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A bilevel model for electricity retailers' participation in a demand response market environment

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ABSTRACT

Demand response programmes are seen as one of the contributing solutions to the challenges posed to power systems by the large-scale integration of renewable power sources, mostly due to their intermittent and stochastic nature. Among demand response programmes, real-time pricing schemes for small consumers are believed to have significant potential for peak-shaving and load-shifting, thus relieving the power system while reducing costs and risk for energy retailers. This paper proposes a game theoretical model accounting for the Stackelberg relationship between retailers (leaders) and consumers (followers) in a dynamic price environment. Both players in the game solve an economic optimisation problem subject to stochasticity in prices, weather-related variables and must-serve load. The model allows the determination of the dynamic price-signal delivering maximum retailer profit, and the optimal load pattern for consumers under this pricing. The bilevel programme is reformulated as a single-level MILP, which can be solved using commercial off-the-shelf optimisation software. In an illustrative example, we simulate and compare the dynamic pricing scheme with fixed and time-of-use pricing. We find that the dynamic pricing scheme is the most effective in achieving load-shifting, thus reducing retailer costs for energy procurement and regulation in the wholesale market. Additionally, the redistribution of the saved costs between retailers and consumers is investigated, showing that real-time pricing is less convenient than fixed and time-of-use price for consumers. This implies that careful design of the retail market is needed. Finally, we carry out a sensitivity analysis to analyse the effect of different levels of consumer flexibility.

1. Introduction

Favoured by ambitious international agreements and national plans, integration of renewable power sources is expected to constantly rise in the years to come in most industrialised countries. Several among the currently or potentially deployable renewable sources, namely wind, solar, tidal and wave, are characterised by an intermittent and stochastic nature. This will pose problems to the operation and management of future power systems, as supply must match demand at all times. Furthermore security of supply will become an issue as the capacity margin is lower during peak-demand hours with low intermittent generation. Finally price volatility is also destined to increase, since it is known that intermittent renewables have an impact on market prices under the current demand conditions (Jónsson et al., 2010; Morales and Conejo, 2011).

As a way to cope with these issues, many propose a revolution of power systems from a structure where supply follows demand to one where demand follows supply. This can be achieved in practice by adopting measures facilitating *demand response*, such as load shedding programmes, time-of-use or real-time based consumer tariffs. While large industrial consumers can participate in spot markets and are already involved in load shedding programmes in many countries, little has been done yet to allow the participation of small end-consumers in demand response programmes, at least within a European context (Torriti et al., 2010). Nevertheless, demand response is receiving increasing attention from governments and policy makers.

In line with this increasing governmental consideration, demand response is being studied intensively by researchers. Several setups have been proposed involving different stakeholders, namely transmission system operators (TSOs), distributing companies (DISCOs) and retailers. In parallel different advantages of demand response have been stressed, in particular the ability to enhance power system security, and the possibility of reducing electricity procurement costs and, at the same time, market risk.

The TSO's perspective on the demand-response problem attracted a fair share of interest since centralising the management of demand response may have a number of advantages. On the one hand, specific stochastic unit commitment approaches were introduced, permitting to account for demand-side reserve bids submitted by an aggregator on the day-ahead market (Parvania and Fotuhi-Firuzabad, 2010), or jointly accounting for wind power generation and demand response based on a linear inverse demand function (Sioshansi, 2010). On the

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Nomenclature

Sets

T	time periods in the optimisation horizon
Ω_2	space of second-stage stochastic variables
Ω_3	space of third-stage stochastic variables

Indices

t	Index of Program Time Unit (PTU) $t \in \{1,2,\ldots N_T\}$
ω_2	scenario index for second-stage stochastic variable $\omega_2 \in \{1,2,\ldots N_{\Omega_2}\}$
ω_3	scenario index for third-stage stochastic variable $\omega_3 \in \{1,2,\ldots N_{\Omega_3}\}$

Random variables

T^a_{t,ω_2}	ambient temperature
π^s_{t,ω_2}	energy price at the spot market
$\pi^\uparrow_{t,\omega_2}$	up-regulation price at the real-time market
$\pi^\downarrow_{t,\omega_2}$	down-regulation price at the real-time market
l^i_{t,ω_3}	consumption from inflexible (must-serve) load
$\psi^\uparrow_{t,\omega_2}$	up-regulation penalty at the real-time market
$\psi^\downarrow_{t,\omega_2}$	down-regulation penalty at the real-time market

Decision variables

E^s_t	energy contracted at the spot market
$\tilde{\pi}_{t,\omega_2}$	dynamic real-time price charged to the end-consumer
l_{t,ω_2}	energy purchased by the consumer
$\Delta E^\uparrow_{t,\omega_2,\omega_3}$	up-regulation energy purchased at the real-time market
$\Delta E^\downarrow_{t,\omega_2,\omega_3}$	down-regulation energy sold at the real-time market
T^r_{t,ω_2}	indoor temperature in the consumer building model
T^f_{t,ω_2}	floor temperature in the consumer building model
T^w_{t,ω_2}	water temperature in the consumer building model
v_{t,ω_2}	deviation from the comfort band for indoor temperature

Parameters

$\underline{\pi}$	minimum dynamic price charged to the end-consumer
$\overline{\pi}$	maximum dynamic price charged to the end-consumer
π^{AVG}	average daily dynamic price charged to the end-consumer
ρ	penalty for deviation from the comfort band for indoor temperature
$\underline{l}$	minimum flexible consumption for the end-consumer
$\overline{l}$	maximum flexible consumption for the end-consumer
$\underline{T^r_t}$	lower bound of the comfort band for indoor temperature
$\overline{T^r_t}$	upper bound of the comfort band for indoor temperature

other hand, different economic dispatch models aiming at integrating demand response and wind power were reviewed and compared in Ilić et al. (2011a) and Ilić et al. (2011b), respectively. These are based on a model using multi-directional information exchange where the TSO chooses a price sequence based on communicated production schedules and corresponding load response to that price sequence.

In parallel in view of their potential leading role in the optimal management of demand response, the DISCOs' point of view was extensively studied with a blend of load shedding and control-based proposals. Indeed, it may be that DISCOs' operating distributed generation and with the capability of interrupting load consider the possibility of optimising their overall operating costs, as in Algarni and Bhattacharya (2009). They may alternatively design optimal bidding strategies with the additional flexibility such that a certain number of load interruptions are allowed by contract as agreed on with the consumers (Oh and Thomas, 2008). In a different paradigm focused on the consumption dynamics, the idea of using price signals for controlling a part of the load has recently appeared, based e.g. on statistical models for the forecasting of the conditional load response to varying prices (Corradi et al., in press), or conversely on the optimisation problem of a load exposed to dynamic market prices (Conejo et al., 2010). Finally as a more global approach involving DISCOs, retailers (which are purchasers of demand response) as well as aggregations of consumers (sellers), specific market designs may be proposed as in Nguyen et al. (2011), the configuration of which should arguably allow maximising social welfare.

In contrast to these proposals mainly focused on TSOs and DISCOs point of views, we take an original path focused on the joint consideration of the economic optimisation problems of a set of consumers and of their electricity-supplying retailer. In this setup, the retailer naturally acts as a buffer between already existing electricity markets and newly-enabled flexible end-consumers. We assume that consumers respond to a dynamic price signal sent by the retailer by shifting part of their consumption to low-price periods, thus minimising the cost of electricity procurement. The fact that the consumption schedule is decided after the communication of the price signal by the retailer implies that there is a leader–follower structure typical of *Stackelberg games*, which were introduced in the original version of the work later translated in von Stackelberg (2011). For simplicity only the load used for heating purposes is considered as flexible, i.e. it can be shifted in time, although this assumption is not binding. In practice consumer flexibility in time is modelled by using a discrete-time state-space model; the reader not familiar with state space models is referred to Madsen (2007). In turn the retailer is also subject to an economic problem, in that it acts as an intermediary by purchasing at wholesale (day-ahead and real-time) markets and selling back to the consumers.

