Chance-Constrained Economic Dispatch of Generic Energy Storage under Decision-Dependent Uncertainty

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***Abstract*—Aggregated and coordinated generic energy storage (GES) resources provide inexpensive, but uncertain flexibility for power grid operations and renewable energy integration. While different types of uncertainty exist, the literature mostly focuses on decision-independent uncertainties (DIUs). This manuscript focuses on two types of newly-introduced decision-dependent uncertainties (DDUs) for dispatching GES resources related to uncertain SOC boundaries and capacity degradation affected by accumulated discomfort and incentive price. To accommodate these different types of uncertainties, we present a novel chance-constrained optimization (CCO) approach for the day-ahead (DA) economic dispatch of GES resources. Two tractable solution methods are developed - a robust approximation and an iterative algorithm - to solve the proposed CCO with both DIUs and DDUs. Reliability indices are further introduced to verify the applicability of the proposed approach with respect to the response reliability of GES resources. Simulation-based analysis shows that the proposed method yields conservative but highly credible strategies by reducing incomplete knowledge of DIUs via data-driven parameter identification and incorporating DDUs in constraints. This effectively reduces costs in reserve markets and demand for corrective dispatch.**

***Index Terms*—generic energy storage, chance-constrained optimization, decision-dependent uncertainty, response reliability.**

I. INTRODUCTION

INCREASING challenges have emerged with the high penetration of renewable energy resources (RES), which mainly manifests itself in frequency and voltage regulation problems, power system stability problems and reliability [1]. Deterministic dispatchable resources, such as conventional power plants and energy storage (ES), have been widely used to overcome these challenges. However, relying on these deterministic resources may not be viable in the future. First, the number of fossil fuel power plants will decrease dramatically due to carbon dioxide emission reduction targets from recent public policies [2]-[3]. Second, direct control of a myriad of ES assets within different sectors (e.g., industrial, commercial, and residential) and geographical locations will be costly and unlikely. Thus, demand response (DR) and other forms of dispatchable demand-side resources represent flexibility that is a less costly alternative and can support power system operations. Such flexibility may be leveraged and controlled via price-based [4] and incentive-based mechanisms [5]. Significant effects on risk hedging and economic operation have been reported [4]-[6], via optimal control and adjustment of energy usage of thermostatically controlled load (TCL), electric vehicle (EV), and battery energy storage (BES). Most of such distributed energy resources have the attributes and abilities of ES devices, hence motivating the term "virtual energy storage" (VES) [7]. In this paper, ES and VES are considered alike to yield generic energy storage (GES) to unify modeling and uncertainty description of both an individual GES resource and a portfolio of GES resources.

The literature related to the modeling and economic dispatch (ED) of GES is vast already. Early research mainly focused on the flexibility modeling of GES associated with multiple factors (e.g., incentive, user preference, etc.) and exploiting flexibility of diverse flexible loads [7]-[9]. Among these works, a virtual battery model is introduced in [9] and describes how to obtain the GES parameters from TCL assets by using first-order energy dynamics, but without considering time-varying and stochastic features. However, the considerable difference between GES and conventional ES mainly lies in the inherent exogenous and endogenous uncertainties of the former [10]. Exogenous uncertainties are uncertainties triggered by factors outside of the system boundary, e.g., from the market or the weather. They are also called decision-independent uncertainties (DIUs), as they are independent of the operation and control strategy (e.g., uncertainties in the power of RES and load). Modeling and probabilistic optimization of GES under diverse DIUs have been widely investigated in previous works. They commonly consider their uncertain power and energy capacity and response probability of GES, derived from *(i)* forecast error of ambient space (e.g., wind and temperature) [11], *(ii)* DR duration [12] and customers' comfort [13], *(iii)* economic effect driven by incentive or price [14], and *(iv)* model reduction error [15] and SOC estimation error [16]. For all the studies presented in [11]-[16], the structure of DIUs can be fully determined before ED with complete information of uncertainties. However, some stochastic properties (e.g., response probability, temperature preference) may actually be affected by the operational and control strategies themselves. These are generally overlooked and simplified as static (and known) probability distributions. This type of stochasticity is

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called endogenous uncertainties, and denoted decision-dependent uncertainties (DDUs).

DDUs are divided into two distinct types which we will refer to as Type 1 and Type 2 [17]. For Type 1-DDUs, decisions influence the parameter realizations by altering the underlying probability distributions for the uncertain parameters. In contrast, for Type 2-DDUs, decisions influence the parameter realizations by affecting the timing or content of the information we observe, which has been addressed in planning problems using, e.g., multi-stage stochastic optimization with adequately defined scenario trees [18]. For power operations, DDUs are represented by Type 1-DDUs, which is the focus herein. Though sparse literature considered DDUs, there are still some closely relevant works using robust optimization (RO) or stochastic optimization (SO) methods under DIUs. The assumption of linear decision-dependency of polyhedral uncertainty sets on decision variables is considered in [19]-[20], rendering a static RO-DDUs model. Adaptive RO-DDUs models that incorporate wait-and-see decisions and endogenous uncertainties are studied in [21]-[22], using iteration algorithms to handle the computational intractability raised by the complex coupling relationship between uncertainties and decisions at two stages. Ref. [23] reformulates the problem into multi-stage SO based on a scenario tree. The related works indicate that the key to the optimization under DDUs is to decouple decisions and uncertainty description through iterative algorithms. And, it becomes necessary to consider DDUs on feasible regions and other potential effects (e.g., loss of reliability), thus, making the problem more complex and intractable. To satisfy decision-makers with different risk preferences and assess the reliability performance, chance-constrained optimization (CCO) may be preferred. To the best our knowledge, no research work has concurrently modeled DDUs of GES in the CCO framework, which is specific to this paper.

To fill in the research gap in both modeling and solution methodologies, this paper addresses the day-ahead chance-constrained economic dispatch of GES with a modified baseline model where both DIUs and DDUs are involved. The main contributions are threefold:

i) Modeling: We propose a modified baseline model and detailed uncertainty description with DIUs and DDUs of GES. Compared with the model from [9], the GES model studied herein incorporates time-varying and rate-limited properties and considers four common device types where parameters can be obtained by a data-driven approach [15]. For the uncertainty description, we consider three types of DIUs (on-off state probability, parameter identification errors and forecast error from the user environment) and two types of DDUs (SOC boundaries and capacity degradation affected by incentive price and accumulated discomfort).

ii) Solution methodology: Two tractable reformulations are proposed to effectively solve the CCO with DDUs by decoupling decisions and uncertainties. For DDUs with incomplete knowledge, a robust approximation approach is introduced to obtain conservative results based on the maximum value of the unknown inversed CDF, by generations of Cantelli's inequality. For specific structures of DDUs, an iterative algorithm allows reducing the optimality gap, while using the robust approximation value as a starting point. The iterative algorithm is guaranteed to converge to the optimum within a finite number of iterations.

iii) Numerical study: We introduce two reliability indices: *loss of response power probability* and *expected response energy not served* to assess the effectiveness and credibility of different strategies and the negative consequence of overlooking various types of DIUs and DDUs in previous works. The case study shows that the proposed models and methods substantially outperform previous approaches in terms of the response reliability due to 1) reduced incomplete knowledge of DIUs via data-driven parameter identification, 2) incorporating DDUs in constraints, which effectively reduces costs in reserve markets and demand for corrective dispatch.

The remainder of this paper is organized as follows. The modified baseline model of GES is proposed in Section II. Uncertainty modeling with DIUs and DDUs is presented in Section III. CCO under DIUs and DDUs and two reformulation methods are proposed in Section IV. A numerical study is provided in Section V to illustrate comparative performance. Extensions of the proposed model are discussed in Section VI. Finally, conclusions are summarized in Section VII.

II. Baseline Model of Generic Energy Storage

The basic model of GES initially presented in [8] is extended herein for four types of commonly used energy storage, i.e., BES, inverter air-conditioner (IVA), and fixed-frequency air-conditioner (FFA), and EV. This manuscript extends the basic GES model to incorporate time-varying and ramp-rate properties. GES constraints are provided in (1a-1f). Updated constraint (1a) defines the relationship between charging and discharging actions, SOC, and energy losses. The newly-introduced constraint (1b) limits the charging/discharging rates on changes in SOC. Updated constraint (1c) limits the time-varying upper and lower SOC. Constraint (1d) ensures a sustainable energy state for the GES over time. Constraints (1e-1f) limit the upper and lower charging and discharging actions. Since sufficient conditions are satisfied (i.e., charging price ("-") is lower than discharging price ("+")), the complementarity constraint for simultaneous charging and discharging, $P_{\mathrm{ch},i,t}^{\mathrm{GES}} P_{\mathrm{dis},i,t}^{\mathrm{GES}} = 0$, is relaxed and can be removed from model [24].

