An Asynchronous Online Negotiation Mechanism for Real-Time Peer-to-Peer Electricity Markets

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***Abstract*—Participants in electricity markets are becoming more proactive because of the fast deployment of distributed energy resources (DERs) and the further development of demand-side management (DSM), which also boosts the emergence of Peer-to-Peer (P2P) market mechanisms. Moreover, the market is also required to operate in a real-time scheme in response to changes in generation and load to maintain power balance. Therefore, a practicable real-time P2P market mechanism is in urgent need. However, it is technically challenging to deploy P2P mechanisms in real-time, since they most often involve a heavy computation burden, while the time available for negotiation in real-time is very short. Our core contribution is hence to design a novel asynchronous online optimization framework to enable the real-time P2P market negotiation mechanism, which can greatly reduce the computation and communication burden from two aspects. First, a novel online consensus alternating direction method of multipliers (ADMM) algorithm is proposed, which can greatly reduce the computation complexity since only one iteration is performed for each agent in every time period. Second, the market operates in an asynchronous mode so that all agents can freely trade without waiting for idle or inactive neighboring agents. The sublinear regret upper bound is proved for our asynchronous online algorithm, which indicates that social welfare can be maximized in the long run on time average. Simulations show that our algorithm enjoys good convergence performance, robustness, and fairness.**

***Index Terms*—Real-time P2P markets, asynchronous online consensus ADMM, forgetting factor, non-stationary regret.**

NOMENCLATURE

Functions

$C(\cdot)$	Production cost or utility function.
$R(\cdot)$	Regret function.

Numbers and Indexes

k	Index for iterations.
N	Cardinal number of agents.
n, m	Indices for agents.
T	Cardinal number of time periods.
t	Indices for time periods.

Parameters

Δt	Duration of each time period.
$\bar{t}$	The last time st before t where agent was active.
ρ, η	Adaptive penalty factors.
τ	Maximal tolerable delay for all agents.
$\underline{E}, \overline{E}$	Boundaries of power.
υ	Forgetting factor.
ε	Allowed maximal violation of trade between agents.
p	Active rate of agent.
S,D,P,L,J,Λ	Positive upper bounded constants in assumptions.
V	Path variation.

Sets and Vectors

$\boldsymbol{\lambda}$	Vector of whole transaction prices.
$\mathbf{E}$	Vector of whole transaction quantities.
$\mathcal{A}$	Set of active neighboring agents.
$\mathcal{A}^c$	Set of inactive neighboring agents.
Ω	Set of agents.
ω	Set of neighboring agents.
$\Omega_p, \Omega_c, \Omega_{ps}$	Set of producers, consumers and prosumers.

Variables

$\hat{E}$	The final trade quantity after projection.
$\hat{E}^{proj}$	The projected total power.
λ	Price for transaction power.
E	Power injection or transaction quantity.
E^*	Optimal power injection or transaction quantity.
F	Consensus variable of trades.

I. INTRODUCTION

The ever-increasing distributed energy resources (DERs) and demand-side management (DSM) characterize the future of electrical power systems and market mechanisms. Market participants undertake a proactive behavior by managing their production and consumption. Therefore, electricity markets are evolving towards more decentralized mechanisms. However, current electricity markets still complete resource allocation and pricing based on the conventional hierarchical and top-down approach [1], which causes participants to be passive receivers. Recently, a novel idea of electricity markets has emerged: these so-called Peer-to-Peer (P2P) electricity markets rely on multi-bilateral trades among participants [2]–[9], who work in a spontaneous and collaborative manner without requiring a central operator to centrally schedule. Employing P2P market mechanisms can yield many advantages, e.g., empowerment of participants, product differentiation, resilience or reliability of power system, and protection of privacy [2], [5]. Existing works about P2P markets mainly focus on the following issues: reallocation of costs [3], product differences [4], [9], dispatch fairness [8], communication properties [6] and costs [7].

Meanwhile, in actual operation, due to changes in the weather, potential power system accidents, renewable power generation uncertainty, demand-side load variations, and other contingencies, the actual generation and load may have a large deviation from the schedule obtained at the day-ahead market stage, power balance hence ought to be restored. Thus, it is required to rapidly reschedule the transactions among participants to maintain power balance in response to these changes, hence calling for real-time markets (RTMs) [10]–[13]. It is necessary to also operate the P2P market mechanism in a real-time scheme, however, which is quite technically challenging. Compared with day-ahead and intraday markets, the negotiation time for RTMs before operation is usually set to 5 minutes or even shorter [14]. Thus, RTMs demand fast computation and communication. However, a P2P market mechanism requires a very large amount of information to be exchanged, much greater than that required for a centralized market mechanism [6], [7], [15]. In a real-time context, such exchanges run the risk of not having enough time to complete the negotiation before the deadline. Eventually, the main challenge for designing a real-time P2P electricity market negotiation mechanism is how to reduce the computation and communication complexity of P2P mechanisms so that it can be deployed in a real-time scheme.

Note that some works already considered deploying P2P mechanisms in real-time operation [16]–[20]. For instance, in [18], bilateral contract networks were proposed as a new market design for P2P energy trading in real-time markets and authors designed a price-adjustment process to implement the negotiation mechanism; A P2P electricity trading model for locally buying and selling electricity among plug-in hybrid electric vehicles was proposed in [16]. An iterative double auction mechanism was proposed to maximize social welfare in day-ahead and real-time stages; The other two works [17], [20] employed blockchain-based approaches within a P2P real-time market, but from the communication network level, prices and payments were still centrally determined by the central coordinator, and not via a P2P structure. However, these works ignored the computational efficiency trouble of the real-time P2P market, and did not consider how one could take advantage of previous negotiations to make current results better.

Table. I gives a comparison between our proposed asynchronous online ADMM and the state-of-the-art algorithms. The standard alternating direction method of multipliers (ADMM) [8], consensus-based ADMM [3], [7], [21], relaxed consensus+innovation (RCI) [6], [9], and primal-dual gradient method [4] all require to operate in a synchronous mechanism to maximize social welfare. But they may need to perform multiple iterations to converge in each time period, and run the risk of exceeding the deadline of negotiation time period before reaching a convergence. The price-adjustment process [18], iterative double auction [16], and continuous double auction (CDA) [22], [23] match buyers and sellers who submit bid prices and amounts in current time period, but they cannot ensure that social welfare is maximized. For the online matching CDA method [24], although it can also implement asynchronous transactions, and each agent only takes one match in every time period, it suffers from sub-optimality and there is a upper-bounded gap between social welfare maximization.

As it may be too expensive to optimally solve a real-time P2P market at every short time period, one needs to think of appropriate and computationally cheaper approaches. In this work, we propose a novel asynchronous online consensus ADMM algorithm to enable the real-time P2P electricity market negotiation mechanism. Compared with above existing methods, we reduce the computation and communication burden of P2P mechanisms via two approaches. First, we innovatively design an online optimization framework to enable the real-time P2P market negotiation mechanism, which can maximize social welfare in the long run on time average – this is computationally lighter and more tractable. Despite many large-scale applications of online optimization, such as network resource allocation [25], [26], demand response [27], [28], and energy management [29], our work is the first application for P2P electricity markets. The number of operations and communications among agents can be heavily reduced, and the complexity of our approach is less than other approaches since the online consensus ADMM algorithm only performs one iteration for every single agent in each time period. Second, the market operates in an asynchronous mode. We first propose the synchronous online consensus ADMM, which requires each agent to wait to receive all bid prices and quantities from neighboring agents to run the algorithm. Then, we improve it to an asynchronous mode, which means that agents can freely trade without waiting to receive bid prices and quantities from inactive or idle neighboring agents, so that the computational efficiency will not be restricted by low-reliability agents. Although the asynchronous online consensus ADMM cannot achieve the optimal solution in every time period, social welfare can be maximized in the long run on time average by proving the sublinear regret upper bound.