The novelty of this approach is fourfold. First of all, it jointly considers the optimisation problem of consumers, consisting of the maximisation of a utility function minus the electricity procurement costs, integrating it into the retailer problem, which is purely economic. Using a game theoretic approach, completely novel compared to the state-of-the-art reviewed above, we are able to capture the conflicting economic interests of the retailer and their end-consumers. Under the assumption that the introduction of real-time prices makes the consumers rational, we quantify the cost/benefit improvement for both the stakeholders involved. Secondly, by incorporating the consumer optimisation problem in the model, our analysis is based on a realistic cost function rather than resorting to models that arbitrarily choose demand elasticities or consumer benefit functions, as in Parvania and Fotuhi-Firuzabad (2010), Sioshansi (2010), Algarni and Bhattacharya (2009) and Nguyen et al. (2011). Thirdly, by using a state-space model for consumer preferences within a game-theoretic approach, we rigorously account for the dynamics of demand response, which are often either heuristically approached, see Ilić et al. (2011a) and Ilić et al. (2011b), or simply discarded by making use of static elastic demand as in Parvania and Fotuhi-Firuzabad (2010), Sioshansi (2010), Algarni and Bhattacharya (2009) and Nguyen et al. (2011). Last but not least, we consider a two-market settlement rather than a single one (Conejo et al., 2010; Corradi et al., in press; Ilić et al., 2011a,b; Nguyen et al., 2011; Oh and Thomas, 2008; Parvania and Fotuhi-Firuzabad, 2010; Sioshansi, 2010). This allows us to quantify the advantages of demand response both with respect to peak-shaving(-shifting) and to the reduction of costs due to imbalances (deviations) of real-time consumption from the day-ahead prognosis.

The paper is structured as follows. Section 2 introduces the mathematical formulation of the retailer and the consumer problems separately. Then, the bilevel problem is linearised and formulated as a single-level optimisation programme in Section 3. Section 4 discusses the results of an illustrative example. Finally, conclusions are drawn in Section 5.

2. Formulation of retailer and consumer optimisation problems

We consider the economic optimisation problem of an energy retailer, which acts as an intermediary between energy wholesalers and end-consumers. Energy is purchased at the wholesale market and, in turn, it is sold to the consumers, who face an economic optimisation problem aimed at minimising the cost of their consumption.

Needless to say, the retailer business can only be profitable if the electricity price charged to the consumers $\tilde{\pi}$ is greater than the purchase (spot) price π^s. This price surcharge is justified by the risk that retailers take when entering into a contract with consumers. Indeed retailers must purchase energy in advance on the wholesale market, at a stochastic price π^s, and sell it to the consumers at a price $\tilde{\pi}$ that is often regulated and/or fixed. Furthermore they might incur penalties when the aggregate consumption from their customers, which is stochastic, deviates from the schedule resulting from the wholesale market clearing process. The most striking example of the risk faced by retailers is undoubtedly the Californian energy crisis in 2000–2001, where several utilities went bankrupt as a result of soaring wholesale prices and regulated retail rates, see Borenstein (2002).

In this model we specifically focus on the interaction between the retailer and a partially flexible consumer, who can decide on the allocation of its heating consumption based on an hourly price schedule communicated by the retailer as well as on weather forecasts (e.g. of the outdoor temperature). The problem exhibits a bilevel structure, where the retailer determines the price schedule delivering the optimal profits (upper-level problem), while the consumer, based on this price schedule, optimises its flexible consumption (lower-level problem). In game theory, hierarchical optimisation problems of this type are usually referred to as Stackelberg games and can be formulated mathematically in the framework of bilevel programmes, which are special instances of *Mathematical Programmes with Equilibrium Constraints* (MPECs). The interested reader is referred to Luo et al. (1996) for a complete treatment of the subject.

In the following, we adopt the general formulation of a bilevel programme

$$\begin{aligned}\text{Maximize}\quad & \phi(\mathbf{x},\mathbf{y})\\ \text{s.t.}\quad & (\mathbf{x},\mathbf{y})\in Z\\ & \mathbf{y}\in\mathcal{S}(\mathbf{x}) = arg\,\min_{\mathbf{y}}\{\theta(\mathbf{x},\mathbf{y}) : \mathbf{y}\in C(\mathbf{x})\}\end{aligned} \tag{1}$$

where $\mathbf{x}\in\mathbb{R}^n$ is the vector of decision variables of the upper-level problem, $\mathbf{y}\in\mathbb{R}^m$ the one of the lower-level problem, $\phi(\mathbf{x},\mathbf{y}) : \mathbb{R}^{n+m}\rightarrow\mathbb{R}$ and $\theta(\mathbf{x},\mathbf{y}) : \mathbb{R}^{n+m}\rightarrow\mathbb{R}$ the objective functions of the upper- and the lower-level problems respectively, Z is the joint feasible region of the upper-level problem and $C(\mathbf{x})$ the feasible region of the lower-level problem induced by $\mathbf{x}$.

From the discussion above, it is clear that the financial risk of the retailer stems from multiple stochastic variables: spot and regulation market prices, weather-related variables that influence heating consumption, fluctuations of the inflexible (must-serve) part of the load as well as inaccuracies in modelling consumer behaviour. The specific market design allows the players (retailers and consumers) to make decisions both day-ahead and real-time. Furthermore energy imbalances are settled ex-post, i.e. after their realisation and the calculation of market prices. Decisions made at the later stages benefit from updated information on the stochastic processes that influence the system, either in the form of more accurate forecasts thanks to a shorter look-ahead time or of realised values of random variables. Specifically, we consider the following situation for the three aforementioned stages:

day-ahead: The retailer decides on the amount E_t^s of energy purchased at the spot market for every Program Time Unit (PTU) t in the optimisation horizon, based on forecast scenarios of the spot market price π_{t,ω_2}^s, of the up- and down-regulation prices, $\pi_{t,\omega_2}^{\uparrow}$ and $\pi_{t,\omega_2}^{\downarrow}$ respectively, of the ambient temperature T_{t,ω_2}^a, of the inflexible load l_{t,ω_3}^i and on its model of consumer behaviour.

real-time: The retailer decides on the price schedule $\tilde{\pi}_{t,\omega_2}$ to be sent out to the consumers for every PTU in the optimisation horizon, given the certain realisation of the spot price π_t^s and the contracted purchase at the spot market E_t^s. At the same time the consumer optimises its heating consumption schedule l_{t,ω_2} based on the price signal received from the retailer and on the realisation of the ambient temperature T_{t,ω_2}^a, which is assumed to be known at this point. This is a simplification of the more realistic, yet intractable, situation where more accurate forecasts are available in real-time than day-ahead, which would result in an exponentially growing scenario-tree.

ex-post: The realisation of the inflexible part of the load l_{t,ω_3}^i becomes known, allowing the calculation of the up- and down-regulation imbalances $\Delta E_{t,\omega_2,\omega_3}^{\uparrow}$ and $\Delta E_{t,\omega_2,\omega_3}^{\downarrow}$, respectively. These imbalances are purchased and sold at the up- and down-regulation price, $\pi_{t,\omega_2}^{\uparrow}$ and $\pi_{t,\omega_2}^{\downarrow}$ respectively, determining the net profit for the retailer.

The proposed model is therefore a stochastic bilevel optimisation model with second- and third-stage recourse. The two levels of the model capture the hierarchical relationship between the retailer and the consumer. The three stages allow us to discriminate between uncertain factors being revealed before real-time operation and those disclosed on an ex-post basis. The remainder of the section is dedicated to the introduction of the upper-level (retailer) and the lower-level (consumer) problems.

2.1. Retailer problem

The objective function of the retailer is the maximisation of the expected market profits, with respect to both the second- and third-stage stochastic variables, given by

$$\phi(\mathbf{x},\mathbf{y}) = \mathbb{E}_{\Omega_2,\Omega_3}\left\{\sum_{t=1}^{N_T}\tilde{\pi}_{t,\omega_2}\left(l_{t,\omega_2}+l_{t,\omega_3}^i\right)-\pi_{t,\omega_2}^s E_t^s-\pi_{t,\omega_2}^{\uparrow}\Delta E_{t,\omega_2,\omega_3}^{\uparrow}+\pi_{t,\omega_2}^{\downarrow}\Delta E_{t,\omega_2,\omega_3}^{\downarrow}\right\} \tag{2}$$

where $\mathbf{x} = \left\{\tilde{\pi}_{t,\omega_2}, E_t^s, \Delta E_{t,\omega_2,\omega_3}^{\uparrow}, \Delta E_{t,\omega_2,\omega_3}^{\downarrow}\right\}$ is the retailer's set of decision variables and $\mathbf{y}\supseteq\left\{l_{t,\omega_2}\right\}$ is the consumer's one.

