposed to replace it with a *monotonicity* axiom, resulting in another unique utility vector and a slightly different vision of fairness.

In contrast to bargaining games, *cooperative games* study games in which coalition formation is allowed: see Osborne and Rubinstein (1994) for a complete introduction. In this theory, it is assumed that players can achieve superior outcomes by cooperating rather than working against each other. Players must establish their common interest and then work together to achieve it, which requires information exchanges. In *transferable utility games*, payoffs are given to the group which then divides among players through a post-allocation scheme. In Shapley (1953), Shapley studied a class of functions that evaluate the participation of players in a coalition. Considering a set of axioms (*i.e.,* symmetry, efficiency and law of aggregation[1]), Shapley showed that there exists a unique value function satisfying those axioms. He derived an explicit formula to compute the value of a player i in a cooperative game with a set N of players:

$$\phi_i(v) = \sum_{S \subset N \setminus \{i\}} \binom{|N|-1}{|S|}^{-1} \big(v(S \cup \{i\}) - v(S)\big),$$

where $v(S)$ gives the total expected sum of payoffs the coalition S can obtain. The values obtained $\{\phi_i(v)\}_{i \in N}$ are called Shapley values. They are considered a fair redistribution of gains in the group. They are, however, hard to compute in practice (as the size of the problem grows, those values are not computable).

A different approach was introduced by John Rawl in Rawls (1971): assuming that a group of individuals has no idea of their rank or situation in society, they will agree on a social contract aiming at maximizing the well-being of the least well-off. If the agents possess distinct characteristics, it might be difficult to compare them and ensure equitable treatment among them. This approach to fairness is often referred to as *minimax fairness*, as this amounts to optimizing the worst objective among agents.

The minimax approach, Shapley values, and Nash bargaining solutions remain the primary methods for addressing fairness in the literature. A detailed review of fairness modeling in optimization can be found in Xinying Chen and Hooker (2023), which provides guidelines for selecting fairness definitions and modeling approaches. The survey covers various fairness criteria and indicators but assumes that fairness can always be represented through a social welfare function, which corresponds to

[1] *i.e.,* when two independent games are combined, their values must be added player by player, see (Shapley, 1953, axiom 3)

utilities in game theory. This assumption implies that agent well-being is comparable through a single value, which may not be suitable for dynamic or stochastic settings. For fairness in resource allocation within communication networks, Ogryczak et al. (2014) provides an overview of relevant methods. In the context of energy systems, we refer to Soares et al. (2024) which presents both game theory and optimization-based fairness approaches, including minimax fairness, egalitarian methods, Shapley values and Nash bargaining.

When fairness is considered in the problem (through the objective or constraints), it comes at a price: a fair solution might not be the most efficient one. Indeed, many articles try to find a balance, or trade-off, between efficiency (have the best objective possible) and fairness (have a fair solution). Bertsimas et al. (2011) established bounds on the price of fairness for resource allocation problems with proportional fairness and minimax approaches.

In this section, we referred to work that laid the foundations of fairness modeling in mathematics. In the following section, we present some applications of aggregations and the way fairness is considered or evaluated.

2.2 Applications of fairness in the literature

In this paper, we focus on a *by-design* approach, meaning that fairness is already accommodated in an optimization model. Although fairness is commonly recognized as crucial, in most articles, the approach adopted derives from act utilitarianism: one should at every moment promote the greatest aggregate happiness, which consists in maximizing social welfare regardless of individual costs. For example, in Xiao et al. (2020), the authors studied an aggregator in charge of multiple agents within a power system. They optimized the total revenue of the aggregation without considering the impact on each agent individually. In Moret and Pinson (2019), a prosumers' aggregator can focus on different indicators (import/export costs, exchange with the system operator, peak-shaving services etc.) to optimize its trades with the energy market, and the trades between prosumers. The indicator to focus on must be agreed on by the prosumers. The authors gave a sensitivity analysis of the problem's parameters to determine what would increase the social acceptability of such an aggregation system. However, the model is utilitarian as it does not consider the allocation of costs among agents.

Other papers have proposed first optimizing the problem and then handling fairness through benefit post-allocation schemes. One way to deal with post-allocation is to model the aggregation as a coalitional game. This is the case of Freire et al. (2015), where the authors studied a risk-averse renewable-energy multi-portfolio problem. In order to get a fair and stable allocation of profits, they chose *the Nucleolus* approach, which finds a vector utility that minimizes the incentive to leave the aggregation for the worst coalition. In particular, this solution is in *the core* of the game, meaning every player gains from staying in the grand coalition. Similarly, in Yang et al. (2021), the authors studied a group of buildings with solar generation that mutually invest in an ESS. The approach is to, first, optimize the problem formulated as a two-stage stochastic coalition game. Then, a fair reallocation of costs is determined by computing the nucleolus allocation, which minimizes the minimal dissatisfaction of agents.

Some papers propose different methods to elaborate post-allocation schemes. For example, in Yang et al. (2023), the authors studied the joint participation of wind farms with shared energy storage. The solution is found by first solving a two-stage stochastic program and then reallocating the lease cost among users in a proportional scheme. They chose to make a wind farm pay depending on its increase of revenue after using the energy storage leasing service. In Wang et al. (2019), the authors valued cooperation in their model, which is another way to look at cost redistribution. They considered an aggregator that participates in the capacity and energy market for a number of energy users. In their model, the aggregator is not in charge of the users' decisions but of the trades with the energy market, therefore he must incentivize users to deviate from their optimal scheduling for minimizing total revenue. They proposed to solve an asymmetric Nash bargaining problem to determine the payoffs each user gets to deviate from their optimal scheduling. In another approach, the authors solved a multi-portfolio problem with fairness considerations in Iancu and Trichakis (2014). Instead of splitting the market impact costs in a pro-rata fashion, they introduced charging variables that are optimized in the model. This approach amounts to having transfer variables, which we avoid in this paper, as they may raise privacy and trust concerns in practical application. Instead, we simplify the approach by designating the aggregator as the sole entity with complete information on the problem, which pays agents directly depending on their actions.

Typically, fairness is dealt with through the objective function or in a post-allocation scheme. However, some researchers proposed constraints to ensure fairness. For example in Argyris et al. (2022), the authors constrained the allocation feasibility set for a resource allocation problem. They introduced a welfare function dominance constraint: the admissible set of social welfare functions must dominate a referenced one. Then, with a utilitarian objective, a trade-off between fairness and efficiency is obtained. An alternative approach, proposed in Oh (2022), is to bound a fairness indicator. The authors studied the energy planning of multiple agents over a virtual energy storage system (VESS), where energy dispatch is managed by an aggregator. They introduced two fairness indicators depending on the energy allocation and added constraints bounding them in a utilitarian model. Then, they compared the results with a minimax approach, where they optimized the minimal fairness indicator over agents.

In many cases, uncertainties are inherent to the problem. If multiple articles have dealt with uncertainties, they rarely have a stochastic take on fairness. For example, in both Yang et al. (2023) and Yang et al. (2021), the authors solved their problem with a two-stage program and then redistributed the costs fairly after uncertainty realization. Thus, there is no stochastic policy for fair redistribution. Other articles accommodated risk-averse profiles to game theory approaches. In Gutjahr et al. (2023), the authors studied a risk-averse extension of the Bargaining Problem. They adapted Nash bargaining axioms to constrain the feasible utility vectors depending on the risk profile of players.

3 A shared-resource allocation problem in the context of a prosumer aggregator

We present here a general framework where a so-called *aggregator* aggregates independent agents' needs (industrial prosumers, residential units, virtual power plants...) and makes economic transactions for the collective. To make aggregation contracts attractive to agents, we encounter two distinct challenges: first, each agent needs to find the contract *acceptable*, ensuring that each agent derives substantial benefits from the aggregation; second, the decisions made by the aggregator, leading to benefits or losses for each agent, should be made fairly. Recall that, for practical reasons, we do not allow money transfers between agents.

In the following, Section 3.1 formalize the setting, Section 3.2 explore various objective functions that model fair decisions, and finally Section 3.3 introduce acceptability constraints.

3.1 Prosumers and market structure

We denote by $x^i \in \mathcal{X}^i$ the set of state and decision variables modeling an agent i. The technical constraints proper to agent i are represented through feasible set $\mathcal{X}^i$, while external constraints (for instance, market exchanges), common to all agents, are represented with feasible set $\mathcal{M}$. Finally, each prosumer wants to minimize a cost function $L^i : \mathcal{X}^i \to \mathbb{R}$, yielding the model (P^i). We denote v^i the optimal value of (P^i). Note that (P^i) can model problems in various contexts. In Section 4, we present the particular application of this framework to prosumers aggregation in energy markets.

We now consider an aggregator in charge of I agents, and denote $x := (x^i)_{i\in[I]}$. The aggregator in problem (A), aggregates agents' decisions into $h(x^1, \dots, x^I)$ to satisfy external constraints $\mathcal{M}$ (see (1c)). Typically, if x^i are purchase variables, h is the sum over agents i of x^i. In addition, the physical constraint of each agent must be conserved (see (1b)), while the external constraints bind all agents' decisions. Finally, on the one hand, constraint (1d) ensures that the cost of an agent i is within an acceptable set $\mathcal{A}^i_\alpha$ they have agreed on prior to optimization, where $\alpha \in [0, 1]$ sets the level of acceptability. On the other hand, $\mathcal{F}_I$ is the agent operator that computes the objective of the aggregator considering the I objective functions of all agents. Depending on the choices of the acceptability sets $\mathcal{A}^i_\alpha$ and the agent operator $\mathcal{F}_I$, discussed, respectively, in Section 3.2 and Section 3.3, we have obtained different approaches to the problem of shared resource allocation.

$$(P^i) \quad \underset{x^i}{\text{Min}} \ L^i(x^i) \quad \text{s.t.} \ x^i \in \mathcal{X}^i, \quad x^i \in \mathcal{M}.$$

$$
\begin{aligned}
(A) \quad \underset{x}{\text{Min}} \quad & \mathcal{F}_I((L^i(x^i))_{i\in[I]}) && && \text{(1a)}\\
\text{s.t.} \quad & x^i \in \mathcal{X}^i && \forall i \in [I] && \text{(1b)}\\
& h(x^1, \dots, x^I) \in \mathcal{M} && && \text{(1c)}\\
& L^i(x^i) \in \mathcal{A}^i_\alpha && \forall i \in [I]. && \text{(1d)}
\end{aligned}
$$

We assume that, for each agent i, the agent's problem (P^i) admits an optimal solution. The *disagreement point* $(L^i(x^i), \dots, L^i(x^i))$ corresponds to each agent operating independently, where x^i is the optimal solution of (P^i). Then if problem (A) has no solution, meaning that there is no aggregation satisfying constraint (1c) or (1d), the agents operate independently. Note that, if $x^i \in \mathcal{M}$ for all agents implies $h(x^1, \dots, x^I) \in \mathcal{M}$, and $\alpha = 1$, then the disagreement point is admissible for (A).

3.2 Fair cost aggregation

First, we focus on the way the aggregator operates to allocate aggregation benefits among participating prosumers. We later establish, in Section 3.3, the conditions under which agents agree to participate using acceptability constraints.

The most natural and efficient method is the so-called *utilitarian approach*:

$$\mathcal{F}_I^U((L^i(x^i))_{i\in[I]}) = \sum_{i\in[I]} L^i(x^i). \tag{2a}$$

This approach aims to minimize total costs independently from the distribution of costs among prosumers: fairness is set aside. Indeed, in case of heterogeneity of the objective functions, it is possible that one of the objective function L^i dominates the others, *i.e.,*

$$L^i(x^i) \geq L^k(x^k), \qquad \forall x^i \in \mathcal{X}^i, \quad \forall x^k \in \mathcal{X}^k,$$

in which case all efforts of the aggregation are focused on minimizing the dominant objective function. A possibility that falls out of the scope of this paper (see Section 2.1) is to solve (A) and then reallocate resources with a fair scheme or put money transfers in place. We study alternative agent operators that ensure fair allocation for various fairness definitions.

First, we consider the *proportional approach* based on Nash bargaining solutions (see Section 2.1). For this approach, we consider the set of reachable (dis)utilities $\mathcal{L} = \big\{(L^1(x^1), \dots, L^I(x^I)) \mid x^i \in \mathcal{X}^i, \forall i \in [I],\ h(x^1, \dots, x^I) \in \mathcal{M}\big\}$, and set the optimal values of (P^i), v^i [2], as the chosen disagreement point. Then, Nash (1950) introduces a set of axioms that must respect a fair distribution of (dis)utilities and show that if $\mathcal{L}$ is convex and compact[3], there exists a unique (dis)utility vector satisfying those axioms. Furthermore, it is proven that Nash's distribution is obtained by maximizing the sum of logarithmic utilities. For our problem, it corresponds to using the agent operator :

$$\mathcal{F}_I^P((L^i(x^i))_{i\in[I]}) := -\sum_{i\in[I]} \log(v^i - L^i(x^i)). \tag{2b}$$

Note that this approach tends to act in favor of smaller participants. Indeed, increasing a slight cost improvement is preferred to increasing an already large cost improvement. In the non-convex case, Nash's solution does not necessarily exist. However,

[2] We implicitly assume here that either there is a unique solution or that we have defined a way to select a solution among the set of optimal solutions.

