Pool Strategy of a Price-Maker Wind Power Producer

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***Abstract*—We consider the problem of a wind power producer trading energy in short-term electricity markets. The producer is a price-taker in the day-ahead market, but a price-maker in the balancing market, and aims at optimizing its expected revenues from these market floors. The problem is formulated as a Mathematical Program with Equilibrium Constraints (MPEC) and cast as a Mixed-Integer Linear Program (MILP), which can be solved employing off-the-shelf optimization software. The optimal bid is shown to deliver significantly improved performance compared to traditional bids such as the forecast conditional mean or median of wind power distribution. Finally, sensitivity analyses are carried out to assess the impact on the offering strategy of the producer's penetration in the market, of the correlation between wind power production and residual system deviation, and of the shape of the forecast distribution of wind power production.**

***Index Terms*—Wind power, price-maker, electricity markets, offering strategies, mathematical programs with equilibrium constraints.**

I. NOMENCLATURE

A. Sets

k	Index for up-regulation block offered at the balancing market, from 1 to N_K
j	Index for down-regulation block offered at the balancing market, from 1 to N_J
ω	Index for scenario, from 1 to N_Ω

B. Constants

c_k	Offered cost for up-regulation block k
b_j	Offered benefit for down-regulation block j
C_k	Production limit for up-regulation block k
C_j	Consumption limit for down-regulation block j
w_ω	Own wind power production in scenario ω
δ_ω	Residual system deviation in scenario ω
$\lambda_\omega^{\mathrm{DA}}$	Day-ahead market price in scenario ω
C^{W}	Installed capacity for wind power producer

C. Lower-Level Variables

$p_{k\omega}$	Up-regulation from block k in scenario ω
$p_{j\omega}$	Down-regulation from block j in scenario ω
$\lambda_\omega^{\mathrm{B}}$	Balancing market price in scenario ω
$\mu_{k\omega}^{\mathrm{S}}$	Dual variable for capacity constraint at the balancing market for block k in scenario ω
$\mu_{j\omega}^{\mathrm{D}}$	Dual variable for capacity constraint at the balancing market for block j in scenario ω

D. Upper-Level Variables

x_ω	Wind power producer's offer in scenario ω

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II. INTRODUCTION

In the recent years, the deployment of wind power into power systems worldwide has increased with impressive pace. In part this expansion has been supported by national governments in the form of market incentives, which resulted in wind power having a competitive advantage with respect to conventional sources of energy. In many cases, wind power producers are granted a fixed feed-in tariff or a minimum price for their production, so as to hedge them from the price fluctuations of electricity markets. Furthermore, they are often relieved of their balance responsibility, which means that the Transmission System Operators (TSOs) bear the costs for the deviations of actual production from the generation schedule, which wind power producers inevitably incur.

As the cost per produced MWh of wind power constantly decreases, wind power producers are forced to participate in electricity markets in the same way as conventional power generators. However, wind generation is characterized by peculiar features that distinguish it from most of the other electricity sources. First of all, it is stochastic, and thus can be forecast only with a certain degree of accuracy [1]. Furthermore, it is non-dispatchable. These features imply that deviations of the actual production from the schedule must be covered by back-up plants.

On the other hand, electricity markets were conceived at a time when the large-scale penetration of wind power was not foreseen. Therefore, their design is better suited to traditional power plants, which are dispatchable and may need a certain time-lag between the submission of production plans and the actual delivery of power. In modern electricity markets, most of the energy trade takes place in so-called *day-ahead* markets, with an advance in time typically in the range between 12–36 hours. Participants are then allowed to contract changes to their day-ahead schedules either in *intra-day* or *balancing* markets. However, prices in such markets may involve penalties and are generally less attractive and more volatile than in the day-ahead market.

In view of the several market floors and of the uncertainty involved, both in production and in market quantities, the problem of determining the optimal bid for a wind power producer is a multi-stage, stochastic optimization problem.

So far the state-of-the-art of research on the topic has focused on the problem of trading wind power as a price-taker. Considering the day-ahead and the balancing market stages only, it can be shown that the optimal day-ahead bid for a price-taker wind power producer is a certain quantile of the forecast wind power distribution, which is a function of the market prices, see [2], as well as [3] and [4] for the case with stochastic market prices. Such quantile-based approach is used to evaluate the performance of wind power forecasts in [5] and [6], both of which employ historical averages of market prices. Furthermore, the performance of this approach is analyzed in [4] in a realistic test-case using state-of-the-art forecasts of both wind production and market prices. Another analytical approach is proposed in [7], where the optimal bid is chosen in a discrete decision space, and the uncertainty in wind power production is modeled using probability tables. Furthermore, an approach based on utility-functions is presented in [8] along with the use of persistence forecasting of wind power production and historical values for market prices. The stochastic programming approach is also popular. In [9], wind power production is modeled using scenarios, and historical averages of prices are used. Furthermore, [10] deals with the participation of wind power producers in multiple market stages (day-ahead, intra-day and balancing). Recently, [3] and [11] have shown further analytical results on the problem of trading wind as a price-taker.

To our knowledge, there are no attempts in the literature to study the optimal bidding for a wind power producer in a price-maker setting. However, the problem is becoming increasingly interesting as, due to its growing penetration into power systems, wind power is more and more capable of influencing market prices [12].

This work models the market participation of a wind power producer that is a price-maker[1] in the balancing market in the framework of *Mathematical Programs with Equilibrium Constraints* (MPECs) [13]. Because a much larger volume is traded in the day-ahead market, we assume that the wind power producer is a price-taker at that stage. Therefore, we can employ scenarios for the day-ahead market price, as well as for wind power production and residual system deviation. We also assume that bids are independent between different trading periods, and therefore consider a single time period in our formulation. The output of the optimization model consists of the optimal day-ahead offer and the balancing market prices for any realization of wind power production and system deviation. Since producers are allowed to bid supply curves in the day-ahead market, the optimal offer is a non-decreasing curve relating quantities of energy to the corresponding minimum accepted prices.

The structure of this paper is the following. Section III introduces the setup of the problem. Then, the mathematical formulation is described in detail in Section IV. Results from a series of case studies are presented in Section V. Finally, Section VI concludes the paper.

[1]We define a producer to be a price-maker when it is capable of impacting the market result through its offer in a broad sense, not necessarily only by marking up its price offer above the marginal cost of production

III. Problem Description

This section introduces the electricity market framework considered and the setup of the problem as a bilevel model.

A. Market Framework

In this work, we consider the short-term trade of electricity in the *day-ahead* and the *balancing* market. In the day-ahead market, wind power producers sell production for each trading period of the following day with a certain advance in time to the actual delivery, typically in the range between 12–36 hours. Since at the time of offering the actual wind power production is uncertain, producers must settle the excess or deficit of production by trading at the balancing market. Notice that *intra-day* markets are not considered in this work. This simplification is realistic since these markets have generally low liquidity [14].