The objective function above is the sum of four terms. The first one represents the revenues from charging the price $\tilde{\pi}_{t,\omega_2}$ to both the flexible and the inflexible load of the consumer, l_{t,ω_2} and l_{t,ω_3}^i respectively. The second term is the cost of purchasing the energy E_t^s at the spot market price π_{t,ω_2}^s. Finally the last two terms represent the cost (profit) of purchasing (selling) up(down)-regulation power $\Delta E_{t,\omega_2,\omega_3}^{\uparrow}$ ($\Delta E_{t,\omega_2,\omega_3}^{\downarrow}$) at the regulation price $\pi_{t,\omega_2}^{\uparrow}$ ($\pi_{t,\omega_2}^{\downarrow}$), where up- and down-regulation are defined as

$$\Delta E_{t,\omega_2,\omega_3}^{\uparrow} = \begin{cases} l_{t,\omega_2}+l_{t,\omega_3}^i-E_t^s, & l_{t,\omega_2}+l_{t,\omega_3}^i-E_t^s\geq 0\\ 0, & \text{otherwise}\end{cases} \tag{3}$$

$$\Delta E_{t,\omega_2,\omega_3}^{\downarrow} = \begin{cases} E_t^s-l_{t,\omega_2}-l_{t,\omega_3}^i, & l_{t,\omega_2}+l_{t,\omega_3}^i-E^s\leq 0\\ 0, & \text{otherwise}\end{cases}. \tag{4}$$

The piecewise definitions (3) and (4) of the up- and down-regulations are necessary only in a two-price market, i.e. if $\pi^{\uparrow}\neq\pi^{\downarrow}$. On the contrary the problem formulation for a single-price real-time market (i.e. a market where $\pi^{\uparrow}=\pi^{\downarrow}$) requires only one variable definition for the imbalance, without piecewise splits. Although we consider here a two-price market for regulation, the model can be easily adapted to the single-price market case, which is simpler to treat owing to the linearity of the definition of the imbalances.

Due to the fact that the model provides no possibility for consumers to switch to a different retailer, i.e. market competition is not modelled, the retailer could increase the end-consumer price possibly up to infinity in order to maximise its profits. On the other hand the process of retailer-switching is rather slow as compared to the optimisation horizon considered here, which makes it hard to consider competition directly in the model. Still, in order to enforce market competitiveness of retailer prices, we choose to introduce constraints that model possible future contracts between retailers and price-responsive consumers. We make the assumption that the two parties will agree on certain characteristics of a variable electricity price, i.e. minimum, maximum and average value during the day, just as today they agree on a fixed rate. In mathematical terms, this assumption implies the following constraints on the consumer-price

$$\tilde{\pi}_{t,\omega_2} \geq \underline{\pi}, \quad \forall t \in T, \forall \omega_2 \in \Omega_2 \tag{5}$$

$$\tilde{\pi}_{t,\omega_2} \leq \overline{\pi}, \quad \forall t \in T, \forall \omega_2 \in \Omega_2 \tag{6}$$

$$\frac{1}{24} \sum_{t=1+24i}^{(1+i)24} \tilde{\pi}_{t,\omega_2} = \pi^{AVG}, \quad i = 0, 1, \ldots, \frac{|T|}{24} - 1, \forall \omega_2 \in \Omega_2. \tag{7}$$

Constraints (5) and (6) ensure that the price charged to the demand is always contained within the range $[\underline{\pi}, \overline{\pi}]$. Constraint (7) enforces that by contract the dynamic price signal must have a fixed daily average. Notice that the latter constraint is necessary in order to ensure a sufficient number of low-price periods. In the absence of this constraint, the retailer would in principle be allowed to always charge the maximum price to the consumer when not faced by high regulation prices. Finally, we underline that constraints (5)–(7) consent a straightforward comparison between retailers in a competitive market, based on few meaningful parameters such as the average hourly price and its maximum and minimum values, i.e. the price level and its volatility.

Following the general formulation of a bilevel programme (1), we write the retailer problem as

$$\underset{\mathbf{x}, \mathbf{y}_{\omega_2}}{\text{Maximize}} \quad \phi\left(\mathbf{x}, \mathbf{y}_{\omega_2}\right) \tag{8a}$$

$$\text{s.t.} \quad (3) - (7) \tag{8b}$$

$$\mathbf{y}_{\omega_2} \in \mathcal{S}_{\omega_2}(\mathbf{x}), \forall \omega_2 \in \Omega_2. \tag{8c}$$

Eq. (8c) enforces that the schedule for flexible load consumption is part of (one of) the solution(s) $\mathcal{S}_{\omega_2}(\mathbf{x})$ of the lower-level optimisation problem for any realisation of the second stage variables $\omega_2 \in \Omega_2$. In practice each consumption schedule l_{t,ω_2} solves a different optimisation problem parameterised in ω_2.

Furthermore, it should be noticed that the objective function in Eq. (2) has two nonlinearities. The first one is introduced by the piecewise linear definition of the imbalances (3) and (4); the second one by the bilinear products $\tilde{\pi}_{t,\omega_2} l_{t,\omega_2}$ in Eq. (2). The former nonlinearity can be worked around through a reformulation of the problem, the latter by enforcing the strong duality theorem, see Luenberger (1984), on the lower-level (consumer) problem. The description of the linearisation is left to Section 3, while the next section introduces the consumer (lower-level) problem.

2.2. Consumer problem

We consider a flexible demand response environment, where the consumer can optimise its future consumption based on a dynamic price schedule communicated by the retailer. We assume here that only the load l_{t,ω_2} necessary for heating is flexible, and treat the remaining, inflexible part of the load l^i_{t,ω_3} as a third-stage stochastic variable. We remark that this limitation to heating load is not critical and other sources of consumer flexibility could be considered. In a similar fashion one could consider more general models akin to the one in Conejo et al. (2010).

Just like the retailer, the end-consumer faces an economic problem, too. With a flexible price, he/she will minimise the cost of the electricity needed for heating by shifting as much consumption as possible to low-price periods, without giving up *too much* on the comfort, i.e. on the indoor temperature of the building. We therefore model the objective of the consumer as a utility function trading-off the cost of electricity procurement and the discomfort for deviating from the reference temperature band.

Two different formulations of the economic optimisation problem of the heating system of a building are introduced in this section. First, a linear programming (LP) formulation is introduced. Then, its equivalent system of Karush–Kuhn–Tucker (KKT) conditions is presented.

2.2.1. LP formulation of the consumer problem

Based on the work in Halvgaard et al. (2012) we consider a three-state, discrete-time state space model for the heating dynamics of a building. The three states of the system are the indoor temperature T^r_{t,ω_2}, the floor T^f_{t,ω_2} temperature and the temperature T^w_{t,ω_2} inside a water tank directly connected to a heat pump. The only input is the electricity consumption l_{t,ω_2}, while the outdoor temperature T^a_{t,ω_2} is a stochastic disturbance. We stress that solar irradiation, an additional disturbance in Halvgaard et al. (2012) is discarded here for the sake of simplicity. Using a matrix formulation, the state space model writes

$$\begin{bmatrix} T^r_{t,\omega_2} \\ T^f_{t,\omega_2} \\ T^w_{t,\omega_2} \end{bmatrix} = \mathbf{A} \begin{bmatrix} T^r_{t-1,\omega_2} \\ T^f_{t-1,\omega_2} \\ T^w_{t-1,\omega_2} \end{bmatrix} + \mathbf{B} l_{t-1,\omega_2} + \mathbf{E} T^a_{t-1,\omega_2} \tag{9}$$

where all the matrices are constant. The output of interest is clearly the indoor temperature T^r_{t,ω_2}, as this is the only variable influencing the consumer comfort. In the following optimisation model, adapted from Halvgaard et al. (2012), the deviation of the output from a reference band $\left[\underline{T}^r_t \quad \overline{T}^r_t\right]$ is linearly penalised in the objective function, where it is summed to the cost of electricity consumption.