Our main contributions include:

- We propose an online optimization framework for P2P real-time electricity markets, meaning that, instead of fully solving a complete P2P market at each time period, we use a recursive approach with a single iteration/optimization performed at each time step based on new information at the current time. The computation complexity of the P2P mechanism is highly reduced since only one iteration is performed for each agent at every time step (while offering some performance guarantees).
- We first propose the online consensus ADMM algorithm to enable the market negotiation mechanism. However, the synchronous mechanism requires each agent to wait to receive all bid prices and quantities from neighboring agents to update, and the computational efficiency may be highly limited by low-reliability agents. To overcome this drawback, we further improve it to an asynchronous mechanism, where agents can freely trade with each other without waiting for idle or slow neighboring agents.
- The sublinear non-stationary regret upper bound for our asynchronous online consensus ADMM algorithm is proved, which implies the social welfare will be maximized in the long run on time average.

The rest of the paper is organized as follows: Section II presents the real-time P2P electricity market model. Section III proposes the synchronous and asynchronous market negotiation mechanisms, followed by the regret and market properties analysis in Section IV. Numerical results and comparisons are presented in Section V. Finally, conclusions and limitations are

TABLE I: Comparison of different algorithms to enable P2P markets

Algorithm	Category	Computational effort per iteration	Number of iterations per time period	Convergence performance for social welfare
Standard ADMM	distributed/synchronous	high	many	maximize in each time period but maybe time out
Consensus-based ADMM	decentralized/synchronous	high	many	maximize in each time period but maybe time out
RCI	decentralized/synchronous	low	many	maximize in each time period but maybe time out
Primal-dual gradient	decentralized/synchronous	high	many	maximize in each time period but maybe time out
Price-adjustment process	decentralized/asynchronous	low	some	cannot guarantee social welfare maximization
Iterative double auction	distributed/asynchronous	high	some	cannot guarantee social welfare maximization
CDA	distributed/asynchronous	low	some	cannot guarantee social welfare maximization
Online matching CDA	distributed/asynchronous	low	one	suboptimal with a upper bounded gap
Asynchronous online consensus ADMM	decentralized/asynchronous	high	one	maximize in the long run on time average

drawn in Section VI.

II. Real-Time P2P Electricity Market Model

We consider a real-time P2P electricity market with a set Ω of N agents, who can be producers, consumers or prosumers over a period of time T. The time period for negotiation before operation and delivery is set to 5 minutes. Compared with a centralized market, a P2P electricity market is much more decentralized, which relies on multi-bilateral direct trades among a community of agents with flexible consumption or production. The network is divided into physical layer and virtual layer. The physical layer includes electrical connection of all agents, whereas the virtual layer is the communication network which is used for information exchange among agents. As it is classically done, agents are supposed rational as in [30], i.e. always objectively taking the most beneficial decisions, and non-strategic, i.e., not anticipating actions and reactions of other agents.

A. Peer-to-Peer Trading

To model the trading process, the total sold or purchased power $E_{n,t}$ of agent $n \in \Omega$ at time period t is split into a summation of bilaterally transaction quantities with a set of neighboring agents $m \in \omega_n$ as

$$E_{n,t} = \sum_{m\in\omega_n} E_{nm,t}, \quad \forall n \in \Omega,\ t = 1, ..., T \tag{1}$$

The neighboring agents of agent n are the agents who are connected to agent n in the communication network. A positive value of $E_{nm,t}$ corresponds to a sale/production and a negative value to a purchase/consumption. To lighten notations, $\mathbf{E}_{n,t} = \{E_{n1,t}, ..., E_{nm,t}\}$ is used to denote the whole transactions of agent n at time period t. The total sold or purchased power of agent n at time period t is constrained as

$$\underline{E}_{n,t} \leq E_{n,t} \leq \overline{E}_{n,t}, \quad \forall n \in \Omega,\ t = 1, ..., T \tag{2}$$

Here, for renewable generators, the upper bound $\overline{E}_{n,t}$ is set to the smaller value of actual power generation and maximal capacity. Each agent can be a producer ($0 \leq \underline{E}_{n,t} \leq \overline{E}_{n,t}$), a consumer ($\underline{E}_{n,t} \leq \overline{E}_{n,t} \leq 0$) or a prosumer ($\underline{E}_{n,t} \leq 0 \leq \overline{E}_{n,t}$). Hence, $E_{nm,t}$ are constrained as

$$\begin{cases} E_{n,t} \geq E_{nm,t} \geq 0, & \forall (n,m) \in (\Omega_p, \omega_n),\ t = 1, ..., T \\ E_{n,t} \leq E_{nm,t} \leq 0, & \forall (n,m) \in (\Omega_c, \omega_n),\ t = 1, ..., T, \\ \underline{E}_{n,t} \leq E_{nm,t} \leq \overline{E}_{n,t}, & \forall (n,m) \in (\Omega_{ps}, \omega_n),\ t = 1, ..., T, \end{cases} \tag{3}$$

where Ω_p, Ω_c and Ω_{ps} are the set of producers, consumers and prosumers, respectively. Finally, the market equilibrium between production and consumption is represented by a set of reciprocity balance constraints as

$$E_{nm,t} + E_{mn,t} = 0, \quad \forall (n,m) \in (\Omega, \omega_n),\ t = 1, ..., T. \tag{4}$$

We denote the production cost and consumer utility of agent n at time period t by convex function $C_{n,t}$. For conventional or renewable generators, the function $C_{n,t}$ reflects the costs by producing the power $E_{n,t} > 0$; while for consumers, the function represents the level of comfort/satisfaction obtained by consumers by consuming the power $E_{n,t} < 0$ [31]. A positive value of $C_{n,t}$ corresponds to a cost, and a negative value to benefit. Fig. 1 gives the illustrative cost functions $C_{n,t}$ for generators and consumers.

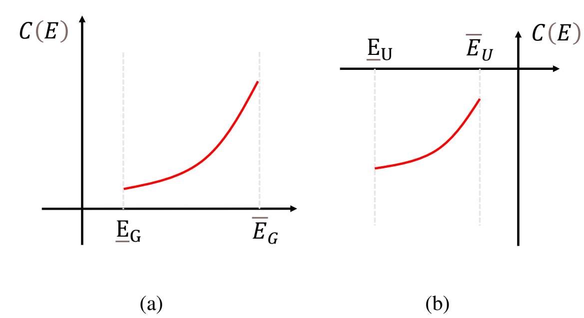


Fig. 1: Illustrative cost functions C for (a) generators, (b) consumers.

B. Social Welfare Maximization Problem

Since agents do not know the information from the future, or in other words, the information is incomplete, they can only make decisions based on current updated and past information. Thus the market has to be running in an online manner instead of optimizing the market for all T. To be more specific, at the beginning of the time period t, each agent will be aware of the current updated information, including cost or utility function coefficients, power boundaries, demand requirements, and renewable generation. After obtaining the information, agents will decide the trading prices and amounts between neighboring agents for later time until the next decision is made. All agents negotiate with each other to reach an agreement on their final transactions while maximizing the social welfare. Since the cost or utility functions of all agents are convex, the equilibrium point will exist and be unique [32]. Therefore, mathematically speaking, the social welfare maximization problem can be regarded as an equivalent minimization problem formulated as

$$\begin{aligned} \min_{\{E_{n\in\Omega}\}} \quad & \sum_{t=1}^{T}\left(\sum_{n\in\Omega} C_{n,t}(E_{n,t})\right) \\ \text{s.t.} \quad & (1)-(4). \end{aligned} \tag{5}$$

If the problem (5) is completed solved in every time period in a decentralized manner, a double loop algorithm is needed, where the function $C_{n,t}$ changes in the outer loop, while the inner loop runs iteratively until the balance constraint (4) is satisfied. However, in a P2P market, the number of communications will increase with the square of the number of agents [7]. Thus, for the problem (5), if we want to obtain the optimal solution in each time period, all agents have to complete many iterations, which results in a very heavy communication and computation burden. Thus, one needs to think of appropriate and computationally cheaper approaches. To this end, we propose an asynchronous online optimization framework for real-time P2P market negotiation mechanisms, which is more practical and applicable.