GES Constraints: $\forall t \in \mathbf{\Omega}_T$, $\forall i \in \mathbf{\Omega}_G$

$$SOC_{i,t+1}^{\mathrm{GES}} = (1-\varepsilon_i^{\mathrm{GES}}) SOC_{i,t}^{\mathrm{GES}} + \eta_{\mathrm{ch},i}^{\mathrm{GES}} P_{\mathrm{ch},i,t}^{\mathrm{GES}} \Delta t / S_i^{\mathrm{GES}} - P_{\mathrm{dis},i,t}^{\mathrm{GES}} \Delta t / \eta_{\mathrm{dis},i}^{\mathrm{GES}} S_i^{\mathrm{GES}} + \alpha_i^{\mathrm{GES}} - \beta_{i,t}^{\mathrm{GES}} \tag{1a}$$

$$-\Delta SOC_{i,\mathrm{down}}^{\mathrm{GES}} \le \Delta SOC_{i,t}^{\mathrm{GES}} \le \Delta SOC_{i,\mathrm{up}}^{\mathrm{GES}} \tag{1b}$$

$$SOC_{i,t,\min}^{\mathrm{GES}} \le SOC_{i,t}^{\mathrm{GES}} \le SOC_{i,t,\max}^{\mathrm{GES}} \tag{1c}$$

$$SOC_{i,T}^{\mathrm{GES}} = SOC_{i,0}^{\mathrm{GES}} \tag{1d}$$

$$0 \le P_{\mathrm{ch},i,t}^{\mathrm{GES}} \le P_{\mathrm{ch},i,\max}^{\mathrm{GES}} \tag{1e}$$

$$0 \le P_{\mathrm{dis},i,t}^{\mathrm{GES}} \le P_{\mathrm{dis},i,\max}^{\mathrm{GES}} \tag{1f}$$

where $\mathbf{\Omega}_T$ and $\mathbf{\Omega}_G$ are sets of time periods and GES units, respectively. Subscripts i and t define GES unit and time period, respectively. Decision variables $P_{\mathrm{ch},i,t}^{\mathrm{GES}}$ and $P_{\mathrm{dis},i,t}^{\mathrm{GES}}$ are the charge, discharge power. Variable $SOC_{i,t}^{\mathrm{GES}}$ defines SOC. Times-step Δt is index used in ED. Parameters $P_{\mathrm{ch},i,\max}^{\mathrm{GES}}$ and $P_{\mathrm{dis},i,\max}^{\mathrm{GES}}$ are the maximum charge and discharge ratings, respectively, while $SOC_{i,t,\max}^{\mathrm{GES}}$ and $SOC_{i,t,\min}^{\mathrm{GES}}$ are the maximum and minimum SOC boundaries, respectively. Up and down permitted rate for changes in SOC are given by $\Delta SOC_{i,\mathrm{up}}^{\mathrm{GES}}$ and $\Delta SOC_{i,\mathrm{down}}^{\mathrm{GES}}$. Parameters $\eta_{\mathrm{ch},i}^{\mathrm{GES}}$ and $\eta_{\mathrm{dis},i}^{\mathrm{GES}}$ are the charge and discharge efficiency, while $\varepsilon_i^{\mathrm{GES}}$ and S_i^{GES} are the self-discharge rate and energy capacity. The newly introduced α_i^{GES} and $\beta_{i,t}^{\mathrm{GES}}$ are constant and energy losses from user environment (ambient space for TCL and self-consumption for EV).

Afterward, the relationship is analyzed between model parameters and the physical parameters of each type, which is summarized in Table 1. Thermal capacity, thermal resistance, and conversion efficiency of TCL are given by C, R, and η, while T^{in}, T^{out} and T^{set} define the indoor, outdoor, and setpoint temperature. These parameters can be obtained by data-driven methods (i.e., load decomposition and parameter identification) [15]. The transformation of TCLs into GES begins with the thermodynamics of a 1st order equivalent thermal parameter (ETP) model, and the difference between IVA and FFA lies in the control mode and power property. Moreover, great differences are witnessed among different types of GES. For instance, the self-discharge rate $\varepsilon_i^{\mathrm{GES}}$ is usually ignored for BES, but VESs tend to embrace a relatively high value. Besides, most of the parameters are constant for BE-S, but time-varying features appear for VES concerning SOC limits, parameters, and energy losses from user environment.

TABLE I COMPARISONS BETWEEN FOUR TYPES OF GES UNITS

GES Type	BES	TCL (IVA/FFA)	EV
SOC_t^{GES}	SOC_t^{BES}	$\dfrac{T_{\max}^{\mathrm{in,TCL}} - T_t^{\mathrm{set,TCL}}}{T_{\max}^{\mathrm{in,TCL}} - T_{\min}^{\mathrm{in,TCL}}}$	SOC_t^{EV}
$P_{\mathrm{ch,max}}^{\mathrm{GES}}$	$P_{\mathrm{ch,max}}^{\mathrm{BES}}$	$P_{\max}^{\mathrm{TCL}} - \dfrac{T_0^{\mathrm{out,TCL}} - T_0^{\mathrm{set,TCL}}}{\eta^{\mathrm{TCL}} R^{\mathrm{TCL}}}$	$P_{\mathrm{ch,max}}^{\mathrm{EV}}$
$P_{\mathrm{dis,max}}^{\mathrm{GES}}$	$P_{\mathrm{dis,max}}^{\mathrm{BES}}$	$\dfrac{T_0^{\mathrm{out,TCL}} - T_0^{\mathrm{set,TCL}}}{\eta^{\mathrm{TCL}} R^{\mathrm{TCL}}} - P_{\min}^{\mathrm{TCL}}$	$P_{\mathrm{dis,max}}^{\mathrm{EV}}$
$SOC_{t,\min}^{\mathrm{GES}}$	$SOC_{\min}^{\mathrm{BES}}$	$\dfrac{T_{\max}^{\mathrm{in,TCL}} - T_{t,\max}^{\mathrm{set,TCL}}}{T_{\max}^{\mathrm{in,TCL}} - T_{\min}^{\mathrm{in,TCL}}}$	$SOC_{t,\min}^{\mathrm{EV}}$
$SOC_{t,\max}^{\mathrm{GES}}$	$SOC_{\min}^{\mathrm{BES}}$	$\dfrac{T_{\max}^{\mathrm{in,TCL}} - T_{t,\min}^{\mathrm{set,TCL}}}{T_{\max}^{\mathrm{in,TCL}} - T_{\min}^{\mathrm{in,TCL}}}$	$SOC_{t,\max}^{\mathrm{EV}}$
$\varepsilon^{\mathrm{GES}}$	$\varepsilon^{\mathrm{BES}}$	$1 - e^{-\Delta t / R^{\mathrm{TCL}} C^{\mathrm{TCL}}}$	$\varepsilon^{\mathrm{EV}}$
S^{GES}	S^{BES}	$\dfrac{\Delta t (T_{\max}^{\mathrm{in,TCL}} - T_{\min}^{\mathrm{in,TCL}})}{\eta^{\mathrm{TCL}} R^{\mathrm{TCL}} \varepsilon^{\mathrm{TCL}}}$	S^{EV}
α^{GES}	0	$\varepsilon^{\mathrm{TCL}} SOC_0^{\mathrm{TCL}}$	0
β_t^{GES}	0	$\varepsilon^{\mathrm{TCL}} \dfrac{T_{t+1}^{\mathrm{out,TCL}} - T_0^{\mathrm{out,TCL}}}{T_{\max}^{\mathrm{in,TCL}} - T_{\min}^{\mathrm{in,TCL}}}$	$SOC_{t,\mathrm{c}}^{\mathrm{EV}}$
$\eta_{\mathrm{ch/dis}}^{\mathrm{GES}}$	$\eta_{\mathrm{ch/dis}}^{\mathrm{BES}}$	1	$\eta_{\mathrm{ch/dis}}^{\mathrm{EV}}$
$\Delta SOC_{\mathrm{up/down}}^{\mathrm{GES}}$	$\Delta SOC_{\max}^{\mathrm{BES}}$	—	$\Delta SOC_{\max}^{\mathrm{EV}}$

III. UNCERTAINTIES IN GENERIC ENERGY STORAGE MODEL

To capture the effect of exogenous (decision-independent) and endogenous (decision-dependent) uncertainties, this section provides modeling for three types of DIUs and two types of DDUs in the modified baseline model. While the results are general, we employ TCL to guide discussions.