$$\underset{\mathbf{y}}{\text{Minimize}} \quad \theta_{\omega_2}(\mathbf{x}, \mathbf{y}) = \sum_{t=1}^{N_T} \tilde{\pi}_{t,\omega_2} l_{t,\omega_2} + \rho v_{t,\omega_2} \tag{10a}$$

$$\text{s.t.} \quad T^r_{t,\omega_2} = a_{11} T^r_{t-1,\omega_2} + a_{12} T^f_{t-1,\omega_2} + a_{13} T^w_{t-1,\omega_2} + b_1 l_{t-1,\omega_2} + e_1 T^a_{t-1,\omega_2} \quad \left(\mu^r_{t,\omega_2}\right) \tag{10b}$$

$$T^f_{t,\omega_2} = a_{21} T^r_{t-1,\omega_2} + a_{22} T^f_{t-1,\omega_2} + a_{23} T^w_{t-1,\omega_2} + b_2 l_{t-1,\omega_2} + e_2 T^a_{t-1,\omega_2} \quad \left(\mu^f_{t,\omega_2}\right) \tag{10c}$$

$$T^w_{t,\omega_2} = a_{31} T^r_{t-1,\omega_2} + a_{32} T^f_{t-1,\omega_2} + a_{33} T^w_{t-1,\omega_2} + b_3 l_{t-1,\omega_2} + e_3 T^a_{t-1,\omega_2} \quad \left(\mu^w_{t,\omega_2}\right) \tag{10d}$$

$$l_{t,\omega_2} \geq \underline{l} \quad \left(\underline{\lambda}_{t,\omega_2}\right) \tag{10e}$$

$$l_{t,\omega_2} \leq \overline{l} \quad \left(\overline{\lambda}_{t,\omega_2}\right) \tag{10f}$$

$$T^r_{t,\omega_2} + v_{t,\omega_2} \geq \underline{T}^r_t \quad \left(\underline{\epsilon}_{t,\omega_2}\right) \tag{10g}$$

$$T^r_{t,\omega_2} - v_{t,\omega_2} \leq \overline{T}^r_t \quad \left(\overline{\epsilon}_{t,\omega_2}\right) \tag{10h}$$

$$v_{t,\omega_2} \geq 0 \tag{10i}$$

The consumer's set of decision variables is $\mathbf{y} = \{l_{t,\omega_2}, v_{t,\omega_2}, T^r_{t,\omega_2}, T^f_{t,\omega_2}, T^w_{t,\omega_2}\}$, while $\mu^r_{t,\omega_2}, \mu^f_{t,\omega_2}, \mu^w_{t,\omega_2}, \underline{\lambda}_{t,\omega_2}, \overline{\lambda}_{t,\omega_2}, \underline{\epsilon}_{t,\omega_2}, \overline{\epsilon}_{t,\omega_2}$ are the dual variables associated with constraints (10b)–(10h). The state space model (9) translates into constraints (10b), (10c) and (10d). Inequalities (10e) and (10f) set the lower and upper limit for electricity consumption, respectively. Variable v_{t,ω_2} represents the absolute value of deviations of the indoor temperature out of the reference band $\left[\underline{T}_t^r \quad \overline{T}_t^r\right]$ through (10g)–(10i). Positive values of this variable are penalised in the objective function, where they are summed with weight ρ to the cost of electricity over the time horizon N_T.

It is stressed that since the dynamic electricity price $\tilde{\pi}_{t,\omega_2}$ enters the consumer problem as a constant vector (it is only a variable in the retailer problem), model (10a)–(10i) is a linear programme. Incidentally, we remark that in this model the retailer must provide the consumer with a price forecast for a certain time-horizon, which resembles the assumption in Ilić et al. (2011a).

Finally, we point out that the objective function (10a) with a linear penalisation of the temperature deviations from a reference band is only one of the possible utility functions for the consumer. However, it has certain characteristics that make it appealing, e.g. its simplicity, and the fact that, as we show in what follows, it leads to a reformulation of the bilevel model as a Mixed-Integer Linear Program (MILP). More sophisticated consumer problems could involve varying upper and lower bounds for the indoor temperature in (10g) and (10h), defined as linear functions of the consumer price $\tilde{\pi}_{t,\omega_2}$, or quadratic penalties for deviations from a reference, which closely relates to Linear Quadratic Regulator (LQR) problems in control theory, see Kwakernaak and Sivan (1972). Such extensions of the model are left for future research.

2.2.2. KKT formulation of the consumer problem

In this section we present the formulation of the consumer problem given by its Karush–Kuhn–Tucker conditions. The equivalence of the KKT formulation and the one in Section 2.2.1 is guaranteed by the linearity of the latter one, which implies that solutions of the optimisation problem are also solution of the KKT system of equations and vice versa, see Conejo et al. (2006).

We begin by stating the stationarity conditions with respect to the decision variables $\mathbf{y} = \left\{l_{t,\omega_2}, v_{t,\omega_2}, T^r_{t,\omega_2}, T^f_{t,\omega_2}, T^w_{t,\omega_2}\right\}$

$$\begin{cases} \tilde{\pi}_{t,\omega_2} - b_1\mu^r_{t+1,\omega_2} - b_2\mu^f_{t+1,\omega_2} - b_3\mu^w_{t+1,\omega_2} + \underline{\lambda}_{t,\omega_2} + \overline{\lambda}_{t,\omega_2} = 0, & t<N_T \\ \tilde{\pi}_{t,\omega_2} + \underline{\lambda}_{t,\omega_2} + \overline{\lambda}_{t,\omega_2} = 0, & t = N_T \end{cases} \tag{11}$$

$$0 \le v_{t,\omega_2} \perp \rho + \underline{\epsilon}_{t,\omega_2} - \overline{\epsilon}_{t,\omega_2} \ge 0 \tag{12}$$

$$\begin{cases} \mu^r_{t,\omega_2} - a_{11}\mu^r_{t+1,\omega_2} - a_{21}\mu^f_{t+1,\omega_2} - a_{31}\mu^w_{t+1,\omega_2} + \underline{\epsilon}_{t,\omega_2} + \overline{\epsilon}_{t,\omega_2} = 0, & t < N_T \\ \mu^r_{t,\omega_2} + \underline{\epsilon}_{t,\omega_2} + \overline{\epsilon}_{t,\omega_2} = 0, & t = N_T \end{cases} \tag{13}$$

$$\begin{cases} -a_{12}\mu^r_{t+1,\omega_2} + \mu^f_{t,\omega_2} - a_{22}\mu^f_{t+1,\omega_2} - a_{32}\mu^w_{t+1,\omega_2} = 0, & t<N_T \\ \mu^f_{t,\omega_2} = 0, & t = N_T \end{cases} \tag{14}$$

$$\begin{cases} -a_{13}\mu^r_{t+1,\omega_2} - a_{23}\mu^f_{t+1,\omega_2} + \mu^w_{t,\omega_2} - a_{33}\mu^w_{t+1,\omega_2} = 0, & t<N_T \\ \mu^w_{t,\omega_2} = 0, & t = N_T \end{cases}. \tag{15}$$

It should be noticed that the stationarity conditions with respect to the variables appearing in the state-update Eqs. (10b)–(10d) have a different formulation at the final step N_T of the optimisation horizon. This is because only one state-update equation includes them, rather than two in the general case, as there is no equation imposing the evolution of the state from N_T to N_{T+1}.

The system of KKT conditions is completed by the (equality and inequality) constraints already included in models (10a)–(10i), along with the complementary slackness conditions associated with the inequality constraints, i.e.

$$T^r_{t,\omega_2} = a_{11}T^r_{t-1,\omega_2} + a_{12}T^f_{t-1,\omega_2} + a_{13}T^w_{t-1,\omega_2} + b_1 l_{t-1,\omega_2} + e_1 T^a_{t-1,\omega_2} \tag{16}$$

$$T^f_{t,\omega_2} = a_{21}T^r_{t-1,\omega_2} + a_{22}T^f_{t-1,\omega_2} + a_{23}T^w_{t-1,\omega_2} + b_2 l_{t-1,\omega_2} + e_2 T^a_{t-1,\omega_2} \tag{17}$$

$$T^w_{t,\omega_2} = a_{31}T^r_{t-1,\omega_2} + a_{32}T^f_{t-1,\omega_2} + a_{33}T^w_{t-1,\omega_2} + b_3 l_{t-1,\omega_2} + e_3 T^a_{t-1,\omega_2} \tag{18}$$

$$0 \ge \underline{\lambda}_{t,\omega_2} \perp l_{t,\omega_2} - \underline{l} \ge 0 \tag{19}$$

$$0 \le \overline{\lambda}_{t,\omega_2} \perp l_{t,\omega_2} - \overline{l} \le 0 \tag{20}$$

$$0 \ge \underline{\epsilon}_{t,\omega_2} \perp T^r_{t,\omega_2} + v_{t,\omega_2} - \underline{T}^r_t \ge 0 \tag{21}$$

$$0 \le \overline{\epsilon}_{t,\omega_2} \perp T^r_{t,\omega_2} - v_{t,\omega_2} - \overline{T}^r_t \le 0. \tag{22}$$

We underline that the system of KKT conditions is linear, with the exception of the complementarity conditions (12) and (19)–(22). In order to linearise these conditions we make use of the Fortuny-Amat linearisation (Fortuny-Amat and McCarl, 1981); for example Eq. (12) can be substituted by the following constraints

$$\rho + \underline{\epsilon}_{t,\omega_2} - \overline{\epsilon}_{t,\omega_2} \ge 0 \tag{23}$$

$$v_{t,\omega_2} \ge 0 \tag{24}$$

$$\rho + \underline{\epsilon}_{t,\omega_2} - \overline{\epsilon}_{t,\omega_2} \le z_{t,\omega_2} M^1 \tag{25}$$

$$v_{t,\omega_2} \le \left(1 - z_{t,\omega_2}\right) M^1 \tag{26}$$

$$z_{t,\omega_2} \in \{0, 1\} \tag{27}$$

where M^1 is a sufficiently large constant. The complementary slackness conditions (19)–(22) can be linearised using the same strategy. Therefore we end up with a (integer linear) system of KKT conditions equivalent to model (10a)–(10i). As a trade-off for introducing additional complexity (i.e. integer variables), we can simply concatenate the KKT conditions as additional constraints of the upper-level problem. This puts the bilevel problem in a tractable formulation. One is finally left with the necessary linearisation of the objective function (2) of the retailer.

