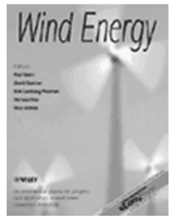

Optimal offering and control policies for wind power in energy and reserve markets

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Optimal offering and control policies for wind power in energy and reserve markets

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Abstract

Proliferation of wind power generation is increasingly making this power source an important asset in designs of energy and reserve markets. Intuitively, wind power producers will require the development of new offering strategies that maximize the expected profit in both energy and reserve markets while fulfilling the market rules and its operational limits. In this paper, we implement and exploit the controllability of the proportional control strategy. This strategy allows the splitting of potentially available wind power generation in energy and reserve markets. In addition, we take advantage of better forecast information from the different day-ahead and balancing stages, allowing different shares of energy and reserve in both stages. Under these assumptions, different mathematical methods able to deal with the uncertain nature of wind power generation, namely stochastic programming, with McCormick relaxation and piecewise linear decision rules are adapted and tested aiming to maximize the expected revenue for participating in both energy and reserve markets, while accounting for estimated balancing costs for failing to provide energy and reserve. A set of numerical examples, as well as a case-study based on real data, allow the analysis and evaluation of the performance and behavior of such techniques.

An important conclusion is that the use of the proposed approaches offers a degree of freedom in terms of minimizing balancing costs for the wind power producer strategically to participate in both energy and reserve markets.

Keywords

Energy and reserve markets; offering strategies; piecewise linear decision rules; stochastic programming; wind power

Nomenclature

The main notation used throughout the paper is stated next for quick reference. Other symbols are defined as required.

Parameters

ε	Coefficient to control the share deviation between both stages

λ	Prices, penalties and unit costs for wind power producers
λ^{cap}	Capacity price for power reserve
λ^{sp}	Spot price for energy
π_w	Probability in each scenario w
G	Lifted support PLDR
L	Lifting operator PLDR
P^{Max}	Maximum total power offer
P^{Min}	Minimum total power offer
Q_w^*	Eventually observed wind power in scenario w
r_i	Line segment PLDR
V	Square matrix with (r+1)*(r+1) dimensions for PLDR
$\hat{W}$	Conditional mean forecast of the wind power distribution
z_j^i	Breakpoints for PLDR

Subscript

w	Wind scenario
i, j	Number of lines

Superscript

$+$	Positive deviation for being long (down-regulation)
$-$	Negative deviation for being short (up-regulation)
$*$	Balancing stage
bpt	Unit penalty cost for the wind power producer
c	Day-ahead stage
pt	Penalty for reserve imbalance

Variables

α	Control strategy (proportional share)
δ	Random variable in PLDR
ΔE	Energy deviation
ΔR	Reserve deviation
μ	Dual variable associated to the inequality constraints in PLDR
E	Energy offer
K	Slope of the linear function for PDLR
Q	Total power offer
R	Power reserve offer

1. Introduction

In many countries, electricity markets are facing the challenge of integrating renewable power (mainly, wind power) into the system while taking into consideration the uncertain production of these type of power plants. Wind power has reached considerable levels of penetration in some power systems, but most of the wind power is not yet fully competitive in the electricity markets because of feed-in tariffs.

Nevertheless, competition in the energy market has been increasing, since wind power plants have grown year after year. Thus, decision-making tools for wind power producers (WPPs) offering in the energy market have been developed in the last few years. In this respect, a large number of studies pursuing an optimal offering strategy for the WPPs in the day-ahead market, accounting with potential balancing costs (assuming a price-taker behavior) can be found in literature. Studies examine the use of strategies for maximizing the expected utility of wind power [1,2], strategies considering risk-analysis and temporal dependencies [3,4], offering in the one-price and two-price system [5], as well as offering under opportunity cost in the imbalance system [6], and many other aspects [7–10]. On the other hand, optimal strategies under the price-maker assumption have emerged [11–16]. Some of these strategies exploit the equilibrium in oligopolistic markets [15] or assume a risk-constrained behavior [16].

Currently, wind power technology allows the WPPs to allocate some of the available power to provide a reserve, thereby enabling them to support some type of ancillary services [17–19]. Thus, different control techniques for the curtailment of the wind power production have arisen in literature, such as proportional and constant control [20], as well as, ΔP and output cap control [21]. Furthermore, this ability will be in new business models for WPPs, since they are now able to initiate their participation in reserve markets. Thus, promising opportunities for increasing revenues by offering in two market products can be exploited. In this new context, with WPPs offering in both energy and reserve markets, it is crucial to develop methods for the optimal offering of wind power in both markets. In this way, some studies have emerged on joint offering of energy and reserve under uncertain production [22–25]. A multi-stage stochastic approach for evaluating under risk analysis the joint participation of wind in both the energy and reserve markets is proposed in [22]. Liang *et al*. [23] developed an analytical approach (based on the multi-newsvendor problem with budget constraint), but assumed that participation in the energy and reserve markets can be independently determined based on a budget constraint. In contrast, an analytical and stochastic approach for determining the wind offer in both energy and reserve markets, assuming wind correlation in both markets (under different control strategies) is proposed in [24] and [25], respectively.

Notwithstanding this, to the best of our knowledge, none of the existing works simultaneously exploits the controllability of control strategies and takes advantage of better forecast information from the different day-ahead and balancing stages in the joint energy and reserve wind offering problem. Thus, this paper exploits the controllability of the proportional control strategy in the day-ahead and balancing markets, taking advantage of the strategy simplicity to implement it in practice (as discussed in [20]). Under this control strategy, we prove that allowing use of different share parameters for energy and reserve between both the day-ahead (first-stage) and balancing markets (second-stage) can result in extra income for the WPPs. Furthermore, this work adapts, tests and validates a number of different approaches (fixed stochastic, flexible stochastic, McCormick relaxation and piecewise linear decision rules – PLDR) for optimal identification of the share of energy and reserve for the WPPs in both the energy and reserve markets. The fixed stochastic approach considers a fixed share of energy and reserve in both market stages and serves as the basis for comparison with the remaining methods. Moreover, all proposed methods are demonstrated, validated and compared on the basis of numerical examples (as well as on a case study based on real-data), while seeking to improve the income of the WPPs, as well as the wind power participation in a wide range of services in electricity markets. This allows increased penetration in the power system.

The paper is organized as follows. Section 2 briefly introduces wind power participation in the energy and reserve markets, detailing the impact and required market changes for allowing WPPs to participate in the reserve market. Section 3 presents the general formulation for wind power revenue in both the energy and reserve markets. Section 4 describes in detail the proposed approaches (i.e. the fixed and flexible stochastic, the McCormick and PLDR methods). Section 5 verifies, tests and compares all approaches on a set of numerical examples, as well as on a case study concerning real data. Finally, conclusions and future work are gathered in Section 6.

2. Wind offering in energy and reserve markets

The current developments in wind-turbine technology have encouraged the WPPs to show interest in the reserve market, thereby seeking extra revenues. New supply of different market products has encouraged the WPPs to rethink their strategic behavior in the wholesale market. On the one hand, the WPPs may split their available wind power into different markets to improve profit and reduce the risk of participating on one single product. On the other hand, potential penalties for power balancing deviations in both market products must be taken into account to avoid significant penalties that may reduce the expected income from both market products.

