

# Online Optimization for Real-Time Peer-to-Peer Electricity Market Mechanisms

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**Abstract**—Owing to the fast deployment of distributed energy resources and the further development of demand-side management, small agents in electricity markets are becoming more proactive. This may boost the development of peer-to-peer (P2P) market mechanisms. Meanwhile, since actual load and power generation may substantially deviate from schedules obtained at the day-ahead (forward) market stage, it is needed to rapidly reschedule the trades among agents to maintain power balance through a real-time market mechanism, also in a P2P framework. However, it is technically challenging to develop and operate such P2P market mechanisms in real-time, since they most often involve a heavy computational burden (e.g., based on iterative distributed optimization approaches), while real-time trading demands fast calculation. Our core contribution is hence to describe and analyse a novel real-time P2P market mechanism within an online optimization framework. It relies on online social welfare maximization using a novel online consensus alternating direction method of multipliers (OC-ADMM) algorithm. The computational complexity is then heavily reduced in order to satisfy real-time requirements. We derive a sublinear non-stationary regret upper bound for our algorithm, which implies that social welfare will be maximized in the long run. Simulations based on a number of case studies show that our algorithm has good convergence performance, tracking ability, and high computational efficiency.

**Index Terms**—Peer-to-peer electricity market, real-time market, online consensus ADMM, primal-dual alternate update, non-stationary regret.

## NOMENCLATURE

### Functions

$\hat{R}(\cdot)$  Non-stationary regret.  
 $C(\cdot)$  Production cost or utility function.  
 $R(\cdot)$  Stationary regret.

### Numbers and Indexes

$k$  Index for iterations.  
 $l$  Index for iterations.  
 $N$  Cardinal number of agents.  
 $n, m$  Indices for agents.  
 $N_\omega$  Cardinal number of neighboring agents.  
 $T$  Cardinal number of time steps.  
 $t$  Indices for time steps.

### Parameters

$\chi^c$  Normalized cost deviation  
 $\epsilon$  Allowed maximal violation of constraint (1).  
 $\rho, \eta$  Penalty factors.  
 $\underline{E}, \bar{E}$  Boundaries of energy.  
 $A$  Allowed maximal violation of trade between agents.  
 $a, b$  Coefficients of the quadratic functions.  
 $G, D, P, L, \Lambda$  Positive upper bounded constants in assumptions

### Sets and Vectors

$\mathbf{E}$  Vector of whole energy transactions.

$\Omega$  Set of agents.  
 $\omega$  Set of neighboring agents.  
 $\Omega_p, \Omega_c, \Omega_u$  Set of producers and consumers.

### Variables

$\hat{E}$  The final trade quantity after projection.  
 $\hat{E}^{proj}$  The projected total energy power.  
 $\lambda$  Energy prices.  
 $\tilde{\lambda}$  Energy prices.  
 $\tilde{\delta}$  Average value of the dual variables  $\delta$  between neighboring agents.  
 $\tilde{\lambda}$  Average value of the prices between neighboring agents.  
 $\tilde{F}$  Average value of the consensus variable between neighboring agents.  
 $\underline{\delta}, \bar{\delta}$  Corresponding dual variables for constraint (3).  
 $\underline{\mu}, \bar{\mu}$  Corresponding dual variables for constraint (2).  
 $\bar{E}$  Power injection or traded quantity.  
 $E^*$  Optimal energy injection or traded quantity.  
 $F$  Consensus variable of trades.

## I. INTRODUCTION

The ever-increasing distributed energy resources and energy system management are changing the way of power system operation. Participants are becoming more proactive in the market, who prefer determining the trading outcomes by themselves. Therefore, electricity markets are evolving towards more decentralized mechanisms. However, current electricity markets still perform resource allocation and pricing based on the conventional hierarchical and top-down approach [1], which makes participants behave as passive receivers. Recently, a novel design of electricity markets has emerged: these so-called peer-to-peer (P2P) electricity markets rely on multi-bilateral trades between each pair of participants [2]–[11]. Employing a P2P market mechanism can yield plenty of advantages, e.g., empowerment of participants, increasing the reliability of power system and protection of privacy [2], [5].

Existing works about P2P markets mainly focus on the following issues: reallocation of the costs [3], product differences [4], [9], dispatch fairness [8], flexibility of battery [10], [11], communication properties [6] and costs [7]. Technically, different decentralized methods are devised to realize market mechanism, for instance, primal-dual gradient [4], relaxed consensus+innovation [6], [9], standard alternating direction method of multipliers (ADMM) [8], stochastic programming [11] and consensus ADMM [3], [12].

Meanwhile, in actual operation, due to changes in the weather, potential power system incidents, renewable power generation uncertainty, demand-side load variations, and other

contingencies, the actual load and power generation may have a large deviation from the schedule obtained at the day-ahead (or more generally forward) market stage – power balance hence ought to be restored. Thus, it is required to rapidly reschedule the trades among market participants to keep power balance in response to these changes – hence calling for real-time markets [13]–[16]. Compared with day-ahead and intraday markets, the time-scale for the real-time market for settlement before operation is usually 5 min or even shorter. Besides, for energy imbalance ancillary services, there exist different kinds of reserve products in real-time, which are classified referring to their responding time scales from 30 min to 15 sec [17]. Thus, the real-time market trading demands fast calculation. However, A P2P market mechanism requires a very large amount of information to be exchanged, much greater than that required for a centralized market [7], [18], [19]. In a real-time market context, such exchanges run a risk of not having enough time enough to succeed if the deadline is reached before the end of the negotiation process. Eventually, the main challenge and requirement for a P2P real-time electricity market is to lower the computational complexity of the P2P mechanism so that it can be deployed in a real-time architecture.

Note that some works already considered P2P mechanisms for real-time electricity markets [20]–[23]. For instance, [20] proposed bilateral contract networks as a new market design for P2P energy trading in real-time markets, but it did not consider how one could take advantage of previous negotiations to make current negotiation faster (also knowing that the agents update their information and preferences). Ref. [21] proposed a P2P local electricity market model incorporating both energy trading and uncertainty trading simultaneously, also based on a contract matching mechanism, though at the day-ahead stage only. Another two works [22], [23] employed blockchain-based approaches within a P2P real-time market, but from the communication network level, prices and payments are still centrally determined by the central coordinator, and not via a P2P structure. However, these works ignored the computational efficiency trouble of a P2P real-time market.

As it may be too expensive to optimally solve a P2P market at each and every time step, one needs to think of appropriate and computationally cheaper approaches. On the basis of the models in these previous related works [3], [5], [9], [24], [25], we innovatively design a real-time P2P market within an online optimization framework. To be specific, instead of solving a complete optimization problem at each and every time, we concentrate on an alternative paradigm aiming to maximize social welfare in the long run – this is computationally lighter and more tractable. The number of operations and communications among agents can be heavily reduced, and the complexity of our approach is less than other approaches for a given setup since we only perform one iteration of the distributed optimization at each time step. Online optimization is a suitable and efficient approach to solve this type of problem where agents’ characteristics are time-varying [26]–[28]. Despite many large-scale applications of online optimization, such as network resource allocation [29], [30], demand response [31], [32], and energy management [33],

our work is the first application for P2P electricity market. The goal of this paper is to design an online decentralized optimization approach with high computational efficiency for the real-time P2P electricity market. To this end, an online consensus ADMM (OC-ADMM) algorithm is proposed, which blends the decentralization property of the consensus method with the superior convergence property of online ADMM.

Moreover, the proposed algorithm can be further improved in three aspects: 1) the local optimization problem can be reduced to analytical formulations by using a primal-dual variable alternate update process, which can greatly improve the computational speed; 2) the traded energy between each pair of two agents can be balanced using a projection-based energy update process. 3) the network constraints can be satisfied by adding a system operator (SO) who behaves as a single agent and helps to complete the calculation of power flows and voltage angles. Then for the improved OC-ADMM, the sublinear regret upper bound is also proved, which implies that social welfare is maximized the long run. In simulations, a simple case is given to illustrate the trading process. Then, real wind power data is employed to show the convergence performance and tracking ability of our algorithm. Finally, the comparison between our algorithm and other methods demonstrates our high computational speed. Our contributions mainly lie as follows.