3. Linearisation and bilevel formulation of the problem

As pointed out in Section 2.1 there are two nonlinearities in the objective function (2) of the upper-level problem. The first one stems from the piecewise definition of negative and positive energy imbalances in Eqs. (3) and (4), and can be linearised through a reformulation of the problem. The second nonlinearity can be overcome by exploiting the strong duality theorem on the lower-level problem. The remainder of this section deals with the linearisation of these terms, and with the presentation of the final formulation of the bilevel problem as a single-level optimisation programme.

3.1. Reformulation of the energy imbalance

In order to reformulate the problem, let us first define the market penalties for up- and down-regulation

$$\psi^{\uparrow}_{t,\omega_2} = \pi^{\uparrow}_{t,\omega_2} - \pi^s_{t,\omega_2} \ge 0 \tag{28}$$

$$\psi^{\downarrow}_{t,\omega_2} = \pi^s_{t,\omega_2} - \pi^{\downarrow}_{t,\omega_2} \ge 0. \tag{29}$$

These values represent the additional cost (or missed revenue) per MWh incurred by the retailer in comparison to the case where it has perfect information on its stochastic consumption. In the latter case, the retailer is charged the spot price for all its consumption. In the former, more realistic, case the retailer will need to adjust its bid on the real-time market, where it is charged $\pi^{\uparrow}_{t,\omega_2} = \pi^s_{t,\omega_2} + \psi^{\uparrow}_{t,\omega_2}$ for any additional consumed MWh, and paid $\pi^{\downarrow}_{t,\omega_2} = \pi^{s}_{t,\omega_2} - \psi^{\downarrow}_{t,\omega_2}$ for any MWh consumed less than the schedule cleared at the spot market. Clearly $\psi^{\uparrow}_{t,\omega_2}$ and $\psi^{\downarrow}_{t,\omega_2}$ can be interpreted as the per-unit penalty for imperfect information on future consumption.

Using the market penalties defined above, the objective function (2) can be reformulated as follows

$$\begin{aligned}\phi(\mathbf{x},\mathbf{y}) = \mathbb{E}_{\Omega_2,\Omega_3}\Bigg\{&\sum_{t=1}^{N_T}\tilde{\pi}_{t,\omega_2}\left(l_{t,\omega_2}+l^{i}_{t,\omega_3}\right)-\pi^{s}_{t,\omega_2}E^{s}_{t}\\ &-\left(\pi^{s}_{t,\omega_2}+\psi^{\uparrow}_{t,\omega_2}\right)\Delta E^{\uparrow}_{t,\omega_2,\omega_3}+\left(\pi^{s}_{t,\omega_2}-\psi^{\downarrow}_{t,\omega_2}\right)\Delta E^{\downarrow}_{t,\omega_2,\omega_3}\Bigg\}=\\ =\mathbb{E}_{\Omega_2,\Omega_3}\Bigg\{&\sum_{t=1}^{N_T}\tilde{\pi}_{t,\omega_2}\left(l_{t,\omega_2}+l^{i}_{t,\omega_3}\right)-\pi^{s}_{t,\omega_2}\left(E^{s}_{t}+\Delta E^{\uparrow}_{t,\omega_2,\omega_3}-\Delta E^{\downarrow}_{t,\omega_2,\omega_3}\right)\\ &-\psi^{\uparrow}_{t,\omega_2}\Delta E^{\uparrow}_{t,\omega_2,\omega_3}-\psi^{\downarrow}_{t,\omega_2}\Delta E^{\downarrow}_{t,\omega_2,\omega_3}\Bigg\}\\ =\mathbb{E}_{\Omega_2,\Omega_3}\Bigg\{&\sum_{t=1}^{N_T}\tilde{\pi}_{t,\omega_2}\left(l_{t,\omega_2}+l^{i}_{t,\omega_3}\right)-\pi^{s}_{t,\omega_2}\left(l_{t,\omega_2}+l^{i}_{t,\omega_3}\right)\\ &-\psi^{\uparrow}_{t,\omega_2}\Delta E^{\uparrow}_{t,\omega_2,\omega_3}-\psi^{\downarrow}_{t,\omega_2}\Delta E^{\downarrow}_{t,\omega_2,\omega_3}\Bigg\} \qquad (30)\end{aligned}$$

where the last line is obtained by noticing that $l_{t,\omega_2}+l^{i}_{t,\omega_3}=E^{s}_{t}+\Delta E^{\uparrow}_{t,\omega_2,\omega_3}-\Delta E^{\downarrow}_{t,\omega_2,\omega_3}$ holds at any time, which is a result of the definitions in Eqs. (3) and (4).

We can now formulate the retailer optimisation problem exploiting the objective function reformulation (30)

$$\underset{x}{\text{Maximize}} \quad \phi(\mathbf{x},\mathbf{y}) \text{ in } (30) \qquad (31a)$$

$$\text{s.t.} \quad \Delta E^{\uparrow}_{t,\omega_2,\omega_3} \geq l_{t,\omega_2}+l^{i}_{t,\omega_3}-E^{s}_{t} \qquad (31b)$$

$$\Delta E^{\downarrow}_{t,\omega_2,\omega_3} \geq E^{s}_{t}-l_{t,\omega_2}-l^{i}_{t,\omega_3} \qquad (31c)$$

$$\Delta E^{\uparrow}_{t,\omega_2,\omega_3}, \Delta E^{\downarrow}_{t,\omega_2,\omega_3} \geq 0. \qquad (31d)$$

$$(5)\text{–}(7), (8a) \qquad (31e)$$

First, it should be emphasised that the maximisation of Eq. (2) with the imbalance definitions in Eqs. (3)–(4) is equivalent to the maximisation of Eq. (30) subject to constraints (31b)–(31d). The latter is a relaxed, yet linear, formulation of the former optimisation problem with a larger feasible space, where the variables $\Delta E^{\uparrow}_{t,\omega_2,\omega_3}$ and $\Delta E^{\downarrow}_{t,\omega_2,\omega_3}$ are allowed to assume greater values than the actual up- and down-regulations. The equivalence of the two optimisation problems is readily proved by noticing that, as long as $\psi^{\uparrow}_{t,\omega_2}, \psi^{\downarrow}_{t,\omega_2} > 0$, all the additional feasible points of Eqs. (31b)–(31d) have a strictly worse objective than at least one feasible point of Eqs. (3)–(4), i.e. the one with the minimal absolute imbalance allowed. In other words, there is no interest for the retailer in artificially pushing up the values of $\Delta E^{\uparrow}_{t,\omega_2,\omega_3}$ and $\Delta E^{\downarrow}_{t,\omega_2,\omega_3}$, as this would contribute negatively to the objective function without any advantages. With similar arguments, it can be shown that $\Delta E^{\uparrow}_{t,\omega_2,\omega_3}$ and $\Delta E^{\downarrow}_{t,\omega_2,\omega_3}$ could assume greater values than the actual up- and down-regulation, but without influencing the other variables, in the case where at least one between $\psi^{\uparrow}_{t,\omega_2}$ and $\psi^{\downarrow}_{t,\omega_2}$ is zero. The actual imbalances can still be calculated by applying Eqs. (3) and (4) to the optimal solution of Eqs. (31a)–(31e).

It is also remarked that the reformulation presented in this section is only needed in a two-price real-time market. Under the single-price market structure, there is no need for a piecewise definition of the imbalances (3) and (4).