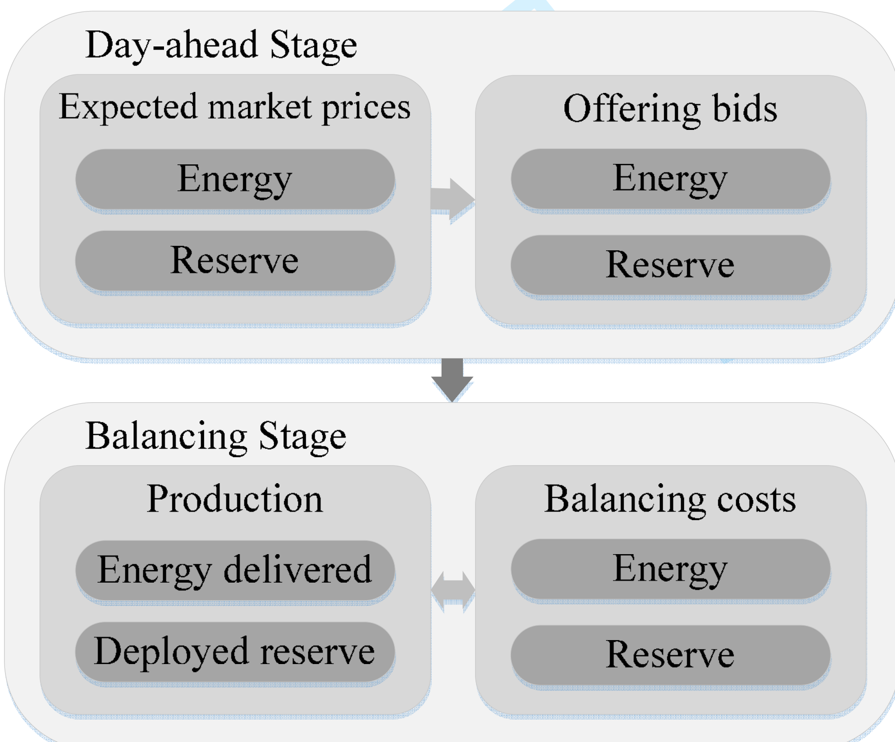

Figure 1. Wind power model in the energy and reserve markets.

In this respect, a strategic market model for the WPPs to participate in the energy and reserve markets is studied. This model allows WPPs to offer their bids for energy and reserve in the day-ahead market while accounting with expected balancing costs for missing expected production of energy and reserve during the balancing stage (Fig.1). Additionally, (through control strategies) the model allows that the share of energy and reserve established in the day-ahead stage (ratio between energy and reserve) can be different in the balancing stage, thereby allowing WPPs to minimize their power deviations while reducing expected balancing costs. This model characteristic is important in that it allows WPPs to use better information about their wind power forecast (closer to the energy delivery) to define the share of energy and reserve assumed in the balancing stage. In more detail, this flexibility allows WPPs to push decisions close to the real-time, thereby improving the quality of their decisions and reducing the lead time effect between the day-ahead decisions and the energy and services delivered.

In terms of system reliability, this flexible characteristic of the model may reduce to some extent the uncertainty of the wind power production, since it uses better forecast information (closer to real-time) to define the energy and reserve share in the balancing stage. Thus, the flexible approach can decrease the energy and reserve deviations between day-ahead and balancing stage in comparison to the approach of same energy and reserve share between the day-ahead and balancing stage. This means that, to some extent, the level of reserve needed in the system can be slightly reduced if the wind power producer can change its energy and reserve share.

However, this strategic offering will require some changes in current market rules. For instance, the WPPs should be allowed to offer in the reserve market in a strategic way. A smart and smooth way of introducing wind power in current reserve markets is to allow the WPPs and conventional generators to jointly offer in the reserve market. Thus, conventional generators can be used to some extent to cover the uncertainty of the wind power producer. However, further changes are still required for full participation of wind power in the reserve market. For example, introducing a new reserve penalty scheme to penalize WPPs for power deviations in the reserve market could force uncertain WPPs to offer their potential available power with some level of certainty, as suggested in [23,24] and implemented in the proposed model. Furthermore, the reserve market design with high penetration of uncertain generation may require operating closer to the delivery, since reserve requirements may dynamically vary on and hourly or even minute-by-minute basis [26]. Thus, WPPs will very probably be called (even forced to some extent) to contribute under these new service conditions. In this context, the flexible characteristic proposed in this work may to some extent cover the participation of WPPs in these new market features, where market decisions are made closer and closer to the delivery.

3. General formulation of market revenues

A general formulation for the revenue of the WPPs in energy and reserve markets is presented in [24]. Following a stochastic programming approach from that, the maximization of the revenue from day-ahead and reserve markets, accounting for the penalties from the balancing market can be expressed as

$$Rev = \lambda^{cap} R^c + \sum_{w \in \Omega} \pi_w \left[\lambda^{sp} E_w^* - T_w^* - O_w^* \right] \tag{1}$$

where λ^{sp} is the expected spot price, E_w^* is the amount of expected delivered energy in scenario w, λ^{cap} is the expected capacity price for contracting reserve, R^c is the expected contracted level of power reserve in day-ahead stage, T_w^* is the balancing costs from the energy deviations, Z_w^* is the expected penalty cost for failing to provide the scheduled reserve and π_w is the probability in each scenario w. Time indices are not used, since all variables and parameters are for the same market time unit.

Additionally, it is assumed that the WPP behaves as a price-taker, which means that the production of the WPP is independent of market prices and penalties. Following that behavior and the certainty equivalent theory [9,27], all the prices are linear in the expressions below.

The balancing costs for energy deviations are usually defined as

$$T_w^* = \begin{cases} \lambda^{*,+} \left(E_w^* - E^c \right) & E_w^* - E^c \geq 0 \\ -\lambda^{*,-} \left(E_w^* - E^c \right) & E_w^* - E^c < 0 \end{cases} \tag{2}$$

where (E_w^* - E^c) is the energy imbalance between the energy delivered E_w^* and the energy contracted (offered) E^c. The variables $\lambda^{*,+}$ and $\lambda^{*,-}$ are the regulation unit costs for positive and negative deviations, i.e.,

$$\begin{aligned} \lambda^{*,+} &= \lambda^{sp} - \lambda^{c,+} \\ \lambda^{*,-} &= \lambda^{c,-} - \lambda^{sp} \end{aligned} \tag{3}$$

where $\lambda^{c,+}$ is the unit down-regulation price for being long, while $\lambda^{c,-}$ is the up-regulation price for being short. Additionally, we consider the two-price settlement rule as in the NordPool for mapping the balancing costs for energy deviations [1]. The settlement rule is part of the balancing mechanism for pricing the deviations from day-ahead contracts to the delivered production at the balancing stage. One-price and two-price system rules are not discussed in detail in this work. Instead, interested readers are referred to [28]. In cases of negative system imbalance (energy surplus – need for downward regulation), it holds that

$$\begin{aligned} \lambda^{c,+} &\leq \lambda^{sp} \\ \lambda^{c,-} &= \lambda^{sp} \end{aligned} \tag{4}$$

Otherwise, when system imbalance is positive (energy deficit – need of upward regulation), it comes to

$$\begin{aligned} \lambda^{c,+} &= \lambda^{sp} \\ \lambda^{c,-} &\geq \lambda^{sp} \end{aligned} \tag{5}$$

In hours of perfect balance, both $\lambda^{c,+}$ and $\lambda^{c,-}$ are equal to the spot price λ^{sp}. In parallel, the costs for the imbalance on the reserve product are formulated based on the one-price system rule (since the penalty for failing to provide this service is directly related to the up/down-regulating price for imbalance of the power reserve), such that

$$O_w^* = \begin{cases} \lambda^{bpt,+}\left(R_w^* - R^c\right) & R_w^* - R^c \geq 0 \\ -\lambda^{bpt,-}\left(R_w^* - R^c\right) & R_w^* - R^c < 0 \end{cases} \tag{6}$$

where (R_w^* - R^c) refers to the reserve power imbalance between the realized level of reserve R_w^* in the balancing stage and the reserve contracted (offered) R^c in the day-ahead stage. $\lambda^{bpt,+}$ is the unit penalty for the WPP when generating more power than that contracted (surplus). In contrast, $\lambda^{bpt,-}$ is the unit penalty cost when the WPP generates less power than that contracted. It holds that

$$\begin{aligned} \lambda^{bpt,+} &= \lambda^{cap} - \lambda^{pt,+} \\ \lambda^{bpt,-} &= \lambda^{pt,-} - \lambda^{cap} \end{aligned} \tag{7}$$

hence $\lambda^{bpt,+}=0$ since (extra) positive reserve is not detrimental to the system's reliability. $\lambda^{pt,-}$ is the penalty for negative reserve imbalance, weighted by the probability that reserve is needed.