- We propose a novel real-time P2P electricity market, which enables the energy trading of multiple proactive dynamic agents. To reduce the computational complexity, an online optimization framework is designed to maximize social welfare in the long run.
- We further improve the online algorithm by devising some techniques: a primal-dual variable alternate update process to improve the computational speed, a projection-based energy update process to guarantee the energy trading balance, and adding a SO who behaves as a single agent to complete the calculation of power flows and voltage angles.
- The sublinear non-stationary regret upper bound for our online algorithm is proved, which implies that social welfare will be maximized in the long run.

The rest of the paper is organized as follows: Section II presents the real-time P2P electricity market model. Section III proposes the OC-ADMM algorithm and improvements for it, followed by the regret and market properties analysis in Section IV. Numerical results and comparisons are presented in Section V. Finally, conclusions are drawn in Section VI.

## II. REAL-TIME PEER-TO-PEER ELECTRICITY MARKET

We consider a real-time P2P electricity market with a set  $\Omega$  of  $N$  dynamic and proactive agents who can be traditional generators, consumers with flexible loads, and renewable generators (wind and PV) over a time horizon  $T$ . The term “dynamic” means the characteristics of agents, e.g., the cost or utility function coefficients, energy demand, and renewable power generation are changing with time. The term “proactive” means the agents are very willing to participate in the market to trade and negotiate with others to determine the electricity

price and quantity. We suppose all agents are rational and truthful as in [34], which means they always make strategic decisions to maximize individual profit.

#### A. Peer-to-Peer Trading

A P2P mechanism for electricity markets is much more decentralized compared with existing centralized markets, which consists of synchronous negotiations over the price and quantity between each pair of two agents. To model the trading process, the net energy power injection  $E_{n,t}$  of each agent  $n \in \Omega$  at time step  $t$  is split into a sum of bilaterally traded quantities with a set of neighboring agents  $m \in \omega_n$  as

$$E_{n,t} = \sum_{m \in \omega_n} E_{nm,t}, \quad \forall n \in \Omega, t = 1, \dots, T. \quad (1)$$

A positive value of  $E_{nm,t}$  corresponds to a sale/production and a negative value to a purchase/consumption. To lighten notations,  $\mathbf{E}_{n,t} = \{E_{n1,t}, \dots, E_{nm,t}\}$  is used to represent the whole energy transactions of agent  $n$  at time step  $t$ . The power set-point of an agent  $n$  is constrained as

$$\underline{E}_{n,t} \leq E_{n,t} \leq \bar{E}_{n,t}, \quad \forall n \in \Omega, t = 1, \dots, T. \quad (2)$$

Here, the energy boundaries of conventional generators and users are usually fixed, while for renewable generators, the upper bound  $\bar{E}_{n,t}$  is set to the actual power generation. Each agent is can be either producer/consumer ( $\underline{E}_{n,t} \bar{E}_{n,t} \geq 0$ ) or prosumer ( $\underline{E}_{n,t} \bar{E}_{n,t} < 0$ ). Hence, the decision variables  $E_{nm,t}$  are constrained as

$$\begin{cases} E_{n,t} \geq E_{nm,t} \geq 0, & \forall (n, m) \in (\Omega_p, \omega_n), t = 1, \dots, T, \\ E_{n,t} \leq E_{nm,t} \leq 0, & \forall (n, m) \in (\Omega_c, \omega_n), t = 1, \dots, T, \\ \underline{E}_{n,t} \leq E_{nm,t} \leq \bar{E}_{n,t}, & \forall (n, m) \in (\Omega_{ps}, \omega_n), t = 1, \dots, T, \end{cases} \quad (3)$$

where  $\Omega_p$ ,  $\Omega_c$  and  $\Omega_{ps}$  are the sets of producers, consumers and prosumers, respectively. Finally, the market equilibrium between energy production and consumption is represented by a set of reciprocity balance constraints defined as

$$E_{nm,t} + E_{mn,t} = 0, \quad \forall (n, m) \in (\Omega, \omega_n), t = 1, \dots, T. \quad (4)$$

For the aim of better formulation, the generator production cost and consumer utility  $C_{n,t}(E_{n,t})$  (positive for producers while negative for consumers) are assumed to be quadratic functions [3], [9], though our mechanism is still applicable for any general convex functions.

$$C_{n,t}(E_{n,t}) = \frac{1}{2} a_{n,t} E_{n,t}^2 + b_{n,t} E_{n,t}. \quad (5)$$

**Remark 1.** *The time-coupling constraints, e.g., the ramp rate limits for conventional generators and total demand requirement for consumers, can also be included. To be specific, since the market is running in an online fashion, the agents do not know future information, they can only make decisions based on current or past information. Then, the time-coupling constraints, e.g., ramping limit or total demand requirement, at time step  $t$  for agent  $n$  can be formulated as*

$$E_{n,t} - E_{n,t-1} \leq \Delta E_n, \quad \forall n \in \Omega_p, t = 1, \dots, T, \quad (6a)$$

$$E_{n,t-1} - E_{n,t} \leq \Delta E_n, \quad \forall n \in \Omega_p, t = 1, \dots, T, \quad (6b)$$

$$E_{n,t} + \sum_{l=1}^{t-1} E_{n,l} \leq D_{n,t}, \quad \forall n \in \Omega_c, t = 1, \dots, T, \quad (6c)$$

where  $E_{n,t-1}$  is the decision variable determined at last time step,  $\Delta E_n$  is the ramp limit for conventional generator  $n$ ;  $D_{n,t}$  is the minimal total required demand for consumer  $n$  in the first  $t$  steps. (6a) and (6b) enforce the ramping up/down constraints for conventional generators, and (6c) implies the total required demand is bounded (remember  $E_{n,t}$  is negative for consumer). Above constraints can be reformulated as

$$E_{n,t-1} - \Delta E_n \leq E_{n,t} \leq E_{n,t-1} + \Delta E_n, \quad \forall n \in \Omega_p, \quad (7a)$$

$$E_{n,t} \leq D_{n,t} - \sum_{l=1}^{t-1} E_{n,l}, \quad \forall n \in \Omega_c. \quad (7b)$$

It is noted that above time-coupling constraints for producers and consumers can be combined with the power level constraints (2) as

$$\begin{aligned} \max\{E_{n,t-1} - \Delta E_n, \underline{E}_{n,t}\} &\leq E_{n,t} \leq \\ \min\{E_{n,t-1} + \Delta E_n, \bar{E}_{n,t}\}, &\quad \forall n \in \Omega_p \end{aligned} \quad (8a)$$

$$\underline{E}_{n,t} \leq E_{n,t} \leq \min\{D_{n,t} - \sum_{l=1}^{t-1} E_{n,l}, \bar{E}_{n,t}\}, \quad \forall n \in \Omega_c. \quad (8b)$$

Therefore, including the time-coupling constraints will not break the model, and the problem is still solvable by using our algorithm.

#### B. Social Welfare Maximization Problem

Since the agents do not know the information from the future, they can only make decisions based on current updated and past information. Thus the market has to be running step by step instead of solving the problem for all  $T$ . To be more specific, at the beginning of time step  $t$ , each agent will be aware of the current updated information, including cost or utility function coefficients, power bounds, demand bounds, and renewable generation. After obtaining these information, the agents will make decisions for the time step  $t$ . All agents negotiate with each other to reach a consensus on their energy trades while maximizing social welfare. Mathematically speaking, this is seen as a minimization problem formulated as

$$\begin{aligned} \min_{\{E_{n \in \Omega}\}} & \sum_{t=1}^T \left( \sum_{n \in \Omega} C_{n,t}(E_{n,t}) \right) \\ \text{s.t.} & (1) - (4). \end{aligned} \quad (9)$$

If the problem (9) is solved exactly at every time step in a decentralized manner, a double loop algorithm is needed where the function  $C_{n,t}$  changes in the outer loop, while the inner loop runs iteratively until the balance constraint (4) is satisfied. However, in a P2P market, the number of communication times will increase with the square of the number of agents [7]. Thus, for the problem (9), if we want to obtain the optimal solution at each time step, all the agents have to complete many iterations and cause very heavy communication and computational burden. Thus, one needs to think of appropriate and computationally cheaper approaches. To this end, we propose an online optimization framework for the real-time P2P market, which is more practicable and applicable.