3.2. Linearisation of bilinear terms

The only nonlinearity still present in objective function (30) consists in the bilinear terms $\tilde{\pi}_{t,\omega_2}l_{t,\omega_2}$. Optimisation problems including bilinear terms are often solved by approximation techniques. For example, Pereira et al. (2005) make use of binary expansion on one of the variables involved in the bilinear term, while a piecewise linear approximation is employed in Vespucci et al. (2013). Using the same approach as in Carrión et al. (2009), we show that this problem allows for an exact linearisation of these terms. By employing the strong duality theorem, see Luenberger (1984), on the lower-level model (10a)–(10i) we enforce that primal and dual objectives are equal at optimality. This implies that

$$\begin{aligned}\sum_{t=1}^{N_T}\tilde{\pi}_{t,\omega_2}l_{t,\omega_2}+\rho v_{t,\omega_2} = &-\mu^{r}_{1,\omega_2}\left(a_{11}T^{r}_{0,\omega_2}+a_{12}T^{f}_{0,\omega_2}+a_{13}T^{w}_{0,\omega_2}+b_1 l_{0,\omega_2}\right)\\ &-\mu^{f}_{1,\omega_2}\left(a_{21}T^{r}_{0,\omega_2}+a_{22}T^{f}_{0,\omega_2}+a_{23}T^{w}_{0,\omega_2}+b_2 l_{0,\omega_2}\right)\\ &-\mu^{w}_{1,\omega_2}\left(a_{31}T^{r}_{0,\omega_2}+a_{32}T^{f}_{0,\omega_2}+a_{33}T^{w}_{0,\omega_2}+b_3 l_{0,\omega_2}\right)\\ &-\sum_{t=1}^{N_T}\Big\{\mu^{r}_{t,\omega_2}e_1 T^{a}_{t-1,\omega_2}+\mu^{w}_{t,\omega_2}e_3 T^{a}_{t-1,\omega_2}+\mu^{f}_{t,\omega_2}e_2 T^{a}_{t-1,\omega_2}\\ &+\underline{\lambda}_{t,\omega_2}\underline{l}+\overline{\lambda}_{t,\omega_2}\overline{l}+\underline{\epsilon}_{t,\omega_2}\underline{T}^{r}_{t}+\overline{\epsilon}_{t,\omega_2}\overline{T}^{r}_{t}\Big\}. \qquad (32)\end{aligned}$$

From the equality between primal and dual objective of the lower-level problem, it follows that the sum of terms $\tilde{\pi}_{t,\omega_2}l_{t,\omega_2}$ is equal to the sum of products between dual variables and parameters of the primal constraints of the lower-level problem, minus $\rho v_{t,\omega_2}$, which are all linear in the bilevel formulation.

By solving Eq. (32) on $\sum_{t=1}^{N_T}\tilde{\pi}_{t,\omega_2}l_{t,\omega_2}$ and taking the expectation with respect to Ω_2 and Ω_3 on both sides of the equation, we are able to replace all the bilinear terms in Eq. (30), thus obtaining the linear reformulation of the objective function that follows

$$\begin{aligned}\phi(\mathbf{x},\mathbf{y}) = \mathbb{E}_{\Omega_2,\Omega_3}\Big\{&-\mu^{r}_{1,\omega_2}\left(a_{11}T^{r}_{0,\omega_2}+a_{12}T^{f}_{0,\omega_2}+a_{13}T^{w}_{0,\omega_2}+b_1 l_{0,\omega_2}\right)\\ &-\mu^{f}_{1,\omega_2}\left(a_{21}T^{r}_{0,\omega_2}+a_{22}T^{f}_{0,\omega_2}+a_{23}T^{w}_{0,\omega_2}+b_2 l_{0,\omega_2}\right)\\ &-\mu^{w}_{1,\omega_2}\left(a_{31}T^{r}_{0,\omega_2}+a_{32}T^{f}_{0,\omega_2}+a_{33}T^{w}_{0,\omega_2}+b_3 l_{0,\omega_2}\right)\\ &-\sum_{t=1}^{N_T}\Big\{\mu^{r}_{t,\omega_2}e_1 T^{a}_{t-1,\omega_2}+\mu^{w}_{t,\omega_2}e_3 T^{a}_{t-1,\omega_2}+\mu^{f}_{t,\omega_2}e_2 T^{a}_{t-1,\omega_2}\\ &+\underline{\lambda}_{t,\omega_2}\underline{l}+\overline{\lambda}_{t,\omega_2}\overline{l}+\underline{\epsilon}_{t,\omega_2}\underline{T}^{r}_{t}+\overline{\epsilon}_{t,\omega_2}\overline{T}^{r}_{t}-\rho v_{t,\omega_2}\\ &+\tilde{\pi}_{t,\omega_2}l^{i}_{t,\omega_3}-\pi^{s}_{t,\omega_2}\left(l_{t,\omega_2}+l^{i}_{t,\omega_3}\right)-\psi^{\uparrow}_{t,\omega_2}\Delta E^{\uparrow}_{t,\omega_2,\omega_3}-\psi^{\downarrow}_{t,\omega_2}\Delta E^{\downarrow}_{t,\omega_2,\omega_3}\Big\}\Big\}. \qquad (33)\end{aligned}$$

3.3. Final problem formulation

As a result of the reformulations above, the bilevel programme can be expressed as the following equivalent single-level MILP

$$\underset{x,y}{\text{Maximize}} \quad \phi(\mathbf{x},\mathbf{y}) \text{ in } (33)$$

$$\text{s.t.} \quad (31b)\text{–}(31d), (5)\text{–}(7), (11), (23)\text{–}(26), (13)\text{–}(15), (16)-(18)$$

$$\left.\begin{aligned}&l_{t,\omega_2}-\underline{l}\geq 0\\ &\underline{\lambda}_{t,\omega_2}\leq 0\\ &l_{t,\omega_2}-\underline{l}\leq z^{2}_{t,\omega_2}M^{2}\\ &\underline{\lambda}_{t,\omega_2}\geq-\left(1-z^{2}_{t,\omega_2}\right)M^{2}\end{aligned}\right\} \text{ linearisation of (19)} \qquad (34a)$$

$$\left.\begin{aligned}&l_{t,\omega_2}-\overline{l}\leq 0\\ &\overline{\lambda}_{t,\omega_2}\geq 0\\ &l_{t,\omega_2}-\overline{l}\geq -z^{3}_{t,\omega_2}M^{3}\\ &\overline{\lambda}_{t,\omega_2}\leq\left(1-z^{3}_{t,\omega_2}\right)M^{3}\end{aligned}\right\} \text{ linearisation of (20)} \qquad (34b)$$

$$\left.\begin{array}{l} T^r_{t,\omega_2} + v_{t,\omega_2} - \underline{T}^r_t \geq 0 \\ \underline{\epsilon}_{t,\omega_2} \leq 0 \\ T^r_{t,\omega_2} + v_{t,\omega_2} - \underline{T}^r_t \leq z^4_{t,\omega_2} M^4 \\ \underline{\epsilon}_{t,\omega_2} \geq -\left(1 - z^4_{t,\omega_2}\right) M^4 \end{array}\right\} \quad \text{linearisation of (21)} \quad (34c)$$

$$\left.\begin{array}{l} T^r_{t,\omega_2} - v_{t,\omega_2} - \overline{T}^r_t \leq 0 \\ \overline{\epsilon}_{t,\omega_2} \geq 0 \\ T^r_{t,\omega_2} - v_{t,\omega_2} - \overline{T}^r_t \geq z^5_{t,\omega_2} M^5 \\ \overline{\epsilon}_{t,\omega_2} \leq \left(1 - z^5_{t,\omega_2}\right) M^5 \end{array}\right\} \quad \text{linearisation of (22)} \quad (34d)$$

$$z^1_{t,\omega_2}, z^2_{t,\omega_2}, z^3_{t,\omega_2}, z^4_{t,\omega_2}, z^5_{t,\omega_2} \in \{0, 1\}. \quad (34e)$$

The bilevel problem can be solved as a single-level one having the same objective function as the upper-level problem (33), and constraints given by the concatenation of

- the constraints (31b)–(31d), (5)–(7) of the upper-level problem
- the stationarity conditions (11), (13)–(15) in the system of KKT conditions for the lower level problem and the linearisation of the stationarity condition (12) i.e. equations (23)–(26)
- the equality constraints (16)–(18) of the lower-level problem
- the linearisation of the complementarity conditions (19)–(22), i.e. equations (34a)–(34d)
- the integrality condition (34e) for the binary variables introduced by the Fortuny-Amat linearisations of the complementarity constraints.