Remark 1. *For simplicity, the power flow model and network constraints are not considered, but our P2P market mechanism can be extended to include the DC power flow model by adding a system operator (SO) into the market, who behaves as a single agent and helps to complete the calculation of power flows and voltage angles. In each time period, the SO will collect total power injections from all agents located at different buses, then determines the power flows and voltage angles by solving an optimization problem. The problem is convex and not difficult to solve, thus the computational efficiency of the market will not alter much.*

III. Asynchronous Real-Time P2P Electricity Market Negotiation Mechanism

In this section, we first propose a synchronous real-time P2P market mechanism, but this mechanism is highly restricted by low-reliability agents. To overcome this drawback, it is further improved into an asynchronous market mechanism.

A. Synchronous Market Mechanism

Since the market operates in an online framework, the problem (5) is decomposed into each single time period. The problem of maximizing social welfare in the time period t is formulated as below

$$\begin{aligned} \min_{\{\mathbf{E}_{n\in\Omega}\}} \quad & \sum_{n\in\Omega}\left(C_{n,t}(E_n)+\sum_{m\in\omega_n}\frac{\eta}{2}(E_{nm,t-1}-E_{nm})^2\right) \\ \text{s.t.} \quad & (1)-(4), \end{aligned} \tag{6}$$

where η is the penalty factor and the Bregman divergence term $\frac{\eta}{2}(E_{nm,t-1}-E_{nm})^2$ is appended to make the results close to previous value $E_{nm,t-1}$ in order to speed up the convergence process [33], [34]. The rationale behind the Bregman divergence term is taking advantage of previous negotiation results to make current results better so that the gap between optimal solutions can be bounded. A novel online consensus ADMM algorithm is proposed to implement the real-time P2P market negotiation mechanism, which produces the following updates:

- **Power Updates**: Each agent n updates their transaction power with neighboring agents by solving the following individual optimization problem with constraints (1)-(3):

$$\begin{aligned} \mathbf{E}_{n,t} = & \operatorname*{argmin}_{\mathbf{E}_n} C_{n,t}(E_n)+\sum_{m\in\omega_n}\lambda_{nm,t-1}(F_{nm,t-1}-E_{nm}) \\ & +\frac{\rho}{2}(F_{nm,t-1}-E_{nm})^2+\frac{\eta}{2}(E_{nm}-E_{nm,t-1})^2, \end{aligned} \tag{7}$$

  where ρ is the penalty factor and $\lambda_{nm,t-1}$ is the dual variable of the reciprocity constraint (4), which also defines the price for transaction quantity $E_{nm,t-1}$. $\boldsymbol{\lambda}_{n,t-1} = \{\lambda_{n1,t-1},...,\lambda_{nm,t-1}\}$ is used to represent the whole transaction prices of agent n to neighboring agents $m \in \omega_n$ for time period $t-1$. $F_{nm,t-1}$ is the consensus variable defined as $\frac{E_{nm,t-1}-E_{mn,t-1}}{2}$ and $F_{n,t-1}=\sum_{m\in\omega_n}F_{nm,t-1}$. Then, each agent broadcasts $\mathbf{E}_{n,t}$ to neighboring agents.
- **Price Updates**: All agents update their prices to neighboring agents $m \in \omega_n$ as:

$$\lambda_{nm,t}=\lambda_{nm,t-1}-\rho(E_{nm,t}+E_{mn,t})/2. \tag{8}$$

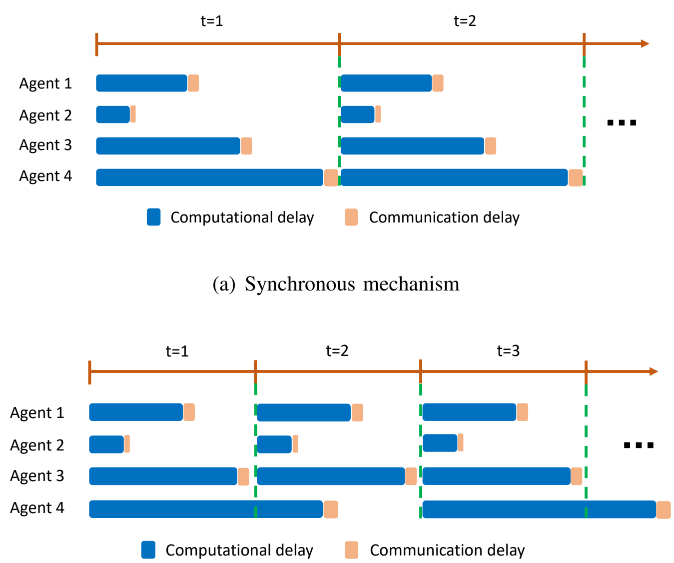
The online consensus ADMM operates in a synchronous manner, which means each agent has to wait to receive all bid prices and quantities from neighboring agents for updates. Under the synchronous mechanism, the market efficiency will be highly restricted by slow and low-reliability agents.

B. Asynchronous Market Mechanism

In order to overcome the drawback of the synchronous mechanism, we propose a novel asynchronous negotiation algorithm for the real-time P2P market. Certain methods can also speed up the market negotiation process, e.g., adaptive penalty factor [35] and alternative stopping criterion [7]. The adaptive penalty factor method focuses on reducing the number of iterations, while the alternative stopping criterion concentrates on reducing the communication times. However, neither of them can deal with the trouble causing by low-reliability and poor-quality agents.

As shown in Fig. 2(a), in the synchronous P2P market, agents have to wait to receive all bid prices and quantities from neighboring agents, and a lot of time is wasted on waiting for the slowest agent. Given that the number of agents in P2P markets is usually very large and the negotiation time in real-time is quite short, if there exists one invalid or damaged agent, the market efficiency will be heavily reduced. Thus, the synchronous mechanism is not suitable for the real-time P2P market. To overcome this issue, we propose a novel asynchronous negotiation mechanism for the real-time P2P market, where agents can freely trade with each other without waiting for inactive or idle neighboring agents.

A simple example is given to show the asynchronous market mechanism. In Fig. 2(b), in the time period $t = 1$, agents 1-3 are active and they will negotiate with each other to determine the transaction prices and quantities without waiting for agent 4, whose transaction prices and quantities between agents 1-3 will remain unchanged. Then, in the time period $t = 2$, all four agents are active, and they can trade with each other. In the asynchronous mechanism, no agent has to be synchronized with all neighboring agents, nor does it need to wait for the slow agents. Compared with the synchronous mechanism, the length of the time period can be shorter, and the market can run more frequently.



(a) Synchronous mechanism

(b) Asynchronous mechanism

Fig. 2: Description diagrams of synchronous and asynchronous mechanisms.