### III. REAL-TIME P2P ELECTRICITY MARKET MECHANISM

Since the market is running in an online manner, problem (9) is decomposed into each single time step. The social welfare maximization problem (written in a cost-minimization form) for time step  $t$  is

$$\begin{aligned} \min_{\{\mathbf{E}_n \in \Omega\}} \quad & \sum_{n \in \Omega} \left( C_{n,t}(E_n) + \sum_{m \in \omega_n} \frac{\eta}{2} (E_{nm,t-1} - E_{nm})^2 \right) \\ \text{s.t.} \quad & (1) - (4), \end{aligned} \quad (10)$$

where  $\eta$  is the penalty factor and the term  $\frac{\eta}{2} (E_{nm,t-1} - E_{nm})^2$  is appended to make the results close to previous value  $E_{nm,t-1}$  in order to speed up the convergence process [35], [36].

#### A. Online Consensus ADMM

Plain ADMM is an algorithm that blends the decomposability of dual ascent with the superior convergence properties of the method of multipliers, but there is a central coordinator for updating the dual variable [37]. Thus the plain ADMM is suitable for a distributed market where there is a market operator help to decide the price. To get rid of the central coordinator, [37] proposed the consensus ADMM method. In this method, all agents can reach a consensus value by only communicating with neighboring agents. Then, [3] proposed a decentralized P2P market based on the consensus ADMM, where each pair of agents will reach a consensus value of the trade quantities. For the plain ADMM, the objective functions are fixed and do not change with time. However, in some practical applications, the objection functions usually change with time. To improve the practicability of the ADMM method, [35] proposed the online ADMM, which is an efficient technique that combines plain ADMM with online learning theory. Compared with plain ADMM, the objective functions are changing with time, and the target is to minimize the objective value over a long period of time. Combining the above approaches, we propose the novel OC-ADMM algorithm to implement a real-time P2P market, which produces the following updates:

- **Energy Updates:** Each agent  $n$  updates their energy transactions with neighboring agents by solving individual optimization problem with constraints (1)-(3) as below:

$$\begin{aligned} \mathbf{E}_{n,t} = \operatorname{argmin}_{\{\mathbf{E}_n\}} \quad & C_{n,t}(E_n) + \sum_{m \in \omega_n} \lambda_{nm,t-1} (F_{nm,t-1} - E_{nm}) \\ & + \frac{\rho}{2} (F_{nm,t-1} - E_{nm})^2 + \frac{\eta}{2} (E_{nm} - E_{nm,t-1})^2, \end{aligned} \quad (11)$$

where  $\rho$  is the penalty factor and  $\lambda_{nm,t-1}$  is the dual variable of the reciprocity constraint (4), which also defines the price for the traded energy  $E_{nm,t-1}$ .  $\lambda_{n,t-1} = \{\lambda_{n1,t-1}, \dots, \lambda_{nm,t-1}\}$  is used to represent the whole energy prices of agent  $n$  to neighboring agents  $m \in \omega_n$  at time step  $t-1$ .  $F_{nm,t-1}$  is the consensus variable defined as  $\frac{E_{nm,t-1} + E_{mn,t-1}}{2}$  and  $F_{n,t-1} = \sum_{m \in \omega_n} F_{nm,t-1}$ . Then, each agent broadcasts  $\mathbf{E}_{n,t}$  to neighboring agents.

- **Price Updates** All agents update their energy prices to neighboring agents  $m \in \omega_n$  as:

$$\lambda_{nm,t} = \lambda_{nm,t-1} - \rho (E_{nm,t} + E_{mn,t}) / 2. \quad (12)$$

The OC-ADMM algorithm is much more efficient since the agents only need to complete one iteration at each time step. However, the algorithm can be further improved. First, the update (11) is still an optimization problem, which can speed up by transforming into analytical expressions; second, since we only run one iteration at each time step, the energy balance between two agents may not be balanced, i.e.,  $E_{nm,t} + E_{mn,t} \neq 0$ , and it is important for a market mechanism to keep the trading balance; third, the network constraints are important in power system and should be considered in the market.

#### B. Improvements for OC-ADMM

To address the above disadvantages, we make three further improvements, which are 1) a primal-dual variables alternate update process to increase the computational speed, 2) a projection-based energy update process to guarantee the energy balance, and 3) adding a SO who behaves as a single agent to complete the calculation of power flows and voltage angles. The procedures of the improved OC-ADMM are summarized in Algorithm 1.

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#### Algorithm 1 Improved OC-ADMM for real-time P2P market

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- 1: **Initialization:**  $t = 1$ ,  $\{\mathbf{E}_n, \forall n \in \Omega\}$ ,  $\rho = \sqrt{T}$ ,  $\eta = \sqrt{T}$  and  $\{\lambda_n, \forall n \in \Omega\}$ .
  - 2: **for**  $t = 1 \rightarrow T$  **do**
  - 3:   **Step 1 : Primal – dual alternate update**
  - 4:   **for**  $n \in \Omega$  **do**
  - 5:     Reveal  $C_{n,t}$  and renewable generation  $\bar{E}_{n,t}$ .
  - 6:     Initial  $k = 1$ ,  $\Delta\mu_n^1 = 0$  and  $\Delta\delta_n^1 = 0$ .
  - 7:     **while**  $E_{n,t}^k - \sum_{m \in \omega_n} E_{nm,t}^k > \epsilon$  **do**
  - 8:       Update  $E_{n,t}^{k+1}$  and  $E_{nm,t}^{k+1}$  by (16) (18-19);
  - 9:       Update  $\Delta\mu_n^{k+1}$  and  $\Delta\delta_n^{k+1}$  by (21) (22);
  - 10:        $k = k + 1$ .
  - 11:     **end while**
  - 12:   **end for**
  - 13:   **Step 2 : Prices update**
  - 14:   Broadcasts  $\mathbf{E}_{n,t}$  to neighboring agents;
  - 15:   Update  $\lambda_{nm,t}$ ,  $m \in \omega_n$  by (12).
  - 16:   **Step 3 : Projection – based energy update**
  - 17:   Initial  $l = 1$ ,  $\hat{E}_{nm,t}^l = E_{nm,t}^k$ ,  $\forall (n, m) \in (\Omega, \omega_n)$ .
  - 18:   **while**  $\exists (n, m) \in (\Omega, \omega_n)$ ,  $\hat{E}_{nm,t}^l + \hat{E}_{mn,t}^l > \epsilon$  **do**
  - 19:     Update  $\hat{E}_{nm,t}^{l+1}$  by (23) (24) and (25);
  - 20:     Broadcasts  $\hat{\mathbf{E}}_{n,t}^{l+1}$  to neighboring agents;
  - 21:      $l = l + 1$ .
  - 22:   **end while**
  - 23:   **Step 4 : Extension to DC power flow model**
  - 24:   Collect  $\hat{E}_{n,t}^l$ ,  $n \in \Omega$ , and calculate  $\{\mathbf{P}_t, \boldsymbol{\theta}_t\}$  by (27).
  - 25: **end for**
- 

1) *Primal-Dual Variables Alternate Update:* Since the individual problem (11) is a convex optimization problem, we can obtain the optimal solution by equivalently solving

its Karush-Kuhn-Tucker (KKT) conditions [38], which is a nonlinear problem with primal  $\{E_{n,t}^{k+1}, \mathbf{E}_{n,t}^{k+1}\}$  and dual variables  $\{\underline{\mu}_n, \bar{\mu}_n\}, \{\underline{\delta}_{nm}, \bar{\delta}_{nm}\}$  for the constraints (2) (3). This problem can be solved in a primal-dual variables alternate update manner. Since the update processes are all analytical expressions, the computational speed can be greatly increased. In this process, iterations are indexed with  $k$ .