Furthermore, we consider a *one-price* balancing market, i.e., all deviations are settled at a unique price, determined according to the marginal pricing rule. The so-called *two-price* or *dual-price* settlement of imbalances, where the day-ahead market price is applied to unwanted deviations in the opposite direction to the overall system imbalance, while the marginal price at the balancing stage is applied to all the other deviations, is not considered in this work. We remark that considering a two-price balancing market in this framework would be possible with some modifications, either by modeling the switch between day-ahead and marginal price with binary variables, or by employing supply curves dependent on the realization of the system deviation. The former option would come at the expense of a higher computational complexity, the latter of an increased modeling burden. We underline that, while some markets (e.g., a part of the Nordic countries in Nord Pool [15] and the Iberian MIBEL [16] in Europe) employ the two-price system for imbalances, there are a number of markets where the one-price scheme is adopted (e.g., Norway [15], the Dutch APX [17] and the German EEX [18] markets).

B. Bilevel Setup

The setup of the problem is sketched in Fig. 1. The Wind Power Producer (WPP) seeks to maximize its total revenues from the day-ahead and the balancing market. Since we assume that the wind power producer is a price-taker at the day-ahead stage, but a price-maker at the balancing market, only the clearing of the latter market is explicitly included in the producer's optimization problem. This is because the day-ahead price is not influenced by the decision of the wind power producer and therefore, it can be modeled exogenously with a discrete number of scenarios. On the contrary, there is a dependence between the balancing market clearing and the optimization problem of the wind power producer. Indeed, the balancing market is cleared with knowledge on the bid of the wind power producer; in turn, the latter optimizes its offer on the basis of the anticipation of the balancing market price, given its offer and a forecast of its production and of the residual system deviation.

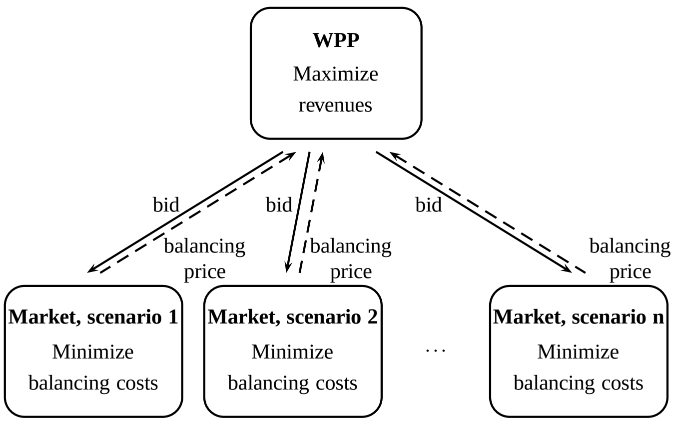

Fig. 1. Sketch of the problem setup

Since we model the uncertainty in future wind power production and residual system deviation with scenarios, we need to solve a balancing market clearing problem for each scenario. Such problem yields, for the particular realization of the uncertainties considered, the optimal dispatch of regulating power and the balancing market price, which enters the upper-level optimization problem (i.e., the producer's one). In the upper-level objective function, the market outcome corresponding to a certain scenario is weighted by the corresponding scenario probability.

Notice that we model exogenously the market participation of players other than the considered wind power producer through scenarios for the residual system deviation. In other words, we make use of a statistical tool able to forecast the aggregate imbalance from other wind power producers, possibly bidding strategically, and the load. However, if production forecasts for all the other wind power producers are available, competition should be modeled through an *Equilibrium Program with Equilibrium Constraints (EPEC)* [19]. We leave this complex topic for future research.

IV. Mathematical Formulation

The bilevel optimization scheme outlined in Section III-B corresponds to a stochastic formulation of an MPEC. We first formulate the problem in the general framework of stochastic MPECs in Section IV-A. Then, we present the formulation of the lower-level problems in Section IV-B, and of the upper-level one in Section IV-C.

A. Stochastic MPEC Formulation

The problem at hand has a bilevel structure where several (lower-level) optimization problems are nested in another (upper-level) one. This can be formulated as a stochastic MPEC as follows.

$$\text{Max.} \quad f(\boldsymbol{x}, \boldsymbol{\lambda}^{\mathrm{B}}) \tag{1a}$$

$$\text{s.t.} \quad h(\boldsymbol{x}, \boldsymbol{\lambda}^{\mathrm{B}}) \leq 0 \;, \tag{1b}$$

$$\left(\boldsymbol{p}_1, \lambda_1^{\mathrm{B}}, \boldsymbol{\mu}_1\right) \in \arg \min_{\boldsymbol{y} \in F_1(\boldsymbol{x})} \left\{ g_1(\boldsymbol{x}, \boldsymbol{y}) \right\} \;, \tag{1c}$$

$$\left(\boldsymbol{p}_2, \lambda_2^{\mathrm{B}}, \boldsymbol{\mu}_2\right) \in \arg \min_{\boldsymbol{y} \in F_2(\boldsymbol{x})} \left\{ g_2(\boldsymbol{x}, \boldsymbol{y}) \right\} \;, \tag{1d}$$

$$\vdots$$

$$\left(\boldsymbol{p}_{N_\Omega}, \lambda_{N_\Omega}^{\mathrm{B}}, \boldsymbol{\mu}_{N_\Omega}\right) \in \arg \min_{\boldsymbol{y} \in F_{N_\Omega}(\boldsymbol{x})} \left\{ g_{N_\Omega}(\boldsymbol{x}, \boldsymbol{y}) \right\} \;. \tag{1e}$$

The upper-level problem consists in the maximization of the objective function $f(\boldsymbol{x}, \boldsymbol{\lambda}^{\mathrm{B}})$ in (1a) subject to the feasibility constraint (1b), and further constrained by the optimality conditions of the lower-level problems (1c)–(1e). For a risk-neutral wind power producer, the objective function $f(\boldsymbol{x}, \boldsymbol{\lambda}^{\mathrm{B}})$ is the expected value of the total revenues in the day-ahead and balancing markets, given the information available at the time of bidding. The decision variables of the upper-level problem are the bid $\boldsymbol{x}$ in the day-ahead market, as well as the variables of the lower-level problems.

The lower-level problems are represented by (1c)–(1e) for all scenarios $\omega = 1, 2, \ldots, N_\Omega$. Such problems aim at the minimization of the objective functions $g_\omega(\boldsymbol{x}, \boldsymbol{y})$, provided that the decision vector $\boldsymbol{y}$ is included in the feasible sets $F_\omega(\boldsymbol{x})$. As we will see in the following section, the objective function of this problem represents the system balancing costs in the realization ω of the uncertainty, which are minimized in the balancing market. The clearing of this market results in the dispatch of balancing power $\boldsymbol{p}_\omega$, primal variable of the lower-level problem, as well as in the dual variables $\lambda_\omega^{\mathrm{B}}$ and $\boldsymbol{\mu}_\omega$. Notice that, as the remainder of the section will clarify, we are particularly interested in the value of $\lambda_\omega^{\mathrm{B}}$. Indeed, this variable indicates the balancing market price in scenario ω, which enters the upper-level optimization problem. Notice also that the lower-level problems are parameterized in the wind power producer's offer in the day-ahead market $\boldsymbol{x}$, which enters such problems as a constant.

Formulation (1) is not suitable for being solved directly by an optimization solver, owing to the nested optimization of the lower-level problems in (1c)–(1e). However, such optimization problems can be replaced by their Karush-Kuhn-Tucker (KKT) conditions, for which a mixed-integer linear formulation exists, under reasonably mild assumptions. Indeed, KKT conditions are necessary and sufficient for optimality if the lower-level problems are convex and their constraints satisfy some regularity conditions [20]. If this holds, bilevel problem (1) can be reformulated as a single-level optimization problem. We derive this formulation explicitly in the remainder of this section.