[3] A sufficient condition for $\mathcal{L}$ to be convex is if $\mathcal{X}^i$ is convex compact, $\mathcal{M}$ convex, h and L^i linear.

if the solution set is comprehensive[4], Conley and Wilkie show in Conley and Wilkie (1996) that, even in the non-convex case, the solution of the optimization problem with objective (2b) satisfies weak-Pareto optimality, symmetry, scale invariance, continuity, and ethical monotonicity. This defines the Nash Extension solution. The setting considered in this paper is not convex but can be made comprehensive by adding a variable for additional losses.

Finally, Rawls' theory of justice leads to the *minimax approach* favoring the least well-off. Here, the operator we obtain is:

$$\mathcal{F}_I^M((L^i(x^i))_{i\in[I]}) := \max_{i\in[I]} \; L^i(x^i). \tag{2c}$$

This method may not be adequate for heterogeneous agents for similar reasons to the utilitarian approach, as it only focuses on minimizing the dominant objective function. To address this issue, we quantify an agent's well-being by looking at the proportional savings made in the aggregation. Then, applying Rawls' principle, we minimize the maximum proportional costs over agents, and we obtain the following agent operator:

$$\mathcal{F}_I^{SM}((L^i(x^i))_{i\in[I]}) := \max_{i\in[I]} \; \frac{L^i(x^i)}{v^i}, \tag{2d}$$

which we refer to as the *Scaled Minimax approach.* Note that in both the scaled minimax approach ($\mathcal{F}_I^{SM}$) and the proportional approach ($\mathcal{F}_I^P$), there are multiple solutions with different aggregated costs. We assume here that we have specified a way to select a solution among them.

3.3 Acceptability constraints

Having defined several methodologies for equitable cost distribution, we must convince agents to be part of the aggregation. We consider agents to be individually rational, that is, a contract cannot be deemed acceptable if at least one agent is not better off independently *i.e.,* $v^i \leq L^i(x^i)$, where $L^i(x^i)$ is the cost of i in the aggregation. We can go one step further and require that, to find the contract acceptable, they benefit from it *i.e.,* $v^i > L^i(x^i)$. We thus define the acceptability set $\mathcal{A}_\alpha^i$ appearing in (1d) as follows:

$$\mathcal{A}_\alpha^i := \{ \; u^i \mid u^i \leq \alpha \; v^i \; \}, \tag{3}$$

where $\alpha \in (0, 1]$ is given. Then, we say a solution is $\alpha-$acceptable if contained in $\mathcal{A}_\alpha^i$. Acceptability sets are independent from one agent to another. We then define *global acceptability* as the Cartesian product of all acceptability sets $\mathcal{A}_\alpha := \mathcal{A}_\alpha^{i_1} \times \cdots \times \mathcal{A}_\alpha^{i_I}$.

Adding acceptability constraints to (A) restricts the set of feasible solutions, which can lead to higher aggregate costs. We define the *price of acceptability* as

$$PoA := \frac{v^\star_{\mathcal{A}_\alpha} - v^\star_\emptyset}{v^\star_\emptyset}, \tag{4}$$

[4] see the definition provided by Conley and Wilkie (1996, pp. 3)

where $v^\star_{\mathcal{A}_\alpha}$ is the optimal value of (A) with acceptability constraints $\mathcal{A}_\alpha$, and $v^\star$ is the optimal value of (A) without them.

Remark 1. *In the scaled minimax model with agent operator $\mathcal{F}_I^{SM}$, the optimal solution is $1-$acceptable. Indeed, if the agents do not take advantage of the aggregation, then $L^i(x^i) = v^i$ and we get a feasible solution with respect to $\mathcal{A}_\alpha$ of optimal value 1. Further, we can see that finding the smallest α such that there exist an α-acceptable solution,* i.e.,

$$
\begin{aligned}
\min_{\alpha} \quad & \alpha && && \text{(5a)}\\
s.t. \quad & x^i \in \mathcal{X}^i && \forall i \in [I] && \text{(5b)}\\
& h(x^1, \dots, x^I) \in \mathcal{M} && && \text{(5c)}\\
& L^i(x^i) \in \mathcal{A}^i_\alpha && \forall i \in [I], && \text{(5d)}
\end{aligned}
$$

is equivalent to problem (A) with agent operator $\mathcal{F}_I^{SM}$ with no acceptability constraints.

Remark 2. *Our problem with the proportional operator $\mathcal{F}_I^P$ necessarily yields a strictly acceptable solution. Indeed, if for agent i, $L^i(x^i) \geq v^i$, then $\log(v^i - L^i(x^i))$ is undefined.*

Note that in this framework, each agent seeks to minimize individual costs, which may not always align with the interests of other participants. By modifying the aggregation objective (*e.g.,* using scaled minimax or proportional operators,) or by relaxing the acceptability requirements (increasing α), the framework provides a range of solutions where agents may act against their individual objectives but ultimately benefit from the group as a whole.

We later discuss how to extend the acceptability constraint to a dynamic (see Section 5.2) and stochastic framework (see Section 6.3). Finally, combining different objective functions with acceptability constraints, we illustrate their impact in the following section.

4 Application to consumer aggregation on the day-ahead and balancing market

In this section, we adapt and illustrate the framework presented in Section 3 to the problem of prosumer aggregation in electricity markets. More specifically, the prosumers have access to: the *day-ahead market*, where every day at 2 pm, prices, and electrical energies are set for all across Europe for the twenty-four hours of the next day; and the *balancing market* on which prosumers must buy or sell electricity at real-time prices to ensure power system balance. A minimum trade of 11 MWh of energy is required to participate in the day-ahead market.

We consider a toy model to illustrate the implications of each model proposed in Section 3. Therefore, we consider a problem with four consumers ($I = 4$) in five stages ($T = 5$). At each stage t, we must decide how much energy $q^{DA}_{t,i}$ (*resp.* $q^{B}_{t,i}$) to purchase from the day-ahead (*resp.* balancing) market for consumer i. Thus, in (P^i), we have

t	**1**	**2**	**3**	**4**	**5**
p_t^{DA}	2	16	1	10	1
p_t^{B}	6	25	5	15	5
$\underline{q}_t^{DA}$	11	11	11	11	11

	A_1	A_2	A_3	A_4
$\underline{q}_i$	0	5	0	2
$\overline{q}_i$	5	5	4	3
Q_i	10	25	8	15

Table 1: Parameters values

$x^i := (q^{DA}_{[T],i}, q^{B}_{[T],i})$. Each consumer has bounds $[\underline{q}_i; \overline{q}_i]$ on its electricity consumption, and a total consumption Q_i to meet at the end of the horizon, amounting to feasible set $\mathcal{X}^i$. Note that the upper bounds on electricity consumption simplify physical constraints that would ensure a finite volume of traded electricity. We introduce binary variables b_t^{DA}, representing the decision to buy in advance, to model the minimum volume requirement for the day-ahead market, which composes the external constraints $\mathcal{M}$. The objective for consumer i is to minimize its electricity costs:

$$L^i(x^i) = \sum_{t=1}^{T} [\, p_t^{DA} q_{t,i}^{DA} + p_t^{B} q_{t,i}^{B} \,], \tag{6a}$$

where p_t^{DA} (*resp.* p_t^B) is the price of electricity at t on the day-ahead (*resp.* balancing) market. We obtain the simple prosumer (P^i) and the aggregated model (A):

$$
\begin{aligned}
(P^i)\ \ \underset{x^i}{\text{Min}}\ \ & L^i(x^i) & (A)\ \ \underset{x}{\text{Min}}\ \ & \mathcal{F}_I\Big((L^i(x^i))_{i\in[I]}\Big) & \text{(6b)}\\
\text{s.t.}\ \ & \underline{q}_i \le q_{t,i}^{DA} + q_{t,i}^{B} \le \overline{q}_i \quad \forall t & \text{s.t.}\ \ & \underline{q}_i \le q_{t,i}^{DA} + q_{t,i}^{B} \le \overline{q}_i \quad \forall t\ \forall i & \text{(6c)}\\
& \sum_{t=1}^{T}(q_{t,i}^{DA} + q_{t,i}^{B}) \ge Q_i & & \sum_{t=1}^{T}(q_{t,i}^{DA} + q_{t,i}^{B}) \ge Q_i \quad \forall i & \text{(6d)}\\
& \underline{q}_t^{DA} b_t^{DA} \le q_{t,i}^{DA} \le M\, b_t^{DA} \quad \forall t & & \underline{q}_t^{DA} b_t^{DA} \le \sum_{i\in[I]} q_{t,i}^{DA} \le M\, b_t^{DA}\ \forall t & \text{(6e)}\\
& b_t^{DA} \in \{0,1\} \quad \forall t, & & b_t^{DA} \in \{0,1\} \quad \forall t, & \text{(6f)}
\end{aligned}
$$

where $\mathcal{F}$ is the chosen agent operator for the aggregation. We solve this small problem with the utilitarian operator $\mathcal{F}_I^U$, with the scaled minimax operator $\mathcal{F}_I^{SM}$ and with the proportional operator $\mathcal{F}_I^P$. For all agent operator , we solve[5] the problem with and without acceptability constraints, with $\alpha = 1$.

We refer to the model with agent operator $f \in \{U, SM, P\}$ (for Utilitarian, Scaled Minimax and Proportional) and acceptability constraints set $a \in \{\emptyset, \alpha\}$ (for no acceptability constraints, or acceptability constraints given by $\mathcal{A}_\alpha$) as m_a^f, and $\mathcal{A} = \emptyset$ corresponds to a model without acceptability constraints. Finally, we compute Shapley's values (see Appendix A for more details), commonly recognized as a fair solution, to compare them to the solutions we obtain with our models.

[5]To guarantee uniqueness of the solution, we select among the set of optimal solutions the one closest to zero *i.e.,* minimizing the sum of the squared variables.

We show on a small artificial illustration how all these models can lead to different solutions. Each model can be evaluated through two metrics: first, the efficiency of the model *i.e.,* the overall costs of the aggregation; second, the fairness of the model *i.e.,* how distributed are the costs over prosumers. For the prosumers' parameters and market prices, we use the data in Table 1. We observe the allocation of costs over consumers in Figure 1, the resulting percentage of savings made by each consumer in Table 2, and the detail of day-ahead and balancing purchases in Table 3. In Appendix B, we test the framework on instances with a smaller gap between day-ahead and balancing prices. The analysis yields similar conclusions, reinforcing the stability of the results.

First, it is worth noting that none of the consumers can individually access the day-ahead market as for any prosumer $\overline{q}_i \leq \underline{q}_t^{DA}$ and thus constraint (6e) excludes any purchase on the day-ahead market. In the utilitarian model $m_\emptyset^U$,the primary focus is to minimize aggregated costs, making it optimal to always consistently access the day-ahead market as a group. To achieve this, consumer A_1 redistributes its energy load across 4 time steps, incurring a higher individual cost (64% higher) than when acting independently. Adding acceptability constraints with $\alpha = 1$ to the model (m_1^U) induces a loss in efficiency but now satisfies individual rationality: $PoA = 0.04$. We observe that the aggregated costs of consumers slightly increase, but now the charge of energy needed to access the day-ahead market is shared between A_1 and A_3, although A_3 does not gain anything from the the aggregation (0% of savings).

Table 2: Percentage of savings $\frac{v^i - L^i(x^i)}{v^i}$ made by A_i in the model m_a^f depending on agent operator $f \in \{U, SM, P\}$ and acceptability set $a \in \{\emptyset, 1\}$ and PoA of the corresponding model.