4. Optimal offer formulation

Although, several wind control techniques have been emerging, few considerations on the strategic implementation of these control techniques in the market perspective have been made [24,25]. As demonstrated in [24], the proportional wind control technique [20] presents a logical and simple strategic behavior in terms of participation in the energy and reserve markets. Thus, we build our different optimization approaches based on this assumption. Furthermore, we have improved the proposed approaches, allowing the WPP to establish different shares of energy and reserve in both the day-ahead and balancing stages.

4.1. Flexible stochastic approach

The full flexible stochastic approach for the revenue of WPPs in the energy and reserve markets is given by

$$max \ \lambda^{cap} R^c + \sum_{w \in \Omega} \pi_w \left[\lambda^{sp} E_w^* - \lambda^{*,+} \Delta E_w^+ - \lambda^{*,-} \Delta E_w^- - \lambda^{bpt,-} \Delta R_w^- \right] \tag{10a}$$

$$s.t. \ \ P^{Min} \leq Q^c \leq P^{Max} \tag{10b}$$

$$E^c + R^c = Q^c \tag{10c}$$

$$E_w^* + R_w^* = Q_w^* \qquad \forall w \in \Omega \tag{10d}$$

$$E^c - E_w^* = \Delta E_w^- - \Delta E_w^+ \qquad \forall w \in \Omega \tag{10e}$$

$$R^c - R_w^* \leq \Delta R_w^- \qquad \forall w \in \Omega \tag{10f}$$

where ΔE^+ is the excess of energy incurred by the WPP, ΔE^- is the deficit of energy incurred by the WPP, ΔR^- is the deficit of reserve incurred by the WPP, P^{Min} and P^{Max} are the bounds of the total power offer in the day-ahead stage. This approach is characterized by its total freedom to choose the energy and reserve share in each stage of the problem, i.e. the WPPs can take advantage of the intermediate information about wind power production, thereby reducing the expected costs at the balancing stage. This means that the WPP can adjust the share of energy and reserve in the balancing stage in line with the expected power production in each scenario w.

4.2. Fixed stochastic approach

The fixed stochastic approach relies on the concept of constraining the use of information in the balancing stage to help in the day-ahead decision. For this purpose, the proportional control strategy is used. The control strategy consists in the proportional split of the energy and reserve given by α^c. In addition to the mathematical formulation from the flexible approach, constraints (11a) to (11e) are included, thus representing the proportional strategy.

$$E^c = \alpha^c Q^c \tag{11a}$$

$$R^c = (1-\alpha^c) Q^c \tag{11b}$$

$$E^*_w = \alpha^*_w Q^*_w \qquad \forall w \in \Omega \tag{11c}$$

$$R^*_w = (1-\alpha^*_w) Q^*_w \qquad \forall w \in \Omega \tag{11d}$$

$$\alpha^*_w = \alpha^c \qquad \forall w \in \Omega \tag{11e}$$

where the energy and reserve offered in the day-ahead market is determined in (11a) and (11b), respectively. Both constraints contain bilinear terms (since two different variables are multiplying, the total power offer Q^c and the control share α^c which split the total power offer into energy and reserve to offer in the day-ahead market), thus forming a system of bilinear equations which is non-convex. The non-convexity of both equations makes the problem more complex, but it is still feasible with proper solvers. The problem has been carried out with CONOPT [29] as a Non-Linear Programming (NLP) solver.

Besides this, constraints (11c) and (11d) determine the energy and reserve share in the balancing stage for each scenario *w*. The fixed behavior of this approach is achieved by assuming that the control parameter for splitting the energy and reserve remains the same in both the day-ahead and balancing stages, i.e., $\alpha^* = \alpha^c$. This constraint (11e) is inferred in the stochastic models as the non-anticipativity constraint, thereby preventing the WPPs from using information close to the balancing stage to influence day-ahead decisions.

4.3. Stochastic approach under McCormick relaxation

A hybrid system between the flexible and fixed approach is proposed in this section, with two distinct goals. The first one aims to turn convex the bilinear constraints from the fixed approach by using McCormick relaxation theory [30]. It is noteworthy that McCormick relaxation theory has been chosen among other relaxation techniques due to its ability to provide a tight approximation gap of the bilinear constraints and easy implementation. The second goal aims to control the influence of balancing stage information in the day-ahead decisions, by means of a coefficient to bound the deviation between the share parameter in the day-ahead and balancing stages. This goal emerges with the perspective of giving the WPP some controllability of the use of information close to the real-time, thereby allowing WPPs to quantify the level of anticipatory decision of the balancing stage, which cannot be provided through the fixed or flexible approaches.

Under the assumptions of the fixed stochastic approach, the McCormick relaxation theory [30] is used to relax the bilinear constraints and turn the problem convex. McCormick's relaxation provides a very good approximation of the bilinear terms, ensuring that the problem is convex, and thereby requiring only traditional Linear Programing (LP) methods to solve the problem, which ensures optimal solutions. Thus, the objective function (10a) is subjected to

$$E^{c} \geq P^{Min}\alpha_{w}^{*} \qquad \forall w \in \Omega \tag{12a}$$

$$E^{c} \leq P^{Min}\alpha_{w}^{*} + Q^{c} - P^{Min} \qquad \forall w \in \Omega \tag{12b}$$

$$E^{c} \leq \alpha_{w}^{*}P^{Max} \qquad \forall w \in \Omega \tag{12c}$$

$$E^{c} \geq \alpha_{w}^{*}P^{Max} + Q^{c} - P^{Max} \qquad \forall w \in \Omega \tag{12d}$$

$$-\varepsilon \leq \alpha^{c} - \alpha_{w}^{*} \leq \varepsilon \qquad \forall w \in \Omega \tag{12e}$$

where (12a) to (12d) is the result of the relaxation of the two bilinear constraints of the fixed approach. On the other hand, the control of the influence of balancing stage information in day-ahead decisions is modeled in (12e), where ε is a coefficient that defines the difference between the share parameter in both the day-ahead and balancing stages. The coefficient varies between 0 and 1, thus influencing the behavior of the split between energy and reserve. If ε is close to 0, the behavior of this approach is close to the fixed stochastic approach. Otherwise, when ε is close to 1, this approach tends to behave similarly to the flexible stochastic approach. Additionally, the model stays complete with the inclusion of (10c) to (10f), and (11c), representing the split of the available power in energy and reserve in both stages, and the deviation of energy and reserve in the balancing stage, respectively.

4.4. Piecewise linear decision rules with axial segmentation

A different and interesting way of modeling the recourse function of the two-stage stochastic problem of WPPs offering in day-ahead market while accounting for balancing costs is through linear decision rules. Linear decision rules are often used to linearly model the uncertainty of the problems, since it can provide tractable upper and lower bounds of the stochastic program. Indeed, the main reason why linear decision rules are used instead of stochastic programing is because it does not need discrete distribution of the uncertain parameter in contrast with stochastic programming. However, the linearization of uncertain variables often comes with rough approximations of the uncertainty, since the uncertainty can behave very differently from linear functions. Thereby, the solution's quality provided by this method can leave much to be desired.