B. Lower-Level Problem

The solution to the problem below for each scenario ω clears the balancing market.

$$\underset{p_{k\omega}, p_{j\omega}}{\text{Min.}} \quad \sum_{k=1}^{N_K} c_k p_{k\omega} - \sum_{j=1}^{N_J} b_j p_{j\omega} \tag{2a}$$

$$\text{s.t.} \quad \sum_{k=1}^{N_K} p_{k\omega} - \sum_{j=1}^{N_J} p_{j\omega} = -(w_\omega - x_\omega) - \delta_\omega \ : \lambda_\omega^{\mathrm{B}} \ , \tag{2b}$$

$$- p_{k\omega} \geq -C_k \ : \mu_{k\omega}^{\mathrm{S}} \qquad \forall k \ , \tag{2c}$$

$$- p_{j\omega} \geq -C_j \ : \mu_{j\omega}^{\mathrm{D}} \qquad \forall j \ , \tag{2d}$$

$$p_{k\omega}, p_{j\omega} \geq 0 \qquad \forall k, j \ . \tag{2e}$$

The decision variables $p_{k\omega}$ and $p_{j\omega}$ represent the dispatch of up- and down-regulation power, respectively, from block offers k and j. The parameter c_k is the price offer (per unit cost) associated with the deployment of supply power from block k. Similarly, b_j is the per unit benefit associated with the power production decrease (down-regulation) from block j. Therefore, objective (2a) is the balancing cost in scenario ω. The balance of supply and demand is enforced by (2b). Indeed, the terms on the right-hand side of the equation are, after a change in sign, the sum of the deviation from the wind power producer (actual production w_ω minus day-ahead bid x_ω) and from all the other market participants (δ_ω). Notice that the residual system deviation and the producer's own imbalance are to be considered as a *perfectly inelastic* demand (or supply) of power, which must be met at any market price. Consequently, these two terms do not appear in the objective function (2a), while their sum is enforced to be equal to the power output of flexible generators at the balancing stage through (2b). Equations (2c) and (2d) ensure that the dispatch of regulating power is not greater than the capacities C_k and C_j, which are the sizes of the block offers in the balancing market. Finally, non-negativity of the power dispatch is enforced by (2e). Notice that the dual variables of the problem are indicated after each constraint preceded by a colon. Variable $\lambda_\omega^{\mathrm{B}}$ is of particular importance, as it indicates the marginal cost of production, which is the balancing market price in a one-price imbalance settlement.

As one can notice, problem (2) is linear and thus it can be equivalently represented by the following set of KKT conditions [20]

$$0 \leq p_{k\omega} \perp c_k - \lambda_\omega^{\mathrm{B}} + \mu_{k\omega}^{\mathrm{S}} \geq 0 \qquad \forall k \ , \tag{3a}$$

$$0 \leq p_{j\omega} \perp -b_j + \lambda_\omega^{\mathrm{B}} + \mu_{j\omega}^{\mathrm{D}} \geq 0 \qquad \forall j \ , \tag{3b}$$

$$\sum_{k=1}^{N_K} p_{k\omega} - \sum_{j=1}^{N_J} p_{j\omega} = -(w_\omega - x_\omega) - \delta_\omega \ , \tag{3c}$$

$$0 \leq \mu_{k\omega}^{\mathrm{S}} \perp C_k - p_{k\omega} \geq 0 \qquad \forall k \ , \tag{3d}$$

$$0 \leq \mu_{j\omega}^{\mathrm{D}} \perp C_j - p_{j\omega} \geq 0 \qquad \forall j \ , \tag{3e}$$

where the $\perp$ operator separating two inequalities implies that at least one of them holds strictly. Conditions (3a) and (3b) are stationarity conditions; the inequalities on the right-hand side define, along with the non-negativity definitions on the left-hand side of (3d) and (3e), the feasible space of the dual problem. Conditions (3d) and (3e) are complementarity slackness conditions; the inequalities on the right-hand side define, along with (3c) and the non-negativity definitions on the left-hand side of (3a) and (3b), the primal feasible space.

Since the $\perp$ operator is equivalent to requiring that the multiplication between two linear expressions be equal to 0, the KKT conditions (3) include nonlinearities. However, it is possible to linearize such conditions by employing binary variables [21], yielding the following set of optimality conditions

$$0 \leq c_k - \lambda_\omega^{\mathrm{B}} + \mu_{k\omega}^{\mathrm{S}} \leq M_1^{\mathrm{SS}} z_{k\omega}^{\mathrm{S1}} \qquad \forall k \ , \tag{4a}$$

$$0 \leq p_{k\omega} \leq M_2^{\mathrm{SS}} \left(1 - z_{k\omega}^{\mathrm{S1}}\right) \qquad \forall k \ , \tag{4b}$$

$$0 \leq -b_j + \lambda_\omega^{\mathrm{B}} + \mu_{j\omega}^{\mathrm{D}} \leq M_1^{\mathrm{SD}} z_{j\omega}^{\mathrm{D1}} \qquad \forall j \ , \tag{4c}$$

$$0 \leq p_{j\omega} \leq M_2^{\mathrm{SD}} \left(1 - z_{j\omega}^{\mathrm{D1}}\right) \qquad \forall j \ , \tag{4d}$$

$$\sum_{k=1}^{N_K} p_{k\omega} - \sum_{j=1}^{N_J} p_{j\omega} = -(w_\omega - x_\omega) - \delta_\omega \ , \tag{4e}$$

$$0 \leq C_k - p_{k\omega} \leq M_1^{\mathrm{S}} z_{k\omega}^{\mathrm{S2}} \qquad \forall k \ , \tag{4f}$$

$$0 \leq \mu_{k\omega}^{\mathrm{S}} \leq M_2^{\mathrm{S}} \left(1 - z_{k\omega}^{\mathrm{S2}}\right) \qquad \forall k \ , \tag{4g}$$

$$0 \leq C_j - p_{j\omega} \leq M_1^{\mathrm{D}} z_{j\omega}^{\mathrm{D2}} \qquad \forall j \ , \tag{4h}$$

$$0 \leq \mu_{j\omega}^{\mathrm{D}} \leq M_2^{\mathrm{D}} \left(1 - z_{j\omega}^{\mathrm{D2}}\right) \qquad \forall j \ , \tag{4i}$$

$$z_{k\omega}^{\mathrm{S1}}, z_{j\omega}^{\mathrm{D1}}, z_{k\omega}^{\mathrm{S2}}, z_{j\omega}^{\mathrm{D2}} \in \{0, 1\} \qquad \forall k, j \ , \tag{4j}$$

where the M constants are large enough to guarantee that the inequalities are never binding when the right-hand side is different from 0. Notice that, as long as such assumption holds and in view of the binary variable definitions in (4j), we have that constraints (4a) and (4b) are equivalent to (3a); (4c) and (4d) to (3b); (4f) and (4g) to (3d); (4h) and (4i) to (3e). Each balancing market clearing problem, i.e., for every scenario, can be replaced by its KKT conditions (4).