	Utilitarian $\mathcal{F}_I^U$					Minimax $\mathcal{F}_I^{SM}$					Proportional $\mathcal{F}_I^P$				
	A1	A2	A3	A4	PoA	A1	A2	A3	A4	PoA	A1	A2	A3	A4	PoA
$\emptyset$	-64	46	72	46	0	60	30	30	30	0	74	21	80	21	0
$\mathcal{A}_1$	48	37	0	37	0.04	60	30	30	30	0	74	21	80	21	0
Shapley	114	20	111	28	0										

Conversely, the proportional solution (from model $m_\emptyset^P$) adopts a more bargaining-oriented approach, resulting in collaboration only during time slots ($t \in \{1, 3, 5\}$) with lower prices. Indeed, as A_1 and A_3 are not forced to consume energy at all times ($\underline{q}_1 = \underline{q}_3 = 0$), they can shift their consumption to time slots with lower prices. On the contrary, A_2 and A_4 must always consume energy ($\underline{q}_2 = 5, \underline{q}_4 = 2$), and the two of them together cannot access the day-ahead market either. Thus, in $m_\emptyset^P$, the solution is for A_1 and A_3 to consume only in time steps $\{1, 3, 5\}$, which leaves A_2 and A_4 to operate independently at $t = 2, t = 4$, resulting in limited savings (21%) compared to the scaled minimax approach ($m_\emptyset^{SM}$). As noticed in Remark 2, the solution is necessarily 1-acceptable. Therefore, the solution is the same in $m_\emptyset^P$ and

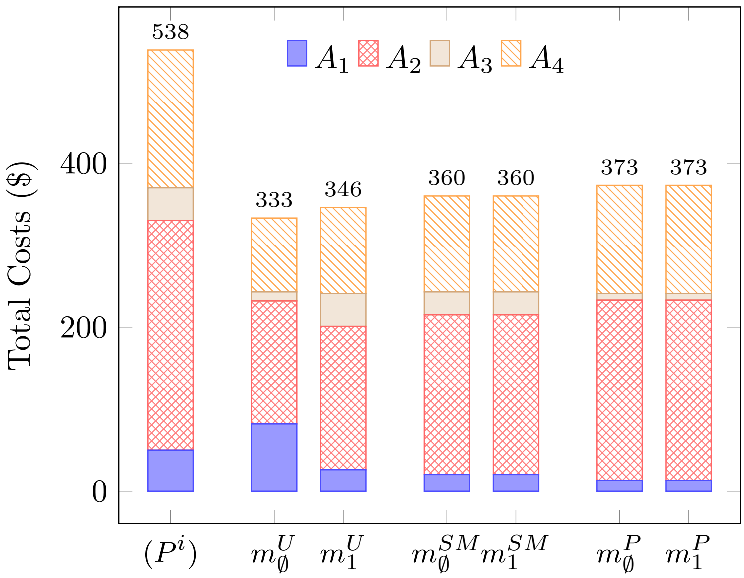

Fig. 1: We observe the result of the static Problem (6f) with parameters given in Table 1. The bars correspond to the outcome of different models, the number above being the total cost. The first bar is the non-aggregated model: we solve each (P^i) independently. Then, there are three groups of two bars, each group corresponding to a choice of agent operator $(\mathcal{F}^U_I, \mathcal{F}^{SM}_, \mathcal{F}^P_I)$. Then, we present the model's results for each objective function, first without and then with acceptability constraints $\mathcal{A}_1$. Each bar is decomposed in 4 blocks corresponding to the cost incurred by each consumer i. At the top of each bar, we can read the sum of aggregated costs in the corresponding model.

m^P_1. Moreover, the proportional solution yields the worst aggregated costs *i.e.,* the less efficient solution.

With the scaled minimax approach, the model $m^{SM}_\emptyset$ yields a trade-off between efficiency and fairness compared to m^U_1: we observe that A_1 and A_3 decide to stop consuming at expensive time steps, thus achieving greater savings. The model also encourages more cooperation than the proportional model $m^P_\emptyset$, as we can observe in Table 3. As a result, in this model, all consumers achieve similar proportional savings, amounting to approximately 30% compared to operating independently, at the exception of A_1 that can save up to 60%. This means that any solution where A_1 shifts its consumption to other time slots to help others access the day-ahead market, would increase its costs too much, and A_3 would save less than 30%: this cannot be an optimal solution of $m^{SM}_\emptyset$. However, the aggregated cost of the aggregation is higher than with $m^U_\emptyset$ and m^U_1. Again, adding acceptability constraints does not change the solution, as the scaled minimax problem is innately 1-acceptable (see Remark 1).

	A1		A2		A3		A4	
t	DA	B	DA	B	DA	B	DA	B
1	***0***	0	5	0	3	0	3	0
2	3	0	5	0	***0***	0	3	0
3	2	0	5	0	1	0	3	0
4	3	0	5	0	***0***	0	3	0
5	2	0	5	0	4	0	3	0

(a) $m_\emptyset^U$

	A1		A2		A3		A4	
t	DA	B	DA	B	DA	B	DA	B
1	***0***	0	5	0	3	0	3	0
2	1.01	0	5	0	1.93	0	3	0
3	3.93	0	5	0	***0***	0	3	0
4	***0***	0	0	***5***	***0***	0	0	***3***
5	5	0	5	0	3	0	3	0

(b) m_1^U

	A1		A2		A3		A4	
t	DA	B	DA	B	DA	B	DA	B
1	***0***	0	5	0	3	0	3	0
2	***0***	0	0	***5***	***0***	0	0	***3***
3	4.44	0	5	0	1.56	0	3	0
4	1.13	0	5	0	1.87	0	3	0
5	4.44	0	5	0	1.56	0	3	0

(c) $m_\emptyset^{SM}$

	A1		A2		A3		A4	
t	DA	B	DA	B	DA	B	DA	B
1	***0***	0	5	0	3	0	3	0
2	***0***	0	0	***5***	***0***	0	0	***3***
3	4.44	0	5	0	1.56	0	3	0
4	1.13	0	5	0	1.87	0	3	0
5	4.44	0	5	0	1.56	0	3	0

(d) m_1^{SM}

	A1		A2		A3		A4	
t	DA	B	DA	B	DA	B	DA	B
1	3	0	5	0	***0***	0	3	0
2	***0***	0	0	***5***	***0***	0	0	***3***
3	3.5	0	5	0	4	0	3	0
4	***0***	0	0	***5***	***0***	0	0	***3***
5	3.5	0	5	0	4	0	3	0

(e) $m_\emptyset^P$

	A1		A2		A3		A4	
t	DA	B	DA	B	DA	B	DA	B
1	3	0	5	0	***0***	0	3	0
2	***0***	0	0	***5***	***0***	0	0	***3***
3	3.5	0	5	0	4	0	3	0
4	***0***	0	0	***5***	***0***	0	0	***3***
5	3.5	0	5	0	4	0	3	0

(f) m_1^P

Table 3: Quantity of energy purchased on the day-ahead and balancing markets per stage for all consumers depending on different models: in bold italic, we highlight purchases on the (more expensive) balancing market and stages where no purchases are made on the day-ahead market.

Lastly, we observe in Table 2 the allocation of savings with a post-allocation rule based on Shapley's values. This approach leverages the efficiency of $m_\emptyset^U$ but then re-allocates costs to obtain a fair and acceptable solution. In this application, A_1 and A_3 save respectively 114% and 111% of their costs compared to operating independently, which amounts to them being paid by the aggregation to participate. Even though this allows A_2 and A_4 to gain from the aggregation, this questions the acceptability of this solution as some would earn money while others have residual costs. Furthermore, due to the extensive computational requirements, the post-allocation rule becomes impractical when dealing with large problems involving many prosumers and time steps. Additionally, adapting it to dynamic and stochastic contexts is unclear, so we do not consider it further.

We test the model with different values of α that represent the targeted gap between a prosumer's cost in the aggregation and its individual optimal cost. Figure 2 presents the proportional savings of prosumers when solving m_α^f for $f \in \{U, SM, P\}$ and $\alpha \in \{0.6, 0.7, 0.8, 0.9, 1.0\}$. Bear in mind that solving $m_\emptyset^{SM}$ is equivalent to maximizing

α (see Remark 1). Hence, if $\alpha^\star$ is the optimal solution of Problem (5), enforcing acceptability constraints with $\alpha \geq \alpha^\star$ results in the same solution, while solving m_α^{SM} with $\alpha < \alpha^\star$ leads to the disagreement point. In our study case, $\alpha^\star = 0.696$.

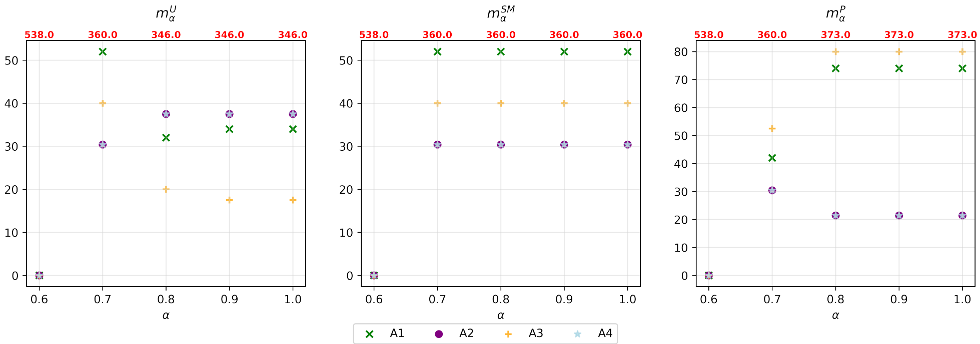

Fig. 2: Proportional savings $\frac{v^i - L^i(x^i)}{v^i}$ of prosumers depending on the chosen fairness operator and the acceptability coefficient α. In red above the figure is the cost of the aggregation in the corresponding model.

In the utilitarian case ($f = U$), increasing α relaxes the model, leading to a reduction in aggregation costs between $m^U_{0.7}$ and $m^U_{0.8}$. In some cases, while the aggregation cost remains unchanged, varying α affects the cost allocation of prosumers. For example, with $\alpha = 0.8$, A_1 and A_3 save respectively 32% and 20%, whereas with $\alpha = 0.9$ they save 34% and 17.5%. Finally, with the proportional operator, we observe a switch in the aggregation cost between $m^P_{0.7}$ and $m^P_{0.8}$ leading to a change in cost allocation. Unlike the utilitarian operator, cost distribution among prosumers remains unchanged for the same aggregation cost. Here, enforcing stronger acceptability results in more evenly distributed proportional savings. Specifically, for $\alpha \geq 0.8$, savings range from 21.4% to 80%, whereas with $\alpha < 0.8$, they range from 30.5% to 52.5%.

We test the utilitarian model m^U_α with a finer discretization of α. As shown in Figure 3, for α between 0.74 and 0.83, the aggregation cost remains unchanged, while cost distribution shifts between A_1 and A_3. In this case, A_1 and A_3 have flexible energy consumption, allowing the aggregation to rely on either for day-ahead market access. A shift in cost distribution between them indicates a change in which consumer is prioritized.

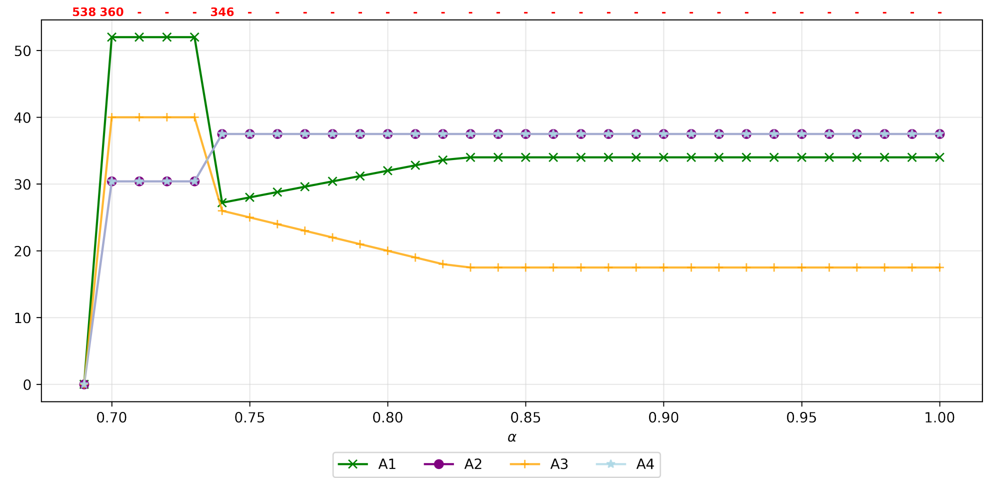

Fig. 3: Proportional savings $\frac{v^i - L^i(x^i)}{v^i}$ of prosumers in the model m^U_α depending on α. Above the figure, the aggregation cost for each model is shown in red, with '$-$' indicating no change.

5 Fairness across time

In most use cases, we can assume that the aggregation of agents is thought to stay in place over long periods. One of the challenges of this long-term setting is incentivizing agents not to leave the aggregation, which requires adjusting the acceptability constraints of the static case.