One way to reduce the approximation gap of traditional linear decision rules is by defining uncertainty through a piecewise linear function. This approach increases the flexibility of the LDR method by approximating to the natural recourse function of the problem, however, the problem size grows significantly. An illustrative example of the decisions made by the PLDR is shown in Fig. 2. One can see that the piecewise approach improves the flexibility of the decisions in comparison with simple linear decision rules.

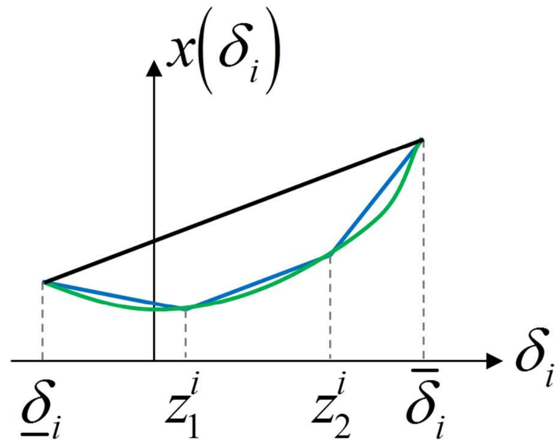

Figure 2. Illustrative example of a natural recourse function (green), linear decision rules approximation (black) and piecewise linear decision rules approximation (blue) under the realization of the uncertain parameter.

In this context, we apply the method of piecewise linear continuous decisions rules with axial segmentation, as developed in [31], to ensure that a good linear approximation to the recourse problem is achieved. The method requires the establishment of breakpoints to model the piecewise function. For instance, quantiles of the wind power distribution can be used to define manually these breakpoints. An improved technique of the PLDR (the PLDR with general segmentation) allows optimal estimation of these breakpoints, although the complexity of the problem increases significantly. Thus, we restrict our attention to the PLDR with axial segmentation, by which specific breakpoints for the piecewise function are defined.

The PLDR with axial segmentation idea is to expand the sample space of the uncertain parameter δ_i into r_i lines with r_i-1 breakpoints z_j^i for $j\in\{1,\ldots,r_i-1\}$ and $i\in\{1,\ldots,k\}$

$$\underline{\delta}_i < z_1^i < \cdots < z_{r_i-1}^i < \overline{\delta}_i \qquad \forall i\in\{2,\ldots,k\} \tag{13a}$$

where $\underline{\delta}_i$ is the lower bound and $\overline{\delta}_i$ the upper bound of δ_i.

Following [31], one can introduce the lifted space $\mathbb{R}^{k'}$ of the piecewise linear parameters $\delta_i' \in \mathbb{R}^{r_i}$ in the lifted support G', where $\delta_i' \in G'$ and $\delta' = (1, \delta_2'^{\top}, \ldots, \delta_k'^{\top})^{\top}$.

Thus, the breakpoints are used to define the lifting operator $L_{i,j}$ as

$$L_{i,j}(\delta)=\begin{cases}\delta_i & if\ r_i=1\\ \min\{\delta_i, z_1^i\} & if\ r_i>1, j=1\\ \max\{\min\{\delta_i, z_j^i\}-z_{j-1}^i, 0\} & if\ r_i>1, j=2,\ldots,r_i-1\\ \max\{\delta_i - z_{j-1}^i, 0\} & if\ r_i>1, j=r_i\end{cases} \tag{13b}$$

where $j\in\{1,\ldots,r_i\}$ and $i\in\{2,\ldots,k\}$. The retraction operator converts the lifted parameters into the original parameters through

$$\hat{G}' = \left\{\delta' \in R^{k'} : V_i' \begin{pmatrix}1\\ \delta_i'\end{pmatrix} \geq 0 \quad \forall\, i \in \{2,\ldots,k\}\right\} \tag{13c}$$

such that V is a square matrix with (r_i +1)x(r_i +1) dimensions, defined as

$$V_i^{'} = \begin{pmatrix} \frac{z_1^i}{z_1^i-\underline{\delta}_i} & -\frac{1}{z_1^i-\underline{\delta}_i} & & & & \\ -\frac{\underline{\delta}_i}{z_1^i-\underline{\delta}_i} & \frac{1}{z_1^i-\underline{\delta}_i} & -\frac{1}{z_2^i-z_1^i} & & & \\ & & \frac{1}{z_2^i-z_1^i} & \ddots & & \\ & & & \ddots & -\frac{1}{z_{r_i-1}^i-z_{r_i-2}^i} & \\ & & & & \frac{1}{z_{r_i-1}^i-z_{r_i-2}^i} & -\frac{1}{\overline{\delta}_i-z_{r_i-1}^i} \\ & & & & & \frac{1}{\overline{\delta}_i-z_{r_i-1}^i} \end{pmatrix} \quad (13d)$$

By applying these theorems to the wind offering problem, it is assumed that the share of energy and reserve in the day-ahead and balancing stages can be different (same assumption of the flexible approach). Thus, the PLDR approach is applied under the flexible approach formulation detailed in section 4.1. Suppose that the wind power uncertainty is expressed in the following piecewise linear form

$$\tilde{W} = \hat{W} + \sum_{i=1}^{r_i} K_i^W \delta_i^{'} \quad (14a)$$

where δ_i' is the random variable in each line i, K_i^W is the slope parameter of the linear function in each line i and $\widehat{W}$ is the conditional mean forecast of the wind power distribution which does not depend on the actual realization of the uncertainty δ. The second stage variables also behave piecewise linearly

$$E^* = \hat{E}^* + \sum_{i=1}^{r_i} K_i^E \delta_i^{'} \quad (14b)$$

$$R^* = \hat{R}^* + \sum_{i=1}^{r_i} K_i^R \delta_i^{'} \quad (14c)$$

$$\Delta E^+ = \Delta\hat{E}^+ + \sum_{i=1}^{r_i} K_i^{\Delta E^+} \delta_i^{'} \quad (14d)$$

$$\Delta E^- = \Delta\hat{E}^- + \sum_{i=1}^{r_i} K_i^{\Delta E^-} \delta_i^{'} \quad (14e)$$

$$\Delta R^- = \Delta\hat{R}^- + \sum_{i=1}^{r_i} K_i^{\Delta R^-} \delta_i^{'} \quad (14f)$$

4.4.1. Equality constraints reformulation

Let us consider the equality constraints (10d) and (10e) of the second stage problem (model in section 4.1. for the flexible approach) for reformulation. This means that only the equality constraints with uncertain variables from the flexible approach formulation are considered for reformulation. By replacing the recourse variables of (10e) with the PLDR previously defined, it gives

$$E^c - \left(\hat{E}^* + \sum_{i=1}^{r_i} K_i^E \delta_i^{'} \right) = \Delta\hat{E}^- + \sum_{i=1}^{r_i} K_i^{\Delta E^-} \delta_i^{'} - \left(\Delta\hat{E}^+ + \sum_{i=1}^{r_i} K_i^{\Delta E^+} \delta_i^{'} \right) \tag{15a}$$