Furthermore, for reasons that will become apparent later in this section, it is interesting to notice that the dual of the lower-level problem (2) is, for every scenario ω,

$$\underset{\mu_{k\omega}^{\mathrm{S}}, \mu_{j\omega}^{\mathrm{D}}, \lambda_\omega^{\mathrm{B}}}{\text{Max.}} \quad - \lambda_\omega^{\mathrm{B}} \left[(w_\omega - x_\omega) + \delta_\omega\right] - \sum_{k=1}^{N_K} C_k \mu_{k\omega}^{\mathrm{S}} - \sum_{j=1}^{N_J} C_j \mu_{j\omega}^{\mathrm{D}} \tag{5a}$$

$$\text{s.t.} \quad \lambda_\omega^{\mathrm{B}} - \mu_{k\omega}^{\mathrm{S}} \leq c_k \qquad \forall k \ , \tag{5b}$$

$$- \lambda_\omega^{\mathrm{B}} - \mu_{j\omega}^{\mathrm{D}} \leq -b_j \qquad \forall j \ , \tag{5c}$$

$$\mu_{k\omega}^{\mathrm{S}}, \mu_{j\omega}^{\mathrm{D}} \geq 0 \qquad \forall k, j \ . \tag{5d}$$

The optimal objective function values of (5) and (2) are equal.

Finally, we stress that the network is not considered in this balancing market clearing model. This simplification, however, is justified in a European context, since the vast majority of European electricity markets employ zonal pricing.

C. Upper-Level Problem

In a one-price system, all deviations from the day-ahead schedule are settled at the marginal cost, i.e., the dual $\lambda_\omega^{\mathrm{B}}$ of the balance equation (2b) at the balancing market. Hence, the optimization problem of a wind power producer writes as

$$\underset{\substack{x_\omega, p_{k\omega}, p_{j\omega}, \\ \lambda_\omega^{\mathrm{B}}, \mu_{k\omega}^{\mathrm{S}}, \mu_{j\omega}^{\mathrm{D}}}}{\text{Max.}} \quad \mathbb{E}\left\{\lambda_\omega^{\mathrm{DA}} x_\omega + \lambda_\omega^{\mathrm{B}} \left(w_\omega - x_\omega\right)\right\} \tag{6a}$$

$$\text{s.t.} \quad 0 \leq x_\omega \leq C^{\mathrm{W}} \quad \forall \omega \ , \tag{6b}$$

$$x_\omega = x_{\omega'} \qquad \omega, \omega' \in \Omega_i, \forall i \ , \tag{6c}$$

$$x_\omega \leq x_{\omega'} \qquad \omega \in \Omega_i, \omega' \in \Omega_j, i < j \ , \tag{6d}$$

KKT conditions of the lower-level problems .

The objective function (6a) is the expectation of the sum of two terms. The first term represents the revenues in the day-ahead market in scenario ω, since it is given by the multiplication of the day-ahead market price $\lambda_\omega^{\mathrm{DA}}$ with the offer x_ω in the same market. In an analogous fashion, the second term represents the revenues at the balancing stage in scenario ω. Therefore, the objective function is the expected total revenues at the two market floors. Notice that, since the wind power producer is a price-taker in the day-ahead market, $\lambda_\omega^{\mathrm{DA}}$ is a parameter and not an optimization variable. Furthermore, since the cleared day-ahead price is disclosed prior to the realization of the stochastic production and the residual system deviation in real-time, scenarios for the day-ahead price can be considered as first-stage scenarios, while the other scenarios can be regarded as second-stage. This implies that the scenario set Ω can be partitioned in a number of subsets Ω_i, across which the day-ahead price is constant, i.e.,

$$\lambda_\omega^{\mathrm{DA}} = \lambda_{\omega'}^{\mathrm{DA}},\ \forall \omega, \omega' \in \Omega_i,\ \forall i\ . \tag{7}$$

Furthermore, since the order of the partitions Ω_i is arbitrary, we assume that they are sorted so that the corresponding day-ahead price is increasing, i.e.,

$$\lambda_\omega^{\mathrm{DA}} \le \lambda_{\omega'}^{\mathrm{DA}},\ \forall \omega \in \Omega_i, \forall \omega' \in \Omega_j, i < j\ . \tag{8}$$

Constraint (6b) enforces that the bid of the wind power producer be included in the range between 0 and the installed capacity C^{W}. Furthermore, market practices usually allow producers to submit bids in the form of non-decreasing supply curves, i.e., price-quantity pairs indicating how much energy the producer is willing to deliver at a certain day-ahead price. Constraints (6c) and (6d) together ensure that the wind power producer's offer is consistent with such practices, based on the partitioning of the scenario set Ω imposed by (7) and (8). Equation (6c) is a *non-anticipativity* constraint, which imposes that a single quantity is offered for every first-stage scenario (realization of the day-ahead price). Constraint (6d) enforces that the offer curve is non-decreasing.

The problem is complicated by the bilinear terms $\lambda_\omega^{\mathrm{B}} x_\omega$ in the objective function (6a), which can be linearized by applying the strong duality theorem on the lower-level (market-clearing) problem. At optimality, the objective value of the primal (2) and the dual (5) problems are equal, i.e.,

$$\begin{aligned}\sum_{k=1}^{N_K} c_k p_{k\omega} - \sum_{j=1}^{N_J} b_j p_{j\omega} = &-\lambda_\omega^{\mathrm{B}}\left[(w_\omega - x_\omega) + \delta_\omega\right] \\ &- \sum_{k=1}^{N_K} C_k \mu_{k\omega}^{\mathrm{S}} - \sum_{j=1}^{N_J} C_j \mu_{j\omega}^{\mathrm{D}}\ .\end{aligned} \tag{9}$$

We can therefore reformulate the term inside the expectation operator in (6a) as follows

$$\begin{aligned}\lambda_\omega^{\mathrm{B}}\left(w_\omega - x_\omega\right) = &-\sum_{k=1}^{N_K}\left(C_k \mu_{k\omega}^{\mathrm{S}} + c_k p_{k\omega}\right) \\ &+ \sum_{j=1}^{N_J}\left(-C_j \mu_{j\omega}^{\mathrm{D}} + b_j p_{j\omega}\right) - \lambda_\omega^{\mathrm{B}}\delta_\omega\ ,\end{aligned} \tag{10}$$

where the expression on the right-hand side is linear.