5.1 Problem formulation

We consider a problem with T stages corresponding to consecutive times where decisions are made. At each stage $t \in [T]$, agent i makes a decision $x^i_t \in \mathcal{X}^i_t$, incurring a cost $L^i_t(x^i_t)$. Those stage costs are aggregated through a time operator $\mathcal{F}^i_T : \mathbb{R}^T \to \mathbb{R}$. Thus, the agent i's problem reads:

$$
\begin{aligned}
(P^i_T) \; := \quad & \underset{x^i_t}{\text{Min}} \quad \mathcal{F}^i_T\Big((L^i_t(x^i_t))_{t\in[T]}\Big) && && \text{(7a)} \\
& \text{s.t} \quad x^i_t \in \mathcal{X}^i_t && \forall t && \text{(7b)} \\
& \qquad\; x^i_t \in \mathcal{M}_t && \forall t. && \text{(7c)}
\end{aligned}
$$

A typical example of time-aggregator $\mathcal{F}^i_T$ is the discounted sum of stage costs *i.e.*, dropping the dependence in x_i for clarity's sake:

$$
\mathcal{F}^i_T((L_t)_{t\in[T]}) = \sum_{t\in[T]} r^t L_t,
$$

for $r \in (0,1]$. Alternatively, $\mathcal{F}_T^i$ can be defined as the maximum stage costs. This might happen for electricity markets where a prosumer aims at *peak shaving i.e.,* minimizing peak electricity demand. Further, time-aggregation operators may vary among agents, who may express different sensitivity to time *i.e.,* the discounted rate r varies among prosumers.

We now write the aggregation problem within this framework. Note that we can cast the current multistage setting into the setting of Section 3, by decomposing each agent into T independent stage-wise sub-agents: then we have $I \times T$ and we can use the methodology of Section 3. Thus we need to define an operator $\mathcal{F}_{I\times T}$ that takes $\{L_t^i\}_{t\in[T],i\in[I]}$ as input.

However, in most settings, it is reasonable to assume that an agent is time-homogeneous, meaning that, in some sense, for all $i \in [N]$, the agents $(i,t)_{t\in[T]}$ are the same, and aggregates the stage costs across time. Consequently, the global aggregation operator $\mathcal{F}_{I\times T}$ can be modeled as aggregating, over agents, their aggregated stage-costs, *i.e.,* $\mathcal{F}_{I\times T} = \mathcal{F}_I \odot \mathcal{F}_T$ where the $\odot$ notation stands for

$$\mathcal{F}_I \odot \mathcal{F}_T\Big((L_i^t)_{i\in[I],t\in[T]}\Big) = \mathcal{F}_I\bigg(\mathcal{F}_T^1\Big((L_t^1)_{t\in[T]}\Big), \dots, \mathcal{F}_T^I\Big((L_t^I)_{t\in[T]}\Big)\bigg). \tag{8}$$

Finally, we obtain the following model for the aggregation of agents in a dynamic framework:

$$(A^T) \;:=\; \underset{x_t^i}{\mathrm{Min}} \quad \mathcal{F}_I \odot \mathcal{F}_T\Big((L_t^i)_{i\in[I],t\in[T]}\Big) \tag{9a}$$

$$\text{s.t.} \quad x_t^i \in \mathcal{X}_t^i \qquad \forall t \tag{9b}$$

$$h(x_t^1, \dots, x_t^I) \in \mathcal{M}_t \qquad \forall t \tag{9c}$$

$$(L_t^i)_{t\in[T]} \in \mathcal{A}^i, \tag{9d}$$

where we recall that we defined $\mathcal{F}_T$ as the sum, and suggest to choose $\mathcal{F}_I$ from the fairness operators $(\mathcal{F}_I^U, \mathcal{F}_I^P, \mathcal{F}_I^{SM})$ introduced in Section 3.2. Thus, we obtain a fair objective function of the aggregated model (A^T). However, we have yet to adapt the notion of acceptability from Section 3.3 to this long-term framework, which is our next topic.

5.2 Dynamic acceptability

In long-term problems, agents should not be tempted to leave the aggregation in between stages for the aggregation to be acceptable. Therefore, we extend our notion of acceptability constraint (3) to a dynamic framework. For simplicity, we set $\alpha = 1$ from now on. First, denote $v_t^i := L_t^i(x_t^{i,\star})$, the optimal independent cost of an agent i at stage t, where $x^{i,\star}$ is the optimal solution of Problem (7).

The acceptability constraint (3) consist in requiring, for each agent i, that its vector of costs $(L_t^i)_{t\in[T]}$ is less than $(v_t^i)_{t\in[T]}$. Unfortunately, there is no natural ordering of $\mathbb{R}^T$, and each (partial) order will define a different extension of the acceptability constraint (3). We present now some extensions of the acceptability constraint derived from standard partial orders.

Maybe the most intuitive choice is the component-wise order (induced by the positive orthant), *i.e.,* comparing coordinate by coordinate. This results in the *stage-wise acceptability* constraint $\mathcal{A}_s$, which enforces that each agent benefits from the aggregation at each stage:

$$\mathcal{A}_s^i = \{ \ (u_t^i)_{t\in[T]} \quad | \quad u_t^i \leq \ v_t^i, \quad \forall t \in [T] \ \}. \tag{10a}$$

As this approach might be too conservative for our model, *i.e.,* constrain the aggregation too much to take advantage of it, we consider two other ordering.

First, we can relax the stage-wise acceptability by considering that at each stage t, each agent benefits from the aggregation if we consider its costs aggregated up to time t. This result in *progressive acceptability* constraint $\mathcal{A}_p^i$:

$$\mathcal{A}_p^i = \{ \ (u_t^i)_{t\in[T]} \quad | \quad \sum_{\tau=1}^{t} u_\tau^i \leq \sum_{\tau=1}^{t} v_\tau^i, \quad \forall t \in [T] \ \}. \tag{10b}$$

Second, we ensure that each agent, aggregating its cost over the whole horizon, benefits from the aggregation (which amounts to the set in (3)) if we consider only the aggregated costs at the end of the horizon. We thus define the *average acceptability* constraint:

$$\mathcal{A}_{av}^i = \{ \ (u_t^i)_{t\in[T]} \quad | \quad \sum_{t=1}^{T} u_t^i \leq \sum_{t=1}^{T} v_t^i \ \}. \tag{10c}$$

Remark 3. *We have that* $\mathcal{A}_s^i \subseteq \mathcal{A}_p^i \subseteq \mathcal{A}_{av}^i$. *The acceptability constraint should be chosen to strike a balance between aggregated cost efficiency (obtained with a less constrained acceptability set), and incentive to stay in the aggregation (obtained with a more constrained acceptability set).*

5.3 Numerical illustration

We take the same example as in Section 4 and try out different combinations of operator $\mathcal{F}_{I\times T}$ ($\mathcal{F}^U, \mathcal{F}^{SM}, \mathcal{F}^P$) and acceptability set $\mathcal{A}$ ($\emptyset, \mathcal{A}_{av}, \mathcal{A}_p, \mathcal{A}_s$). We denote m_a^f the model with agent operator $f \in \{U, SM, P\}$ and acceptability set $a \in \{\emptyset, av, p, s\}$. Figure 4 represent the distribution of prosumers' costs for these different cases, while Table 4 report their proportional savings. Finally, on Table 5, we report the day-ahead and balancing purchases of the different models with progressive and stagewise acceptability, while the results with no acceptability and average acceptability are in Section 4 on Table 3. In this section, we do not compare the cost allocation to Shapley values, as their computation in a dynamic setting is not straightforward. A naive approach could be to compute Shapley values over subproblems on the horizon $[1, t]$ at each stage t. However, this method does not align with our approach, which explicitly accounts for future stages.

We observe in Figure 4 that increasing acceptability constraints (from none to average, progressive, and stage-wise) come at a price but give stronger guarantees to

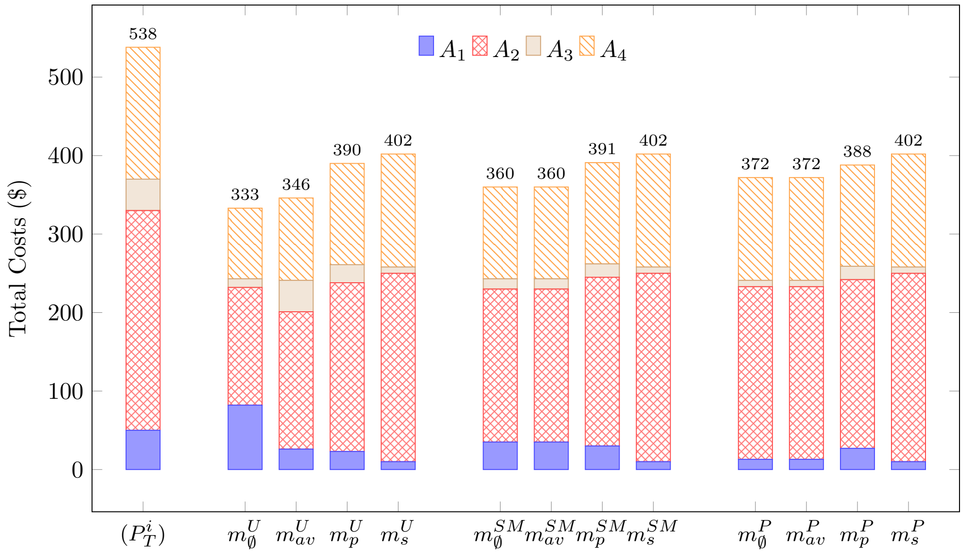

Fig. 4: Each bar corresponds to a different model. The first one is the independent model: we solve each (P_T^i) independently. Then, there are three groups of four bars, each group corresponding to a choice of agent operator $(\mathcal{F}^U, \mathcal{F}^{SM}, \mathcal{F}^P)$. Then, given an operator, we have the model first without then with different acceptability constraints $(\emptyset, \mathcal{A}_{av}, \mathcal{A}_p, \mathcal{A}_s)$. Each bar is decomposed in 4 blocks corresponding to the share of each consumer i. At the top of each bar, we can read the sum of aggregated costs in the corresponding model.

Table 4: Percentage of savings $\frac{v^i - L^i(x^i)}{v^i}$ achieved by A_i in the model m_a^f depending on agent operator f and acceptability set a and PoA of the corresponding model.

	Utilitarian $\mathcal{F}_I^U$					Minimax $\mathcal{F}_I^{SM}$					Proportional $\mathcal{F}_I^P$				
	A1	A2	A3	A4	PoA	A1	A2	A3	A4	PoA	A1	A2	A3	A4	PoA
$\emptyset$	-64	46	73	46	0	30	30	67	30	0	74	21	80	21	0
$\mathcal{A}_{av}$	48	37	0	37	0.04	30	30	67	30	0	74	21	79	21	0
$\mathcal{A}_p$	55	23	44	23	0.17	44	23	57	23	8.6	45	23	56	23	0.04
$\mathcal{A}_s$	80	14	80	14	0.21	80	14	80	14	12	80	14	80	14	0.08

each user. In fact, we have seen that $m_\emptyset^U$ is the most efficient model but yields solutions in contradiction to individual rationality. We can correct this defect by enforcing average acceptability, but this is not enough to ensure everyone gains from the aggregation, as A_3 makes 0% of savings. With more constrained acceptability, $\mathcal{A}_p$ and $\mathcal{A}_s$, we enforce individual rationality over time or at all times. This leads to solutions where the savings are shared among prosumers in fairer proportions- at the loss of efficiency.

	A1		A2		A3		A4		A1		A2		A3		A4	
t	DA	B	DA	B	DA	B	DA	B	DA	B	DA	B	DA	B	DA	B
1	***0***	0	0	***5***	***0***	0	0	***3***	***0***	0	0	***5***	***0***	0	0	***3***
2	***0***	0	0	***5***	***0***	0	0	***3***	***0***	0	0	***5***	0	***0***	0	***3***
3	5	0	5	0	4	0	3	0	5	0	5	0	4	0	3	0
4	1.4	0	5	0	1.6	0	3	0	***0***	0	0	***5***	0	***0***	0	***3***
5	3.6	0	5	0	2.4	0	3	0	5	0	5	0	4	0	3	0

(a) $m^U_{\mathcal{A}_p}$ (b) $m^U_{\mathcal{A}_s}$

	A1		A2		A3		A4		A1		A2		A3		A4	
t	DA	B	DA	B	DA	B	DA	B	DA	B	DA	B	DA	B	DA	B
1	***0***	0	0	***5***	***0***	0	0	***3***	***0***	0	0	***5***	***0***	0	0	***3***
2	***0***	0	0	***5***	***0***	0	0	***3***	***0***	0	0	***5***	0	***0***	0	***3***
3	5	0	5	0	4	0	3	0	5	0	5	0	4	0	3	0
4	2	0	5	0	1	0	3	0	***0***	0	0	***5***	0	***0***	0	***3***
5	3	0	5	0	3	0	3	0	5	0	5	0	4	0	3	0

(c) $m^{SM}_{\mathcal{A}_p}$ (d) $m^{SM}_{\mathcal{A}_s}$

	A1		A2		A3		A4		A1		A2		A3		A4	
t	DA	B	DA	B	DA	B	DA	B	DA	B	DA	B	DA	B	DA	B
1	***0***	0	0	***5***	***0***	0	0	***3***	***0***	0	0	***5***	***0***	0	0	***3***
2	***0***	0	0	***5***	***0***	0	0	***3***	***0***	0	0	***5***	0	***0***	0	***3***
3	3.8	0	5	0	3.45	0	3	0	5	0	5	0	4	0	3	0
4	1.94	0	5	0	1.06	0	3	0	***0***	0	0	***5***	0	***0***	0	***3***
5	4.25	0	5	0	3.5	0	3	0	5	0	5	0	4	0	3	0

(e) $m^P_{\mathcal{A}_p}$ (f) $m^P_{\mathcal{A}_s}$

Table 5: Quantity of energy purchased on the day-ahead and balancing markets per stage for all consumers depending on different models: in bold italic, we highlight purchases on the (more expensive) balancing market and stages where no purchases are made on the day-ahead market.