A. Decision-independent Uncertainties

(a) On-off State Probability (DIU, Single Time)

GES units can only respond to DR orders when they operate in on-state or normal state. In the general case, they can be fully controlled by local controllers. However, they can not maintain 100% reliability which depends on their own usage pattern. The probability distribution of on-off state $\tilde{\omega}_{i,t}$ can be modeled as:

$$f(\tilde{\omega}_{i,t}) = \begin{cases} p_{i,t} & \tilde{\omega}_{i,t} = 1 \\ 1 - p_{i,t} & \tilde{\omega}_{i,t} = 0 \end{cases}, \forall t \in \mathbf{\Omega}_T \ , \forall i \in \mathbf{\Omega}_S \tag{2}$$

i.e., as a Bernoulli distribution, with the on-state probability $p_{i,t}$ (for unit i and time t) obtained from a data-driven approach. This DIU (a) necessarily affects the reliability of the response.

(b) Parameter Identification Errors (DIU, Single Time)

Identification errors of GES parameters (e.g., R, C, $T_{t,\max}^{\mathrm{in}}$, $T_{t,\min}^{\mathrm{in}}$, T_0^{set}, $P_{\max}$, and $P_{\min}$ for TCL assets, $SOC_{t,\max}$ and $SOC_{t,\min}$ for EV assets) are inevitable since high-resolution dynamics are ignored for simplification. Though it highly depends on the quality of data, as analyzed in [15], the identification errors are usually within ± 10%. Parameter identification errors can be modeled as:

$$\tilde{\xi}_i \sim \boldsymbol{N}(\mu_{\xi_i}, \sigma_{\xi_i}, \mu_{\xi_i}(1 - r_{\xi_i}), \mu_{\xi_i}(1 + r_{\xi_i})), \forall \xi_i \in \mathbf{\Omega}_E \tag{3}$$

i.e., using a truncated normal distribution. $\tilde{\xi}_i$ is the random parameter for ETP model, and the set of ETP parameters are defined as $\mathbf{\Omega}_E = \left\{R_i, C_i, T_{i,t,\max}^{\mathrm{in}}, T_{i,t,\min}^{\mathrm{in}}, T_{i,0}^{\mathrm{set}}, P_{i,\max}, P_{i,\min}\right\}, \forall i \in \mathbf{\Omega}_S$. The mean and standard deviation of parameters are given by μ_{ξ_i} and σ_{ξ_i}, while r_{ξ_i} is the maximum error ratio. This DIU (b) mainly relates to power and SOC boundaries of GES units.

(c) Forecast Error of User Environment (DIU, Single Time)

Constraint (1a) is for the dynamical evolution of the GES SOC, also accounting for losses stemming from the user environment. For instance, a forecast of ambient temperature is necessary to appraise the SOC evolution for TCL assets. Forecast errors are usually within ±5% [12], and can also be modeled by a truncated normal distribution, i.e.

$$\tilde{T}_t^{\mathrm{out}} \sim \boldsymbol{N}(\mu_{T_t^{\mathrm{out}}}, \sigma_{T_t^{\mathrm{out}}}, \mu_{T_t^{\mathrm{out}}}(1 - r_{T_t^{\mathrm{out}}}), \mu_{T_t^{\mathrm{out}}}(1 + r_{T_t^{\mathrm{out}}})) \tag{4}$$

where $\tilde{T}_t^{\mathrm{out}}$ is the random parameter for ambient temperature. $\mu_{T_t^{\mathrm{out}}}$ and $\sigma_{T_t^{\mathrm{out}}}$ are the related mean and standard deviation. $r_{T_t^{\mathrm{out}}}$ is the maximum error ratio of ambient temperature. This

DIU (c) directly related to energy losses originating from the user environment, and affecting the real-time SOC of GES units.

B. Decision-dependent Uncertainties

(a) SOC Boundary Expansion Effect Driven by Incentive Price (DDU, Single Time) and Contract Effect Driven by Accumulated Discomfort (DDU, Across Time)

The ability of GES to respond during DR events is another important uncertainty to consider. Previous works using refused DR probability and load recovery of customers concerning the response comfort [13][25] will introduce discrete uncertainties and exponential decay in the optimization model, which will render the formulation nonconvex. Instead, we focus on the changes of real-time SOC boundaries to describe this kind of uncertainty. The uncertainty of the SOC boundaries is fixed as described in section III.A when referring to DIU. With respect to DDU, the customers' decision, which will impact on the SOC boundaries, comes as a trade-off between corresponding inconvenience costs (i.e., the accumulated disutility or discomfort they sustained during DR periods) and the expected earnings (i.e., incentive price) from GES. Thus, SOC boundaries of GES are not static within their comfort zones shown in the blue zones in Fig. 1. Conversely, GES energy bounds on the SOC will either expand or contract based on increases in incentive payments (expansion) or accumulated discomfort (contraction). Please see (5a-5b) for an illustration. Consider g to be a non-decreasing function of the incentive payment and represents the SOC boundaries with maximum comfort. $\tilde{\lambda}_{i,t}$ is the random parameters for normalized net comfort with distribution $f(\mu_{\lambda_{i,t}},\sigma_{\lambda_{i,t}})$. Let h be the monotonic decreasing function between $\mu_{\lambda_{i,t}}$ and accumulated discomfort $RD_{i,t}$ described as (5c). Normalized accumulated response power $RD_{i,t}$ is defined as (5d).

$$\widetilde{SOC}_{i,t,\max}^{\mathrm{DDU}}=\frac{g(\widetilde{SOC}_{i,t,\max}^{\mathrm{DIU}},c_{\mathrm{ch},i,t}^{\mathrm{GES}})-\widetilde{SOC}_{i,t,\mathrm{central}}^{\mathrm{DIU}}}{2}\tilde{\lambda}_{i,t}+\frac{g(\widetilde{SOC}_{i,t,\max}^{\mathrm{DIU}},c_{\mathrm{ch},i,t}^{\mathrm{GES}})+\widetilde{SOC}_{i,t,\mathrm{central}}^{\mathrm{DIU}}}{2} \tag{5a}$$

$$\widetilde{SOC}_{i,t,\min}^{\mathrm{DDU}}=\frac{g(\widetilde{SOC}_{i,t,\min}^{\mathrm{DIU}},c_{\mathrm{dis},i,t}^{\mathrm{GES}})-\widetilde{SOC}_{i,t,\mathrm{central}}^{\mathrm{DIU}}}{2}\tilde{\lambda}_{i,t}+\frac{g(\widetilde{SOC}_{i,t,\min}^{\mathrm{DIU}},c_{\mathrm{dis},i,t}^{\mathrm{GES}})+\widetilde{SOC}_{i,t,\mathrm{central}}^{\mathrm{DIU}}}{2} \tag{5b}$$

$$\tilde{\lambda}_{i,t}\sim f(\mu_{\lambda_{i,t}},\sigma_{\lambda_{i,t}}),\ \mu_{\lambda_{i,t}}=h(RD_{i,t}) \tag{5c}$$

$$RD_{i,t}=\sum_{\tau=1}^{t}(P_{\mathrm{dis},i,\tau}^{\mathrm{GES}}/P_{\mathrm{dis},i,\max}^{\mathrm{GES}}+P_{\mathrm{ch},i,\tau}^{\mathrm{GES}}/P_{\mathrm{ch},i,\max}^{\mathrm{GES}})/T \tag{5d}$$

The visualization of section III.B DDU (a) and section III.A DIU (b) is shown in Fig. 1. The different boundaries between DIU and DDU show how the feasible region for DDUs can be enlarged in the expansion stage and reduced in the contract stage. Further discussion on the DDUs is presented next:

1) Intuition on comfort: as the net comfort declines from "+1" to "0" (i.e., during the expansion stage), the mode of $\widetilde{SOC}_{i,t,\max}^{\mathrm{DDU}}$ shifts from $g(\widetilde{SOC}_{i,t,\max}^{\mathrm{DIU}},c_{\mathrm{ch},i,t}^{\mathrm{GES}})$ to $\widetilde{SOC}_{i,t,\max}^{\mathrm{DIU}}$. During the contraction stage, as the net comfort declines further from "0" to "-1", the mode continues to decline from $\widetilde{SOC}_{i,t,\max}^{\mathrm{DIU}}$ to central value $\widetilde{SOC}_{i,t,\mathrm{central}}^{\mathrm{DIU}}$.