The final optimization problem, incorporating the linearization of the bilinear terms in (10) and of the KKT conditions of the lower-level problem in (4), as well as using a finite number of scenarios for describing the uncertainty, so that the expectation operator in (6a) reduces to a sum weighted by probabilities, writes as

$$\begin{aligned}\underset{\Theta}{\mathrm{Max.}}\quad & \sum_{\omega=1}^{N_\Omega}\pi_\omega\Bigg\{\lambda_\omega^{\mathrm{DA}} x_\omega - \sum_{k=1}^{N_K}\left(C_k\mu_{k\omega}^{\mathrm{S}} + c_k p_{k\omega}\right) \\ &\quad + \sum_{j=1}^{N_J}\left(-C_j\mu_{j\omega}^{\mathrm{D}} + b_j p_{j\omega}\right) - \lambda_\omega^{\mathrm{B}}\delta_\omega\Bigg\} \qquad (11\mathrm{a}) \\ \text{s.t.}\quad & \text{(6b)–(6d)}\ , \\ & \text{(4a)–(4j)} \qquad \forall\omega\ .\end{aligned}$$

The set of decision variables includes variable x_ω of the upper-level problem, the variables of the primal and the dual lower-level problems, as well as the binary variables needed for the linearization of the complementarity conditions, i.e.,

$$\begin{aligned}\Theta = \big\{& x_\omega, p_{k\omega}, p_{j\omega}, \lambda_\omega^{\mathrm{B}}, \mu_{k\omega}^{\mathrm{S}}, \mu_{j\omega}^{\mathrm{D}}, \\ & z_{k\omega}^{\mathrm{S1}}, z_{j\omega}^{\mathrm{D1}}, z_{k\omega}^{\mathrm{S2}}, z_{j\omega}^{\mathrm{D2}},\ \forall k, j, \omega\big\}\ .\end{aligned} \tag{12}$$

Notice that model (11) is a Mixed-Integer Linear Problem (MILP), which can be solved employing off-the-shelf optimization software.

V. Application Studies

This section describes a series of studies on the application of the presented model in a realistic setup. At first, the models employed in the examples for random variables are described in Section V-A. Then, results obtained in a single example are commented on in Section V-B. Finally, Sections V-C, V-D and V-E present the results of sensitivity analyses assessing the impact of the producer's market penetration, of the correlation between its output and the residual system deviation, and of the shape of the forecast wind power probability density function, respectively.

A. Modeling the Uncertainty

The uncertainties in the system, i.e., day-ahead price, wind power production and system deviation, are modeled using a discrete set of scenarios. It is assumed that the first-stage variable (day-ahead price) is independent of the second-stage ones (wind power production and residual system deviation). This basically means that we can build the scenario tree by generating first-stage and second-stage scenarios independently, and associating a copy of the second-stage scenarios to each first-stage scenario. Note that this assumption implies no loss of generality for the proposed method, since it could be overcome by employing a scenario generation method accounting for the possible dependency structure between first-stage and second-stage variables. We underline that this could be achieved without increasing the size of the optimization problem, and therefore, it is merely an issue linked to the scenario generation method, which is out of the scope of this paper. Finally, we point out that this independence assumption might be not valid in practice in markets with high penetration of wind power production, especially as far as the relationship between the day-ahead price and the forecast wind power distribution is concerned [12]. In this regard, however, we would like to underline that this simplification does not result in an overestimation of the economic improvement obtained with the proposed offering model, but quite the opposite. In fact, neglecting the possible correlation between these variables would result in conservative performance results in comparison to more "traditional" trading strategies (e.g., offering the forecast mean or a certain quantile), which do not allow differentiated offers on the basis of the realization of the day-ahead price, as the proposed method does.

Day-ahead price scenarios were generated by random sampling from the probabilistic forecast of the spot price in Nord Pool for the 12th trading period of the 7th September 2011. The probabilistic forecast is obtained by employing the semi-parametric approach extensively described in [22]. That method combines a nonparametric description of the central part of predictive distributions based on quantile regression for quantiles with nominal proportion between 5% and 95%, and a parametric (exponential) description of the distribution tails. The quantile regression models use the predicted conditional expectations of day-ahead price and load as input. The parameters in the quantile regression models are adaptively estimated using the method of [23], while the parameters for the exponential tails are estimated once and for all under the maximum likelihood criterion.

As far as the second-stage variables are concerned, we employ Beta distributions to model wind power generation, as advocated in [24]. In practical applications it would be desirable to make use of a state-of-the-art forecasting tool employing a non-parametric model for the distribution of wind power production [25]. However, Beta distributions are sufficiently realistic to the purpose of this paper. Furthermore, notice that this assumption implies no loss of generality, as drawing scenarios from a non-parametric distribution would result in no additional complexity for the proposed optimization method.

For the residual system deviation we consider a Student's t-distribution, which provides a good fit for the hourly data for net system deviation in Western Denmark (DK-1 area price in Nord Pool) during the year 2011, which are available at [26]. A histogram of the actual data and an illustration of the parametric fit are provided in Fig. 2. Once again, in a practical application it would be desirable to employ a state-of-the-art probabilistic model to describe this stochastic variable, possibly getting rid of the stationarity assumption implicit in our approach.

The parameters employed for these distributions are shown in Table I. Furthermore, notice that when sampling scenarios for the system deviation from the Beta distribution, we discarded scenarios lower (greater) than the 0.001 (0.999) quantile. This is done because arbitrarily low (or high) values of system deviation could be sampled, which is not realistic and could potentially destabilize the results of the analysis. Furthermore, it should be noticed that the standard deviation of the Student's t-distribution used for the residual system imbalance is 217.57 MWh, which is comparable to the installed

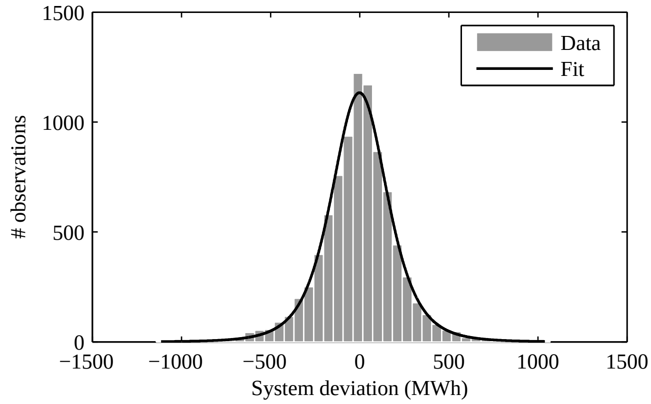

Fig. 2. Histogram for system deviation in Western Denmark (DK-1 price area of Nord Pool) during 2011, and fit using a Student's t distribution

wind power capacity $C^{\mathrm{W}} = 300\,\mathrm{MW}$ owned by the producer. On the contrary, the total installed capacity in Denmark is approximately $14\,\mathrm{GW}$. These two figures are in line with the assumption that the producer is a price-taker at the day-ahead market, where a significant share of the total installed capacity is supposed to participate, and a price-maker at the balancing market, whose trading volume corresponds to the total system imbalance.

TABLE I
INFORMATION ON STOCHASTIC INPUT PARAMETERS

Stochastic variable	Distribution type	Parameters	# scenarios original	# scenarios reduced
λ^{DA}	non-parametric	-	10 000	12
w	Beta	$\alpha = 3.78$ $\beta = 1.62$ $C^{\mathrm{W}} = 300$	10 000	100
δ	Student's t	$\mu = -0.96$ $\sigma = 161.14$ $\nu = 4.43$	10 000	100

As one can see in Table I, 10 000 scenarios were generated independently for each stochastic variable. In order to impose different rank correlation levels between wind power production and system deviation, we employed the method in [27]: first, we generated two random permutations of 10 000 Normal scores; then, we imposed the desired correlation (notice that Pearson and Spearman correlation almost coincide for Gaussian variables) by multiplying the permutations by the Cholesky factor of the desired rank correlation matrix; finally, we reordered the random samples for wind power production and system deviation according to the order of this product.