**Primal Updates:** Relaxing the constraint (1) and based on the KKT conditions of (11) for a trade with  $m \in \omega_n$ , we have

$$\begin{aligned} & \partial C_{n,t}(E_{n,t}) - \lambda_{nm,t-1} + \rho(E_{nm,t} - F_{nm,t-1}) \\ & + \eta(E_{nm,t} - E_{nm,t-1}) - \underline{\mu}_n + \bar{\mu}_n - \underline{\delta}_{nm} + \bar{\delta}_{nm} = 0. \end{aligned} \quad (13)$$

Summing up (13) for all  $m \in \omega_n$ , we have

$$\begin{aligned} & N_{\omega_n}(a_{n,t}E_{n,t} + b_{n,t}) \\ & = \sum_{m \in \omega_n} \lambda_{nm,t-1} - (\rho + \eta)E_{n,t} + \rho F_{n,t-1} + \eta E_{n,t-1} \\ & + N_{\omega_n}(\underline{\mu}_n - \bar{\mu}_n) + \sum_{m \in \omega_n} (\underline{\delta}_{nm} - \bar{\delta}_{nm}). \end{aligned} \quad (14)$$

For simplicity,  $\underline{\mu} - \bar{\mu}$  and  $\underline{\delta} - \bar{\delta}$  are denoted by  $\Delta\mu$  and  $\Delta\delta$ , respectively. Given the results from last iteration  $k$ ,  $E_{n,t}^{k+1}$  is updated as

$$E_{n,t}^{k+1} = \mathcal{V}_{n,t} + \frac{\Delta\mu_n^k + \Delta\delta_n^k}{a_{n,t} + \frac{\rho + \eta}{N_{\omega_n}}}, \quad (15)$$

where  $\mathcal{V}_{n,t} = (\tilde{\lambda}_{n,t-1} + \rho\tilde{F}_{n,t-1} + \eta\tilde{E}_{n,t-1} - b_{n,t}) / (a_{n,t} + \frac{\rho + \eta}{N_{\omega_n}})$ ,  $\tilde{\lambda}_{n,t-1}$ ,  $\tilde{F}_{n,t-1}$ ,  $\tilde{E}_{n,t-1}$  and  $\Delta\delta_n^k$  are the average values. Based on the complementary condition,  $E_{n,t}^{k+1}$  can be obtained in another equivalent form as

$$E_{n,t}^{k+1} = \max \left\{ \min \left\{ \mathcal{V}_{n,t} + \frac{\Delta\delta_n^k}{a_{n,t} + \frac{\rho + \eta}{N_{\omega_n}}}, \bar{E}_{n,t} \right\}, \underline{E}_{n,t} \right\}. \quad (16)$$

According to (13), the trade to  $m \in \omega_n$  is updated as

$$E_{nm,t}^{k+1} = \mathcal{W}_{nm,t} + \frac{\Delta\mu_n^k + \Delta\delta_n^k - a_{n,t}E_{n,t}^{k+1}}{\rho + \eta}, \quad (17)$$

where  $\mathcal{W}_{nm,t} = (\lambda_{nm,t-1} + \rho F_{nm,t-1} + \eta E_{nm,t-1} - b_{n,t}) / (\rho + \eta)$ . Similarly, we can obtain  $E_{nm,t}^{k+1}$  for a producer, in another form as

$$\max \left\{ \min \left\{ \mathcal{W}_{nm,t} + \frac{\Delta\mu_n^k - a_{n,t}E_{n,t}^{k+1}}{\rho + \eta}, E_{n,t}^{k+1} \right\}, 0 \right\}. \quad (18)$$

If agent  $n$  is a consumer, the  $E_{nm,t}^{k+1}$  is updated as

$$\max \left\{ \min \left\{ \mathcal{W}_{nm,t} + \frac{\Delta\mu_n^k - a_{n,t}E_{n,t}^{k+1}}{\rho + \eta}, 0 \right\}, E_{n,t}^{k+1} \right\}. \quad (19)$$

If agent  $n$  is a prosumer, the  $E_{nm,t}^{k+1}$  is updated as

$$\max \left\{ \min \left\{ \mathcal{W}_{nm,t} + \frac{\Delta\mu_n^k - a_{n,t}E_{n,t}^{k+1}}{\rho + \eta}, \bar{E}_{n,t} \right\}, \underline{E}_{n,t} \right\}. \quad (20)$$

**Dual Updates:** From (15), the dual variables  $\mu_n^{k+1}$  is updated as

$$\Delta\mu_n^{k+1} = \left( a_{n,t} + \frac{\rho + \eta}{N_{\omega_n}} \right) (E_{n,t}^{k+1} - \mathcal{V}_{n,t}) - \Delta\delta_n^k. \quad (21)$$

From (17), the average dual variable  $\delta_n^{k+1}$  is updated as

$$\Delta\delta_n^{k+1} = (\rho + \eta) \left( \tilde{E}_{n,t}^{k+1} - \tilde{\mathcal{W}}_{n,t} \right) - \Delta\mu_n^{k+1} + a_{n,t}E_{n,t}^{k+1}. \quad (22)$$

After updating the dual variables  $\{\Delta\mu_n^{k+1}, \Delta\delta_n^{k+1}\}$ , we go back to update the primal variables  $\{E_{n,t}^{k+1}, \mathbf{E}_{n,t}^{k+1}\}$ . This process repeats until  $|E_{n,t}^k - \sum_{m \in \omega_n} E_{nm,t}^k| < \epsilon$ , where  $\epsilon$  is the allowed maximal violation.

In conclusion, instead of solving the optimization problem (11), the primal and dual variables are updated in an alternate iteration process, which is more fast since they are all simple analytical calculations.

2) *Projection-based Energy Update:* In the electricity market, the energy balance for trade between agents is necessary. To this end, we design a projection-based energy update process to determine the final quantity by projecting the consensus value  $F_{nm,t} = (E_{nm,t} - E_{mn,t})/2$  into feasible region (2). In this process, iterations are indexed with  $l$ .

First, we set the trade  $\hat{E}_{nm,t}^{l+1}$  to be the consensus value of the trade between agents  $n$  and  $m$  from last iteration as

$$\hat{E}_{nm,t}^{l+1} = \hat{F}_{nm,t}^l = (\hat{E}_{nm,t}^l - \hat{E}_{mn,t}^l)/2. \quad (23)$$

Here, the initial value  $\hat{E}_{nm,t}^1$  is obtained after the primal-dual alternate update process, i.e.,  $\hat{E}_{nm,t+1}^1 = E_{nm,t+1}^k$ . Then the total energy power is obtained as  $\hat{E}_{n,t}^{l+1} = \sum_{m \in \omega_n} \hat{E}_{nm,t}^{l+1}$ . After projecting into the feasible region (2), the projected total energy power  $\hat{E}_{n,t}^{proj,l+1}$  is updated as

$$\hat{E}_{n,t}^{proj,l+1} = \max \left\{ \min \left\{ \hat{E}_{n,t}^{l+1}, \bar{E}_{n,t} \right\}, \underline{E}_{n,t} \right\}. \quad (24)$$

Then the trade  $\hat{E}_{nm,t}^{l+1}$  to neighboring agent  $m$  is updated as

$$\hat{E}_{nm,t}^{l+1} = \hat{E}_{nm,t}^{l+1} \frac{\hat{E}_{n,t}^{proj,l+1}}{\hat{E}_{n,t}^{l+1}}. \quad (25)$$

Finally, all agents send the new trade quantities  $\hat{E}_{nm,t}^{l+1}$  to its neighboring agents, and check if the balance equation  $\hat{E}_{nm,t}^{l+1} + \hat{E}_{mn,t}^{l+1} < \epsilon$  is satisfied, where  $\epsilon$  is the allowed maximal violation; if not, repeat the processes (23)-(25) until all trades are balanced.

3) *Extension to DC Power Flow Model:* For simplicity, the power flow model and network constraint are not considered, but our P2P market mechanism can be extended to include the DC power flow model by adding a system operator (SO), who behaves as a single agent and helps to complete the calculation of power flows and voltage angles.