2) Distribution: unimodal distribution (e.g., Log-normal distribution) can be used as initialization of distribution *f*, and the real distribution can be determined by the Kolmogorov-Smirnov test from real actions of SOC and power of GES units.

3) Structure: the intuitive linear form of DDUs is designed in this paper to capture reasonable relations between comfort and incentives, while providing a convex formulation. However, the functions *g* and *h* can be generalized further via learning from test data. Specifically, function *g* could also be an affine function according to the electricity elastic model [14] and function *h* can be a polynomial function.

4) Parameters: function *g* and *h* have parameters that can be determined by regression methods using live measurements. And the values can differ for charging and discharging usage, which are driven by customers' preference.

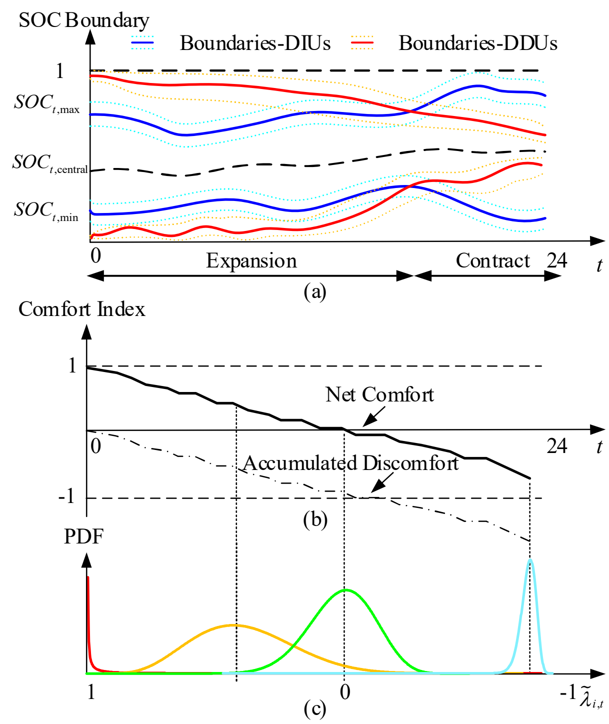

Fig. 1. Visualization of DDUs in SOC boundaries: (a) comparison of boundaries-DIUs and boundaries-DDUs, (b) changes of net comfort, (c) changes of probability distribution function of normalized net comfort.

(b) Capacity Degradation (DIU & DDU)

Capacity degradation represents the reduction of response capacity (both the quantity and unit capacity) of VES, which has been considered in DDU (a) and DIU (a). For DDU (a), degraded capacity manifests itself in SOC boundaries and affects the response capacity of GES, while DDU (b) mainly affects the response reliability of GES. In an individual physical resource, such as battery-based storage, degradation is a common and complex chemical aging process during the charge and discharge of ES, which involves the nonlinear relationship between reduced capacity with multi-factors (e.g., average SOC, average depth of discharge, operating temperature, etc.), and most of these are indeed DDUs. The nonlinear aging process can be changed due to different stages of aging (e.g., SEI formation stage with a constant degradation rate, steady degradation stage with a linearized rate, etc.). Additional details can be found in [26], which are not detailed herein. Next, we incorporate GES into optimization formulation.

IV. Chance-constrained Optimization under DIUs and DDUs

A. Original Problem Formulation

In this paper, we consider DA-ED problem for a microgrid. The microgrid system operator aggregates GESs assets (e.g., TCL-GES and BES-GES), RES assets (i.e., wind generation and solar generation), and conventional loads. The goal of the system operator of the microgrid is to supply a DA dispatch of the assets to minimize operational costs while maintaining the power balance. The formulation is as follows:

Objective function:

$$\min_{y} F(\boldsymbol{y},\tilde{z}) = \sum_{t\in\boldsymbol{\Omega}_T} (C_t^{\text{GES}} + C_t^{\text{Grid}}) \tag{6}$$

$$C_t^{\text{GES}} = \sum_{i\in\boldsymbol{\Omega}_G} (c_{\text{dis},i,t}^{\text{GES}} P_{\text{dis},i,t}^{\text{GES}} + c_{\text{ch},i,t}^{\text{GES}} P_{\text{ch},i,t}^{\text{GES}})\Delta t \tag{7a}$$

$$C_t^{\text{Grid}} = c_t^{\text{Grid}} P_t^{\text{Grid}} \Delta t \tag{7b}$$

The operational cost includes the incentive cost of GESs C_t^{GES} and the cost of power bought from the grid C_t^{Grid}. The power imported from the grid is denoted P_t^{Grid}. Charging, discharging, and DA time of use (ToU) prices are given by $c_{\text{ch},i,t}^{\text{GES}}$, $c_{\text{dis},i,t}^{\text{GES}}$, and c_t^{Grid}, respectively. The marginal costs of PV and WT assets are zero. Parameter $\tilde{z}$ is a set of random values, set of decision variables is given by $\boldsymbol{y} = \left\{P_{\text{dis},i,t}^{\text{GES}}, P_{\text{ch},i,t}^{\text{GES}}, P_t^{\text{Grid}}, SOC_{i,t}^{\text{GES}}, RD_{i,t}\right\}$. The objective function is subject to the following constraints:

(a) GES Chance Constraints: $\forall t \in \boldsymbol{\Omega}_T$, $\forall i \in \boldsymbol{\Omega}_G$

$$\mathbb{P}(P_{\text{ch},i,t}^{\text{GES}} \le \tilde{P}_{\text{ch},i,\max}^{\text{GES}}) \ge 1-\gamma \tag{8a}$$

$$\mathbb{P}(P_{\text{dis},i,t}^{\text{GES}} \le \tilde{P}_{\text{dis},i,\max}^{\text{GES}}) \ge 1-\gamma \tag{8b}$$

$$\mathbb{P}(\widetilde{SOC}_{i,t,\min}^{\text{GES}} \le SOC_{i,t}^{\text{GES}}) \ge 1-\gamma \tag{8c}$$

$$\mathbb{P}(SOC_{i,t}^{\text{GES}} \le \widetilde{SOC}_{i,t,\max}^{\text{GES}}) \ge 1-\gamma \tag{8d}$$

where $1-\gamma$ represents the security level of the chance constraints and ensures that the probability of a constraint violation is smaller than γ.

(b) Power Balance Chance Constraints: $\forall t \in \boldsymbol{\Omega}_T$

$$\mathbb{P}\Big(\sum_{i\in\boldsymbol{\Omega}_{RES}} \tilde{P}_{i,t}^{\text{RES}} + \sum_{i\in\boldsymbol{\Omega}_G} (P_{\text{dis},i,t}^{\text{GES}} - P_{\text{ch},i,t}^{\text{GES}}) + P_t^{\text{Grid}} \ge \tilde{P}_t^{\text{LD}}\Big) \ge 1-\gamma \tag{8e}$$

where $\tilde{P}_{i,t}^{\text{RES}}$ and $\tilde{P}_t^{\text{LD}}$ are the stochastic parameters for RES and load powers, respectively. RES includes the wind and the solar, while $\boldsymbol{\Omega}_{RES}$ is the set of RES units.

(c) Other Constraint: $\forall t \in \boldsymbol{\Omega}_T$

$$0 \le P_t^{\text{Grid}} \le P_{\max}^{\text{Grid}} \tag{8f}$$

where $P_{\max}^{\text{Grid}}$ is the maximum power import from the grid. Next, we present the complete formulations for DIUs and DDUs.