After this, we made use of the fast-forward scenario reduction technique [28] to decrease the number of first-stage and second-stage scenarios to 12 and 100, respectively. This procedure is based on a heuristic that iteratively adds scenarios to a reduced set, so as to minimize the maximum mutual distance between elements. Then, probabilities of the reduced scenarios are determined by assigning the probability of each scenario in the original set to the closest element in the reduced set.

The last variables to be set are the ones characterizing the bids of participants at the balancing market, i.e., the per unit costs (benefits) of offered production increase (decrease), c_k (b_j), and the size of the respective blocks C_k (C_j). Data of individual bids at the balancing market are hardly available, owing to the confidentiality policies of market and transmission system operators. However, Nord Pool Spot publishes historical supply curves for the day-ahead Scandinavian market [29].

We employed the supply curve for the 12th trading period on the 7th September 2011 and the scenarios generated for the day-ahead price in Nord Pool for the same day and time to build bids for up- and down-regulation. First of all, we halved the capacity of the day-ahead bids in order to account for the fact that not all the generators trading in the day-ahead market are participating at the balancing market. This results in the marginal cost curve illustrated in Fig. 3. Assuming that all the producers whose marginal cost is below the day-ahead price scenario are dispatched at the day-ahead market, and the ones whose cost is above such price are not, we consider that down-regulation (production decrease from schedule) is supplied by the former participants and up-regulation by the latter ones. This way, we obtain a set of balancing market bids that is dependent on the first-stage scenario, i.e., the realization of the day-ahead price. In total we employed $N_K + N_J = 159$ offer blocks in the case study, with variable total numbers of up- and down-regulation blocks depending on the level of the day-ahead price, as a result of the splitting of the curve explained above.

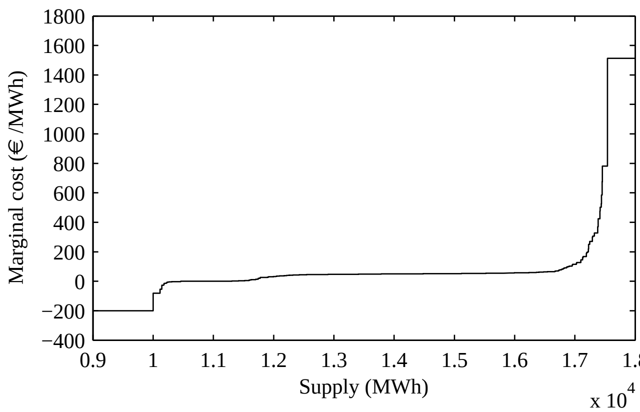

Fig. 3. Marginal cost curve at the balancing market

Notice that, despite we derive the bids from a supply curve, demand could provide regulation as well at the balancing market by increasing or decreasing the scheduled consumption.

B. Results with Optimal Bidding

In the first case study, we employ the dataset generated as described in the previous section and impose a correlation $\rho = 0.3$ between the out-turn of the wind power producer and the residual system deviation using the method in [27], which is briefly sketched in Section V-A.

Defining the producer's penetration in the balancing market, ψ, as the ratio between the standard deviation of the wind power distribution and its sum with the standard deviation of the residual system imbalance, we obtain

$$\psi = \frac{\sigma_w}{\sigma_w + \sigma_\delta} = 19.88\,\% \; . \tag{13}$$

Notice that this definition of penetration is only one among several possible ones. However, as clarified later, it is intuitive as an increase in ψ is obtained by scaling up the wind power producer's capacity, and scaling down the total system deviation.

In this case, a totally price-insensitive day-ahead offer is optimal, consisting of the following optimal quantity

$$x = 76.69\,\mathrm{MWh} \; . \tag{14}$$

First of all, it seems that the possibility of offering a curve does not lead to improved market results in this case, as the producer prefers a single quantity bid. We link this feature to the choice of a mostly convex supply curve, see Fig. 3. Indeed, for increasing prices, the penalty given by the price spread between the day-ahead and the balancing markets tends to be higher for a "short" producer (i.e., producing less than the day-ahead offer). This implies that the higher the day-ahead price, the lower the optimal bid for the producer. However, this is not possible since constraint (6d) enforces that the bid curve be not decreasing.

Second, the optimal quantity bid in (14) appears to be a rather low quantile of the wind power distribution. Indeed, it lays just below the lowest scenario for wind power production. However, notice that this bid is far from being trivial. Indeed, for a price-taker wind power producer in a market with one-price settlement of imbalances, the optimal bid would be either 0 or the nominal capacity, depending on whether the expectation of the balancing price is higher or lower than the day-ahead price [3]. Besides, wind power producers often bid the forecast conditional mean of wind power distribution in practice, which is perceived as a "safe" strategy. Among the reasons for this is the fact that there is a well established literature on point forecasting for wind power production, and the fact that point forecasts have been used for years since wind generation became a contributor to the electricity generation mix in power systems. Furthermore, point forecasts such as the conditional mean are recognized as risk-averse, as it minimizes the expected squared deviation from actual production [4]. Remarkably, none of these possible offers are optimal.

Table II reports the main financial results obtained by bidding the optimal quantity (14). The first and second columns represent the improvement in average market revenues as compared to the strategies of offering the conditional mean and median of wind power distribution in the day-ahead market. The third column, instead, compares with the case where the actual production is traded exclusively at the balancing market. Notice that the nominal capacity offer is not included in the table, as this offer is far from being optimal with a hockey-stick supply curve such as the one depicted in Fig. 3.

The improvement is above 3% as compared to bidding the mean or the median, and above 1.5% better than with a null day-ahead bid. The last column reports the average energy price (€ 54.26/MWh) obtained by averaging the ratio between revenues and wind power production over the scenario set.

TABLE II
FINANCIAL RESULTS OBTAINED USING THE OPTIMAL BID

Profit improvement w.r.t. mean (%)	median (%)	zero (%)	Average price (€/MWh)
3.08	3.25	1.58	54.26

Finally, it should be noticed that, while the offer in (14) is aimed at maximizing the expected revenues, no account is taken of the possible impact on the producer's imbalance. Indeed, this offer results in an expected average imbalance (in absolute value) equal to 122.06 MWh. On the other hand, a known result is that the expected absolute value of the imbalance is minimized by offering the forecast median, which in this case would yield an expected imbalance of 44.82 MWh. We refer the reader to [4] for further discussion on the topic as well as for quantitative results obtained in a price-taker setting.

The optimization described above was performed using CPLEX 12 in GAMS. The model size is reported in Table III. The algorithm converged in 1680 s on a laptop equipped with a 4-core processor clocking at 2.66 GHz. Despite the model size, the problem was solved relatively fast. In this respect, it is worth mentioning that the algorithm was warm-started by setting the binary variables to the values resulting from the market-clearing procedure when the wind power producer's offer is set to the mean of the scenario set for production at any price level. Notice that the latter problem is an LP, and therefore solves rather quickly.

TABLE III
REDUCED MODEL SIZE IN CPLEX

	Size
Rows	57 353
Columns	35 748
Non-zeros	150 797
Binaries	17 885

C. Sensitivity Analysis: Market Penetration

As mentioned in the previous section, we expect the optimal bid for a small wind power producer to be either 0 or the nominal capacity when the latter quantity is small compared to the residual system deviation. On the contrary, in the idealized situation where the producer is the only participant incurring deviations from the day-ahead schedule, we would expect that the optimal bid be close to the median of the conditional wind power distribution. This is because the resulting balancing market price would always be less favorable than the day-ahead market price. In the cases in between these two extremes, we expect the bid to have an intermediate behavior.