Let the power network consist of a set  $\mathcal{N}$  of buses and a set  $\mathcal{L}$  of lines. We consider the DC power flow model to characterize the line limits and the generation-load balance, which is widely used in market studies [39], [40]. Let  $P_{ij,t}$  be the real power flow in the line connecting bus  $i$  and  $j$ ,  $Y_{ij}$  is the susceptance of the line,  $\theta_{i,t}$  is the voltage angle at bus  $i$ , and  $P_{ij}^{min}, P_{ij}^{max}$  are the line thermal limit. Without the loss of generality, let bus 1 be the slack bus, i.e.,  $\theta_{1,t} = 0$ . Then, the network constraint and power balance can be represented as follows,

$$P_{ij}^{min} \leq P_{ij,t} = Y_{ij}(\theta_{j,t} - \theta_{i,t}) \leq P_{ij}^{max}, \forall (i, j) \in \mathcal{L} \quad (26a)$$

$$\sum_{n \in \mathcal{N}_i} E_{n,t} = \sum_{(i,j) \in \mathcal{L}} P_{ij,t}, \quad \forall i \in \mathcal{N} \quad (26b)$$

In transmission networks, the real power flows  $P_{ij,t}$  are proportional to the difference of voltage angles between the two ends of the line. To avoid any damage to transmission lines, the power flows are bounded by thermal capacity related to the heat they can dissipate. Moreover, the power balance (26b) must be kept at each bus of the grid between line flows and power injections of agents connected to it.  $\mathbf{P}_t = \{P_{ij,t}, (i,j) \in \mathcal{L}\}$  and  $\boldsymbol{\theta}_t = \{\theta_{i,t}, i \in \mathcal{N}\}$  are used to represent all the power flows in lines and voltage angles of buses at time step  $t$ .

The SO will be added as a single agent to complete the power flows and voltage angles calculation. The network constraints (26a)-(26b) can be converted into an optimization problem as

$$\begin{aligned} \{\mathbf{P}_t^{l+1}, \boldsymbol{\theta}_t^{l+1}\} &= \underset{\{\mathbf{P}, \boldsymbol{\theta}\}}{\operatorname{argmin}} \sum_{i \in \mathcal{N}} \left\| \sum_{n \in \mathcal{N}_i} \hat{E}_{n,t}^l - \sum_{(i,j) \in \mathcal{L}} P_{ij,t} \right\|_2^2 \\ \text{s.t. } P_{ij}^{min} &\leq P_{ij,t} = Y_{ij}(\theta_j - \theta_i) \leq P_{ij}^{max} \quad (i,j) \in \mathcal{L} \end{aligned} \quad (27a)$$

At each iteration, the SO will collect the total power injection  $\hat{E}_{n,t}^l$   $n \in \Omega$  from all agents located at different buses, then determines the power flow  $\mathbf{P}_t$  and voltage angles  $\boldsymbol{\theta}_t$  by solving above problem (27).

Although extending the P2P real-time market to include the DC power flow model requires the SO to solve the optimization problem (27), it is easy to solve, and the computational efficiency will not be highly reduced as demonstrated in the simulations.

#### IV. MARKET ANALYSIS

In this section, we will first analyze the convergence performance for our proposed online algorithm, which is usually measured by the regret. Regret is the accumulated gap between the online solution and the best solution in hindsight [35], [41]–[43]. Under some standard assumptions, the sublinear regret upper bound for our algorithm is proved, which implies that social welfare will be maximized in the long run. Then, the four desirable properties of our proposed market mechanisms are analyzed.

##### A. Non-Stationary Regret Analysis

The stationary regret is commonly adopted, which compares with the cost induced by the best and fixed decisions in hindsight:

$$R(T) = \sum_{t=1}^T \left( \sum_{n \in \Omega} C_{n,t}(\hat{E}_{n,t}) \right) - \sum_{t=1}^T \left( \sum_{n \in \Omega} C_{n,t}(E_n^*) \right), \quad (28)$$

where  $E_n^*$  is the best and fixed decisions. Several sublinear regret upper bounds measured by stationary regret have been established in early works [35], [41], [42]. However, the stationary regret requires the decisions to remain unchanged throughout the period, which is not possible for the real-time

electricity market, since the renewable energy generation is usually changing with time, and the power balance will not be satisfied with fixed decisions. Recently, a new performance metric known as the non-stationary regret was proposed by [43]. The non-stationary regret compares the cumulative gap between the online decisions and the best solutions at each time step as defined below

$$\tilde{R}(T) = \sum_{t=1}^T \left( \sum_{n \in \Omega} C_{n,t}(\hat{E}_{n,t}) \right) - \sum_{t=1}^T \left( \sum_{n \in \Omega} C_{n,t}(E_{n,t}^*) \right), \quad (29)$$

where  $E_{n,t}^*$  is the optimal solution of agent  $n$  at time step  $t$ .

Before presenting the results, we introduce the some standard assumptions to derive the sublinear regret upper bound as presented later in Theorem 1.

##### Assumption 1.

- Functions  $C_{n,t}$  are convex with bounded subgradients, i.e.,  $\partial C_{n,t}(E_{n,t}) \leq G$ ,  $\forall n \in \Omega$ , with  $G$  being a positive constant.
- The initial values are set to zero, i.e.,  $\lambda_{nm,1} = 0$  and  $E_{nm,1} = 0$ ,  $\forall (n,m) \in (\Omega, \omega_n)$ .
- For any  $t$ , we assume the gap between the optimal and initial solutions are bounded, i.e.,  $(E_{nm,t}^* - E_{nm,1})^2 \leq D_1$  and  $(F_{nm,t}^* - F_{nm,1})^2 \leq D_2$ ,  $\forall (n,m) \in (\Omega, \omega_n)$ , with  $D_1$  and  $D_2$  being positive constants.
- Path variation is defined as the temporal change of the optimal solutions sequence, i.e.,  $P_{nm,T} = \sum_{t=1}^T |E_{nm,t}^* - E_{nm,t+1}^*|$ . We assume that the optimal solutions do not change dramatically, in other words, the path variation is bounded as  $P_{nm,T} \leq P$ ,  $\forall (n,m) \in (\Omega, \omega_n)$ , with  $P$  being a positive constant called variation budget.
- The electricity prices do not change dramatically, or in mathematical sense, the price variation between two time steps is bounded as  $|\lambda_{nm,t+1} - \lambda_{nm,t}| \leq \Lambda$ , with  $\Lambda$  being a positive constant.

In Assumption 1, (a)-(c) are generally required in the online optimization setting. For Assumption 1(d), [43] proved that if there is no restriction on the path variation, the non-stationary regret is linear in  $T$  regardless of the strategies. Thus we assume that the path variation is bounded, which is also proposed in early works [43]–[45]. Finally, for Assumption 1(e), it is reasonable that the electricity price cannot be infinite in the electricity market background.

Bearing all above in mind, the following theorem establishes the regret bound for our online algorithm.

**Theorem 1.** *OC-ADMM algorithm has the following sublinear non-stationary regret upper bound by setting  $\rho = \sqrt{T}$  and  $\eta = \sqrt{T}$*

$$\tilde{R}(T) \leq N(N-1) \left( \frac{D_1 + D_2 + \Lambda^2 + G^2}{2} + LP \right) \sqrt{T}, \quad (30)$$

where  $L = \max_{n \in \Omega} 2 * (\bar{E}_n - \underline{E}_n)$ .

*Proof.* See Appendix A.  $\square$

According to the above theorem, since the regret has  $\mathcal{O}(\sqrt{T})$  upper bound, we have  $\lim_{T \rightarrow \infty} \frac{\hat{R}(T)}{T} = 0$ , which indicates the gap of the social welfare (seen as a cost) between the optimal solutions are approaching to zero in the long run on time average. In the next subsection, we will analyze the desirable properties for a market-clearing mechanism based on Theorem 1.

### B. Desirable Properties of Market Mechanism

It is necessary and important to evaluate the quality of a market-clearing mechanism by checking the four desirable properties, which are market efficiency<sup>1</sup>, incentive compatibility<sup>2</sup>, cost recovery<sup>3</sup> and revenue adequacy<sup>4</sup> [46]. Based on the Hurwicz theorem [46], no mechanism is capable of achieving all those properties at the same time.