Complete CCO-DIUs & CCO-DDUs formulations:

The overall problem with CCO-DIUs and CCO-DDUs can be formulated as

$$\begin{aligned} &\min_{y} F(\boldsymbol{y},\tilde{z}) \\ &\text{s.t.}\ \ (1\text{a}-1\text{f}),\ (8\text{a}-8\text{f})\ \ (\text{CCO-DIUs}) \\ &\qquad (5\text{a}-5\text{d}) \qquad\quad\ (\text{CCO-DDUs}) \end{aligned} \tag{9}$$

The difference mainly lies in the consideration of DDUs (5a-5d), besides traditionally considered DIUs. The CCO-DIU is a convex optimization formulation, however, the CCO-DDU is only guaranteed to be convex, if it satisfies that

i) $F^{-1}(1-\gamma,\mu)$ is a convex function of μ,

ii) *h* from Eq. (5c) is a concave function.

The proof is developed in Appendix A.

B. Necessary Problem Reformulation

(a) Chance Constraints Reformulation under DIUs:

Without using scenario-based methods, chance constraints (8a-8f) admit a deterministic and tractable reformulation. We introduce the distribution-free reformulation in (10). Affine functions of the decision variables are given by *c* and *d*. Mean and standard deviation of the distribution are given by μ and σ Normalized inverse cumulative distribution function F^{-1} can be obtained by Monte Carlo sampling [27] of any kind of distribution (e.g., normal distribution, beta distribution). The reformulation is described as (11).

$$\begin{aligned} &\mathbb{P}\Big(c(y)^T \tilde{z} + d(y) \le e\Big) \ge 1-\gamma \Rightarrow \\ &c(y)^T \mu + d(y) + F^{-1}(1-\gamma)\left\|c(y)\sigma\right\|_2 \le e \end{aligned} \tag{10}$$

$$P_{\text{ch},i,t}^{\text{GES}} \le \mu_{P_{\text{ch},i,\max}^{\text{GES}}} - F_{P_{\text{ch},i,\max}^{\text{GES}}}^{-1}(1-\gamma)\sigma_{P_{\text{ch},i,\max}^{\text{GES}}} \tag{11a}$$

$$P_{\text{dis},i,t}^{\text{GES}} \le \mu_{P_{\text{dis},i,\max}^{\text{GES}}} - F_{P_{\text{dis},i,\max}^{\text{GES}}}^{-1}(1-\gamma)\sigma_{P_{\text{dis},i,\max}^{\text{GES}}} \tag{11b}$$

$$SOC_{i,t}^{\text{GES}} \le \mu_{SOC_{i,t,\max}^{\text{GES}}} - F_{SOC_{i,t,\max}^{\text{GES}}}^{-1}(1-\gamma)\sigma_{SOC_{i,t,\max}^{\text{GES}}} \tag{11c}$$

$$\mu_{SOC_{i,t,\min}^{\text{GES}}} + F_{SOC_{i,t,\min}^{\text{GES}}}^{-1}(1-\gamma)\sigma_{SOC_{i,t,\min}^{\text{GES}}} \le SOC_{i,t}^{\text{GES}} \tag{11d}$$

$$\begin{aligned} &\sum_{i\in\boldsymbol{\Omega}_{RES}} (\mu_{P_{i,t}^{\text{RES}}} - F_{P_{i,t}^{\text{RES}}}^{-1}(1-\gamma)\sigma_{P_{i,t}^{\text{RES}}}) + P_{\text{b},t}^{\text{Grid}} \\ &+ \sum_{i\in\boldsymbol{\Omega}_G} (P_{\text{dis},i,t}^{\text{GES}} - P_{\text{ch},i,t}^{\text{GES}}) \ge \mu_{P_t^{\text{LD}}} + F_{P_t^{\text{LD}}}^{-1}(1-\gamma)\sigma_{P_t^{\text{LD}}} \end{aligned} \tag{11e}$$

(b) Chance Constraints Reformulation under DDUs:

(R1) Robust Approximation

For reformulation under DDUs, the value of $F^{-1}(1-\gamma,\boldsymbol{y})$ is unknown before optimization. Thus, generalizations of the Cantelli's inequality can be used to estimate the best probability bound (i.e., the maximum value of $F^{-1}(1-\gamma,\boldsymbol{y})$) according to the availability of different information about the distribution, and both mean and variance. The maximum value $F^{-1}(1-\gamma)$ of

$F^{-1}(1-\gamma, \boldsymbol{y})$ for six widely used distributions are derived and listed in Table II and these may be readily employed in any CCO-DDUs problems. The supporting proofs are provided in APPENDIX. B. The value is decreased with increasing security levels. Besides, the upper listed in Table II are with less available information but with higher value, which will further lead to higher security levels and tighter constraints (i.e., $F_{NA}^{-1}(1-\gamma) > F_S^{-1}(1-\gamma) > F_U^{-1}(1-\gamma) > F_{SU}^{-1}(1-\gamma)$). Since we do not know the exact distribution of DDUs in advance, however, at least we can obtain the approximate shape of the distribution (e.g., unimodal or symmetric, etc.) through some live measurements or prior knowledge of the GES. For instance, if the unknown distribution is a Beta-like distribution, the unimodal function (Type 3) can be used to yield a robust reformulation that is less conservative than types 1 and 2. To optimize with the specific distribution for DDUs, we present next an iterative algorithm.

TABLE II APPROXIMATION OF WIDELY USED NORMALIZED INVERSE CUMULATIVE DISTRIBUTION

Type & Shape	$F^{-1}(1-\gamma)$	γ
1) No distribution assumption	$\sqrt{(1-\gamma)/\gamma}$	$0<\gamma\le 1$
2) Symmetric distribution	$\sqrt{1/2\gamma}$	$0<\gamma\le 1/2$
	0	$1/2<\gamma\le 1$
3) Unimodal distribution	$\sqrt{(4-9\gamma)/9\gamma}$	$0<\gamma\le 1/6$
	$\sqrt{(3-3\gamma)/(1+3\gamma)}$	$1/6<\gamma\le 1$
4) Symmetric & unimodal distribution	$\sqrt{2/9\gamma}$	$0<\gamma\le 1/6$
	$\sqrt{3}(1-2\gamma)$	$1/6<\gamma\le 1/2$
	0	$1/2<\gamma\le 1$
5) Student's *t* distribution	$t_{v,\sigma}^{-1}(1-\gamma)$	$0<\gamma\le 1$
6) Normal distribution	$\Phi^{-1}(1-\gamma)$	$0<\gamma\le 1$

Algorithm1: Iterative algorithm for CCO-DDUs

Input: Probability level $1-\gamma$, convergence criterion δ , deterministic and reformulated random parameters under DIUs of CCO.

Output: Value of decision variables $\boldsymbol{y}$ and cost function $F(\boldsymbol{y},\tilde{z})$.

Sep 1 Initialization: Set k=1, $F^{-1}(1-\gamma,\boldsymbol{y}_0)$ with robust reformulation value. Compute CCO-DDUs with $F^{-1}(1-\gamma,\boldsymbol{y}_0)$ to obtain initial value of $\boldsymbol{y}_0$ and update value of $F^{-1}(1-\gamma,\boldsymbol{y}_1)$ using value of $\boldsymbol{y}_0$, eps = $\left|F^{-1}(1-\gamma,\boldsymbol{y}_1)-F^{-1}(1-\gamma,\boldsymbol{y}_0)\right|$.

Sep 2 While eps> δ **do**

Compute CCO-DDUs with $F^{-1}(1-\gamma,\boldsymbol{y}_k)$ to obtain $\boldsymbol{y}_k$ and update value of $F^{-1}(1-\gamma,\boldsymbol{y}_{k+1})$ using value of $\boldsymbol{y}_k$, eps = $\left|F^{-1}(1-\gamma,\boldsymbol{y}_{k+1})-F^{-1}(1-\gamma,\boldsymbol{y}_k)\right|$.

$k \leftarrow k+1$

End

Sep 3 Return $\boldsymbol{y}=\boldsymbol{y}_k$, $F(\boldsymbol{y},\tilde{z})=F(\boldsymbol{y}_k,\tilde{z})$

(R2) Iterative Algorithm

We propose an iterative algorithm in Algorithm 1 for more precise structure of DDUs, if live measurement/data about GES are provided. Robust reformulation (R1) is used as starting point of $F^{-1}(1-\gamma,\boldsymbol{y})$ and the convergence of the algorithm is determined by the convexity of CCO-DDUs, also shown in APPENDIX. A.