Following [32–34], this equality constraint can be reformulated in a way to eliminate the random variable δ and assure finite cardinality, hence

$$E^c - \hat{E}^* = \Delta\hat{E}^- - \Delta\hat{E}^+ \tag{15b}$$

$$-K_i^E = K_i^{\Delta E^-} - K_i^{\Delta E^+} \qquad \forall i \tag{15c}$$

Similar reformulation is performed for the other equality constraint (10d), where its final form is assumed as

$$\hat{E}^* + \hat{R}^* = \hat{W} \tag{15d}$$

$$K_i^E + K_i^R = K_i^W \qquad \forall i \tag{15e}$$

4.4.2. Inequality constraint reformulation

Let us consider the inequality constraint (10f) of the second stage problem for reformulation. By replacing the PLDR for the recourse decision variables, this inequality can be reformulated as

$$\min_{\delta^{'}} \left\{ \begin{array}{l} \sum_{j=1}^{r} K_j^R \delta_j^{'} + \sum_{j=1}^{r} K_j^{\Delta R^-} \delta_j^{'} \\ s.t. \sum_{j=1}^{r} V_{i,(j+1)}^{'} \delta_j^{'} \geq -V_{i,1}^{'} : \mu_i \qquad \forall i \end{array} \right\} \geq R^c - \hat{R}^* - \Delta\hat{R}^- \tag{16a}$$

where μ_i is the dual variable associated to the i-th inequality constraint. By applying duality theory [32,35] on the minimization problem on the left-hand-side of the above inequality, one can transform it to the following maximization problem

$$\max_{\mu} \left\{ \begin{array}{ll} \sum_{i=1}^{r+1} -V_{i,1}^{'} \mu_i & \\ s.t. \sum_{i=1}^{r+1} V_{i,(j+1)}^{'} \mu_i = K_j^R + K_j^{\Delta R^-} & \forall j \\ \mu_i \geq 0 & \forall i \end{array} \right\} \geq R^c - \hat{R}^* - \Delta\hat{R}^- \tag{16b}$$

The equivalent representation of the above problem in a system of constraints is

$$\sum_{i=1}^{r+1} -V_{i,1}^{'} \mu_i \geq R^c - \hat{R}^* - \Delta\hat{R}^- \tag{17a}$$

$$\sum_{i=1}^{r+1} V_{i,(j+1)}^{'} \mu_i = K_j^R + K_j^{\Delta R^-} \qquad \forall j \tag{17b}$$

$$\mu_i \geq 0 \qquad \forall i \tag{17c}$$

where the set of inequalities (17) has finite cardinality. Moreover, it is noteworthy that all recourse decision variables (in its piecewise linear form 14b – 14f) from the second stage problem are positive variables. Thus, it is required performing similar reformulations to all these inequality constraints affected by the uncertainty.

4.4.3. Objective function reformulation

The reformulation of the objective function leads to the employment of the PLDR as expressed in (14). This yields

$$\begin{aligned}\rho = {} & \lambda^{cap} R^{c} + \lambda^{sp}\left(\hat{E}^{*} + \sum_{i=1}^{r_i} K_i^{E}\,\mathrm{E}\left[\delta_i'\right]\right) \\ & -\lambda^{*,+}\left(\Delta\hat{E}^{+} + \sum_{i=1}^{r_i} K_i^{\Delta E^{+}}\,\mathrm{E}\left[\delta_i'\right]\right) - \lambda^{*,-}\left(\Delta\hat{E}^{-} + \sum_{i=1}^{r_i} K_i^{\Delta E^{-}}\,\mathrm{E}\left[\delta_i'\right]\right) \\ & -\lambda^{bpt,-}\left(\Delta\hat{R}^{-} + \sum_{i=1}^{r_i} K_i^{\Delta R^{-}}\,\mathrm{E}\left[\delta_i'\right]\right)\end{aligned} \tag{18a}$$

The expectation over the uncertain parameter for the PLDR is a model based on the lifting operator matrix $L_{i,j}$. and the probability for each line segment π^r

$$\mathrm{E}\left[\delta_i'\right] = \sum_{j=1}^{r+1}\left(\frac{L_{j(z_i)} + L_{j(\bar{z}_i)}}{2}\right)\pi_i^{r} \tag{18b}$$

Thus, by replacing the expectation calculus (18b) of the uncertain parameter δ_i' in the objective function (18a), one can obtain the final form of the objective function for the PLDR model.

4.4.4. Final model with compact formulation

Finally, the wind offering problem under PLDR assumes its piecewise linear form as

$$\begin{aligned} & \max \ (18a) \\ & \quad s.t. \ (10b), (10c) \\ & \qquad (15b)-(15e) \\ & \qquad (17a)-(17c) \\ & \qquad E^{*}, P^{*}, \Delta E^{+}, \Delta E^{-}, \Delta P^{-} \geq 0 \ \text{ in form of (17)} \end{aligned} \tag{19}$$

where (10b) and (10c) represents the constraints for the first stage decision making process, detailed in section 4.1. Furthermore, all positive second stage decision variables must be in their piecewise linear form and represented by the set of inequalities as in (17).

5. Evaluation of wind offering methods

5.1. Illustrative case

An illustrative example to test and evaluate the proposed approaches has been performed under specific assumptions and parameters. Supposing a wind power plant of 12 MW with a set of 100 wind power scenarios from [36]. It is assumed that all the wind power scenarios have equal probability. Additionally, a minimum power offer of 3 MW is established as a minimum requirement to participate in the market, ensuring some proper profit to the wind power producer.

Furthermore, a set of prices for energy and reserve, as well as unit costs for energy and reserve deviations during the balancing stage is gathered in Table I.

Table 1

In order to design the piecewise function, a set of breakpoints needs to be defined. For this numerical example, three breakpoints have been established, corresponding to the quantiles of 25%, 50% and 75% of the wind power distribution, respectively.

The behavior of each approach under this test case for offering in the energy market is depicted in Fig. 3. Observe the different behavior for each approach. The fixed stochastic approach places a high offer of energy in the day-ahead market (about 7.8 MW), while the flexible, McCormick and the PLDR methods place small energy offers, thereby allocating most of the available power to the reserve market. In this case, the fixed approach is the most risk-averse method by offering a high bid in the energy market, to the detriment of participating in the reserve market. The behavior of the fixed method can be explained by the inclusion of the non-anticipativity constraint (fixing the share parameter in both day-ahead and market stages), since it blocks the proper use of the better information about the expected production during the balancing stage. The PLDR approach tends to follow the behavior shown by the flexible and McCormick approaches, however, one can observe the slope change of the linear function around the breakpoints.

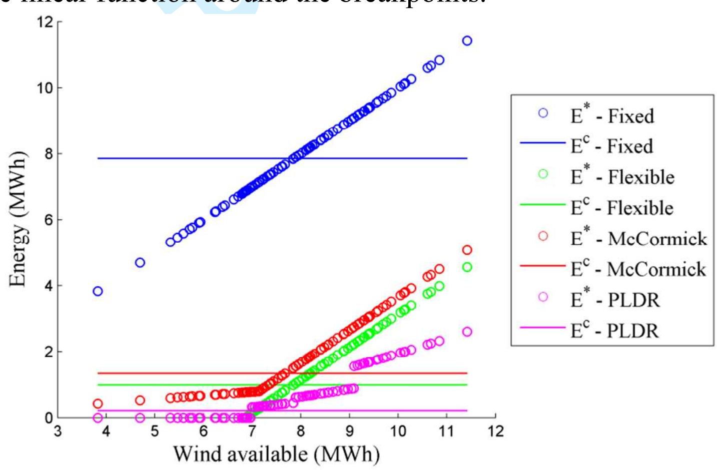

Figure 3. Behavior of energy offered (E^c) and delivered (E^*) in the market for fixed, flexible, McCormick with ε=1 and piecewise linear decision rule methods.

The participation of the wind power producer in the reserve market for each approach is presented in Fig. 4. One can verify that there is no participation in the reserve market through the fixed approach. The fixed approach presents an all-or-nothing behavior, where all the available power is submitted to one single market (energy or reserve, depending on the relation between prices and penalties for deviations). On the other hand, the remaining approaches reserve some power to participate in the reserve market. Thus, the flexible approach is the method that allows a full degree of freedom for the decision making process under the participation in both markets, taking into account penalties in the balancing stage. Furthermore, the McCormick approach presents a very similar behavior to the flexible approach, however the differences in the performance are due to the relaxation of the non-linearity in the proportional strategy.