1) *Market efficiency*: For our real-time P2P market, regret can be regarded as the cumulative total cost gap between our online algorithm and the optimal solution. From the Theorem 1, we have  $\lim_{T \rightarrow \infty} \frac{\hat{R}(T)}{T} = 0$ , which implies the cost is minimized or that social welfare and market efficiency are maximized in the long run.

2) *Incentive compatibility*: A market participant may gain profit by not trustfully offering in terms of price or quantity, but we assume that they are all truthful in this work.

3) *Cost recovery*: The individual profit for a producer at time  $t$  is

$$\sum_{m \in \omega_n} \lambda_{nm,t} E_{nm,t} - C_{n,t}(E_{n,t}). \quad (31)$$

Since the quadratic cost function (5) is convex, monotonically increasing, and passing through the origin, the agent can always set  $E_{n,t} = E_{nm,t} = 0$  to avoid a negative profit. Thus the cost recovery is satisfied.

4) *Revenue adequacy*: From (12), the prices between agents are identical, i.e.,  $\lambda_{nm,t} = \lambda_{mn,t}$ , and after the projection, the energy between agents are balanced, i.e.,  $\hat{E}_{nm,t} + \hat{E}_{mn,t} = 0$ . Thus, the revenue adequacy is satisfied.

Summing up the above, our real-time P2P market mechanism satisfies most of the desirable properties.

## V. SIMULATION RESULTS

As a basis for illustration and discussion, we first consider a simple case to show the trading prices and quantities. Then our algorithm is tested on a dataset that is taken every five minutes of wind power generation from 20 wind farms in Australia [47] for 30 days, to show the convergence and tracking performance. Finally, the computational efficiency is demonstrated by comparing with other methods in terms of computational time. To better show the performance, uniform distribution stochastic parameter settings are applied. We perform simulations using Matlab R2017b on a PC with 1.6 GHz Intel Core 4 Duo CPU and 8 GB memory, and the convex optimization problem is solved by CVX Sedumi solver [48].

<sup>1</sup>Market efficiency is maximized when outcomes maximize social welfare.

<sup>2</sup>A mechanism is called incentive-compatible if every participant can maximize its objective just by acting according to its true preferences.

<sup>3</sup>Cost recovery implies that individual profit is non-negative.

<sup>4</sup>Revenue adequacy implies that there is no financial deficit in the market.

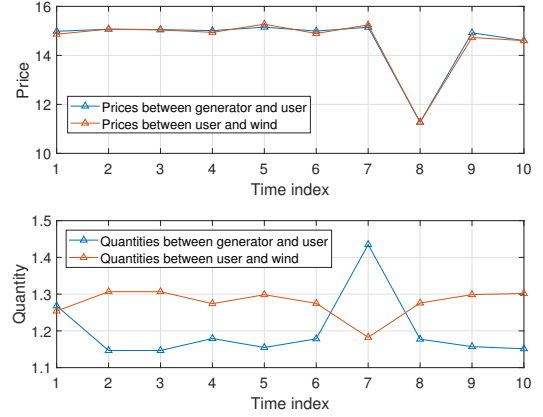


Fig. 1: The prices and quantities between agents.

### A. Illustrative Case Study

We first take a simple case with only one conventional generator, one wind generator, and one user, to show the trading process in terms of prices and quantities under the changing wind power generation and utility function. The initial parameters of agents are summarized in Table I. We set the utility function of user and wind power generation are changing with time, and the values are shown in Table. II. We also include the time-coupling constraints into the case, i.e., the ramp rate limit for conventional generator and total demand requirement for user. The ramp rate limit is set to 0.5 kW, and the total demand is set to 20 kW for 10 time steps.

The final traded prices and quantities between agents in 10 time steps are shown in Fig. 1. The values are changing with time due to the varying user's characteristics and wind power generation. We give a comparison of the total cost between our online P2P mechanism with the centralized mechanism, and the price between with pool-based mechanism. As seen in Fig. 2, the total cost of the online P2P mechanism is higher than the centralized mechanism at some time steps, but the gap is very small. While for the price, in the pool-based market, the price is identical for all agents, while in our online P2P mechanism, the prices are different between each pair of agents, and changing with time.

### B. Application to Real Data

We build a large market composed of 20 conventional generators, 20 flexible residential users, and 20 wind power generators with the real wind generation data in Australia for 30 days. The convergence performance and tracking ability of our algorithm are tested.

1) *Convergence Performance*: Fig. 3 shows the convergence performance of our online algorithm before and after the projection. We can see  $\hat{R}(T)/T$  decreases quite fast both before and after projection, and the former one is lower. The reason for that is before projection, the balance constraint is relaxed and the solution is not feasible, thus the cost may be lower than that of the optimal solution. While after projection, since the solution is feasible, it will always exist a small positive gap from the optimal solution. It is shown that after

TABLE I: Agents' initial characteristics of simple case study

Agent	$a_{n,t}$ [\$/kW <sup>2</sup> ]	$b_{n,t}$ [\$/kW]	$E_n$	$\bar{E}_n$
Generator	0.0210	15.0413	0	4.9014
User	0.0144	6.4149	-5.3252	-2.4533
Wind	0.0100	5	1.3021	1.3021

TABLE II: Agents' changing characteristics of simple case study

Parameter	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	t=10
$a_{n,t}^u$	0.0144	0.0177	0.0119	0.0145	0.0171	0.0128	0.0166	0.0112	0.0196	0.0159
$b_{n,t}^u$	6.9078	8.9760	7.4488	8.2316	8.7734	8.3985	5.8131	7.4918	6.7019	6.1191
$E_{n,t}^w$	1.3021	1.3067	1.3067	1.2739	1.2985	1.2748	1.3012	1.2758	1.3018	1.3018

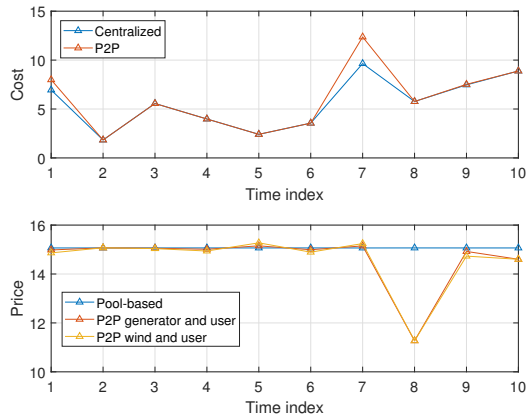
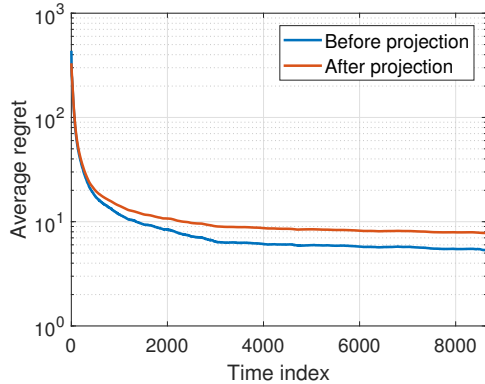


Fig. 2: The comparison of cost/price between P2P mechanism with centralized/pool-based mechanism.

Fig. 3: Average regret  $\tilde{R}(T)/T$  before and after projection.

about 2000 time steps, the average regret decreases much slower but still keeps going down.

2) *Tracking Ability*: Then we focus on the ability of tracking the optimal solution, a normalized index is proposed to measure the cost deviation

$$\chi_t^c \triangleq \frac{\sum_{n \in \Omega} |C_{n,t}(E_{n,t}) - C_{n,t}(E_{n,t}^*)|}{\sum_{n \in \Omega} |C_{n,t}(E_{n,t}^*)|}, \quad (32)$$

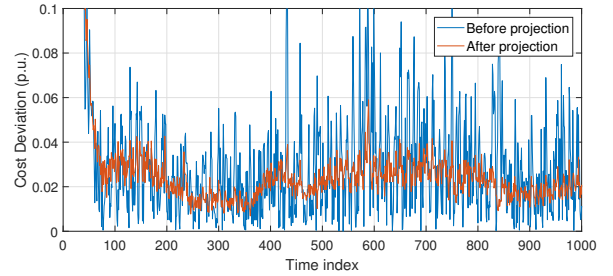


Fig. 4: The normalized cost deviation.