On the other hand, with an agent operator reflecting fairness (like scaled minimax or proportional), we obtain solutions that already aim at a fairer distribution of savings. Consequently, if we can observe solutions changing with increasing acceptability constraints, those changes are more apparent with a utilitarian operator. Indeed, in the utilitarian model, A_2 achieves savings ranging from 14% to 46% of his independent cost. In contrast, under the scaled minimax approach, the savings range from 14% to 30%, and with the proportional approach, the savings fall between 14% and 21%.

Note that even though the acceptability constraints and the agent operator are two distinct tools, they both drive the model to fairer solutions for all agents in the aggregation.

6 Accommodating fairness to uncertainties with stochastic optimization

Problems with energy generation, especially from renewable sources, and prices on energy markets are inherently uncertain as we have decisions to make over time, and the future is uncertain. Then, in addition to acceptability and fairness, we must tackle the challenge of handling uncertainties (while being fair about how we handle those). We want to address this issue by extending the problem presented in Section 3 to a stochastic framework. To that end, we introduce random variable $\boldsymbol{\xi}$ and probability space $(\Omega, \mathcal{A}, \mathbb{P})$, which gathers all sources of uncertainties in the problem. We assume that Ω is finite, a common simplification in stochastic optimization to make problems more tractable. If Ω is not finite, we rely on sample average approximation.

In the same way that we decomposed the problem in Section 5 with T time steps, we can decompose the problem here with Ω scenarios. Thus, there are similarities with the previous section. The main difference is that the set of time-step $\{1, \dots, T\}$ has a natural ordering, while the set of scenario Ω does not, which leads to discussing different partial orders on $\mathbb{R}^{\Omega}$ than on $\mathbb{R}^{T}$.

6.1 Static stochastic problem formulation

The problem at hand is naturally formulated as a multistage stochastic problem. For simplicity reasons, we first consider a $2-$stage relaxation of the problem: in the first stage, *here-and-now* decisions x_0^i must be made before knowing the noise's realization; in the second stage, once the noise's realization is revealed, *recourse* actions $\boldsymbol{x_1^i(\xi)}$ can be decided. To alleviate notations, we write $\boldsymbol{x^i(\xi)} := (x_0^i, \boldsymbol{x_1^i(\xi)})$ and add the non-anticipative constraints– that ensure first-stage decisions are taken with no knowledge of the future– in feasibility set $\mathcal{X}^i$.

We first adapt the individual model (P^i) to a stochastic framework:

$$
\begin{aligned}
(P^{i,\rho}) \ := \quad & \min_{\boldsymbol{x}^i(\boldsymbol{\xi})} \quad \rho\left[L^i(\boldsymbol{x}^i(\boldsymbol{\xi}), \boldsymbol{\xi})\right] && \text{(11a)}\\
& \text{s.t.} \quad \boldsymbol{x}^i(\boldsymbol{\xi}) \in \mathcal{X}^i \qquad \text{a.s.} && \text{(11b)}\\
& \qquad\quad \boldsymbol{x}^i(\boldsymbol{\xi}) \in \mathcal{M} \qquad \text{a.s.,} && \text{(11c)}
\end{aligned}
$$

where ρ is a (coherent) risk-measure *i.e.,* a function which gives a deterministic cost equivalent to a random cost, reflecting the risk of a decision for prosumer i, see *e.g.,* Artzner et al. (1999). The choice of ρ depends on the attitude of i towards risk. For example, the risk measure associated with a risk-neutral approach is the mathematical expectation $\mathbb{E}_{\xi}$. Alternatively, a highly risk-averse profile will opt for the worst-case measure $\sup_{\xi}$. Another widely used risk measure is the Average Value at Risk (a.k.a Conditional Value at Risk, or expected shortfall, see Rockafellar et al. (2000)), or a convex combination of expectation and Average Value at Risk.

Now, we adapt the deterministic aggregation model (A). We face the same challenge as in Section 5. With multiple scenarios, we can consider that we have $I \times \Omega$

prosumers and we need to choose an operator $\mathcal{F}_{I\times\Omega} : \mathbb{R}^{I\times\Omega} \to \mathbb{R}$, leading to:

$$
\begin{aligned}
(A^\rho) \ := \quad & \underset{\boldsymbol{x}}{\text{Min}} \quad \mathcal{F}_{I\times\Omega}\Big((L^i(\boldsymbol{x}^i(\boldsymbol{\xi}),\boldsymbol{\xi}))_{i\in[I]}\Big) && && \text{(12a)}\\
& \text{s.t.} \quad \boldsymbol{x}^i(\boldsymbol{\xi}) \in \mathcal{X}^i && \forall i \in [I] \quad \text{a.s.} && \text{(12b)}\\
& \qquad h(\boldsymbol{x}^1(\boldsymbol{\xi}),\dots,\boldsymbol{x}^I(\boldsymbol{\xi})) \in \mathcal{M} && \text{a.s.} && \text{(12c)}\\
& \qquad L^i(\boldsymbol{x}^i(\boldsymbol{\xi}),\boldsymbol{\xi}) \in \mathcal{A}^i && \forall i \in [I] \quad \text{a.s..} && \text{(12d)}
\end{aligned}
$$

We assume the aggregator knows risk measures and prosumers' objectives. As in Section 5.1, there are multiple possible choices for such operators. We assume that this operator $\mathcal{F}_{I\times\Omega}$ results from the composition of two operators: an uncertainty-operator $\mathcal{F}^i_\Omega$ dealing with the scenarios, which can differ from one prosumer to another; and an agent operator $\mathcal{F}_I$, as defined in Section 3.2. However, contrary to Section 5, it is not clear if we should aggregate first with respect to uncertainty (meaning that a prosumer manages its own risk) or with respect to prosumers (meaning that the risks are shared). We next discuss reasonable modeling choices of aggregation operators and acceptability constraints.

6.2 Stochastic objective

For the sake of conciseness, we are going to consider two possible uncertainty aggregators: a risk-neutral choice, where $\mathcal{F}^i_\Omega$ is the mathematical expectation $\mathbb{E}_\xi$, and a worst-case operator where $\mathcal{F}^i_\Omega$ is the supremum over the possible realization $\sup_\xi$. For the agent operator $\mathcal{F}_I$, which reflects the way to handle fairness, we consider either the utilitarian $\mathcal{F}^U_I$ or the scaled minimax $\mathcal{F}^{SM}_I$ options (see Section 3.3 for definitions).

We suggest four different compositions of $\mathcal{F}^i_\Omega$ and $\mathcal{F}_I$ to construct the aggregation operator $\mathcal{F}_{I\times\Omega}$. Again, for simplicity of notations, we write $\boldsymbol{L^i}$ instead of $L^i(\boldsymbol{x}^i(\boldsymbol{\xi}),\boldsymbol{\xi})$.

First, we introduce the risk-neutral and utilitarian operator $\mathcal{F}^{US}_{I\times\Omega}$, which aims at minimizing the aggregated expected costs of prosumers:

$$
\begin{aligned}
\mathcal{F}^{US}_{I\times\Omega}\Big((\boldsymbol{L^i})_{i\in[I]}\Big) &= \mathcal{F}^U_I \odot \mathbb{E}_\Omega\Big((\boldsymbol{L^i})_{i\in[I]}\Big) && \text{(13a)}\\
&= \sum_{i=1}^{I} \sum_{\xi\in\Omega} \pi_\xi \; L^i(x^i(\xi),\xi). && \text{(13b)}
\end{aligned}
$$

Alternatively, considering a robust approach to uncertainties, we have the operator $\mathcal{F}^{UR}_{I\times\Omega}$ which minimizes the worst-case aggregated costs of prosumers:

$$
\begin{aligned}
\mathcal{F}^{UR}_{I\times\Omega}\Big((\boldsymbol{L^i})_{i\in[I]}\Big) &= \sup_{\xi\in\Omega} \odot \mathcal{F}^U_I\Big((\boldsymbol{L^i})_{i\in[I]}\Big) && \text{(14a)}\\
&= \sup_{\xi\in\Omega} \Big\{ \sum_{i=1}^{I} L^i(x^i(\xi),\xi) \Big\}. && \text{(14b)}
\end{aligned}
$$

Remark 4. *We claim that* $\sup_{\xi\in\Omega} \odot \mathcal{F}_I^U$ *makes more sense than* $\mathcal{F}_I^U \odot \sup_{\xi\in\Omega}$ *as the later aggregates each prosumer's worst costs. Indeed, if the worst-case costs for different prosumers occur in different scenarios, the aggregated costs calculated might never happen or happen in a scenario* ξ*, not in* Ω*.*

On the other hand, we have $\mathbb{E}_{\xi\in\Omega} \odot \mathcal{F}_I^U = \mathcal{F}_I^U \odot \mathbb{E}_{\xi\in\Omega}$*, by associativity of sums. Similarly, by associativity of supremum, we have* $\sup_{\xi\in\Omega} \odot \mathcal{F}_I^{SM} = \mathcal{F}_I^{SM} \odot \sup_{\xi\in\Omega}$*.*

As the first two operators do not model fairness considerations into the model, we now look for a fair distribution by using $\mathcal{F}_I^{SM}$ to aggregate prosumers' costs. First, let $\boldsymbol{x}^{i,\star}(\boldsymbol{\xi})$ be the[6] optimal solution of $(P^{i,\rho})$, and denote $v_\xi^{i,\rho} := L^i(x^{i,\star}(\xi), \xi)$, the cost incurred by i when operating alone under uncertainty realization ξ. Finally, $\boldsymbol{v}^{i,\rho}$ is the random variable taking values $v_\xi^{i,\rho}$ for the respective realization ξ.

Results given in Sections 4 and 5.3 suggest that the scaled minimax approach suits our problem more than the proportional approach. Thus, in a stochastic framework, we propose the operator $\mathcal{F}_{I\times\Omega}^{SMS}$:

$$\mathcal{F}_{I\times\Omega}^{SMS}\Big((\boldsymbol{L^i})_{i\in[I]}\Big) = \mathcal{F}_I^{SM} \odot \mathbb{E}_\Omega\Big((\boldsymbol{L^i})_{i\in[I]}\Big) \tag{15a}$$

$$= \max_{i\in[I]} \left\{ \frac{\mathbb{E}\big[\boldsymbol{v}^{i,\mathbb{E}}\big] - \sum_{\xi\in\Omega} \pi_\xi \, L^i(x^i(\xi), \xi)}{\mathbb{E}\big[\boldsymbol{v}^{i,\mathbb{E}}\big]} \right\}. \tag{15b}$$

Finally, combining the robust and the scaled minimax approaches, we obtain the operator $\mathcal{F}_{I\times\Omega}^{SMR}$, which focuses on the prosumer having the worst worst-case proportional costs:

$$\mathcal{F}_{I\times\Omega}^{SMR}\Big((\boldsymbol{L^i})_{i\in[I]}\Big) = \sup_{\xi\in\Omega} \odot \mathcal{F}_I^{SM}\Big((\boldsymbol{L^i})_{i\in[I]}\Big) \tag{16a}$$

$$= \sup_{\xi\in\Omega} \left\{ \max_{i\in[I]} \left\{ \frac{v_\xi^{i,\mathbb{E}} - L^i(x^i(\xi), \xi)}{v_\xi^{i,\mathbb{E}}} \right\} \right\}. \tag{16b}$$

Remark 5. *Note that here, depending on the sense of the combination between uncertainty-operator and agent-operator, we could have a model with different risk-measure profiles for the prosumers.*

We now turn to extending the acceptability constraint (3) to a stochastic setting.

6.3 Stochastic dominance constraints

As in Section 5.2, to induce acceptability, we require that, for each prosumer i, its random cost $L^i(\boldsymbol{x}^i(\boldsymbol{\xi}), \boldsymbol{\xi})$ is less than the random cost of the independent model $\boldsymbol{v}^{i,\mathbb{E}}$. Unfortunately, there is no natural ordering of random variable (or equivalently of $\mathbb{R}^\Omega$), and each (partial) order will define a different extension of the acceptability constraint (3).

We now present four acceptability constraints, using various ordering on the space of random variable, leveraging the stochastic dominance theory (see Dentcheva and

[6] We assume uniqueness of a way of selecting an optimal solution, as in Section 3)

Ruszczynski (2003) for an introduction in the context of stochastic optimization). In this section, we give the mathematical expression of acceptability constraints, but a mixed integer formulation can be found in Appendix C.