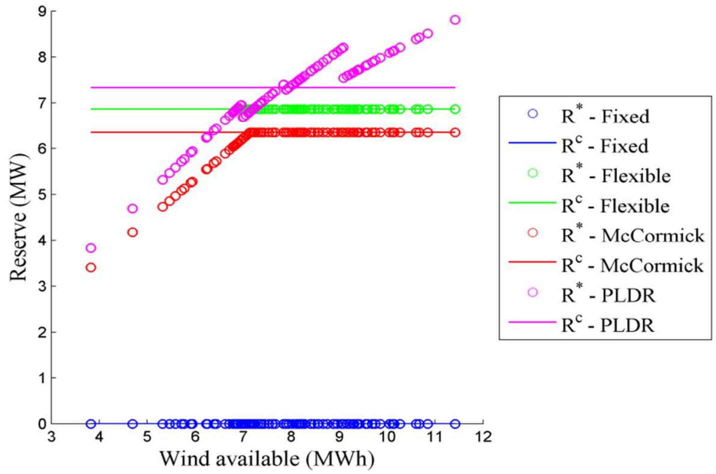

Figure 4. Behavior of reserve offered (R^c) and deployed (R^*) in the market for fixed, flexible, McCormick with ε=1 and piecewise linear decision rules methods.

It is noteworthy that, for lower levels of wind power available, both flexible and the PLDR approaches fully allocate the available wind power to the reserve market, thereby reducing the expected costs in the balancing stage. i.e. by allocating all the available power to the reserve market, the expected costs in this market will be lower, since the penalty for failing to provide power reserve ($\lambda^{bpt,-}$) is much higher than the penalty for failing to provide energy ($\lambda^{*,-}$). On the other hand, for levels of available wind power higher than the bid offered in the reserve market (P^c), both the flexible and McCormick approaches establish the deployed reserve (P^*) as equal to that offered in the day-ahead market, then allocating the remaining available power to the energy market. In contrast, the PLDR approach reserves more power in the balancing stage (P^*) than is offered in the reserve market (P^c), thus assuming a loss opportunity cost for ensuring this robustness.

The parameter representing the share between energy and reserve at day-ahead and balancing stage is shown in Fig. 5. The flexible, McCormick and the PLDR methods have a behavior in which most of the available power is submitted to the reserve market in the day-ahead stage, while in the balancing stage there is a trend to increase the share for providing energy in cases of medium-high levels of the available power (above 7 MW). Permission to establish different share parameters among the trading floors increases the freedom for decision-making, thereby increasing the potential profit.

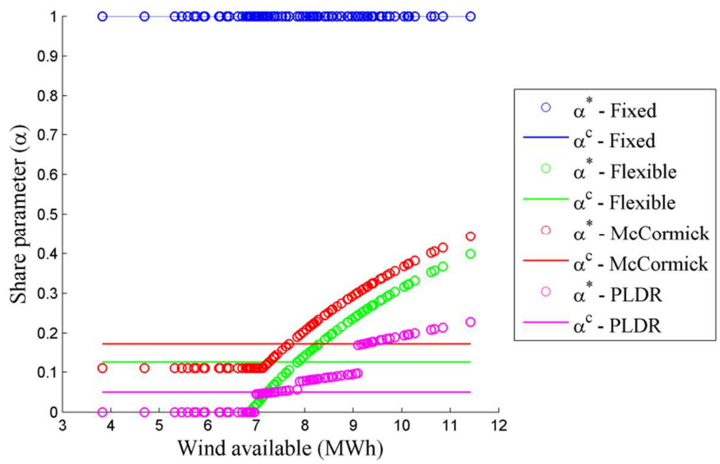

Figure 5. Share parameter in the day-ahead (α^c) and balancing stage (α^*) for fixed, flexible, McCormick with ε=1 and piecewise linear decision rules methods.

The expected revenue for each method is shown in Table 2. It is noteworthy that the flexible approach has the highest expected revenue, while in the opposite side is the fixed approach. The approach with McCormick relaxation has been performed for different ε (ε=1 and ε=0.01). In more detail, the approach with ε=1 obtains an expected revenue and behavior close to the flexible approach, while the approach with ε=0.01 performed closely to the fixed approach results, as expected. This is true, since ε defines the deviation between the share parameter in both the day-ahead and balancing stages. The PLDR model obtains expected revenue very close to the fixed approach, although with distinct energy and reserve solutions. The PLDR model could get better expected revenue by allocating more available power (for middle-high levels of available wind power) to the energy market, thereby reducing the loss opportunity cost (i.e. reducing the excess of available power that is reserved).

Table 2

In order to analyze the energy and reserve distributions under potential realization of the wind scenarios, box plots for energy (E^*) and reserve (R^*) are illustrated in Fig. 6 and Fig. 7, respectively. Thus, Fig. 6 displays the distribution in a standardized way of the expected delivered energy. One can see that each approach has a certain pattern for distributing the available wind power to energy in each scenario. The PLDR approach is the approach with the smaller interquartile range, which means that the dispersion of the data set is closer to the median of the distribution compared with the other approaches. It is noteworthy that the flexible, McCormick with ε=1 and PLDR approaches just focus on a small part of the available wind power to produce energy than the fixed and McCormick (ε=0.01) approaches. In contrast, the remaining available wind power is allocated to provide a power reserve, as can be seen in Fig. 7.

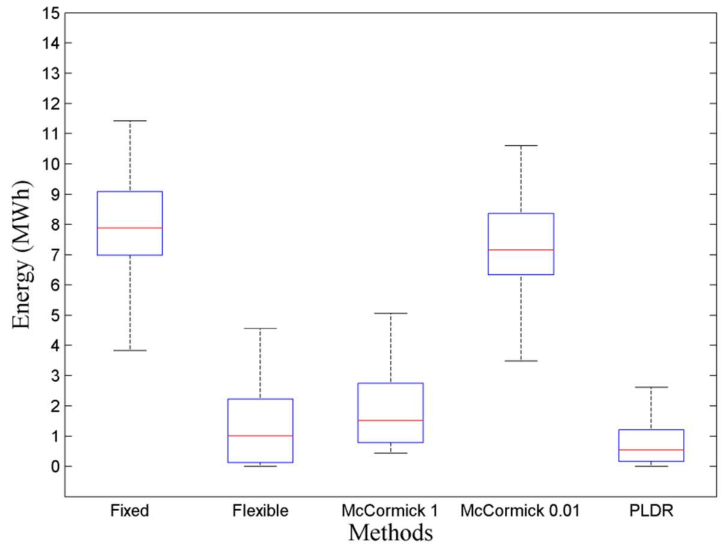

Figure 6. Box plot for energy amounts settled through the balancing stage by the proposed methodologies (fixed, flexible, McCormick with ε=1 and ε=0.01, and piecewise linear decision rules).

In what concerns the supply of power reserve, the fixed approach does not provide reserve, so the distribution data set of the variable R^* is 0. From Fig. 7, one can observe a small range of variation in providing reserve, which makes sense, since the penalty for failing to provide reserve is substantially higher than the case of the energy penalties. Thus, the interquartile range is close to zero, i.e. the distribution data is concentrated in the median of the distribution. However, some of the approaches (e.g. flexible, McCormick with ε=1 and PLDR) scheduled low values of reserve in a few scenarios, being such scenarios represented by the suspected outliers and outliers depicted in Fig. 7.