Different levels of penetration ψ, as defined in (13), of the wind power producer in the balancing market can be obtained simply by scaling the wind power production (15) and the residual system deviation (16), so as to satisfy (17). Since there is one degree of freedom left, we can choose the scaling factors A and B that leave unchanged the sum between the installed wind power capacity and the maximum absolute value of system deviation, as enforced by (18).

$$w^i_\omega = A w_\omega \ , \tag{15}$$

$$\delta^i_\omega = B \delta_\omega \ , \tag{16}$$

$$\psi^i = \frac{\sigma_{w^i}}{\sigma_{w^i} + \sigma_{\delta^i}} = \frac{A\sigma_w}{A\sigma_w + B\sigma_\delta} \ , \tag{17}$$

$$A C^{\mathrm{W}} + \max_\omega \left\{ \left| \delta^i_\omega \right| \right\} = C^{\mathrm{W}} + \max_\omega \left\{ \left| \delta_\omega \right| \right\} \ . \tag{18}$$

This makes comparisons more consistent, since we can expect similar prices with similar total deviation levels in the balancing market. We consider penetration levels spanning from 10% to 25% with an interval of 2.5%.

Fig. 4 shows the optimal bids obtained for the penetration levels mentioned above. As one can notice, the optimal offer in the day-ahead market is 0 with the lowest value of penetration $\psi = 10\%$. Then, the curve tends to increase with the value of ψ, as we expected from our intuitive analysis.

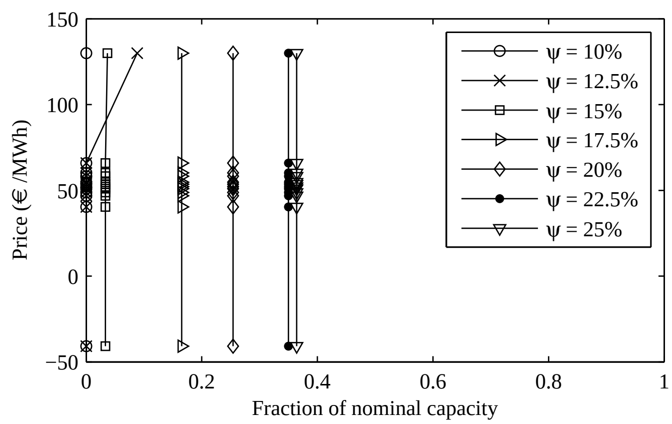

Fig. 4. Day-ahead offer curves with different levels of market penetration of the producer

The main financial results are summarized in Table IV. It is important to notice that, as ψ increases, the improvement obtained using the optimal bid versus the conditional mean and median drops from over 7% to about 2%. On the contrary, the improvement compared to the zero day-ahead offer rises from 0% to around 2.5%. Finally, the average price obtained decreases by € 3.5/MWh. This result is also in line with the expectations, since an increasing penetration implies that the total imbalance will tend to be in general of the same sign as the producer's deviation, thus leading to less favorable prices.

D. Sensitivity Analysis: Correlation

For the study in this section, we reorder the second-stage scenarios so as to impose a rank correlation level of -0.7, -0.3, 0, 0.3 and 0.7, using the method in [27], which is sketched in Section V-A.

The optimal offering curves are depicted in Fig. 5. As it appears, there is a decreasing trend in the day-ahead offer, which drops from roughly 170 MWh ($\rho = -0.7$) to about 40 MWh ($\rho = 0.7$). Apparently, the producer takes better advantage of

TABLE IV
FINANCIAL RESULTS WITH DIFFERENT LEVELS OF MARKET PENETRATION OF THE PRODUCER

Producer penetration (%)	Profit improvement w.r.t. mean (%)	median (%)	zero (%)	Average price (€/MWh)
10	7.34	7.58	0.00	56.90
12.5	5.97	6.20	0.03	56.10
15	4.56	4.79	0.16	55.34
17.5	3.63	3.81	1.26	54.68
20	3.05	3.23	1.58	54.24
22.5	2.53	2.70	2.18	53.76
25	1.96	2.13	2.53	53.40

the negative correlation with the residual system deviation by bidding closer to its median. Indeed, such a bid implies that the producer's deviation is more frequently of opposite sign compared to the system imbalance, and will therefore result in more favorable balancing market prices. With increasing correlation, a low bid better hedges the producer from the highest balancing prices, which occurs when the system is short of power.

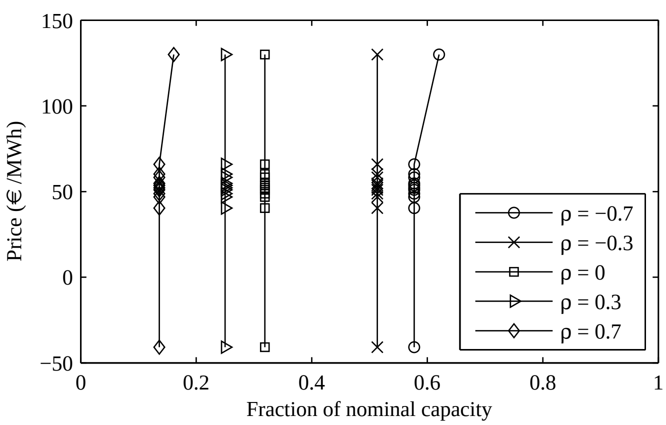

Fig. 5. Day-ahead offer curves with different levels of correlation between the wind power output of the producer and the residual system deviation

Table V reports the main financial results in this sensitivity study. The higher the correlation, the larger the improvement with respect to bidding the median. Contrarily, the improvement with respect to the zero day-ahead bid drops. Finally, the average price diminishes with increasing correlation, which is an intuitive result, since a high correlation between own and system deviations implies less favorable prices in the balancing market.

TABLE V
FINANCIAL RESULTS WITH DIFFERENT LEVELS OF CORRELATION BETWEEN THE WIND POWER OUTPUT OF THE PRODUCER AND THE RESIDUAL SYSTEM DEVIATION

Correlation	Improvement w.r.t. mean (%)	median (%)	zero bid (%)	Average price (€/MWh)
-0.7	0.19	0.27	4.72	54.84
-0.3	0.69	0.95	3.57	54.47
0	1.89	2.59	2.33	54.36
0.3	3.08	3.25	1.58	54.26
0.7	7.43	8.05	0.13	52.93

E. Sensitivity Analysis: Distribution Shape

In this section, we consider different Beta distributions modeling the forecast probability density function (pdf) of wind power production. To this end, we consider four different values ($[1.89, 3.78, 5.67, 7.56]$) for the parameter α of the Beta distribution, and four different values ($[1.62, 3.24, 4.86, 6.48]$) for β. To assess the effect of a changing distribution on the performance of the proposed strategy, we consider the 16 possible combinations of these parameter values.

The considered parameter space covers a wide range of cases of wind power production. Qualitatively speaking, the chosen parameters give rise to pdfs with low mean and positive skewness when $\alpha < \beta$, with mean around half the installed wind power capacity and skewness close to 0 when $\alpha \approx \beta$, and to distributions with high mean and negative skewness when $\alpha > \beta$. Fig. 6 illustrates three examples of Beta distributions, one for each group described above, obtained with parameter values employed in the simulation.