In a very conservative perspective, we consider the almost-sure order, comparing random variables scenario by scenario:

$$\mathcal{A}^{i,\rho}_{a.s} := \{ \boldsymbol{u}^{i,\rho} \mid u^{i,\rho}_{\xi} \leq v^{i,\rho}_{\xi}, \quad \forall \xi \}. \tag{17a}$$

We can relax the almost-sure ordering by not requiring the benefit of aggregation for all scenarios but distributionally. For example, if we have two scenarios ξ and ζ, with the same probability, we consider that it is acceptable to lose on ξ if we do better on ζ, that is such that $u^{i,\rho}_{\xi} \leq v^{i,\rho}_{\xi}$ and $u^{i,\rho}_{\zeta} \geq v^{i,\rho}_{\zeta}$. To formalize this approach, we turn to *stochastic first-order dominance constraints* (see Dentcheva and Ruszczynski (2003)), and leverage 1^{st} order acceptability:

$$\begin{aligned} \mathcal{A}^{i,\rho}_{(1)} := & \ \{ \boldsymbol{u}^{i,\rho} \mid \boldsymbol{u}^{i,\rho} \preceq_{(1)} \ \boldsymbol{v}^{i,\rho} \} & \text{(17b)} \\ := & \ \{ \boldsymbol{u}^{i,\rho} \mid \mathbb{P}(\boldsymbol{u}^{i,\rho} > \eta) \ \leq \ \mathbb{P}(\boldsymbol{v}^{i,\rho} > \eta), \qquad \forall \eta \in \mathbb{R} \} & \\ := & \ \{ \boldsymbol{u}^{i,\rho} \mid \mathbb{E}\big[g(\boldsymbol{u}^{i,\rho})\big] \ \leq \ \mathbb{E}\big[g(\boldsymbol{v}^{i,\rho})\big] \quad \forall g : \mathbb{R} \to \mathbb{R}, \ \text{non-decreasing}\}. & \end{aligned}$$

One downside of this acceptability constraint is that the modeling entails numerous binary variables, posing practical implementation challenges.

We can thus consider a relaxed, less risk-averse version of 1^{st} order acceptability, relying on *stochastic second-order dominance constraints*, also known as increasing convex acceptability, which is equivalent to :

$$\begin{aligned} \mathcal{A}^{i,\rho}_{(ic)} & := \{ \boldsymbol{u}^{i,\rho} \mid \boldsymbol{u}^{i,\rho} \preceq_{(ic)} \ \boldsymbol{v}^{i,\rho} \} & \text{(17c)} \\ & = \{ \boldsymbol{u}^{i,\rho} \mid \mathbb{E}\big[(\boldsymbol{u}^{i,\rho} - \eta)^+ \big] \leq \mathbb{E}\big[(\boldsymbol{v}^{i,\rho} - \eta)^+ \big] \qquad \forall \eta \in \mathbb{R}\} & \\ & = \{ \boldsymbol{u}^{i,\rho} \mid \mathbb{E}\big[g(\boldsymbol{u}^{i,\rho}) \big] \leq \mathbb{E}\big[g(\boldsymbol{v}^{i,\rho}) \big], \forall g : \mathbb{R} \to \mathbb{R}, \text{convex, non-decreasing} \}. & \end{aligned}$$

Moreover, increasing convex acceptability is also easier to implement than 1^{st} order acceptability (see Appendix C).

Finally, the risk-neutral acceptability constraint compares two random variables through their expectation:

$$\mathcal{A}^{i,\rho}_{\mathbb{E}} := \{ \boldsymbol{u}^{i,\rho} \mid \mathbb{E}_{\mathbb{P}}[\boldsymbol{u}^{i,\rho}] \leq \ \mathbb{E}_{\mathbb{P}}[\boldsymbol{v}^{i,\rho}] \}. \tag{17d}$$

We can use another convex risk measure instead of the expectation in (17d).

Remark 6. *We have that $\mathcal{A}^{i,\rho}_{a.s} \subseteq \mathcal{A}^{i,\rho}_{(1)} \subseteq \mathcal{A}^{i,\rho}_{(ic)} \subseteq \mathcal{A}^{i,\rho}_{\mathbb{E}}$. Therefore, in the same way as in Remark 3, the acceptability constraint yields a balance between risk-neutral ($\mathcal{A}^{i,\rho}_{\mathbb{E}}$) and a robust approach on risk ($\mathcal{A}^{i,\rho}_{a.s}$), with intermediary visions on risk ($\mathcal{A}^{i,\rho}_{(ic)}$, $\mathcal{A}^{i,\rho}_{(1)}$)*

6.4 Numerical illustration

We consider the stochastic version of the example presented in Section 4, where balancing prices $\{\boldsymbol{p}_t^B\}_{t\in[T]}$ are random variables with uniform, independent, distribution over $[0.35p_t^{DA}, 5p_t^{DA}]$. The problem can be formulated as a multistage program, where day-ahead purchases are decided in the first stage, and then each stage corresponds to a time slot where we can buy energy on the balancing market at a price $\boldsymbol{p}_t^B$.

We solve and discuss the sample average approximation of the two-stage approximation of this problem. More precisely, we draw 50 prices scenario, and solve a two-stage program where the first stage decisions are the day-ahead purchases, and the second stage decisions are the balancing purchases from time slot 1 to T. We set $I = 4, T = 10$, and we draw $\Omega = 50$ scenarios of balancing prices. For the prosumers' parameters and market prices, we use the data on Tables 1 and 6.

Table 6: Prices on the day-ahead market

t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
p_t^{DA}	3	3	7	4	2	10	7	4	7.5	8
$\underline{q}_t^{DA}$	12	12	12	12	12	12	12	12	12	12

We solve the problem with different combinations of aggregation operators and acceptability sets and can compare the impact of each combination on the solution. We denote m_a^f the model with aggregation operator $f \in \{US, SMS, UR, SMR\}$ and acceptability set $a \in \{\emptyset, \mathbb{E}, (ic), (1), (a.s)\}$. We read prosumers' expected percentage of savings with risk-neutral and worst-case approaches on Table 7. For example, in model $m_{(1)}^{US}$, we read that A_1 (resp. A_2, A_3, A_4) saves 36% (resp. $24\%, 35\%, 18\%$) of its costs. The expected cost of the aggregation is 882\$, thus asking for first-order acceptability costs 198\$. Moreover, we can observe the distribution of prosumers' expected costs with a risk-neutral (resp. worst-case) approach on Figure 5 (resp. Figure 6).

Our first comment is that the problems previously identified from a utilitarian perspective with no acceptability constraints still exist in a stochastic framework. Indeed, both with the risk-neutral utilitarian $\mathcal{F}_{I\times\Omega}^{US}$ and worst-case utilitarian $\mathcal{F}_{I\times\Omega}^{UR}$ operators, we observe on Table 7 that some prosumers can pay more in the aggregation compared to being alone (A_1 pays $+30\%$ in the stochastic approach, and $+28\%$ in the robust approach). This highlights the necessity for either acceptability constraints or an aggregation operator.

If we choose a fair approach through the objective (operators $\mathcal{F}_{I\times\Omega}^{SMS}$ and $\mathcal{F}_{I\times\Omega}^{SMR}$), we guarantee a higher percentage of savings to all prosumers than with the utilitarian approach, regardless of the chosen acceptability set. For example, with no acceptability constraints, all prosumers save at least 32% of their costs in a risk-neutral approach and 12% in a robust approach, compared to respectively -30% and -28% with the utilitarian approach. These guarantees come at the price of efficiency: for example, the expected aggregated cost of the scaled minimax approach is 13% higher than with the utilitarian approach in the risk-neutral case.

Table 7: Percentage of expected savings $\frac{\mathbb{E}\left[\boldsymbol{v}^{i,\rho}-\mathbb{E}\left[L^i(\boldsymbol{x}^i(\boldsymbol{\xi}))\right]\right]}{\mathbb{E}\left[\boldsymbol{v}^{i,\rho}\right]}$ made by A_i, expected aggregated costs $\mathbb{E}\left[\mathcal{F}_I(L^1(\boldsymbol{x}^{i,\star}(\boldsymbol{\xi}))_{i\in[I]})\right]$ and PoA in the corresponding model.

	Utilitarian Stochastic $\mathcal{F}^{US}_{I\times\Omega}$						Scaled Minimax Stochastic $\mathcal{F}^{SMS}_{I\times\Omega}$					
	A_1	A_2	A_3	A_4	(A)	PoA	A_1	A_2	A_3	A_4	(A)	PoA
$\emptyset$	-30	53	2	53	684	0	32	36	32	32	770	0
$\mathcal{A}^{\mathbb{E}}_{\mathbb{E}}$	0	53	0	45	684	0	32	36	32	32	770	0
$\mathcal{A}^{\mathbb{E}}_{(ic)}$	0	53	0	45	684	0	32	36	32	32	770	0
$\mathcal{A}^{\mathbb{E}}_{(1)}$	36	24	35	18	882	0.29	23	21	29	19	917	0.19
$\mathcal{A}^{\mathbb{E}}_{a.s}$	40	17	43	14	930	0.36	33	17	38	16	938	0.22
	Utilitarian Robust $\mathcal{F}^{UR}_{I\times\Omega}$						Scaled Minimax Robust $\mathcal{F}^{SMR}_{I\times\Omega}$					
	A_1	A_2	A_3	A_4	(A)	PoA	A_1	A_2	A_3	A_4	(A)	PoA
$\emptyset$	-28	53	4	52	686	0	41	12	41	102	974	0
$\mathcal{A}^{sup}_{\mathbb{E}}$	0	53	0	45	686	0	41	12	41	102	974	0
$\mathcal{A}^{\sup}_{(ic)}$	0	53	0	45	686	0	41	12	41	102	974	0
$\mathcal{A}^{\sup}_{(1)}$	26	25	26	17	898	0.31	41	12	41	102	974	0
$\mathcal{A}^{\sup}_{a.s}$	40	17	43	14	930	0.35	41	12	41	102	974	0

Conversely, when solving this problem with a utilitarian approach (operators $\mathcal{F}^{US}_{I\times\Omega}$ and $\mathcal{F}^{UR}_{I\times\Omega}$), we can increase the guaranteed percentage of savings by constraining more the acceptability. Indeed, with $\mathcal{F}^{US}_{I\times\Omega}$, all prosumers save at least from 0% with risk-neutral acceptability to 18% with first-order acceptability, and with $\mathcal{F}^{UR}_{I\times\Omega}$, it is from 0% to 17%. Notably, first-order acceptably has a price: the aggregation costs increase from 770\$ with risk-neutral acceptability to 917\$ with first-order acceptability. However, increasing the acceptability to almost-sure does not improve this guarantee, as the problem is now getting too constrained. In particular, with the robust scaled-minimax operator $\mathcal{F}^{UR}_{I\times\Omega}$, the problem is so constrained from the beginning that the choice of acceptability set is inconsequential: the distribution of costs is always the same.

In this case study, models with increasing-convex acceptability yield the same solution then for the case with risk-neutral acceptability. This suggests that the additional constraints imposed by increasing-convex acceptability do not eliminate the optimal solution found under risk-neutral acceptability. However, in the general case, one could expect to obtain different solutions, since increasing-convex acceptability defines a more restrictive feasible set (see Remark 6).