Let $\mathcal{A}_{n,t} \subseteq \omega_n$ denote the set of active neighboring agents of agent n at time period t. For example, in Fig. 2, $\mathcal{A}_{1,1} = \{2, 3\}$ and $\mathcal{A}_{2,2} = \{1, 3, 4\}$. We use $\mathcal{A}^c_{n,t}$ to denote the complementary set of $\mathcal{A}_{n,t}$, i.e., $\mathcal{A}_{n,t} \cap \mathcal{A}^c_{n,t} = \varnothing$ and $\mathcal{A}_{n,t} \cup \mathcal{A}^c_{n,t} = \omega_n$. We define $\mathbf{E}^{\mathcal{A}}_{n,t}$ to be the whole transaction quantities of agent n for active neighboring agents $m \in \mathcal{A}_{n,t}$ at time period t.

Based on the synchronous online consensus ADMM, we further propose the asynchronous online consensus ADMM. Let $\bar{t}_n$ be the last time period before t where agent n was active. $\upsilon \in (0, 1]$ is the forgetting factor, which has been proposed and developed in [36]–[39]. The forgetting factor is given to control the amount of older information that is used for current computation. A value of 1 results in no forgetting while decreasing values increase the amount of forgetting. Values slightly less than 1 are generally preferred. The asynchronous online consensus ADMM algorithm produces the following updates.

- **Power Updates**: The active agent n will update the transaction power for active neighboring agent $m \in \mathcal{A}_{n,t}$ by solving the following individual optimization problem:

$$
\begin{aligned}
\mathbf{E}^{\mathcal{A}}_{n,t} = \underset{\mathbf{E}^{\mathcal{A}}_n}{\operatorname{argmin}} & \sum_{l=\bar{t}_n+1}^{t} \upsilon^{t-l} C_{n,l}(E_n) \\
& + \sum_{m\in\mathcal{A}_{n,t}} \lambda_{nm,t}(F_{nm,t-1} - E_{nm}) \\
& + \frac{\rho}{2}(F_{nm,t-1} - E_{nm})^2 + \frac{\eta}{2}(E_{nm} - E_{nm,t-1})^2 \quad \text{(9a)} \\
\text{s.t.} \quad & E_n = \sum_{m\in\mathcal{A}_{n,t}} E_{nm,t} + \sum_{m\in\mathcal{A}^c_{n,t}} E_{nm,t} \quad \text{(9b)} \\
& \text{(2) and (3),}
\end{aligned}
$$

while for the idle neighboring agents $m \in \mathcal{A}^c_{n,t}$, the quantities will remain unchanged, i.e.,

$$E_{nm,t} = E_{nm,t-1}, \ m \in \mathcal{A}^c_{n,t}. \quad (10)$$

To ease the notation, we define $G_{n,t}(E_n) = \sum_{l=\bar{t}_n+1}^{t} \upsilon^{t-l} C_{n,l}(E_n)$. Then, each agent broadcasts $\mathbf{E}_{n,t}$ to neighboring agents.

- **Price Updates**: All agents update their prices to neighboring agents $m \in \omega_n$ as:

$$
\lambda_{nm,t} = \begin{cases} \lambda_{nm,t-1} - \rho\frac{E_{nm,t}+E_{mn,t}}{2}, \ \forall m \in \mathcal{A}_{n,t} \\ \lambda_{nm,t-1}, \ \forall m \in \mathcal{A}^c_{n,t} \end{cases} \quad (11)
$$

However, since each pair of two agents only have one negotiation in each time period, the power balance between two agents may not be balanced, i.e., $E_{nm,t} + E_{mn,t} \neq 0$. To address this problem, we design a projection-based power update process to ensure the power balance.

C. Projection-based Power Update Process

In the electricity market, it is necessary to ensure the balance for trade among agents. Thus, a projection-based power update process is designed to determine the final transaction quantities. This process is executed after the power and price updates (9)-(11). In this process, iterations are indexed with k.

First, $\hat{E}^{k+1}_{nm,t}$ is set to be the intermediate value of the transaction quantities between active agents n and m from last iteration as

$$\hat{E}^{k+1}_{nm,t} = (\hat{E}^{k}_{nm,t} - \hat{E}^{k}_{mn,t})/2, \ \forall m \in \mathcal{A}_{n,t}. \quad (12)$$

Here, the initial value $\hat{E}^1_{nm,t}$ is obtained after the power update (9), i.e., $\hat{E}^1_{nm,t+1} = E_{nm,t+1}$. Then the total power is obtained as

$$\hat{E}^{k+1}_{n,t} = \sum_{m\in\mathcal{A}_{n,t}} \hat{E}^{k+1}_{nm,t} + \sum_{m\in\mathcal{A}^c_{n,t}} E_{nm,t}. \quad (13)$$

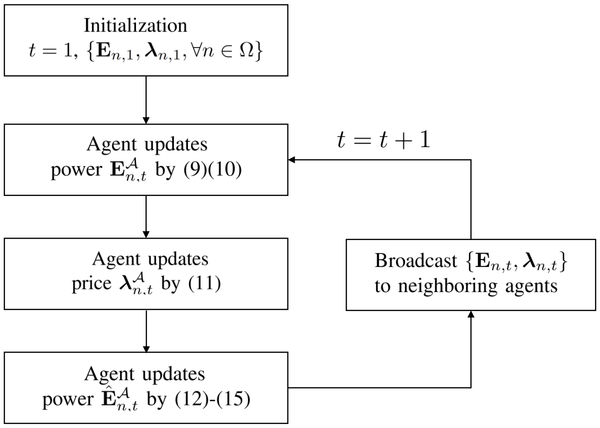


Fig. 3: Flowchart of the asynchronous negotiation mechanism for real-time P2P electricity market

After projecting into the feasible region (2), the projected power $\hat{E}^{proj,k+1}_{n,t}$ is updated as

$$\hat{E}^{proj,k+1}_{n,t} = \max\left\{\min\left\{\hat{E}^{k+1}_{n,t}, \overline{E}_{n,t}\right\}, \underline{E}_{n,t}\right\}. \quad (14)$$

Then the trade $\hat{E}^{k+1}_{nm,t}$ to active neighboring agent m is updated as

$$\hat{E}^{k+1}_{nm,t} = \hat{E}^{k+1}_{nm,t}\frac{\hat{E}^{proj,k+1}_{n,t}}{\hat{E}^{k+1}_{n,t}}, \ \forall m \in \mathcal{A}_{n,t}. \quad (15)$$

Finally, all active agents send the updated quantities $\hat{E}^{k+1}_{nm,t}$ to its active neighboring agents, and check if the balance equation $\hat{E}^{k+1}_{nm,t}+\hat{E}^{k+1}_{mn,t} < \varepsilon$ is satisfied, where ε is the allowed maximal violation; if not, repeat the processes (12)-(15) until balance.

For the projection-based power update process, there is only one situation where agents cannot reach transaction balance, which is that the maximum power generation of all generators still cannot meet the minimum demand of all consumers. However this situation probably will not happen in actual operation unless there is a serious malfunction occurs the power system. Therefore, as long as the market has feasible solutions, the power update process will ensure balances for trading among agents.

The asynchronous negotiation mechanism for the real-time P2P electricity market is detailed in Fig. 3.

IV. Market Analysis

In this section, we first analyze the convergence performance of the online algorithm. Under some standard assumptions, the sublinear regret upper bound for the asynchronous online consensus ADMM algorithm is proved, which indicates that social welfare will be maximized in the long run on time average. After that, the four desirable properties of market mechanisms are analyzed.