The value of the index ranges from 0 to infinity, and the smaller value denotes better tracking ability. For better presentation, we only show the first 1000 time steps. Fig. 4 shows that the cost deviation keeps at a low level mostly under 0.08 before projection and 0.04 after projection, which implies our algorithm can track the optimal solution well.

### C. Computational Efficiency

In this section, we will show the computational efficiency of our algorithm, measured by the computational time. Since our algorithm and OC-ADMM are fully decentralized, the computational time of one iteration is determined by the slowest agent. Simulations have been performed 30 times for the centralized algorithm (CVX Sedumi solver), OC-ADMM, OC-ADMM with DC and improved OC-ADMM at a different scale of agents with different setups per iteration. As seen in Fig. 5, the OC-ADMM is quite efficient whose computational time is kept lower than 1s, and the improved OC-ADMM can further highly improve the performance to about  $10^{-4}$ s with a little higher variance. For our proposed decentralized algorithms, the computational time almost remains unchanged, however, for the centralized algorithm, e.g., the interior point method, the computation time is exponentially increasing with the number of agents.

### D. Extension to DC Optimal Power Flow

We first consider a small market with 10 agents on the IEEE 9-bus test system as shown in Fig. 6. The susceptance  $Y$  and line capacity limit  $C$  are setting to 3 and 10 for all lines. Agent



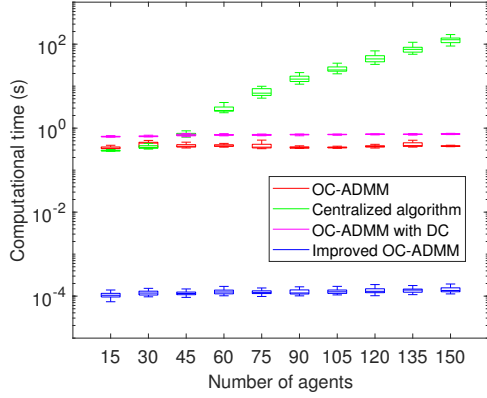


Fig. 5: The computational time of different methods.

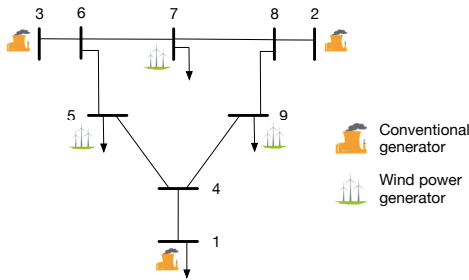


Fig. 6: IEEE 9-bus test system used for case study.

G1-G3 are conventional generators, agent U1-U4 are users, and agent R1-R3 are wind power generators. The power flows in different lines at the first 12 time steps are shown in Fig. 7. The results indicate that the power flows are also changing with time, and are always not larger than the maximum line capacity.

Then we run our online algorithm with the DC power flow model in a 57-bus system, which is a larger system, to show the computational efficiency at a different scale of agents. As shown in Fig. 5, the computational time of OC-ADMM with DC also almost remains unchanged and below 1 sec. The time gap between OC-ADMM with and without DC is due to the time consumed by the SO, who helps to calculate the power flows and phase angles, and the time is only affected by the network scale. We further give a simulation to show the impact of network scale on the computational time. As shown in Fig. 8, the computational time will increase with the number of buses in network, but the time is still much lower than the requirement in the real-time market (5 min).

## VI. CONCLUSION

P2P markets are considered as an evolution of the future electricity markets driven by distributed energy resources and demand response management development. How to design an efficient real-time P2P electricity market mechanism with changing environment and agents remains a challenge. To reduce the computational and communication burden, we turn to optimize the market in an online mechanism and propose a novel OC-ADMM algorithm that incorporates the online

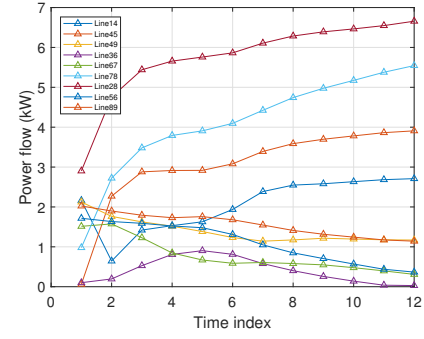


Fig. 7: Power flows in different lines at different time steps.

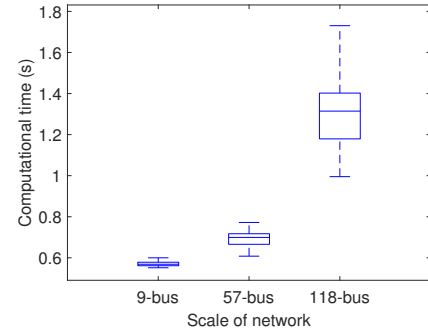


Fig. 8: Computational time as a function of network scale.

optimization approach. Besides, some further improvements are made to increase performance and practicability. Finally, we give a proof of the sublinear non-stationary regret upper bound for our algorithm, which indicates that social welfare will be maximized in the long run on average. Simulation results show that the improved OC-ADMM has good convergence performance, tracking ability, and high computational efficiency.

The main limitation of our approach is that the market mechanism has to be running in a synchronous manner. Each agent has to wait to receive all bidding prices and quantities from neighboring agents, and the computational efficiency of the market is highly restricted by the slowest agents. To overcome this problem, we are working on designing an asynchronous mechanism to make it more practical and applicable in the real world.

## APPENDIX A

### PROOF OF THE SUBLINEAR REGRET UPPER BOUND

Let  $\frac{\partial C_{n,t}(E_{n,t})}{\partial E_{nm,t}}$  be the gradient of  $C_{n,t}(E_n)$  at  $E_{nm,t}$ . Since  $E_{n,t}$  minimizes (11), combining (12), we have

$$\begin{aligned} \frac{\partial C_{n,t}(E_{n,t})}{\partial E_{nm,t}} &= \lambda_{nm,t} + \rho(F_{nm,t-1} - F_{nm,t}) \\ &+ \eta(E_{nm,t-1} - E_{nm,t}), \quad \forall m \in \omega_n. \end{aligned} \quad (33)$$

Since  $C_{n,t}$  is a convex function and its subgradient at  $E_{nm,t}$  is given in (33), for optimal solution  $E_{n,t}^*$  we have

$$C_{n,t}(E_{n,t}) - C_{n,t}(E_{n,t}^*)$$

$$\begin{aligned}
&\leq \sum_{m \in \omega_n} \frac{\partial C_{n,t}(E_{n,t})}{\partial E_{nm,t}} (E_{nm,t} - E_{nm,t}^*) \\
&= \sum_{m \in \omega_n} \lambda_{nm,t} (E_{nm,t} - F_{nm,t}^*) \\
&\quad + \sum_{m \in \omega_n} \rho (F_{nm,t-1} - F_{nm,t}) (E_{nm,t} - F_{nm,t}^*) \\
&\quad + \sum_{m \in \omega_n} \eta (E_{nm,t-1} - E_{nm,t}) (E_{nm,t} - E_{nm,t}^*) \\
&= \sum_{m \in \omega_n} \lambda_{nm,t} (E_{nm,t} - F_{nm,t}^*) \\
&\quad + \sum_{m \in \omega_n} \frac{\rho}{2} [(F_{nm,t-1} - F_{nm,t}^*)^2 - (F_{nm,t} - F_{nm,t}^*)^2] \\
&\quad + (E_{nm,t} - F_{nm,t})^2 - (E_{nm,t} - F_{nm,t-1})^2 \\
&\quad + \frac{\eta}{2} [(E_{nm,t}^* - E_{nm,t-1})^2 - (E_{nm,t}^* - E_{nm,t})^2] \\
&\quad - (E_{nm,t} - E_{nm,t-1})^2. \tag{34}
\end{aligned}$$