All in all, we obtain various solutions with different balances between efficiency and fairness and different risk visions. In this example, in the stochastic case, if we want to give the same guarantees to every prosumer, the natural choice is operator $\mathcal{F}^{SMS}_{I\times\Omega}$. This approach costs 13% more than in $m_\emptyset^{\mathcal{F}^{US}_{I\times\Omega}}$, but guarantees at least 32% of savings to each prosumer. However, with a risk-averse approach, the robust scaled-minimax operator If prosumers are risk-averse, we could opt for a robust operator $\mathcal{F}^{SMR}_{I\times\Omega}$; however this approach might overly reduce the potential gains from aggregation. Alternatively, the

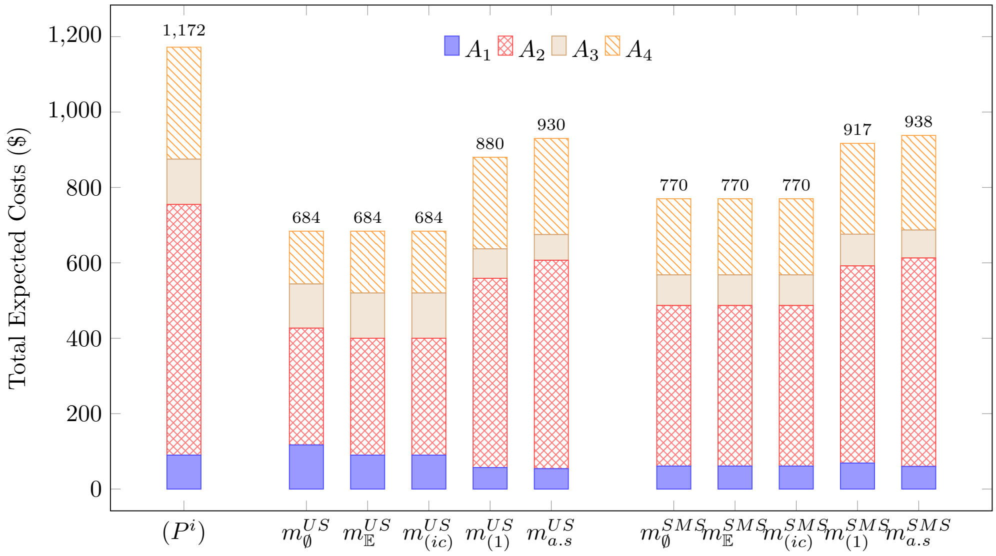

Fig. 5: The bars correspond to the results of different models we solve with a stochastic approach. The first one is the model without aggregation: we solve each $(P^{i,\mathbb{E}})$ independently. The second bar corresponds to the problem solved with operator $\mathcal{F}_{I\times\Omega}^{US}$ without acceptability constraints. Then, the four following bars correspond to the same problem with increasingly strong acceptability $(\mathcal{A}_{\mathbb{E}}^{\mathbb{E}}, \mathcal{A}_{(ic)}^{\mathbb{E}}, \mathcal{A}_{(1)}^{\mathbb{E}}, \mathcal{A}_{a.s}^{\mathbb{E}})$. The following bar is for the problem solved with operator $\mathcal{F}_{I\times\Omega}^{SMS}$ without acceptability constraints, followed by four bars with different acceptability sets. Each bar is decomposed in 4 blocks corresponding to the expected share $\mathbb{E}[L^i(\boldsymbol{x}^i(\boldsymbol{\xi}), \boldsymbol{\xi})]$ of each consumer i. At the top of each bar, we can read the sum of expected aggregated costs in the corresponding model.

utilitarian oper ator $(\mathcal{F}_{I\times\Omega}^{US})$ combined with first-order acceptability accounts for the risks faced by consumers while being less restrictive than a robust approach. In this case, it ensures at least 18% of savings to each prosumer and induces 29% of efficiency loss.

Finally, we test the framework using different samples of scenarios drawn from the same probability distribution (see Appendix D). The results remain consistent across different scenario sets: while costs and proportional savings may fluctuate, the relative impact of the fairness operator and acceptability constraints remains unchanged. Furthermore, increasing the variability of the scenarios drawn for these tests does not significantly impact the empirical conclusions we have drawn previously.

Conclusion

In this paper, we provide a framework including dynamic and stochastic cases to accommodate fairness in aggregation problems like prosumer aggregation, virtual power plant, portfolio management in energy markets, ancillary service provision, etc.

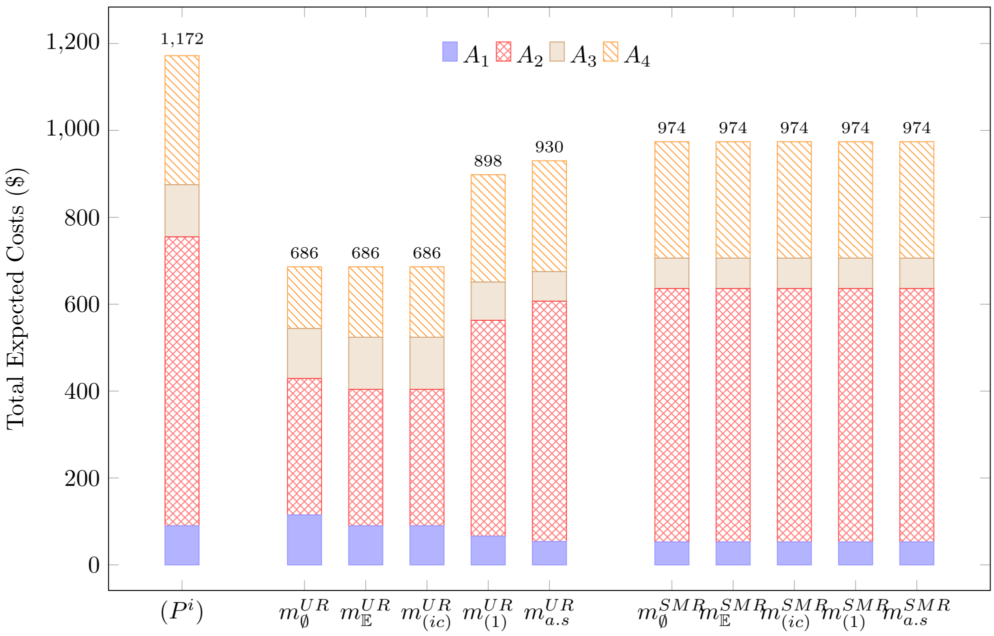

Fig. 6: This figure can be read like Figure 5, except that the two considered operators are $\mathcal{F}^{UR}_{I\times\Omega}$ and $\mathcal{F}^{SMR}_{I\times\Omega}$.

A salient point is to consider, on one hand, acceptability constraints that ensures that each participant has interest in participating in the aggregation, and on the other hand objective function (*e.g.,* utilitarian, scaled minimax, and proportional) that fairly share additional gains. Through the discussion in Section 2, we emphasized the importance of fairness and the need to carefully consider how to model it and be aware of the different approaches available.

In our numerical example, we obtained a spectrum of options from various combinations of acceptability sets and objective functions, ranging from the most efficient models (with the lowest aggregated costs) to the fairest models (where agents' gains are more comparable). Too-restrictive acceptability sets or a bargaining approach (proportional operator $\mathcal{F}^P_I$) can significantly reduce efficiency, while an intermediate approach leverages aggregation benefits without excessively favoring certain prosumers. Thus, we recommend the scaled min-max agent aggregator with progressive acceptability constraint in the dynamic case (resp. increasing convex acceptability in the stochastic case), which balances efficiency and fairness well.

In future work, we plan to discuss the extension of the aggregation problem to a multistage stochastic program, where we would have to combine the partial orders presented in the dynamic framework in Section 5 and the stochastic orders of the stochastic framework in Section 6. This will require a discussion of possible aggregators $\mathcal{F}_{T\times\Omega\times I}$ over agents, time, and uncertainty simultaneously. Although we can easily assume a factorization of the form $\mathcal{F}_I \odot \mathcal{F}_{T\times\Omega}$, it would not be realistic to describe $\mathcal{F}_{T\times\Omega}$ as the composition of a time aggregator and an uncertainty aggregator.

Indeed, such a factorization would not guarantee time-consistency of the problem and might not even preserve non-anticipativity. Acceptability constraints must be defined using multivariate stochastic order (see Dentcheva and Ruszczyński (2009))whose mathematical programming representations are more involved.

Further, all problems in Sections 4, 5.3 and 6.4 are solved using generic MILP (Gurobi) or MINLP (Juniper.jl Kröger et al. (2018)) solvers, which perform well for problems of reasonable size. However, as complexity increases—particularly in dynamic and stochastic settings—scalability becomes a challenge. Since this paper focuses on the methodological framework for fairness rather than numerical resolution, we tested the framework on small instances that did not require specialized algorithms. For larger or multistage stochastic problems, where the size grows exponentially, decomposition methods should be considered to improve computation.

Finally, it would be interesting to investigate potential bounds on the *price of acceptability*.

Statements and Declarations

The authors declare they have no financial interests.

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A Computing Shapley's values

In this appendix, we give more details on how to compute Shapley's values for the application in Section 4. First, we introduce the general formulas and definitions required to compute Shapley's values, then we apply them to our example.

A.1 Definitions and Formulas

Shapley's values are commonly seen as a fair distribution of costs (or revenues) when agents cooperate in a cooperative game. The Shapley Value returns each player's fair share of the total gains by averaging their contributions across all possible ways they can join the coalition. Let N be the number of agents in the game. We denote $w : 2^{|N|} \to \mathbb{R}$, the worth function associating to a coalition S, the expected payoff obtained by cooperation.

To compute Shapley's values, first, we compute $\delta_i(S)$, the marginal contribution of agent i to coalition $S \subset N$:

$$\delta_i(S) = w(S \cup \{i\}) - w(S) \tag{18a}$$

Then, the Shapley value of agent i, given a characteristic function w, is $\phi_i(w)$:

$$\phi_i(w) = \frac{1}{n} \sum_{S \subset N \setminus \{i\}} \binom{n-1}{|S|}^{-1} \delta_i(S) \tag{18b}$$

A.2 Application to our example

We consider an example with 4 agents. For each coalition $S \subset [4]$, we solve the aggregated problem with utilitarian operator $\mathcal{F}_S^U$ and acceptability constraints $\mathcal{A}_1$- as in cooperation games we must satisfy individual rationality properties- and obtain optimal solution c_S Then, we introduce the characteristic function assessing the worth of coalition S as:

$$w(S) = \sum_{i \in S} v^i - c_S, \tag{19a}$$

where v^i is the optimal value of problem (P^i). Thus $w(S)$ designates the savings made by coalition S when they cooperate. On Tables 8 and 9, we detail the intermediate computations, and in Equations (19b) to (19e) we compute shapley's values.

$$\begin{aligned}\phi_1(w) = \frac{1}{4}\Bigg[&\binom{3}{1}^{-1} \big(\delta_1(\{2\}) + \delta_1(\{3\}) + \delta_1(\{4\})\big) \\ &+ \binom{3}{2}^{-1} \big(\delta_1(\{2,3\}) + \delta_1(\{2,4\}) + \delta_1(\{3,4\})\big)\end{aligned}$$

	{1}	{2}	{3}	{4}	{1,2}	{1,3}	{1,4}	{2,3}	{2,4}	{3,4}	{1,2,3}	{1,2,4}	{1,3,4}	{2,3,4}	{1,2,3,4}
$\sum_{i\in S} v^i$	50	280	40	168	330	90	218	320	448	208	370	498	258	488	538
c_S	50	280	40	168	330	90	218	320	448	208	244	365	162	392	333
$w(S)$	0	0	0	0	0	0	0	0	0	0	126	133	96	96	205

Table 8: Costs with and without cooperation and worth w of each coalition S.

	{1}	{2}	{3}	{4}	{1,2}	{1,3}	{1,4}	{2,3}	{2,4}	{3,4}	{1,2,3}	{1,2,4}	{1,3,4}	{2,3,4}
$\delta_1(S)$	-	0	0	0	-	-	-	126	133	96	-	-	-	109
$\delta_2(S)$	0	-	0	0	-	126	133	-	-	96	-	-	109	-
$\delta_3(S)$	0	0	-	0	126	-	96	-	96	-	-	72	-	-
$\delta_4(S)$	0	0	0	-	133	96	-	96	-	-	79	-	-	-

Table 9: Marginal contributions of agent i to each coalition.

$$+ \binom{3}{3}^{-1} \delta_1(\{2,3,4\})\Bigg]$$

$$\phi_1(w) = \frac{1}{4}\left[\frac{1}{3} \times 0 + \frac{1}{3}(126 + 133 + 96) + 1 \times 109\right] = 56.8 \tag{19b}$$

$$\phi_2(w) = \frac{1}{4}\left[\frac{1}{3} \times 0 + \frac{1}{3}(126 + 133 + 96) + 1 \times 109\right] = 56.8 \tag{19c}$$

$$\phi_3(w) = \frac{1}{4}\left[\frac{1}{3} \times 0 + \frac{1}{3}(126 + 96 + 96) + 1 \times 72\right] = 44.5 \tag{19d}$$

$$\phi_4(w) = \frac{1}{4}\left[\frac{1}{3} \times 0 + \frac{1}{3}(133 + 96 + 96) + 1 \times 79\right] = 46.8 \tag{19e}$$

As the values computed here represent the way to distribute savings, we must subtract $\phi_i(w)$ from the cost of agent i to obtain its costs in the aggregation after fair allocation through Shapley's values in Table 10.

	A_1	A_2	A_3	A_4
$\phi_i(w)$	56.8	56.8	44.5	46.8
v^i	50	280	40	168
$L^i(x^i)$	-6.8	223.2	-4.5	121.2
$\frac{100(v^i - L^i(x^i))}{v^i}$	114	20	111	28

Table 10: Costs and proportional savings of the agents with Shapley's post-allocation scheme.

B Additional results for Section 4

We report tests on various instances that show that our observations are not solely problem-dependent. Specifically, we consider cases with different gaps between day-ahead and balancing prices.

As a reference, we use the balancing prices in Table 11. We then evaluate the models with day-ahead prices set as $p_t^{DA} = \gamma p_t^B$, where $\gamma \in \{0.5, 0.6, 0.7, 0.8, 0.9\}$. A lower γ increases the benefit of aggregation. The parameters for the minimum and maximum energy consumption of prosumers are given in Table 1. Since we set $T = 10$, the total demand is adjusted to $Q = [20, 50, 16, 27]$.