A. Regret Analysis

The convergence performance of an online optimization algorithm is usually measured by the regret, which is the accumulated gap between the online solutions and the best solutions in hindsight [33], [40]–[42]. The regret for our problem is formulated as below.

$$R(T) = \sum_{t=1}^{T}\left(\sum_{n\in\Omega} C_{n,t}(\hat{E}_{n,t})\right) - \sum_{t=1}^{T}\left(\sum_{n\in\Omega} C_{n,t}(E^*_{n,t})\right) \quad (16)$$

where $\hat{E}_{n,t}$ is the online solution of agent n in the time period t using our asynchronous online consensus ADMM, and $E^*_{n,t}$ is the optimal solution. The goal of the online algorithm is to generate decisions making the regret $R(T)$ small. If the regret is bounded by a sublinear function of T, e.g., $R(T) = \mathcal{O}(\sqrt{T})$ [43], then the average regret converges to zero as $T \to \infty$, which indicates that the sequence of the decisions converges to the best fixed decisions in hindsight.

Before presenting the results, some needed rational assumptions are introduced to derive the sublinear regret upper bound to be presented later in Theorem 1.

Assumption 1.

(a) *$G_{n,t}$ are convex with bounded subgradients for $m \in \omega_n$, i.e., $\frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}} \le S$, $\forall (n,m) \in (\Omega, \omega_n)$, with S being a positive constant.*

(b) *The initial values are set to zero, i.e., $\lambda_{nm,1} = 0$ and $E_{nm,1} = 0$, $\forall (n,m) \in (\Omega, \omega_n)$.*

(c) *The gap between optimal solutions and initial ones are bounded, i.e., $(E^*_{nm,t} - E_{nm,1})^2 \le D_1$ and $(F^*_{nm,t} - F_{nm,1})^2 \le D_2$, $\forall (n,m) \in (\Omega, \omega_n)$, with D_1 and D_2 being positive constants.*

(d) *The cost gap between online solutions and optimal ones are bounded, i.e., $C_{n,t}(E_{n,t}) - C_{n,t}(E^*_{n,t}) \le J$, $\forall n \in \Omega$, with J being a positive constant.*

(e) *Path variation is defined as the temporal change of the optimal solutions sequence, i.e., $V_{nm,T} = \sum_{t=1}^{T}\left|E^*_{nm,t} - E^*_{nm,t+1}\right|$. The optimal solutions do not change dramatically, or in mathematical sense, the path variation is bounded as $V_{nm,T} \le P$, $\forall (n,m) \in (\Omega, \omega_n)$, with P being a positive constant called variation budget.*

(f) *The electricity prices do not change dramatically, or in mathematical sense, the price variation between two time periods is bounded as $|\lambda_{nm,t+1} - \lambda_{nm,t}| \le \Lambda$, $\forall (n,m) \in (\Omega, \omega_n)$, with Λ being a positive constant.*

(g) *Let $\tau > 1$ be a maximal tolerable delay for all agents. For all $(n,m) \in (\Omega, \omega_n)$ and time periods $t \ge 0$, it must be that $m \in \mathcal{A}_{n,t} \cup \mathcal{A}_{n,t-1} \cdots \cup \mathcal{A}_{n,\max\{t-\tau+1,1\}}$.*

In Assumption 1, (a) is an assumption that many may see as simplifying the reality of prosumers' marginal utility functions. The issue of potential non-convexity in market clearing approaches is general in the electricity market literature, and not restricted to the case of the P2P market studied here. It should be noted the fact that function $G_{n,t}$, which acts as a smoother over the previous cost and utility functions, may actually help in having more well-behaved functions as input to the current optimization process. (b) is the settings of initial values. (c) and (d) are also reasonable since the online solutions and optimal solutions cannot be infinite. $(a)-(d)$ are usually required in the online optimization settings. Generally, in the field of online optimization research, it is inevitable to make some assumptions about the real problems, and our assumptions as similar in essence to, e.g., [42], [44]–[47]. For the specific case of (e), it is reasonable to think that the optimal solutions do not change drastically or infinitely from one step to the next, otherwise, the power system would probably break down. Thus, we place mild restrictions on the possible variation of the optimums, as is also proposed in previous works [42], [45], [46]. (f) is reasonable in the context of electricity markets since electricity prices cannot be infinite. Finally for (g), any unbounded delay will jeopardize the market social welfare convergence. Therefore, throughout this paper, we assume that the asynchronous delay in this market is bounded. If it was not, this would mean an agent is not active anymore, and should be removed from the market. We then follow the popular asynchronous model [48] to make that assumption. It implies that every agent n will trade with neighboring agents within the period $[t-\tau+1,t]$. In another word, the bid prices and quantities from neighboring agents received by each agent n must be at most τ time periods delayed.

Bearing all the above in mind, the following theorem establishes the sublinear regret upper bound for the asynchronous online consensus ADMM algorithm.

Theorem 1. *The asynchronous online consensus ADMM algorithm has the following sublinear regret upper bound by setting $\rho=\sqrt{T}$ and $\eta=\sqrt{T}$.*

$$R(T)\leq\frac{N(N-1)}{\upsilon^{\tau}}\left(\frac{D_1+D_2+\Lambda^2+S^2}{2}+LP\right)\sqrt{T}+\frac{NJ}{\upsilon^{\tau}(1-\upsilon)} \tag{17}$$

where $L=\max_{n\in\Omega}2*(\overline{E}_n-\underline{E}_n)$.

Proof. See Appendix A. □

Since the regret has $\mathcal{O}(\sqrt{T})$ upper bound, we have $\lim_{T\to\infty}\frac{R(T)}{T}=0$, which implies the accumulated gap between our online solutions and optimal ones are approaching to zero in the long run on time average. In the market context, regret can be regarded as the cumulative total cost gap between online solutions and optimal ones. Thus, the sublinear regret upper bound also indicates that social welfare will be maximized in the long run on time average.

B. Desirable Properties of Market Mechanism

It is important and necessary to evaluate a market mechanism by checking the four desirable properties, which are market efficiency[1], incentive compatibility[2], cost recovery[3] and revenue adequacy[4]. Based on the Hurwicz theorem [49], no mechanism is capable of achieving all those properties at the same time.

1) Market efficiency: According to Theorem 1, we have $\lim_{T\to\infty}\frac{R(T)}{T}=0$, which implies that social welfare and market efficiency will be maximized in the long run on time average.

2) Incentive compatibility: A market agent can gain profit by not trustfully bidding price or quantity in our mechanism. However, a blockchain-based platform is efficient to prevent that from happening. Since a blockchain-based platform is adaptive for a P2P market to ensure both operational feasibility and fair payments to all agents, we can combine our real-time P2P market negotiation mechanism with the blockchain-based platform. First, the trading prices and quantities among agents are determined via the asynchronous online consensus ADMM algorithm, and then the actual production and consumption are verified using the blockchain-based platform. Based on the platform, each agent can check whether other agents generate or consume the same amount of power as the transaction. If the agent does not obey the transaction to generate or consume power, the deal will be canceled, and the agent will not receive payment, or even be fined.

3) Cost recovery: The individual profit for an agent n in time period t is

$$\sum_{m\in\omega_n}\lambda_{nm,t}E_{nm,t}-C_{n,t}(E_{n,t}) \tag{18}$$

Since the quadratic cost function is convex, monotonically increasing, and passing through the origin, the agent can always set $E_{n,t}=E_{nm,t}=0$ to avoid a negative profit. In other words, the agent does not participate in the market during the time period t. Thus, the cost recovery is satisfied.

4) Revenue adequacy: From (11), the prices between each pair of two agents are identical, i.e., $\lambda_{nm,t}=\lambda_{mn,t}$, and after the projection-based power update process, the quantities between each pair of two agents are balanced, i.e., $\hat{E}_{nm,t}+\hat{E}_{mn,t}=0$. Thus, the revenue adequacy is satisfied.

Summing up the above, our proposed real-time P2P market mechanism satisfies most of the desirable properties.

[1]Market efficiency is maximized when outcomes maximize social welfare.

[2]A mechanism is called incentive-compatible if every participant can maximize its objective just by acting according to its true preferences.