The resulting problem can be solved as a single-level MILP due to the linearity of both the objective function and the constraints, while integer variables are introduced by the Fortuny-Amat linearisation of the complementarity conditions. Problems of this type can be solved using commercial off-the-shelf optimisation software. In this work the problem is formulated in the GAMS environment and solved by employing the CPLEX solver.

4. Numerical results and discussion

We describe here the numerical results obtained by running the model in Section 3.3 on a small test-case based on real-world data.

The example simulates a single bidding round at the spot market for the retailer, which optimises its bid and real-time market operation using a 48-hour horizon. Uncertainties on the future realisation of spot and real-time market prices, outdoor temperature and inflexible load are modelled through scenarios.

For the sake of simplicity, we limit the number of consumers to three. Indeed, this number is sufficient to draw quantitative conclusions on the behaviour of the model, at the same time allowing the visualisation of relevant variables for each consumer. Incidentally, we stress that although there is no theoretical limit on the number of consumers that can be considered in the model – adding one consumer translates into adding one set of lower-level KKT conditions to the constraints of the upper-level programme – there is a certain computational burden implied by the increasing number of integer variables.

Aggregation of consumers into classes characterised by similar building dynamics, behaviour and therefore consumption is paramount for obtaining a tractable, yet realistic, model for the retailer problem. In general, clustering of consumers is widely applied in decision making problems. For instance, clustering techniques for modelling electricity consumption have been proposed in Chicco et al. (2004), where their importance for electricity providers is also underlined. Clustering the driving behaviour of electric car owners is proposed in Kristoffersen et al. (2011) for optimising their charging and discharging. Similarly, the three consumers included in this example can be regarded as three classes each grouping a number of consumers with similar behaviour, i.e. building dynamics, heating preferences, etc. Indeed we will treat the three consumers as groups by assigning them different probabilities, i.e. by varying the distribution (or proportion) of consumers belonging to a certain class.

In the following section, we present the parameters chosen to model consumer heating dynamics. Then, we describe how scenarios have been generated in order to model uncertainties. Finally, the results of the example are discussed.

4.1. Parameters in the model of building dynamics

The consumer optimisation problem described in Section 2.2 includes among its constraints a state space model of consumer building dynamics.

Table 1 summarises the values used for the parameters as well as their units. The chosen parameter values are the ones used in Halvgaard et al. (2012), exception made for a lower hourly consumption limit $\overline{l}$ for electricity and lowered b_1, b_2 and b_3 values, due to the choice of a smaller gain for the heat pump, which is decreased by a factor of 3. These changes aim at better spreading the electricity consumption over the day, rather than having few daily consumption spikes as in Halvgaard et al. (2012). We do not discuss here the physical meaning of the parameters, and just refer the interested reader to Halvgaard et al. (2012) and Madsen and Holst (1995) for discussion on the physical interpretation of the parameters and on how they can be estimated.

Besides, we consider time-varying comfort bands $\left[\underline{T}^r_t \quad \overline{T}^r_t\right]$, so that there is a higher reference for indoor temperature during the day and a lower one during the night. In order to model different consumer preferences, we assume that the three consumer groups have different comfort bands. As one can see in Fig. 1, the first consumer is the most flexible, as it accepts temperatures in a range of 5 °C, while the range is narrowed down to 2 °C for the third consumer. It is worth mentioning that these temperature ranges need not be constant as in this example, but could e.g. be wider during working hours and narrower when consumers are expected to be at home.

4.2. Scenario generation

This section describes the methodology employed for generating scenarios for the market quantities (spot and regulation prices), inflexible load and temperature required as inputs to the model. All the employed methodologies are rather simplistic answers to complicated problems,

Table 1
Parameter values considered for the LP model representing the consumer's heating dynamics.

Parameter	Value	Unit
a_{11}	0.4103	–
a_{12}	0.5586	–
a_{13}	0.0028	–
a_{21}	0.1092	–
a_{22}	0.8801	–
a_{23}	0.0078	–
a_{31}	0.0022	–
a_{32}	0.0310	–
a_{33}	0.9668	–
b_1	0.0044	°C/kWh
b_2	0.0173	°C/kWh
b_3	4.2332	°C/kWh
e_1	0.0284	–
e_2	0.0029	–
e_3	0	–
$\underline{l}$	0	kWh
$\overline{l}$	0.33	kWh
ρ	30	€/°C

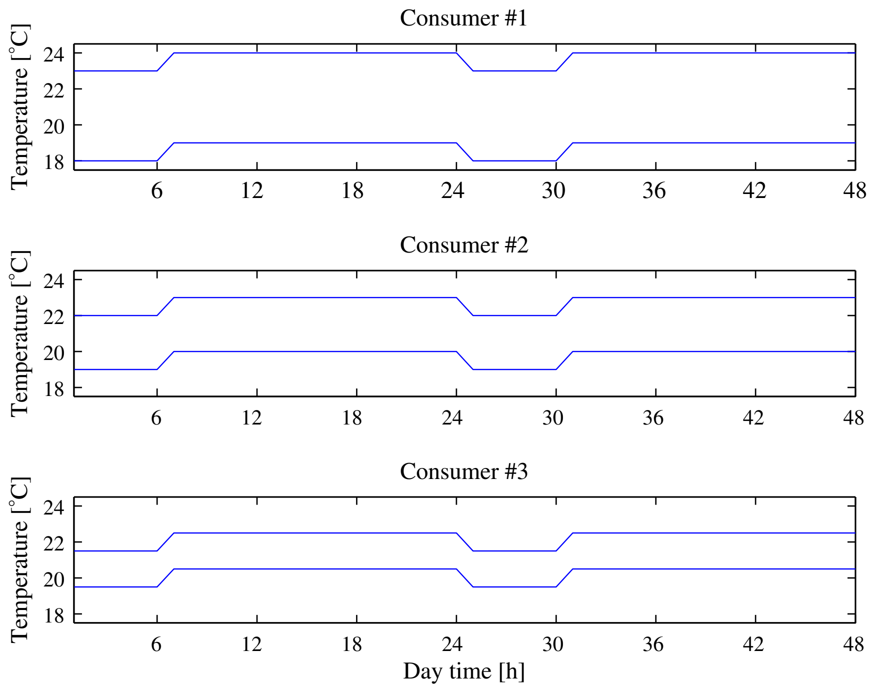

Fig. 1. Comfort bands $\left[\underline{T}_t^r \quad \overline{T}_t^r\right]$ for the three consumer groups. The consumer flexibility decreases from top to bottom.

i.e. modelling of weather- and market-related stochastic processes, which are out of the scope of this paper. The interested reader is referred to Dubrovsky (1997) and Madsen (1985) and to Weron (2006) and Jónsson (2012), respectively for an introduction to modelling of stochastic processes related to weather and electricity markets.

As far as the spot market price π^s_{t,ω_2} is concerned, we use the observed spot market prices in the DK-2 (Eastern Denmark) market area of NordPool, the Scandinavian power exchange, as mean value for the scenarios. We choose arbitrarily to consider prices pertaining to the 15–16th March 2011, which are available at Energinet website (2011) along with other market data for NordPool. In order to generate scenarios, we simulate a multivariate Gaussian process with an exponentially decreasing covariance structure, i.e. the (i, j)-th element of the covariance matrix is given by

$$C(i,j) = \sigma^2 e^{-|i-j|/\tau} \tag{35}$$

The parameter σ is the standard deviation of the process. We consider a constant standard deviation $\sigma = €\,6.67$, which is the approximate RMSE value for the spot market price forecasting model in the work in Jónsson et al. (2013).[1] Furthermore we point out that the time-lags considered for these scenarios are at least 13 h, which is the look-ahead time of the scenarios for the first hour of the first day considered. The parameter τ sets the exponential decay of correlation with respect to the time lag. We choose the value $\tau = 7$ h in the example. The choice of model (35) is justified by the fact that, despite being relatively simple, it allows us to consider the dynamics of market prices and to easily enforce a realistic value for the standard deviation of the forecast error.

Finally, scenarios are generated by adding the coloured Gaussian noise to the observed spot market price. Fig. 2 shows both the observed spot market price and the obtained scenarios.

[1] This work considers the DK-1 (i.e. Western Denmark) price area of NordPool. Generally the price difference between DK-1 and DK-2 is negligible.