Table 11: Balancing prices per stage

t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
p_t^B	34	35	30	33	24	25	38	35	39	25

To compare the different instances, we consider three indicators. First, the **minimal proportional savings**:

$$\alpha^\star := \min_{i \in [I]} \Big\{ \frac{v^i - L^i(x^i)}{v^i} \Big\},$$

representing the smallest proportional savings received by any agent. Then, the **dispersion of proportional savings**, denoted δ:

$$\delta := \max_{i \in [I]} \Big\{ \frac{v^i - L^i(x^i)}{v^i} \Big\} - \alpha^\star,$$

that measures the difference between the highest and lowest proportional savings. Finally, **global aggregation gains**:

$$v^+ := \sum_{i \in [I]} v^i - v(m_\alpha^f),$$

quantifying the overall benefit of aggregation.

Figure 7 presents the different indicators for $\gamma \in \{0.5, 0.6, 0.7, 0.8, 0.9\}$ across models m_1^f with $f \in \{U, SM, P\}$ and $\alpha = 1$. Several general observations hold across the five instances.

First, as expected, the minimax operator consistently yields the highest minimal proportional savings $\alpha^\star$, regardless of the acceptability level. It also minimizes the dispersion of proportional savings δ. Second, for a given acceptability set, the utilitarian operator achieves the highest overall gains. As γ decreases, the gap between day-ahead and balancing prices widens, making access to the day-ahead market more advantageous. Consequently, aggregation becomes more beneficial for each agent, leading to higher global gains and $\alpha^\star$. However, as γ increases from 0.5 to 0.8, the dispersion of proportional savings grows under the utilitarian operator, while it decreases under the proportional one.

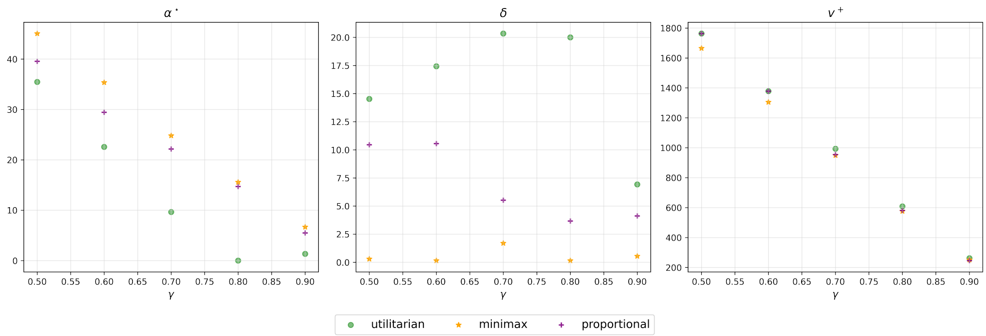

Fig. 7: Minimal proportional saving $\alpha^\star$, dispersion of proportional savings δ and global aggregation gains v^+ obtained by solving m_1^f for $f \in \{U, SM, P\}$ for instances depending on the gap γ between day-ahead and balancing prices.

C Modeling of stochastic dominance constraints

We present here practical formulas to implement the stochastic orders dominance constraints introduced in Section 6.3. Those constraints establish a dominance between $\boldsymbol{v}^{i,\rho}$, the random variable representing i independent costs, and $\boldsymbol{u}^{i,\rho}$, the random variable representing i costs in the aggregation.

C.1 First-order dominance constraint model

The first-order dominance constraints (17b) model is based on Gollmer et al. (2008).

Lemma 1. *In Problem* (A^ρ), *acceptability constraints* $\boldsymbol{u}^{i,\rho} \preceq_{(1)} \boldsymbol{v}^{i,\rho}$ *can be modeled with:*

$$b^i_{\xi,\eta} \in \{0,1\} \qquad \forall \eta \in [\Omega],\ \forall \xi \in \Omega \tag{20a}$$
$$u^{i,\rho}_\xi - v^{i,\rho}_\eta \leq M\ b^i_{\xi,\eta} \qquad \forall \eta \in [\Omega],\ \forall \xi \in \Omega \tag{20b}$$
$$\sum_{\xi=1}^{\Omega} \pi_\xi b^i_{\xi,\eta} \leq a_\eta \qquad \forall \eta \in [\Omega]. \tag{20c}$$

We denote $a_\eta := \mathbb{P}(\boldsymbol{v}^{i,\rho} > v^{i,\rho}_\eta)$, *which is a parameter for the aggregation problem.*

Proof. As Ω is assumed to be finite, $\boldsymbol{v}^{i,\rho}$ follows discrete distribution with realizations $v^{i,\rho}_\eta$ for $\eta \in \Omega$. Then,

$$\begin{aligned} \boldsymbol{u}^{i,\rho} \preceq_{(1)} \boldsymbol{v}^{i,\rho} &\iff \mathbb{P}(\boldsymbol{u}^{i,\rho} > \eta) \ \leq\ \mathbb{P}(\boldsymbol{v}^{i,\rho} > \eta) && \forall \eta \in \mathbb{R} \\ &\iff \mathbb{P}(\boldsymbol{u}^{i,\rho} > v^{i,\rho}_\eta) \ \leq\ \mathbb{P}(\boldsymbol{v}^{i,\rho} > v^{i,\rho}_\eta) && \forall \eta \in \Omega. \end{aligned}$$

Then, using $\mathbb{P}(\boldsymbol{X} > x) = \mathbb{E}[\mathbb{1}_{\boldsymbol{X}>x}]$, and introducing binary variables $b^i_{\xi,\eta} = \mathbb{1}_{u^{i,\rho}_\xi > v^{i,\rho}_\eta}$, we get:

$$\left(\mathbb{P}(\boldsymbol{u}^{i,\rho} > v^{i,\rho}_\eta) \;\le\; \mathbb{P}(\boldsymbol{v}^{i,\rho} > v^{i,\rho}_\eta) \iff \sum_{\xi=1}^{\Omega} \pi_\xi b^i_{\xi,\eta} \le a_\eta\right) \qquad \forall \eta \in \Omega.$$

To linearize the definition of $b^i_{\xi,\eta}$, we rely on big-M constraint:

$$\begin{aligned} & b^i_{\xi,\eta} \in \{0,1\} && \forall \eta \in \Omega, \forall \xi \in \Omega \\ & u^{i,\rho}_\xi - v^{i,\rho}_\eta \le M b^i_{\xi,\eta} && \forall \eta \in \Omega, \forall \xi \in \Omega. \end{aligned}$$

□

C.2 Increasing convex dominance constraint model

The increasing convex dominance constraints (17c), is based on Carrión et al. (2009).
Lemma 2. *In problem (A^ρ), the acceptability constraint $\boldsymbol{u}^{i,\rho} \preceq_{(ic)} \boldsymbol{v}^{i,\rho}$ can be modeled with:*

$$s^i_{\xi,\eta} \ge 0 \qquad \forall \eta \in [\Omega],\ \forall \xi \in \Omega \tag{21a}$$

$$s^i_{\xi,\eta} \ge u^{i,\rho}_\xi - v^{i,\rho}_\eta \qquad \forall \eta \in [\Omega],\ \forall \xi \in \Omega \tag{21b}$$

$$\sum_{\xi=1}^{\Omega} \pi_\xi s^i_{\xi,\eta} \le a^{ic}_\eta \qquad \forall \eta \in [\Omega]. \tag{21c}$$

We denote $a^{ic}_\eta := \mathbb{E}[(\boldsymbol{v}^{i,\rho} - v^{i,\rho}_\eta)^+]$.

Proof. As in Appendix C.1, we know that $\boldsymbol{v}^{i,\rho}$ follows a discrete distribution with realizations $v^{i,\rho}_\eta)$ for $\eta \in \Omega$. Then,

$$\begin{aligned} \boldsymbol{u}^{i,\rho} \preceq_{(ic)} \boldsymbol{v}^{i,\rho} &\iff \mathbb{E}\big[\,(\boldsymbol{u}^{i,\rho} - \eta)^+\,\big] \le \mathbb{E}\big[\,(\boldsymbol{v}^{i,\rho} - \eta)^+\,\big] && \forall \eta \in \mathbb{R} \\ &\iff \mathbb{E}\big[\,(\boldsymbol{u}^{i,\rho} - v^{i,\rho}_\eta)^+\,\big] \le \mathbb{E}\big[\,(\boldsymbol{v}^{i,\rho} - v^{i,\rho}_\eta)^+\,\big] && \forall \eta \in \Omega. \end{aligned}$$

We introduce positive variables $s^i_{\xi,\eta} = (u^{i,\rho}_\xi - v^{i,\rho}_\eta)^+$, for $\eta \in \Omega$. Thus, we can model the increasing convex dominance constraints as: follows

$$\left(\mathbb{E}\big[\,(\boldsymbol{u}^{i,\rho} - v^{i,\rho}_\eta)^+\,\big] \le \mathbb{E}\big[\,(\boldsymbol{v}^{i,\rho} - v^{i,\rho}_\eta)^+\,\big] \iff \sum_{\xi=1}^{\Omega} \pi_\xi s^i_{\xi,\eta} \le a^{ic}_\eta\right) \qquad \forall \eta \in [\Omega].$$

□

D Additional results for Section 6

We present additional tests for different instances of the problem described in Section 6.4. We consider instances where balancing prices $\{\boldsymbol{p_t^B}\}$ are random variables with uniform, independent distribution over $[0.3p_t^{DA}, 3p_t^{DA}]$. From this distribution, we randomly generate 20 samples of 50 scenarios. Prosumers' parameters and day-ahead market prices are taken from Tables 1 and 6.

For each scenario sample, we solve m_a^f with $f \in \{US, SMS\}$ and $a \in \{\emptyset, \mathbb{E}, (ic), (1), (a.s)\}$. As in Appendix B, we report the minimum proportional savings $\alpha^\star$, dispersion of proportional savings δ and global aggregation gains v^+ for each model. For each indicator, we compute the expectation, standard deviation, minimum, and maximum values across the 20 instances and summarize the results in Tables 12 and 13.

	$\alpha^\star$				δ				v^+			
	average	std	min	max	average	std	min	max	average	std	min	max
$m_\emptyset^{US}$	-39.8	16	-75	-9.5	69	18.8	30.8	108.5	148.1	12.9	123.7	172.7
$m_\mathbb{E}^{US}$	0	0	0	0	27	2.9	21.4	31.4	147.7	13.1	123.7	171.1
$m_{(ic)}^{US}$	0	0	0	0	27	2.9	21.4	31.4	147.7	13.1	123.7	171.1
$m_{(1)}^{US}$	6.8	3.4	0	11.6	10.8	5.3	0	17	80.5	41.5	0	135.7
$m_{(a.s)}^{US}$	0.7	2.2	0	8.3	1.5	4.5	0	15.3	8.4	26.3	0	97.4

Table 12: Average value, standard deviation, minimum and maximum values of the minimal proportional saving $\alpha^\star$, the dispersion of proportional savings δ and global aggregation gains v^+ obtained by solving the models with the risk-neutral utilitarian operator $\mathcal{F}_{I\times\Omega}^{US}$ over 20 different samplings of scenarios.

	$\alpha^\star$				δ				v^+			
	average	std	min	max	average	std	min	max	average	std	min	max
$m_\emptyset^{SMS}$	16.8	1.2	13.8	18.7	0.6	0.9	0	4.2	134.7	13	107.2	159.6
$m_\mathbb{E}^{SMS}$	16.8	1.2	13.8	18.7	0.6	0.9	0	4.2	134.7	13	107.2	159.6
$m_{(ic)}^{SMS}$	16.8	1.2	13.8	18.7	0.6	0.9	0	4.2	134.7	13	107.2	159.6
$m_{(1)}^{SMS}$	7.2	4.3	0	13.6	8.2	5.4	0	14	72.6	41.5	0	119.2
$m_{(a.s)}^{SMS}$	0.7	2.2	0	8.3	1.4	4.5	0	15.1	8.4	26.3	0	97.3

Table 13: Average value, standard deviation, minimum and maximum values of the minimal proportional saving $\alpha^\star$, the dispersion of proportional savings δ and global aggregation gains v^+ obtained by solving the models with the risk-neutral scaled-minimax operator $\mathcal{F}_{I\times\Omega}^{SMS}$ over 20 different samplings of scenarios.

From these tables, we observe that although the values fluctuate between different samples, the comparison between models remains consistent. Notably, under the risk-neutral scaled-minimax operator $\mathcal{F}^{SMS}_{I\times\Omega}$, $\alpha^\star$ and δ almost do not fluctuate. Moreover, when comparing the utilitarian approach ($\mathcal{F}^{US}_{I\times\Omega}$) with the minimax approach ($\mathcal{F}^{SMS}_{I\times\Omega}$), the latter consistently yields the highest $\alpha^\star$ and minimal δ. Finally, increasing the level of acceptability reduces the dispersion of proportional savings but also leads to a decrease in global gains.