[3]Cost recovery implies that individual profit is non-negative.

[4]Revenue adequacy implies that there is no financial deficit in the market.

V. Simulation Results

Our online algorithms are tested on a dataset of wind power generation of 20 wind farms in Australia [50] to show the convergence performance. In order to better display performance, uniformly distributed stochastic parameter settings are applied. We use Matlab R2017b on a PC with 1.6 GHz Intel Core 4 Duo CPU and 8 GB memory to perform simulation, and solve the convex optimization problem by CVX Sedumi solver. Since MATLAB cannot implement parallel computing, the agents run the procedure sequentially, i.e., one agent will start running after the previous one has completed. We record the computational time of each agent, and the time for all agents to complete an iteration is the maximal value of the computational time of all agents. The decentralized mechanism is achieved in this way.

A. Convergence Performance

We establish a market consisting of 20 conventional generators, 20 users, and 20 wind generators with actual wind power generation data. The convergence performance of the synchronous and asynchronous mechanisms is tested. At any given moment, an agent has only two states, i.e., it can only be either active or inactive, and these two states are mutually

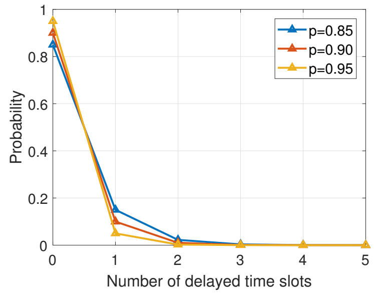
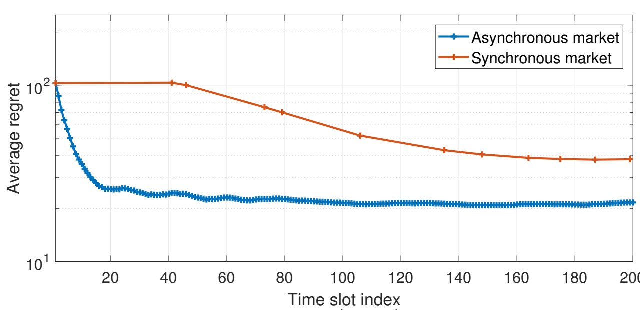
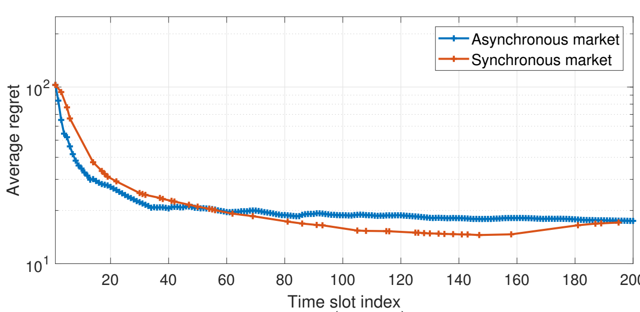


Fig. 4: The probability of the number of consecutive delayed time periods under different active rate.

exclusive. Thus we believe the Bernoulli distribution is suitable for modeling the active rates of agents. We set the active rate is p_n and the idle rate is $1 - p_n$. The active rates of agents can be collected according to the type of agents. For conventional thermal generators, the active rates can be obtained according to the generation plans; for consumers, active rates can be obtained based on electric usage habits; for renewable generators, when to enter the market depends on the weather condition, which can be roughly estimated from historical data. The probability of the number of consecutive delayed time periods is shown in Fig. 4 under different active rates. It can be seen from the figure that agents are extremely unlikely to be infinitely delayed.



(a) $p_n \in (0.9, 1)$



(b) $p_n \in (0.95, 1)$

Fig. 5: Average regret $R(T)/T$ of synchronous and asynchronous markets with different active rate ranges.

We operate the synchronous and asynchronous market mechanisms in two different active rate ranges, i.e., $p_n \in (0.9, 1)$ and $p_n \in (0.95, 1)$, lasting 200 time periods, and the convergence performance is shown in Fig. 5. For the asynchronous mechanism, it can be seen that agents can freely trade in each time period and the average regret $R(T)/T$ drops rapidly in about the first 40 time periods. Since then, the average regret declines much slower, but it still keeps going down with small oscillations, mainly due to the large uncertainty in renewable power generation. The change of the active rate only has a slight impact on the convergence performance, which shows the robustness of the asynchronous mechanism.

Compared with the asynchronous mechanism, the active rate has a greater impact on the convergence performance of the synchronous mechanism. As the active rate increases, the probability of all agents being active at each time period will increase, and the number of transactions will increase accordingly. The market can be running more frequently, thereby increasing the average regret convergence speed, even better than the asynchronous mechanism. Therefore, if the reliability of all agents is high, the asynchronous mechanism may require more iterations (time periods). However, it should be noticed that in the real world P2P market, the number of agents is usually very large, which means as long as there exists one agent who has a very low active rate, the synchronous mechanism could be very inefficient. In the real world, it is less likely that all agents are efficient and have high reliability. Thus, the asynchronous mechanism probably will not take more iterations (time periods) than the synchronous mechanism in practical applications.

B. *Market Transaction Frequency*

We operate the synchronous and asynchronous market mechanisms under five different active rate ranges to investigate the frequency of market transactions, which is measured by the average number of time periods between two market transactions. The markets run 50 times with different settings in 200 time periods. Fig. 6 shows the average number of time periods between two transactions. The figure shows that, as the lower bound of active rates decreases, the average number of time periods between two transactions for synchronous mechanism increases exponentially. However, for the asynchronous market, since agents can trade with each other in each time period, the value remains at 1. Therefore, the asynchronous market negotiation mechanism can operate more frequently and better catch up with the uncertainty of renewable power generation.

C. *Fairness of Individual Profit*

We build a small market with 5 conventional generators, 5 users, and 5 wind generators. The parameters of the same type of agents are identical. The active rates of the five conventional generators are set to 0.6, 0.7, 0.8, 0.9, and 1, respectively, while the active rates of the other agents are all set to 1. Intuitively, a more reliable agent should get more profit than a low-reliability agent in the market. By displaying the accumulated individual profits of conventional generators and users in 200 time periods, we compare the fairness of synchronous and asynchronous mechanisms. As shown in Fig. 7, the result indicates that the profits of the same type of

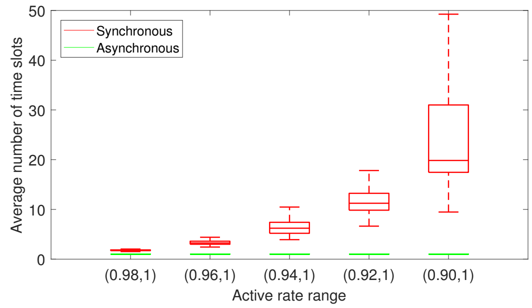


Fig. 6: The number of time periods between two transactions under different active rate ranges.

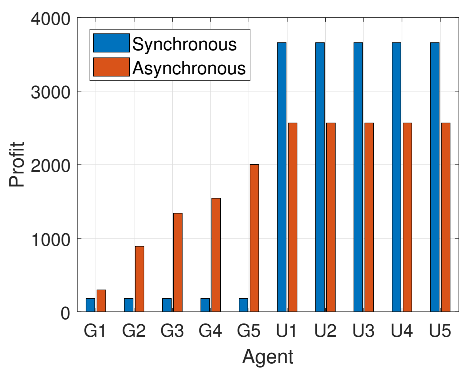


Fig. 7: The profits of agents under two mechanisms

agents are identical in the synchronous market since agents have to trade together. In the asynchronous market, agents with a higher active rate can trade more frequently and earn more profits. Therefore, the asynchronous market mechanism is more fair, and the deployment of it can encourage agents to improve the quality and reliability.