V. NUMERICAL ANALYSIS

A. Set-Up

Ground truth data obtained from the Pecan Street dataset is used for the data-driven analysis of TCL-GES. And 100 TCL-GES units are selected for the case study. Historical data of RES and load are collected from the urban distribution area of Jiangsu province, China in 2020. The tiered electricity price of Jiangsu province, China is used for day-ahead electricity price. All the data used in this paper can refer to [28]. Optimization problems are coded in MATLAB with YALMIP interface and solved by GUROBI 9.5 solver. The programming environment is Core i7-1165G7 @ 2.80GHz with RAM 16 GB.

B. Baseline Results Compared with Different Models

We next compute and compare solutions of four test models M1-M4 that differ in uncertainties of optimization:

(M1) MILP-CP: baseline model of GES was proposed in [9], considering no uncertainties and no time-varying parameters (maximum of SOC boundaries) and inputs from user environment (mean of ambient temperature), rendering a MILP problem with constant parameters.

(M2) MILP-TP: modified baseline model of GES is proposed in this paper, considering no uncertainties but with time-varying parameters and inputs from user environment, rendering a MILP problem with time-varying parameters.

(M3) CCO-DIUs: CCO used in most of the state of the arts is with DIUs [11], [16], rendering a decision-independent chance-constrained optimization problem.

(M4) CCO-DDUs: CCO used in this paper is with both the DIUs and DDUs, rendering a decision-dependent chance-constrained optimization problem.

We first adopt the linear function to describe the DDUs shown as (12). $c_{i,\max}^{\mathrm{GES}}=2$, $c_{\mathrm{ch},i,t}^{\mathrm{GES}}$=0.3 , $c_{\mathrm{dis},i,t}^{\mathrm{GES}}$=0.6 , $a_i = 2.5$, $b_i = 7$. The different set of charging and discharging prices lies in the different requirement of flexibility (more requirement for discharging with higher price), and different set of discomfort effect coefficient a_i and b_i lies in different temperature preference (more uncomfortable with higher setpoint temperature). The ToU pricing is set to be 0.5-0.9-1.4 (CNY/kW.h). Security level 1-γ is set to be 0.95.

$$g(\widetilde{SOC}_{i,t,\max}^{\mathrm{DIU}}, c_{\mathrm{ch},i,t}^{\mathrm{GES}}) = \widetilde{SOC}_{i,t,\max}^{\mathrm{DIU}}(1-\frac{c_{\mathrm{ch},i,t}^{\mathrm{GES}}}{c_{i,\max}^{\mathrm{GES}}})+\frac{c_{\mathrm{ch},i,t}^{\mathrm{GES}}}{c_{i,\max}^{\mathrm{GES}}} \quad (12a)$$

$$g(\widetilde{SOC}_{i,t,\min}^{\mathrm{DIU}}, c_{\mathrm{dis},i,t}^{\mathrm{GES}}) = \widetilde{SOC}_{i,t,\min}^{\mathrm{DIU}}(1-c_{\mathrm{dis},i,t}^{\mathrm{GES}}/c_{i,\max}^{\mathrm{GES}}) \quad (12b)$$

$$\mu_{\lambda_{i,t,\max}} = 1-a_i RD_{i,t}, \mu_{\lambda_{i,t,\min}} = 1-b_i RD_{i,t} \quad (12c)$$

Comparisons of M1 & M2 and M3 & M4 are shown in Fig. 2 and Fig. 3, respectively. Key metrics are listed in Table III and summarize the results. Great difference has been observed between M1 & M2 concerning the SOC distribution and power of charge and discharge. GES units have been discharging except for the last moment in M1, while GES units have to charge after 12 am in M2. It is because the SOC boundaries are reduced because of compressed temperature ranges and reduced feasible region in M2, while SOC boundaries are overestimated in M1 ranging from "0" to "1" and produce over-optimistic optimization results. Comparing different net-energy actions (e.g., net charge power, energy losses from self-discharge, energy losses from ambient space) of M2 can be seen in Fig. 4, where self-discharge levels change with SOC and (ambient)

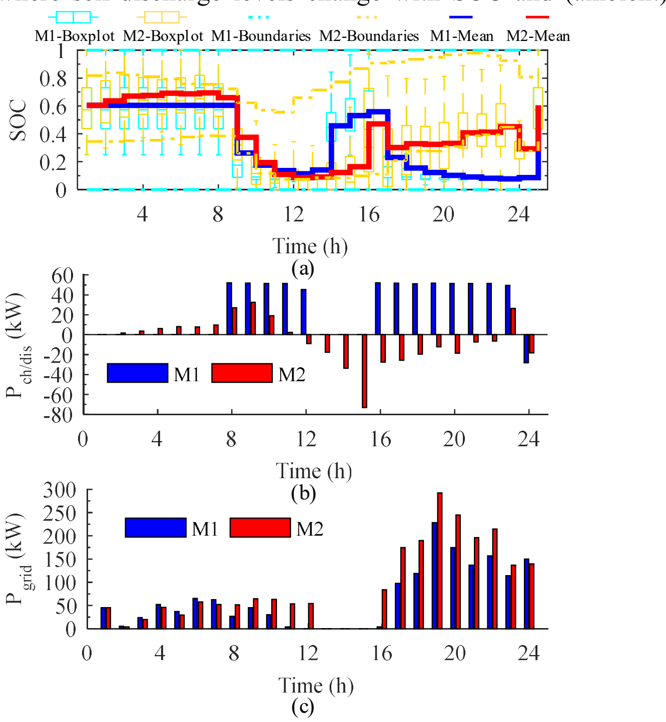

Fig. 2. Comparison between model (1) & (2): (a) SOC distribution, (b) total power of charge and discharge, (c) power of grid

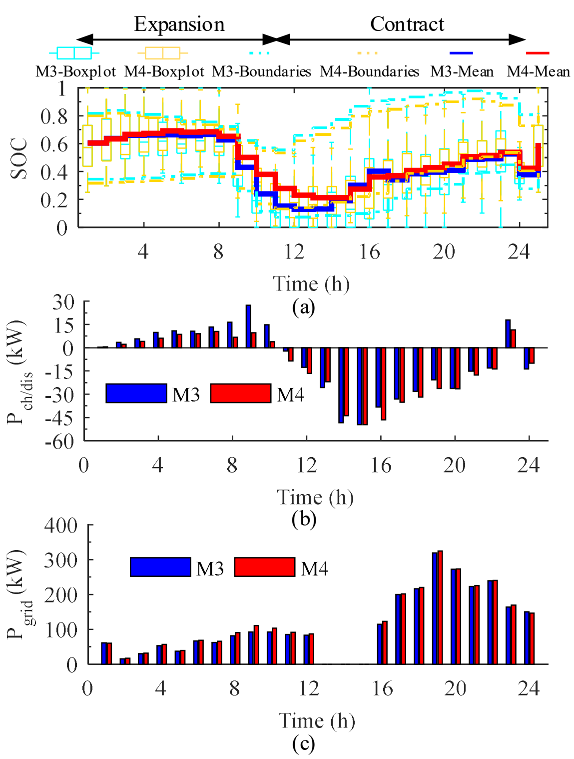

Fig. 3. Comparison between model (3) & (4): (a) SOC distribution, (b) total power of charge and discharge, (c) power of grid

TABLE III Optimization Results with Different Models and Uncertainties

Metric	M1	M2	M3	M4
Cost (CNY)	2096.47	2721.07	3214.12	3255.55
$\sum P_{\mathrm{dis},i,t}^{\mathrm{GES}}$ (kW)	662.88	209.79	190.18	96.70
$\sum P_{\mathrm{ch},i,t}^{\mathrm{GES}}$ (kW)	28.19	334.43	386.73	372.44
$\sum P_{\mathrm{b},t}^{\mathrm{Grid}}$ (kW)	1578.95	2213.58	2654.56	2741.98

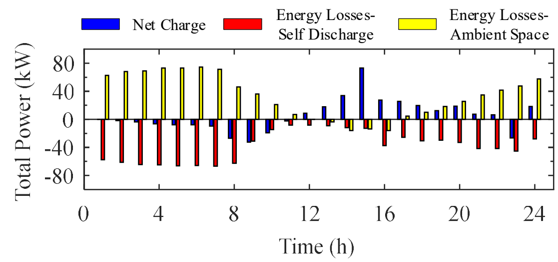

Fig. 4. Comparison between different parts of energy input and output

energy losses, which are changed with ambient temperature conditions. Thus, even if the GES unit is supplied by the grid, SOC will still be reduced by the other two outputs. Additionally, compared with M3, the expansion effect of SOC boundaries is witnessed in M4 before 12 am and the following contract effect results in a relatively higher SOC level. Concerning charge/discharge power, dispatched power is declined at the expansion stage of M4 to reduce the overall discomfort, and feasible region is reserved for the charging period due to the redundant output of RES energy and peak load. The rapidly compressed SOC boundaries caused by the accumulated discomfort indicates that it is not suitable to fully exploit the flexibility of VES throughout the day, but it is better to use VES for short-time period with less negative impact on real-time flexibility. In terms of optimality, M4 performs the highest operational costs because a trade-off should be made between the overall comfort and the profit.