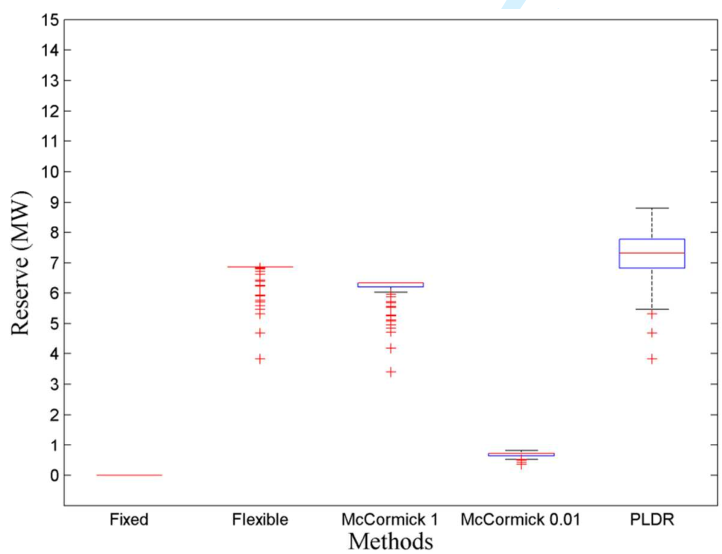

Figure 7. Box plot for reserve deployment in the balancing stage by the proposed methodologies (fixed, flexible, McCormick with ε=1 and ε=0.01, and piecewise linear decision rules).

5.2. Sensitivity analysis for McCormick stochastic approach

A sensitivity analysis for the stochastic approach under the McCormick relaxation is performed. We analyze different values for the deviation (ε) between the day-ahead and balancing share parameters. Thus, Fig.8 shows the behavior of the energy offered in the day-ahead stage and delivered in the balancing stage for this methodology under different values of ε. One can observe that, for small deviations of ε (α^c-α_w^*), the energy offered and delivered approximates to the results of the fixed stochastic approach shown in Fig.4, as expected. Besides that, intermediate results are ensured by the methodology with ε=0.05. It is noteworthy that, as long as the coefficient ε increases, the methodology results tend to converge to the flexible stochastic approach presented in Fig.4.

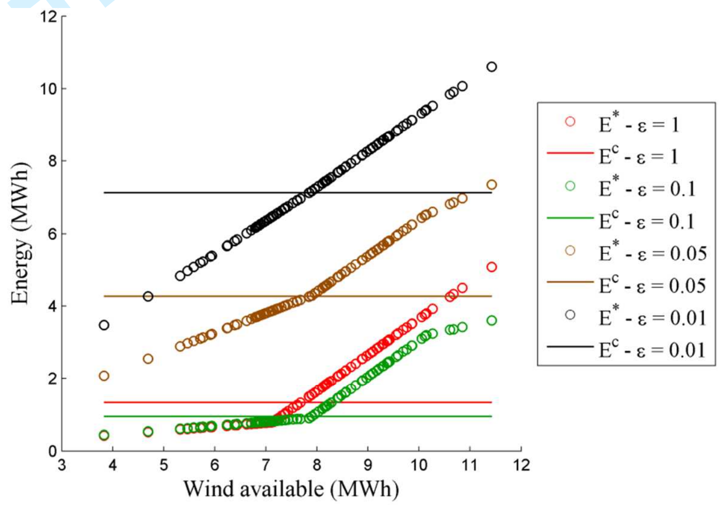

Figure 8. Energy offered (E^c) and delivered (E^*) for McCormick approach under different share deviations (ε).

The expected offer and deployed reserve for the McCormick approach with different values of ε, are depicted in Fig.9. As expected, the behavior of this approach in the reserve market is similar to what is presented in Fig.5. As long as the value of ε decreases, the power offered in the reserve market also decreases. It is noteworthy that there is similar behavior between the approach with ε=0.1 and ε=1, where a higher level of reserve offer is settled for the approach with ε=0.1. In a closer view of high levels of available wind power, the approach with ε=0.1 deploys more reserve than that offered in the day-ahead stage, resulting in a loss opportunity cost to the wind power producer.

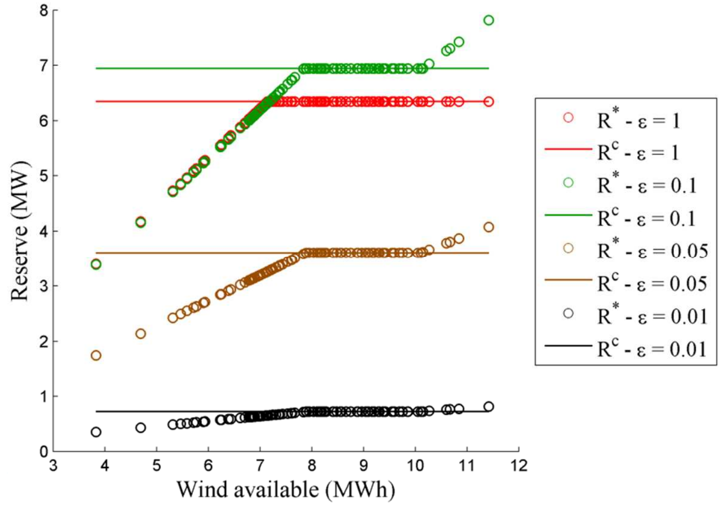

Figure 9. Reserve offered (R^c) and deployed (R^*) for McCormick approach under different share deviations (ε).

The performance of the share parameters in both stages is shown in Fig.10. Note that α_w^* tends to follow the share established in the day-ahead stage. Additionally, it is interesting to note that the performance of the share parameter based on ε=1 and ε=0.1 is very similar. In fact, for lower levels of available wind power (lower than 7 MW), both conjectures get similar results, which means that the constraints concerning ε are not bidding. Moreover, for the conjecture with ε=1, it is clear that α_w^* does not closely follow α^c due to its degree of freedom in the methodology, while the opposite occurs for the conjecture of ε=0.1 with available wind power lower than 8 MW.

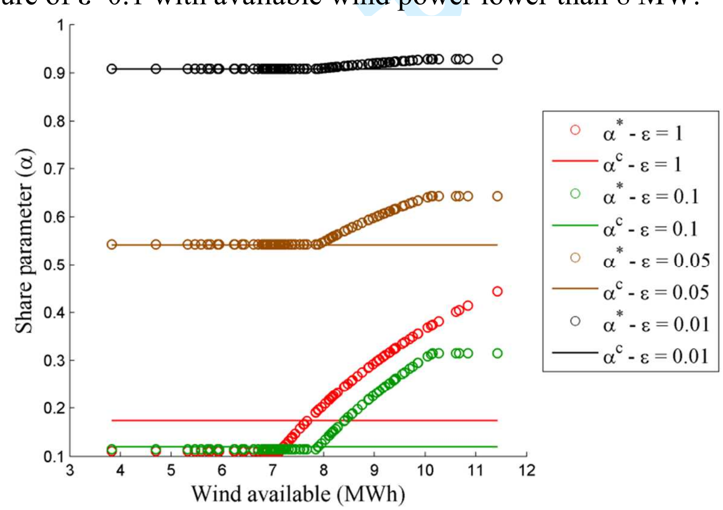

Figure 10. Share parameter in the day-ahead (α^c) and balancing stages (α^*) for the McCormick approach under different share deviations (ε).