According to the Fenchel-Young's inequality [49], we have

$$\begin{aligned}
&C_{n,t}(\hat{E}_{n,t}) - C_{n,t}(E_{n,t}) \\
&\leq \sum_{m \in \omega_n} \frac{\partial C_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}} (\hat{E}_{nm,t} - E_{nm,t}) \\
&\leq \sum_{m \in \omega_n} \frac{1}{2\rho} \left( \frac{\partial C_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}} \right)^2 + \frac{\rho}{2} (\hat{E}_{nm,t} - E_{nm,t})^2 \\
&\leq \sum_{m \in \omega_n} \frac{1}{2\rho} \left( \frac{\partial C_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}} \right)^2 + \frac{\rho}{2} (F_{nm,t} - E_{nm,t})^2 \\
&\leq \sum_{m \in \omega_n} \frac{1}{2\rho} \left( \frac{\partial C_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}} \right)^2 + \frac{(\lambda_{nm,t-1} - \lambda_{nm,t})^2}{2\rho}, \tag{35}
\end{aligned}$$

For the penultimate inequality, because  $\hat{E}_{nm,t}$  begins updating from  $F_{nm,t}$ , if the projection update finishes at the first time, which also means both agents do not touch the bound, the term  $(\hat{E}_{nm,t} - E_{nm,t})^2$  reaches the maximal value  $(F_{nm,t} - E_{nm,t})^2$ ; The worst case is that the final trade reaches at the initial value  $E_{nm,t}$  or  $-E_{mn,t}$ , which means at the beginning, one of the agents  $n$  or  $m$  has reached the bound, then the term  $(\hat{E}_{nm,t} - E_{nm,t})^2$  reaches the minimal value zero.

Combining (34)-(35), we have

$$\begin{aligned}
&C_{n,t}(\hat{E}_{n,t}) - C_{n,t}(E_{n,t}^*) \\
&\leq \sum_{m \in \omega_n} \lambda_{nm,t} (E_{nm,t} - F_{nm,t}^*) + \frac{\rho}{2} (E_{nm,t} - F_{nm,t})^2 \\
&\quad + \sum_{m \in \omega_n} \frac{\rho}{2} [(F_{nm,t-1} - F_{nm,t}^*)^2 - (F_{nm,t} - F_{nm,t}^*)^2] \\
&\quad + \sum_{m \in \omega_n} \frac{\eta}{2} [(E_{nm,t}^* - E_{nm,t-1})^2 - (E_{nm,t}^* - E_{nm,t})^2] \\
&\quad + \sum_{m \in \omega_n} \frac{1}{2\rho} \left( \frac{\partial C_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}} \right)^2 + \frac{(\lambda_{nm,t-1} - \lambda_{nm,t})^2}{2\rho} \tag{36}
\end{aligned}$$

We notice that for the second term

$$(F_{nm,t-1} - F_{nm,t}^*)^2 - (F_{nm,t} - F_{nm,t}^*)^2$$

$$\begin{aligned}
&= (F_{nm,t-1} - F_{nm,t-1}^*)^2 - (F_{nm,t} - F_{nm,t}^*)^2 \\
&\quad + (F_{nm,t-1} - F_{nm,t}^*)^2 - (F_{nm,t-1} - F_{nm,t-1}^*)^2 \\
&\leq (F_{nm,t-1} - F_{nm,t-1}^*)^2 - (F_{nm,t} - F_{nm,t}^*)^2 \\
&\quad + (2F_{nm,t-1} - F_{nm,t-1}^* - F_{nm,t}^*) (F_{nm,t-1} - F_{nm,t}^*) \\
&\leq (F_{nm,t-1} - F_{nm,t-1}^*)^2 - (F_{nm,t} - F_{nm,t}^*)^2 \\
&\quad + L |E_{nm,t-1}^* - E_{nm,t}^*|. \tag{37}
\end{aligned}$$

Similarly, for the last term

$$\begin{aligned}
&(E_{nm,t}^* - E_{nm,t-1})^2 - (E_{nm,t}^* - E_{nm,t})^2 \\
&\leq (E_{nm,t-1} - E_{nm,t-1}^*)^2 - (E_{nm,t} - E_{nm,t}^*)^2 \\
&\quad + L |E_{nm,t-1}^* - E_{nm,t}^*|. \tag{38}
\end{aligned}$$

For the first term, using  $E_{nm,t} - F_{nm,t} = \frac{\lambda_{nm,t-1} - \lambda_{nm,t}}{\rho}$ ,  $F_{nm,t}^* + F_{mn,t}^* = 0$ ,  $\lambda_{nm,t} = \lambda_{mn,t}$  and summing up for all  $n \in \Omega$  yields

$$\begin{aligned}
&\sum_{n \in \Omega} \sum_{m \in \omega_n} \lambda_{nm,t} (E_{nm,t} - F_{nm,t}^*) + \frac{\rho}{2} (E_{nm,t} - F_{nm,t})^2 \\
&= \frac{1}{2} \sum_{\forall (n,m) \in (\Omega, \omega_n)} \lambda_{nm,t} [E_{nm,t} + E_{mn,t} - (F_{nm,t}^* + F_{mn,t}^*)] \\
&\quad + \frac{\rho}{2} (E_{nm,t} - F_{nm,t})^2 + \frac{\rho}{2} (E_{mn,t} - F_{mn,t})^2 \\
&= \frac{1}{2} \sum_{\forall (n,m) \in (\Omega, \omega_n)} 2\lambda_{nm,t} (E_{nm,t} - F_{nm,t}) \\
&\quad + \frac{1}{2\rho} (\lambda_{nm,t-1} - \lambda_{nm,t})^2 + \frac{1}{2\rho} (\lambda_{mn,t-1} - \lambda_{mn,t})^2 \\
&= \sum_{\forall (n,m) \in (\Omega, \omega_n)} \frac{1}{2\rho} (\lambda_{nm,t-1}^2 - \lambda_{nm,t}^2). \tag{39}
\end{aligned}$$

Combining (36)-(39) and based on Assumption 1,  $\tilde{R}(T)$  is bounded as follows

$$\begin{aligned}
&\sum_{t=1}^T \left( \sum_{n \in \Omega} C_{n,t}(\hat{E}_{n,t}) \right) - \sum_{t=1}^T \left( \sum_{n \in \Omega} C_{n,t}(E_{n,t}^*) \right) \\
&\leq \sum_{\forall (n,m) \in (\Omega, \omega_n)} \frac{1}{2\rho} (\lambda_{nm,1}^2 - \lambda_{nm,T}^2) \\
&\quad + \sum_{n \in \Omega} \sum_{m \in \omega_n} \frac{\rho}{2} [(F_{nm,1} - F_{nm,1}^*)^2 - (F_{nm,T} - F_{nm,T}^*)^2] \\
&\quad + \sum_{n \in \Omega} \sum_{m \in \omega_n} \frac{\eta}{2} [(E_{nm,1} - E_{nm,1}^*)^2 - (E_{nm,T} - E_{nm,T}^*)^2] \\
&\quad + \sum_{t=1}^T \sum_{n \in \Omega} \sum_{m \in \omega_n} \frac{(\rho + \eta)L}{2} |E_{nm,t}^* - E_{nm,t}^*| \\
&\quad + \sum_{t=1}^T \sum_{n \in \Omega} \sum_{m \in \omega_n} \frac{1}{2\rho} \left( \frac{\partial C_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}} \right)^2 \\
&\quad + \sum_{t=1}^T \sum_{n \in \Omega} \sum_{m \in \omega_n} \frac{(\lambda_{nm,t-1} - \lambda_{nm,t})^2}{2\rho} \\
&\leq N(N-1) \left( \frac{\rho D_2}{2} + \frac{\eta D_1}{2} + \frac{(\rho + \eta)LP}{2} + \frac{TG^2}{2\rho} + \frac{T\Lambda^2}{2\rho} \right). \tag{40}
\end{aligned}$$

Setting  $\rho = \sqrt{T}$  and  $\eta = \sqrt{T}$  yields sublinear regret  $\tilde{R}(T)$ .

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