C. *Response reliability assessment*

Although the results seem better using models (M1-M3) in terms of response power of GES and operational cost, the optimization strategy will not work in practice, which was pointed in the previous work [9, 12] and explained it as kind of "reliability loss". Thus, we introduce two reliability indices in this paper to assess reliability loss of response of GES units (i.e., the difference between strategies and real actions): loss of response power probability (*LORP*) and expected response energy not served (*ERNS*). They are defined in (13), $X_k \mid \boldsymbol{y}, \tilde{\boldsymbol{z}}$ represents the reliability loss events under strategy $\boldsymbol{y}$ and uncertainty $\tilde{\boldsymbol{z}}$. R is the function of response energy losses.

$$LORP = \sum_k \mathbb{P}(X_k \mid \boldsymbol{y}, \tilde{\boldsymbol{z}}) \tag{13a}$$

$$ERNS = \sum_k \mathbb{P}(X_k \mid \boldsymbol{y}, \tilde{\boldsymbol{z}}) R(X_k \mid \boldsymbol{y}, \tilde{\boldsymbol{z}}) \tag{13b}$$

Fig. 5 and Fig. 6 show the reliability performance comparison between M1 & M2 and M3 & M4 with respect to weighted average SOC boundary and reliability indices, respectively. Compared with theoretical SOC boundaries, the practical ones compress deeply in M1, while barely difference appears in M4, which provides a convincing explanation of the huge violation and reliability loss during the second half of DR using previous models. In terms of reliability indices, lower reliability (i.e., higher LORP & ERNS) are revealed in models (M1-M3) due to the overestimation of the feasible region. The positive value of ERNS represents the under-response of GES units, while the negative one represents the over-response of GES units. Moreover, the expected reliability results shown in Table IV indicate that the reliability indices are constant for M1 & M2 regardless of the security level, while the reliability performance becomes worse when utilizing the strategies under a lower security level. Moreover, the reliability indices shown

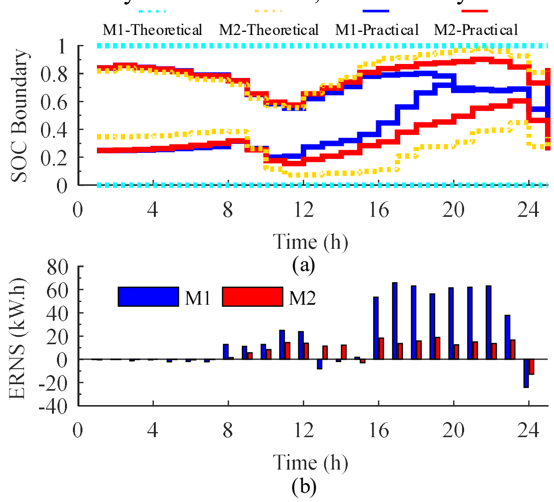

Fig. 5. Performance comparison between M1 & M2 with respect to (a) weighted average SOC boundary and (b) ERNS

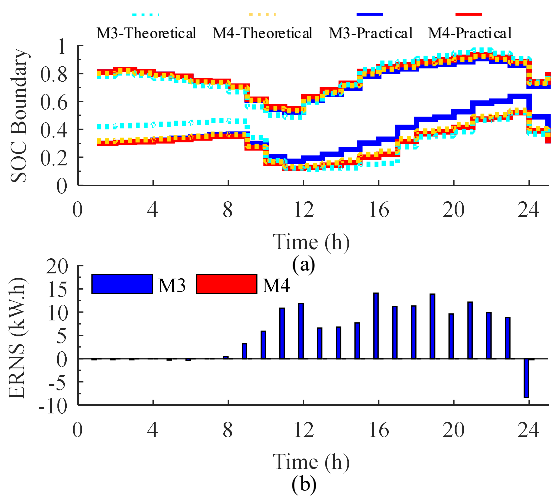

Fig. 6. Performance comparison between M3 & M4 with respect to (a) weighted average SOC boundary and (b) ERNS

TABLE IV Reliability Performance between Different Models and Security Level

γ	Indices	M1	M2	M3	M4
0.05	*LORP*	*LORP* 0.54	*LORP* 0.35	0.28	0.00
	ERNS			6.42	0.01
0.25	*LORP*			0.32	0.00
	ERNS	*ERNS* 24.68	*ERNS* 8.69	7.64	0.86
0.45	*LORP*			0.34	0.01
	ERNS			8.48	1.03

in M1-M3 are far beyond the security level $1-\gamma$, while results in M4 are maintained within the security level. The better performance proved that optimization under DDUs can offer a more credible, but conservative strategy, which reduces the demand for real-time corrective dispatch and extra costs for the reserve market.

D. Performance of two reformulation methods

In this subsection, the convergence performance is shown in Fig. 7 with two common types of distribution (Beta and Log-normal distribution). It can be seen that the iterative algorithm converges within 3 iterations. Moreover, the optimization results compared with two reformulated methods are shown in Table V, where both distributions adopt the robust value of the unimodal type. The solution time depends on the complexity of obtaining the parameters and inverse CDF of the distribution (more time for Beta distribution). And it is obvious that R1 outperforms the other in terms of computational efficiency, but it will produce more conservative strategies, while results are more optimal in terms of cost by using R2. It is observed that the optimality gap between the two methods is less than 1% and can be tolerated, which indicates that R1 can be widely used if the structure of distribution of DDUs is unknown.

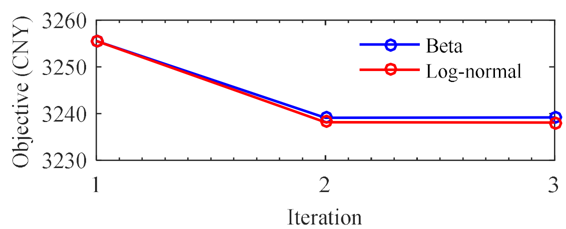

Fig. 7. Convergence performance under Beta and Log-normal distribution

TABLE V Optimization Compared with Two Reformulation Methods

γ	Distribution Type	R1 Cost (CNY)	R1 Time (s)	R2 Cost (CNY)	R2 Time (s)
0.05	Beta Distribution	3255.55	18.50	3239.18	2757.16
0.25		2954.22	17.83	2946.95	2586.58
0.45		2793.55	17.66	2783.10	3211.78
0.05	Log-normal Distribution	3255.55	18.50	3238.09	58.49
0.25		2954.22	17.83	2945.54	57.56
0.45		2793.55	17.66	2781.16	53.08

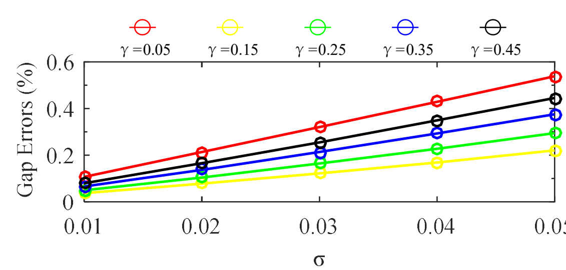

Fig. 8. Sensitivity of gap errors with different standard deviations of DDUs

Afterward, sensitivity analysis is computed to compare the gap errors between the usage of two reformulation methods with different standard deviations of DDUs (Log-normal distribution). The results shown in Fig. 8 indicates that the sensitivity of gap errors is low and it will decline at first and then increase with the decrease of security level. Moreover, a linear increasing function between gap errors and variance can be found regardless of security level, which is helpful to estimate errors without knowing the accurate distribution.