5.3. Analysis of the methodologies under real data

A case study based on a wind power plant with 15 MW of installed capacity participating in the Nord Pool is assumed. The wind data is based on power measurements and a series of 48 h-ahead point predictions between March 2001 and April 2003 taken from [1]. This data set contains the quantiles of the wind power distribution, as well as the measurement data for 48 h-ahead. To use this data for validation of the proposed methodologies, scenarios need to be generated. Thus, the quantiles for the point predictions were used to generate 100 scenarios for each time interval based on the scenario generation process described in [37]. Besides, prices and penalties for energy and reserve are required. Advance knowledge of the expected prices and penalties is assumed. However and for this specific case, we consider the Nord Pool prices and penalties for the same period of the wind data (between March 2001 and April 2003). It is noteworthy that traditional electricity markets have no penalties for wind failing to provide the reserve, since current market rules allow no participation of wind in the reserve market. In this context, a reserve penalty for failing to provide reserve market must be assumed. Thus, the reserve penalty for negative reserve imbalance ($\lambda^{pt,-}$) is assumed to be three times higher than the capacity price (λ^{cap}) from the reserve market, since the deficit of reserve in real-time may reduce significantly the proper levels of security and reliability of the system.

Besides the common assumptions for all methodologies used in this work, the PLDR approach requires the definition of breakpoints. Thus, the definition of breakpoints for the PLDR approach follows the same assumptions as the numerical example, i.e., three breakpoints for modelling the piecewise function based on the 25th, 50th and 75th quantile of the wind distribution function for each hour and day are considered.

The cumulative results for energy production and revenue over the two years for each methodology are shown in Table 3. The results depict the cumulative offering bids in day-ahead stage under forecast scenarios. Overall, one can see that the fixed approach offers most of the expected available wind power to the energy market. In contrast, the remaining methods offer the expected available wind power in both markets in a balanced way. i.e., they try improving the expected profit through participating in the reserve market, accounting with a high reserve penalty for failing to provide the contracted reserve. In terms of expected revenue, the flexible approach is the one with higher revenue, followed by the McCormick approach with ε=1. The worst expected revenue comes from the fixed approach, presenting a conservative behavior, since it practically only offers in the energy market. Thus, the flexible approach can improve the expected revenue by about 3% over the fixed approach. Additionally, the McCormick approach with ε=0.01 presents a behavior closer to the fixed approach, as expected. This is due to the parameter ε that controls the deviation of the share in the energy and reserve markets between the day-ahead and the balancing stage, which in this case is too restrictive.

Table 3

In contrast, the deployed wind power as energy and reserve and respective revenue is shown in Table 4. The results are based on an evaluation of the offered bids and share from the different approaches under the wind measurement data. Thus, the expected revenue (in Table 4) represents the evaluation of the offered bids (of each approach) under the realization of wind power. The wind measurement data concerns the realization of wind power generation for the two-year data set. Furthermore, the energy and reserve share of the realized wind power have been determined using the share parameter from the second stage problem of each method.

Table 4

One can see that the behavior of the methods under the realization of wind power is similar to the expected results (from Table 3). However, the total amount of realized power is lower than the total expected power (from Table 3), which is also reflected in the energy and reserve share, as well as in the expected revenue. In more detail, the expected revenue under the measurement data of wind power is significantly lower than the expected revenue from the methods under expectation, which makes sense, since deviations of energy and reserve production from day-ahead and balancing take into account the balancing penalties for energy and reserve. It is noteworthy, that the deviation between the expected revenue from the optimization (Table 3) and validation (Table 4) process for each method is 34.30%, 18.36%, 18.25%, 30.07% and 31.96%, respectively.

Nevertheless, the flexible approach is the one with higher expected revenue, while the fixed approach is the method with worst expected revenue. In fact, the difference between the flexible and fixed approach in terms of expected revenue exceeds 22%. Furthermore, it is noteworthy that the decrease of power is higher in the energy share than in the reserve share for most of the methods, which is understandable, since the penalties for energy deviations are considerably lower than the reserve penalty for missing the contracted offer.

6. Conclusions

The current participation of the wind power producers in electricity markets will constantly be changing, since wind turbines are now able to provide energy and reserve services in the electricity market.

In this work, four different approaches (fixed and flexible stochastic, under McCormick relaxation and piecewise linear decision rules) for wind power producer's offering in the energy and reserve market are formulated. All the approaches are based on the proportional control paradigm that allocates proportional share of the available wind power to the energy and reserve markets. The fixed stochastic approach shows a risk-averse behavior by fixing the share parameter between the day-ahead and balancing stages. On the opposite direction, the flexible stochastic approach presents a risk-neutral behavior (thereby, increasing the expected revenue), since this approach allows the share parameter assuming different values in both the day-ahead and balancing stages, however, requiring a certain level of perfect information on the balancing stage. On the other hand, piecewise linear decision rules incorporates a more conservative trend by allocating almost all the available wind power to single market participation. This method will always compete with the fixed stochastic approach, since it contemplates robust characteristics, which limits the economic performance of the method. Although, the method is likely to improve by considering better decision of the breakpoints, its improvement will most likely be small, thereby the interest of using piecewise linear decision rules is to some extent limited. In contrast, the stochastic approach with McCormick relaxation gives full degree of freedom to the wind power producers to impose a risk-averse or risk-neutral behavior by just adjusting the coefficient that defines the difference between the share parameter in both the day-ahead and balancing stages. An important conclusion from this work is that all the proposed approaches provide a certain range of solutions that may cover different goals and behaviors of the wind power producer, e.g. maximizing revenue and participation in the energy and reserve market. Thus, the use of such proposed methods can improve the expected revenue of wind power producers compared to the revenue with current energy-only market participation.

Future work will focus on improving the performance of the piecewise linear decision rules by optimizing the number and value of the breakpoints in the piecewise function. This may be done by studying the version of piecewise linear decision rules with general segmentation.

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Table list

Table 1. Prices and units costs for energy and reserve

Energy	Price (€/MWh)	Reserve	Price(€/MW)
λ^{sp}	40	λ^{cap}	41
$\lambda^{c,+}$	30	$\lambda^{bpt,+}$	0
$\lambda^{c,-}$	50	$\lambda^{pt,-}$	96

Table 2. Expected energy and reserve bids, as well as expected revenue of the simulation for all proposed approaches (fixed, flexible, McCormick and PLDR).

Method	FIXED	Flexible	McCormick (ε=1)	McCormick (ε=0.01)	PLDR
Energy bid (MW)	7.84	0.98	1.33	7.13	0.21
Reserve bid (MW)	0	6.86	6.34	0.72	7.33
Total expected power (MW)	7.84	7.84	7.67	7.85	7.54
Expected revenue (€)	305.74	311.75	310.95	306.13	305.92

Table 3. Expected cumulative simulation results of two years of data for all proposed approaches (fixed, flexible, McCormick and PLDR).

Method	FIXED	Flexible	McCormick (ε=1)	McCormick (ε=0.01)	PLDR
Energy bid (MW)	77 116.14	43 467.19	43 826.89	75 870.59	46 802.13
Reserve bid (MW)	27.14	33 898.43	28 703.61	1 260.12	32 892.09
Total expected power (MW)	77 143.28	77 365.62	72 530.50	77 130.71	79 694.22
Expected revenue (€)	1 905 292.08	1 965 930.58	1 951 840.02	1 907 032.48	1 919 319.43

Table 4. Deployed cumulative simulation results of two years of data for all proposed approaches (fixed, flexible, McCormick and PLDR).

Method	Fixed	Flexible	McCormick (ε=1)	McCormick (ε=0.01)	PLDR
Energy share (MW)	66 075.92	33 608.11	38 309.53	64 989.56	44 622.15
Reserve share (MW)	21.42	32 489.23	27 787.81	1 107.78	21 475.19
Total expected power (MW)	66 097.34	66 097.34	66 097.34	66 097.34	66 097.34
Expected revenue (€)	1 251 807.04	1 604 984.40	1 595 615.68	1 333 659.36	1 305 892.66