D. The Impact of Forgetting Factor

The impact of the value of forgetting factor on the convergence performance is tested. The forgetting factor is given to control the amount of older information that is used for current computation. By giving less weight to older data, the agent focuses more on the updated new information. Fig. 8 shows that the reduction of the forgetting factor value will increase the amount of forgetting and improve the convergence performance.

VI. Conclusion

P2P markets are considered as an evolution of the future electricity markets driven by the development of DERs and DSM. However, it is technically challenging to operate P2P market mechanisms in real-time, since they usually involve a heavy computation burden, while real-time trading demands fast calculations. How to reduce the computation complexity of P2P mechanisms to be within a real-time architecture remains a challenge. To this end, we propose a novel real-time P2P electricity market that integrates online optimization approaches and asynchronous mechanism, where each agent only performs one iteration and can freely trade with neighboring agents without waiting for the idle or inactive ones. Finally, we give proof of the sublinear regret upper bound for our asynchronous online consensus ADMM algorithm, which indicates that social welfare can be maximized in the long run on time average. Simulation results show that our asynchronous market mechanism has good convergence performance, robustness, and fairness compared with the synchronous mechanism.

The main drawback of the asynchronous mechanism are that if most agents are reliable, it may take a very long period of time to converge below an acceptable level. However, it is less likely that all agents are efficient and have high reliability in actual markets.

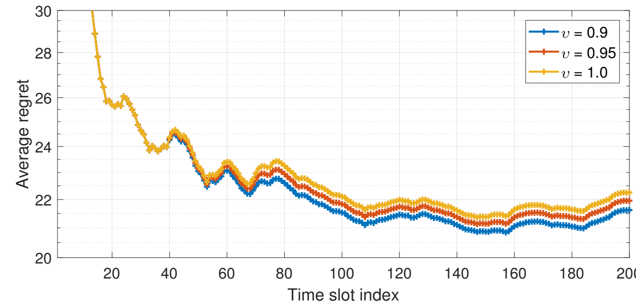


Fig. 8: The impact of forgetting factor value.

Appendix A
Proof of the Sublinear Regret Upper Bound

Let $\frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}}$ be the gradient of $G_{n,t}(E_n)$ at $E_{nm,t}$. Since $\mathbf{E}^{\mathcal{A}}_{n,t}$ minimizes (9), combining (11), we have

$$\begin{aligned}\frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}} &= \lambda_{nm,t} + \rho(F_{nm,t-1} - F_{nm,t}) \\ &+ \eta(E_{nm,t-1} - E_{nm,t}),\ \forall m \in \mathcal{A}_{n,t}\end{aligned} \tag{19}$$

Since $G_{n,t}$ is a convex function and its subgradient at $E_{nm,t+1}$ is given in (19), for optimal solution $E^*_{n,t}$ we have

$$\begin{aligned}&G_{n,t}(E_{n,t}) - G_{n,t}(E^*_{n,t}) \\ &\leq \sum_{m\in\mathcal{A}_{n,t}} \frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}}(E_{nm,t} - E^*_{nm,t}) \\ &= \sum_{m\in\mathcal{A}_{n,t}} \lambda_{nm,t}(E_{nm,t} - F^*_{nm,t}) \\ &\quad + \sum_{m\in\mathcal{A}_{n,t}} \rho(F_{nm,t-1} - F_{nm,t})(E_{nm,t} - F^*_{nm,t}) \\ &\quad + \sum_{m\in\mathcal{A}_{n,t}} \eta\left(E_{nm,t-1} - E_{nm,t}\right)\left(E_{nm,t} - E^*_{nm,t}\right) \\ &= \sum_{m\in\mathcal{A}_{n,t}} \lambda_{nm,t}(E_{nm,t} - F^*_{nm,t}) \\ &\quad + \sum_{m\in\mathcal{A}_{n,t}} \frac{\rho}{2}\left[(F_{nm,t-1} - F^*_{nm,t})^2 - (F_{nm,t} - F^*_{nm,t})^2\right. \\ &\quad \left. + (E_{nm,t} - F_{nm,t})^2 - (E_{nm,t} - F_{nm,t-1})^2\right] \\ &\quad + \frac{\eta}{2}\left[(E^*_{nm,t} - E_{nm,t-1})^2 - (E^*_{nm,t} - E_{nm,t})^2\right.\end{aligned}$$

$$-(E_{nm,t}-E_{nm,t-1})^2\Big] \quad (20)$$

According to the Fenchel-Young's inequality [51], we have

$$\begin{aligned}
&G_{n,t}(\hat{E}_{n,t})-G_{n,t}(E_{n,t})\\
&\leq \sum_{m\in\mathcal{A}_{n,t}} \frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}}(\hat{E}_{nm,t}-E_{nm,t})\\
&\leq \sum_{m\in\mathcal{A}_{n,t}} \frac{1}{2\rho}\left(\frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}}\right)^2+\frac{\rho}{2}(\hat{E}_{nm,t}-E_{nm,t})^2\\
&\leq \sum_{m\in\mathcal{A}_{n,t}} \frac{1}{2\rho}\left(\frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}}\right)^2+\frac{\rho}{2}(F_{nm,t}-E_{nm,t})^2\\
&\leq \sum_{m\in\mathcal{A}_{n,t}} \frac{1}{2\rho}\left(\frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}}\right)^2+\frac{(\lambda_{nm,t-1}-\lambda_{nm,t})^2}{2\rho}
\end{aligned} \quad (21)$$

For the penultimate inequality, because $\hat{E}_{nm,t}$ begins updating from $F_{nm,t}$, if the projection update finishes at the first time, which also means both agents do not touch the bound, the term $(\hat{E}_{nm,t}-E_{nm,t})^2$ reaches the maximal value $(F_{nm,t}-E_{nm,t})^2$; The worst case is that the final trade reaches at the initial value $E_{nm,t}$ or $-E_{mn,t}$, which means at the beginning, one of the agents n or m has reached the bound, then the term $(\hat{E}_{nm,t}-E_{nm,t})^2$ reaches the minimal value zero.

Combining (20)-(21), we have

$$\begin{aligned}
&G_{n,t}(\hat{E}_{n,t})-G_{n,t}(E^*_{n,t})\\
&\leq \sum_{m\in\mathcal{A}_{n,t}} \lambda_{nm,t}(E_{nm,t}-F^*_{nm,t})+\frac{\rho}{2}(E_{nm,t}-F_{nm,t})^2\\
&\quad+\sum_{m\in\mathcal{A}_{n,t}} \frac{\rho}{2}\left[(F_{nm,t-1}-F^*_{nm,t})^2-(F_{nm,t}-F^*_{nm,t})^2\right]\\
&\quad+\sum_{m\in\mathcal{A}_{n,t}} \frac{\eta}{2}\left[(E^*_{nm,t}-E_{nm,t-1})^2-(E^*_{nm,t}-E_{nm,t})^2\right]\\
&\quad+\sum_{m\in\mathcal{A}_{n,t}} \frac{1}{2\rho}\left(\frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}}\right)^2+\frac{(\lambda_{nm,t-1}-\lambda_{nm,t})^2}{2\rho}
\end{aligned} \quad (22)$$