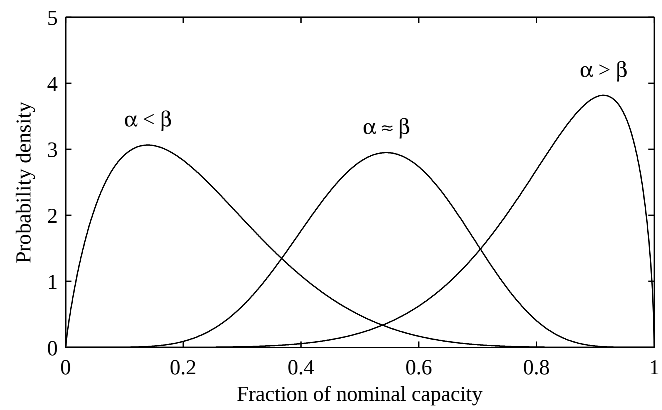

Fig. 6. Examples of Beta distributions as parameters change: $\alpha = 1.89$ and $\beta = 6.48$ result in the low-mean distribution, $\alpha = 7.56$ and $\beta = 6.48$ in the mid-mean distribution, $\alpha = 7.56$ and $\beta = 1.62$ in the high-mean distribution

To analyze the performance improvement brought by the proposed optimal strategy, we test it against two usual benchmarks for day-ahead market offer: the zero-offer and the conditional mean of wind power distribution.

In Fig. 7, the improvement in expected profit with respect to the zero offer is shown as a surface for the 16 combinations of the α and β parameters of the Beta distribution considered. As one can notice, the improvement lies between 0 and 3%, which is consistent with the magnitude of the improvement observed in the previous studies. Remarkably, the zero-offer is basically optimal for low values of α and high values of β, which result in low-mean Beta distributions. Furthermore, there is a rather visible increasing trend of the performance improvement toward the right-hand side of the figure, where we find gradually higher values of α and lower values of β. Indeed, the combinations of parameters located on the right corner of the figure result in distributions with the highest mean and most negative skewness. Intuitively, it is reasonable that the zero offer becomes less and less efficient as the power distribution shifts closer to the installed capacity.

Fig. 8 illustrates the improvement with respect to offering the forecast conditional mean of wind power production. Once

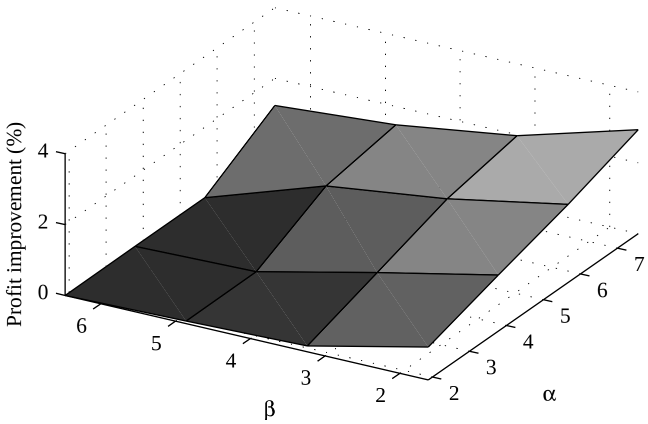

Fig. 7. Profit improvement with respect to offering zero as a function of the parameters α and β of forecast wind power distribution

again, the magnitude of the improvement is consistent with the results obtained so far. Besides, there is a trend specular to the one observed in Fig 7. Indeed, the performance improvement decreases as we move from the left to the right-hand side of the figure. This trend highlights that, with combinations of α and β yielding distributions with high mean and negative skewness, the margin for improvement of the optimal strategy compared to offering the conditional mean decreases.

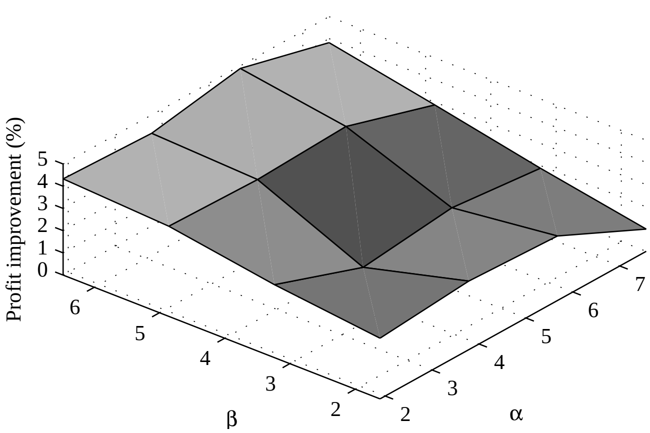

Fig. 8. Profit improvement with respect to offering the forecast mean as a function of the parameters α and β of forecast wind power distribution

Finally, let us point out that the surfaces in Figs. 7 and 8 are obtained from a single simulation per set of parameter values. These surfaces correspond to one of the potential realizations from a stochastic process, the stochasticity coming from how representative these particular sets of scenarios may be. If one wanted to have a full (though very costly) picture of the potential variations in these surfaces, one would have to repeat the simulations several times in a Monte Carlo fashion, therefore obtaining their empirical probabilistic description

VI. Conclusion

This paper considers the optimization problem of a wind power producer being a price-taker at the day-ahead market, but a price-maker at the balancing market. We model this problem as a Mathematical Program with Equilibrium Constraints (MPEC) and cast it as a Mixed-Integer Linear Program (MILP). Uncertainty in day-ahead price, wind power production, and system deviation is modeled by employing scenarios.

Through a case study built from Nord Pool, the Scandinavian electricity market, and considering a one-price settlement of imbalances, we show that the optimal day-ahead bid is different from the zero and the nominal capacity offer, as well as from the forecast conditional mean and median of wind power distribution. This result is non trivial, since for a price-taker producer the optimal bid is either zero or the nominal capacity. The improvement in expected revenues with respect to these strategies amounts to between 1.5% and 3%.

Furthermore, we assess the impact of the producer's market penetration, correlation with the system imbalance and shape of forecast distribution of power production. We find that the optimal offer in the day-ahead market is increasing with market penetration and decreasing with correlation. Besides, the average market value of the energy traded by the wind power producer is a decreasing function of both parameters. Finally, we show a consistent performance improvement up to 5% with respect to offering zero or the mean at the day-ahead market, under a number of different distributions of forecast wind power production.

This work opens up several directions for future research. First of all, it would be interesting to assess the impact of market design on the optimal offering strategy and on the market results for the wind power producer, by modeling e.g., the two-price imbalance settlement. This would shed light on the current debate on the optimal design of balancing markets. Furthermore, the model could be extended so as to allow trading in the intraday market. Besides, considering the offering problem of a wind power producer that is a price-maker at all market stages would be a relevant extension. Modeling the electricity network could be another important upgrade of the model presented. Another topic of research consists in devising a method to solve this problem by using decomposition techniques, capable of exploiting its structure. Finally, modeling competition between wind power producers in the framework of *Equilibrium Problems with Equilibrium Constraints* (EPECs) would be particularly interesting.

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