VI. Discussion of Extended Problems

Compared with ES, the above case studies reveal the great challenges when using VESs as an alternative to ES: (1) non-trivial energy losses due to self-discharge and user environment, (2) time-varying properties, DIUs and DDUs of SOC boundaries, (3) on-off state due to users' behavior. The above case study accurately mitigates effects of challenges (1-2), so in this subsection, we further propose two solution methods to deal with (3).

(S1) Portfolio with Deterministic Reserve

The on-state probability tends to be quite low especially for night-time and working hours in the residential sector, so system operators could participate in DA reserve market to enhance the security level under the support of deterministic reserve, such as ES, CPP, etc. For those units below the security level, deterministic reserves will work as reliability support, and constraints for deterministic reserve are added as (14).

$$P_{i,t,\min}^{\mathrm{RE}} \le P_{i,t}^{\mathrm{RE}} \le P_{i,t,\max}^{\mathrm{RE}} \quad (14a)$$

$$-P_{i,\mathrm{down}}^{\mathrm{RE}} \le P_{i,t}^{\mathrm{RE}} - P_{i,t-1}^{\mathrm{RE}} \le P_{i,\mathrm{up}}^{\mathrm{RE}} \quad (14b)$$

$$P_{i,t}^{\mathrm{RE}} = P_{\mathrm{dis},i,t}^{\mathrm{GES}} - P_{\mathrm{ch},i,t}^{\mathrm{GES}} \quad (14c)$$

(S2) Portfolio with Probabilistic Reserve

It is redundant and costly to use 100% reliable reserve, so the probabilistic reserve is another alternative [29]. Considering the effect of reliability combination, the requirement of combined probabilistic reserve and its cost are shown in (15). $P_{i,t}^{\mathrm{RE}}$ is the reliability of probabilistic reserve, $c_{i,t}^{\mathrm{RE}}$ is corresponding price.

$$(1-p_{i,t})(1-R_{i,t}^{\mathrm{RE}})=\gamma,\ c_{i,t}^{\mathrm{RE}}=a^{\mathrm{RE}}(R_{i,t}^{\mathrm{RE}})^{b^{\mathrm{RE}}} \quad (15)$$

First, the on-state probability of 100 TCL-GESs is analyzed and the maximum and minimum average on-state probability are 0.999 and 0.830, respectively. The corresponding time periods are 7 pm and 9 am, respectively, following the customers' behavior. Therefore, the real-time security level for DR produced by TCL-GES units can be described as $p_{i,t}(1-\gamma)$, so system operators should use other reserves to guarantee the security requirement $(1-\gamma)$. For reserve price, we set $a^{\mathrm{RE}}=1$ (100% reliability), $b^{\mathrm{RE}}=2$, and we compute the modified results with different solution methods. Results for S1 and S2 shown in Table VI are more credible but less optimal just considering challenges (1-2). The power demand of GESs is reduced and transferred to demand of reserve instead. And the demand of reserve is gradually reduced with the decrease of security level. Moreover, portfolio with probabilistic reserves overperforms deterministic ones in terms of optimality of cost.

TABLE VI Modified Results Compared with Two Solution Methods

γ %	S1			S2		
	Cost (CNY)	$\sum P_{\mathrm{ch/dis},i,t}^{\mathrm{GES}}$ (kW)	$\sum P_{i,t}^{\mathrm{RE}}$ (kW)	Cost (CNY)	$\sum P_{\mathrm{ch/dis},i,t}^{\mathrm{GES}}$ (kW)	$\sum P_{i,t}^{\mathrm{RE}}$ (kW)
20	3053.41	337.32	85.64	3028.66	335.09	88.32
50	2771.26	368.63	25.79	2758.60	366.38	29.30
80	2516.43	366.69	0.00	2516.43	366.69	0.00

VII. Conclusion

In this paper, a novel CCO formulation has been proposed for the day-ahead economic dispatch of uncertain GES units, which fully incorporates dynamic properties and various types of DIUs and DDUs. The numerical results show that the dynamic flexibility of GES units is reduced and limited by inconvenience effect and time-varying users' preferences. And considering DDUs enable decision-makers to make a trade-off between overall profit and customers' discomfort, thus, producing more conservative, but more credible strategies. This effectively reduce demand for real-time corrective dispatch and extra costs in the reserve market. In addition, the computational performance shows that the two proposed reformulation methods can be used for any CCO problem under distribution-free DDUs, where the robust approximation outperforms the other one in computational efficiency (with few seconds) while maintaining a good performance (within 1% optimality gap). And the major attraction is that robust approximation can be applicable in CCO problem without complete knowledge of DDUs.

Future work will focus on further reducing the potential risk of GES units and achieving a trade-off between expected profit and risk via investment portfolio optimization in multiple markets (e.g., energy market, reserve market, ancillary market, etc.). In addition, measurements/data from GES units should be further analyzed to infer the structure of DDUs.

Appendix

A. Convexity and Convergence conditions

For DDUs designed in this paper, the mean changes with decision variables, but the variance is fixed. Thus, $F^{-1}(1-\gamma,\boldsymbol{y})$ is equal to $F^{-1}(1-\gamma,\mu)$, and the convexity condition of the proposed CCO-DDUs is as follows.

i) $F^{-1}(1-\gamma,\mu)$ is a convex function of μ.

ii) h is a concave function.

The numerical simulation of most of the distributions (e.g., Log-normal, Gamma, Beta, etc.) shows that $F^{-1}(1-\gamma,\mu)$ is a convex function of μ when μ is within [-1,1] (the range of discomfort). And the convergence of the iterative algorithm is guaranteed when the convexity condition is satisfied [30].

B. Proof of the value of robust approximation

We define F as the CDF function, $\mathbb{P}$ as PDF function, $k(k\ge 0)$ as constant, ξ as the probabilistic parameter with zero mean and unit variance under the following distributions.

1) No distribution assumption: using classical Cantelli inequality:

$$F_{NA}(k) = 1 - \sup_{\mathbb{P}\in NA} \mathbb{P}[\xi \geq k] = k^2 / 1 + k^2 \tag{16a}$$

$$F_{NA}^{-1}(1-\gamma) = \sqrt{(1-\gamma)/\gamma} \tag{16b}$$

2) Symmetric distribution: using Chebyshev's inequality:

$$F_S(k) = 1 - \sup_{\mathbb{P}\in S} \mathbb{P}[\xi \geq k] = 1 - \frac{1}{2}\sup_{\mathbb{P}\in S} \mathbb{P}[|\xi| \geq k] = 1 - \frac{1}{2k^2} \tag{16c}$$

$$F_S^{-1}(1-\gamma) = \sqrt{1/2\gamma} \tag{16d}$$

3) Unimodal distribution: using one-sided Vysochanskij–Petunin inequality:

$$F_U(k) = 1 - \sup_{\mathbb{P}\in U} \mathbb{P}[\xi \geq k] = \begin{cases} 1 - 4/(9k^2+9) & k \geq \sqrt{5/3} \\ 1-(3-k^2)/(3+3k^2) & 0 \leq k \leq \sqrt{5/3} \end{cases} \tag{16e}$$

$$F_U^{-1}(1-\gamma) = \begin{cases} \sqrt{(4-9\gamma)/9\gamma} & 0 < \gamma \leq 1/6 \\ \sqrt{(3-3\gamma)/(1+3\gamma)} & 1/6 < \gamma \leq 1 \end{cases} \tag{16f}$$

4) Symmetric & unimodal distribution: using Gauss's inequality:

$$F_{SU}(k) = 1 - \sup_{\mathbb{P}\in SU} \mathbb{P}[\xi \geq k] = 1 - \frac{1}{2}\sup_{\mathbb{P}\in U} \mathbb{P}[|\xi| \geq k] = \begin{cases} 1 - 2/9k^2 & k \geq 2/\sqrt{3} \\ 1/2 + k/2\sqrt{3} & 0 \leq k \leq 2/\sqrt{3} \end{cases} \tag{16g}$$

$$F_{SU}^{-1}(1-\gamma) = \begin{cases} \sqrt{2/9\gamma} & 0 < \gamma \leq 1/6 \\ \sqrt{3}(1-2\gamma) & 1/6 < \gamma \leq 1/2 \end{cases} \tag{16h}$$

5-6) Student's t and Normal distribution: we can use normalized CDF $t_{v,\sigma}^{-1}(1-\gamma)$ and $\Phi^{-1}(1-\gamma)$ without introducing approximation errors.

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