For the first term, using $E_{nm,t}-F_{nm,t}=\frac{\lambda_{nm,t-1}-\lambda_{nm,t}}{\rho}$, $F^*_{nm,t}+F^*_{mn,t}=0$, $\lambda_{nm,t}=\lambda_{mn,t}$ and summing up for all $n\in\Omega$ yields

$$\begin{aligned}
&\sum_{n\in\mathcal{A}_t}\sum_{m\in\mathcal{A}_{n,t}} \lambda_{nm,t}(E_{nm,t}-F^*_{nm,t})+\frac{\rho}{2}(E_{nm,t}-F_{nm,t})^2\\
&=\frac{1}{2}\sum_{\forall(n,m)\in(\Omega,\mathcal{A}_{n,t})} \lambda_{nm,t}\left[E_{nm,t}+E_{mn,t}-(F^*_{nm,t}+F^*_{mn,t})\right]\\
&\quad+\frac{\rho}{2}(E_{nm,t}-F_{nm,t})^2+\frac{\rho}{2}(E_{mn,t}-F_{mn,t})^2\\
&=\frac{1}{2}\sum_{\forall(n,m)\in(\Omega,\mathcal{A}_{n,t})} 2\lambda_{nm,t}(E_{nm,t}-F_{nm,t})\\
&\quad+\frac{1}{2\rho}(\lambda_{nm,t-1}-\lambda_{nm,t})^2+\frac{1}{2\rho}(\lambda_{mn,t-1}-\lambda_{mn,t})^2\\
&=\sum_{\forall(n,m)\in(\Omega,\mathcal{A}_{n,t})} \frac{1}{2\rho}(\lambda^2_{nm,t-1}-\lambda^2_{nm,t})
\end{aligned} \quad (23)$$

For the second term

$$\begin{aligned}
&(F_{nm,t-1}-F^*_{nm,t})^2-(F_{nm,t}-F^*_{nm,t})^2\\
&=(F_{nm,t-1}-F^*_{nm,t-1})^2-(F_{nm,t}-F^*_{nm,t})^2\\
&\quad+(F_{nm,t-1}-F^*_{nm,t})^2-(F_{nm,t-1}-F^*_{nm,t-1})^2\\
&\leq(F_{nm,t-1}-F^*_{nm,t-1})^2-(F_{nm,t}-F^*_{nm,t})^2\\
&\quad+(2F_{nm,t-1}-F^*_{nm,t-1}-F^*_{nm,t})(F^*_{nm,t-1}-F^*_{nm,t})\\
&\leq(F_{nm,t-1}-F^*_{nm,t-1})^2-(F_{nm,t}-F^*_{nm,t})^2\\
&\quad+L\left|E^*_{nm,t-1}-E^*_{nm,t}\right|
\end{aligned} \quad (24)$$

Similarly, for the third term

$$\begin{aligned}
&(E^*_{nm,t}-E_{nm,t-1})^2-(E^*_{nm,t}-E_{nm,t})^2\\
&\leq(E_{nm,t-1}-E^*_{nm,t-1})^2-(E_{nm,t}-E^*_{nm,t})^2\\
&\quad+L\left|E^*_{nm,t-1}-E^*_{nm,t}\right|
\end{aligned} \quad (25)$$

Combining (22)-(23) and based on Assumption 1, we have

$$\begin{aligned}
&\sum_{t=1}^{T}\left(\sum_{n\in\mathcal{A}_t} G_{n,t}(\hat{E}_{n,t})\right)-\sum_{t=1}^{T}\left(\sum_{n\in\mathcal{A}_t} G_{n,t}(E^*_{n,t})\right)\\
&\leq \sum_{\forall(n,m)\in(\mathcal{A}_t,\mathcal{A}_{n,t})} \frac{1}{2\rho}(\lambda^2_{nm,1}-\lambda^2_{nm,T})\\
&\quad+\sum_{n\in\mathcal{A}_t}\sum_{m\in\mathcal{A}_{n,t}} \frac{\rho}{2}\left[(F_{nm,1}-F^*_{nm,1})^2-(F_{nm,T}-F^*_{nm,T})^2\right]\\
&\quad+\sum_{n\in\mathcal{A}_t}\sum_{m\in\mathcal{A}_{n,t}} \frac{\eta}{2}\left[(E_{nm,1}-E^*_{nm,1})^2-(E_{nm,T}-E^*_{nm,T})^2\right]\\
&\quad+\sum_{t=1}^{T}\sum_{n\in\mathcal{A}_t}\sum_{m\in\mathcal{A}_{n,t}} \frac{(\rho+\eta)L}{2}\left|E^*_{nm,t}-E^*_{nm,t}\right|\\
&\quad+\sum_{t=1}^{T}\sum_{n\in\mathcal{A}_t}\sum_{m\in\mathcal{A}_{n,t}} \frac{1}{2\rho}\left(\frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}}\right)^2\\
&\quad+\sum_{t=1}^{T}\sum_{n\in\mathcal{A}_t}\sum_{m\in\mathcal{A}_{n,t}} \frac{(\lambda_{nm,t-1}-\lambda_{nm,t})^2}{2\rho}\\
&\leq N(N-1)\left(\frac{\rho D_2}{2}+\frac{\eta D_1}{2}+\frac{(\rho+\eta)LP}{2}+\frac{TS^2}{2\rho}+\frac{T\Lambda^2}{2\rho}\right)
\end{aligned} \quad (26)$$

Now, for each $n\in\mathcal{A}^c_T$, let $\tilde{t}_n$ be the last iteration where agent n was active. Using Assumption 1(g), we have that

$$\begin{aligned}
&\sum_{t=1}^{T}\left(\sum_{n\in\mathcal{A}_t} G_{n,t}(\hat{E}_{n,t})\right)-\sum_{t=1}^{T}\left(\sum_{n\in\mathcal{A}_t} G_{n,t}(E^*_{n,t})\right)\\
&=\sum_{t=1}^{T}\sum_{n\in\mathcal{A}_t}\sum_{l=\tilde{t}_n+1}^{t} \upsilon^{t-l}\left(C_{n,l}(\hat{E}_{n,t})-C_{n,l}(E^*_{n,t})\right)\\
&\geq\sum_{t=1}^{T}\sum_{n\in\Omega} \upsilon^{\tau}\left(C_{n,t}(\hat{E}_{n,t})-C_{n,t}(E^*_{n,t})\right)\\
&\quad-\sum_{n\in\mathcal{A}^c_T}\sum_{l=\tilde{t}_n+1}^{T} \upsilon^{T-l}\left(C_{n,l}(\hat{E}_{n,t})-C_{n,l}(E^*_{n,t})\right)
\end{aligned} \quad (27)$$

Noting that $\hat{E}_{n,l} = \hat{E}_{n,t}$ for $l = \bar{t}_n + 1, ..., t$ and using Assumption 1(f), the last term in (27) can be bounded as follows:

$$\sum_{n \in \mathcal{A}_T^c} \sum_{l=\tilde{t}_n+1}^{T} \upsilon^{T-l} \left(C_{n,l}(\hat{E}_{n,t}) - C_{n,l}(E^*_{n,t}) \right) \leq NJ \frac{1-\upsilon^{T-\tilde{t}_n}}{1-\upsilon} \leq \frac{NJ}{1-\upsilon} \tag{28}$$

where we have used $\mathcal{A}_t^c \in \Omega$. By using (28) and (27) in (26), we finally have

$$R(T) = \sum_{t=1}^{T} \sum_{n \in \Omega} \left(C_{n,l}(\hat{E}_{n,t}) - C_{n,t}(E^*_{n,t}) \right) \leq \frac{N(N-1)}{\upsilon^\tau} \left(\frac{\rho D_2}{2} + \frac{\eta D_1}{2} + \frac{(\rho+\eta)LP}{2} + \frac{TS^2}{2\rho} + \frac{T\Lambda^2}{2\rho} \right) + \frac{NJ}{\upsilon^\tau(1-\upsilon)} \tag{29}$$

Setting $\rho = \sqrt{T}$ and $\eta = \sqrt{T}$ yields sublinear regret $R(T)$.

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