Scenarios for the real-time market prices $\pi^{\uparrow}_{t,\omega_2}$ and $\pi^{\downarrow}_{t,\omega_2}$ are generated from the spot price scenarios using a model based on the average values of the ratios

$$\alpha^{\uparrow}_t = \frac{\pi^{\uparrow}_t}{\pi^s_t} \quad \alpha^{\downarrow}_t = \frac{\pi^{\downarrow}_t}{\pi^s_t}. \tag{36}$$

These averages are calculated for the three winter months in the DK-2 price area of NordPool using data from Energinet website (2011), resulting in the values $\tilde{\alpha}^{\uparrow} = 1.19$ and $\tilde{\alpha}^{\downarrow} = 0.95$. Scenarios can then be generated from the model as functions of the spot price scenarios

$$\pi^{\uparrow}_{t,\omega_2} = \tilde{\alpha}^{\uparrow}\pi^s_{t,\omega_2} \quad \pi^{\downarrow}_{t,\omega_2} = \tilde{\alpha}^{\downarrow}\pi^s_{t,\omega_2}. \tag{37}$$

As a consequence of the use of this model, there is a single regulation price scenario associated to each spot price realisation. This is clearly a simplified model for the regulation prices. We point out, though, that there is no obstacle in using third-stage scenarios in the proposed model, besides that of modelling the stochastic regulation prices. Furthermore, this simplification does not introduce significant distortions in the results of the model, since both regulation penalties $\psi^{\uparrow}_{t,\omega_2}$ and $\psi^{\downarrow}_{t,\omega_2}$ are different from 0 at any time and for all scenarios. In other words, the scenarios $\pi^{\uparrow}_{t,\omega_2}$ and $\pi^{\downarrow}_{t,\omega_2}$ represent the expected real-time market prices conditioned on the realisation of the spot market price π^s_{t,ω_2}. Furthermore, it should be noticed that model (36) is not a very good predictor of the balancing market prices, especially as far as the up-regulation price is concerned (the standard deviations of the ratios in Eq. (36) are 0.76 and 0.12, respectively). While developing a state-of-the-art forecasting tool for the regulation prices is out of the scope of this paper, one should keep in mind that more sophisticated forecasting models should be used in realistic applications. The reader should notice that the choice of model (36) implies no loss of generality, as the scenarios for the regulation prices are exogenous to the optimisation model.

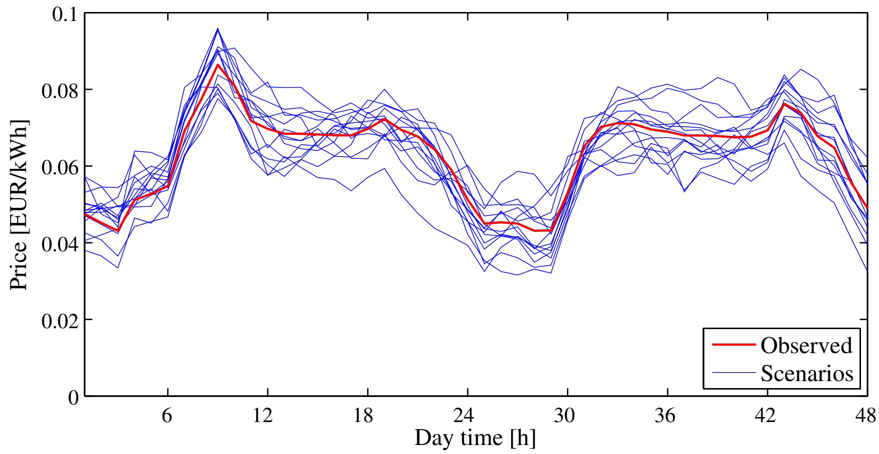

Fig. 2. Observed spot market price in the DK-2 price area of NordPool and generated scenarios for the period 15–16th March 2011.

Scenarios for T^a are formed by gathering temperature observations available at the Iowa Environmental Mesonet website (2011).[2] Measurements are picked from a single location during different days of March 2011 with similar temperature patterns. The obtained scenarios are shown in Fig. 3. In total $N_{\Omega_2} = 14$ second stage scenarios are considered in this example, for reasons of data availability. In a more realistic setup one would want to make use of more advanced modelling of weather-related variables. We refer the reader interested in the subject to Dubrovsky (1997) for a presentation of scenario-generation techniques applied to weather-related variables.

Finally the third-stage scenarios for the inflexible load l^i_{t,ω_3} are generated with a model similar to the one used for the spot price π^s_{t,ω_2}. The observed load in DK-2, available at Energinet website (2011), is scaled and used as mean value of the process. The scaling is done so that the inflexible load is approximately about 85% of the total load, which is the share of the residual household load after subtracting the consumption due to heating in Denmark (Danish Energy Association, 2010). We emphasise, though, that the share of total consumption represented by heating varies from country to country. Coloured Gaussian noise with a covariance structure of the same form of Eq. (35) is then added to the load pattern. The standard deviation is here set to $\sigma = 0.0075$ kWh. As Fig. 4 highlights, the variance of the inflexible load is relatively smaller than the variance of the spot price, reflecting the easier predictability of the load compared to market quantities. Ideally the standard deviation of the inflexible load would be a function of the size of the considered customer group, decreasing in relative terms with respect to the size owing to smoothing of the errors. In total a number of third-stage scenarios $N_{\Omega_3} = 10$ are selected. Note that the number of scenarios should be large enough to guarantee a faithful representation of the uncertainties involved in the problem. Once more we stress that developing refined models for the uncertainty is outside the scope of this paper.

4.3. Numerical results

The results of the illustrative example are discussed in this section. First, we assess the differences in consumer behaviour and market performance of the retailer between the cases of fixed-price, Time-Of-Use (TOU) price and dynamic-price contracts between retailer and consumers. Then, different distributions of consumer groups are considered in order to discuss how dynamic prices imposed by the retailer impact the market players, and how this impact is influenced by consumer behaviour.

4.3.1. Advantages of dynamic pricing

In order to compare the fixed-, TOU- and the dynamic-price case, the model is run three times on the same dataset. In the first run the price charged by the retailer is set to be constant over time to the value € 0.2/kWh, which amounts to replacing Eqs. (5)–(7) with the equation $\tilde{\pi}_{t,\omega_2} = € \ 0.2$/kWh.

In the second run, the Time-Of-Use (TOU) pricing scheme illustrated in Table 2 is employed. In this scheme, consumption is charged € 0.3/kWh during peak hours, € 0.2/kWh during flat hours, and € 0.1/kWh during valley hours. Notice that, since there are 8 h for each group, the average TOU price is equal to the price in the fixed-price scheme, i.e. € 0.2/kWh.

In the third run, the original model in Section 3.3 is simulated. We remind the reader that the price in this model is dynamic, but must have a daily mean $\pi^{AVG} = €$ 0.2/kWh, which is equal to the fixed price and to the average of the TOU price. Furthermore the price must always fall within the range [0.1, 0.3]€/kWh. In all the cases considered the distribution of the consumer groups is set to [0.3, 0.4, 0.3], which means that 30% of the consumers have highly flexible behaviour, 40% are balanced and 30% have low flexibility.

The dynamics of $\tilde{\pi}_{t,\omega_2}$ and of the flexible load l_{t,ω_2} are shown in Figs. 5, 6 and 7 for each consumer type, in the fixed-, TOU- and dynamic-price case, respectively. Mean, median and range (i.e. maximum and minimum value) across scenarios are shown for these variables, which, except for $\tilde{\pi}_{t,\omega_2}$ in the fixed- and TOU-price case, are scenario-dependent.

In the fixed-price case, there is no economic incentive for the consumer to modify his/her consumption schedule according to the price signal sent by the retailer. In practice the optimisation consists of a trade-off between consumption (and therefore cost) minimisation and aversion to deviations from the comfort band. In this example, the consumer chooses to allocate all of its consumption during the first hours of the simulation horizon, as shown in Fig. 5.

The situation changes in the TOU-price case, where the consumers prefer to allocate their flexible consumption during valley hours, which are characterised by low prices. Clearly, consumption takes place during peak hours only when necessary, i.e. during few hours for consumer type 2 and for the least flexible consumer type 3.

In the dynamic price case, the consumer adapts to the price signal submitted by the retailer. Remarkably, the price plotted in Fig. 7 is on average lower during night time, i.e. hours 0–8 and 21–32. The consumer response follows the price signal: indeed, flexible consumption

[2] The geographical displacement between the locations of the temperature and market datasets is justified by reasons of data accessibility. This displacement is equivalent to considering that temperature and market price scenarios are independent from each other. We assume that the results obtained in this paper would hold, at least qualitatively, using consistent datasets.

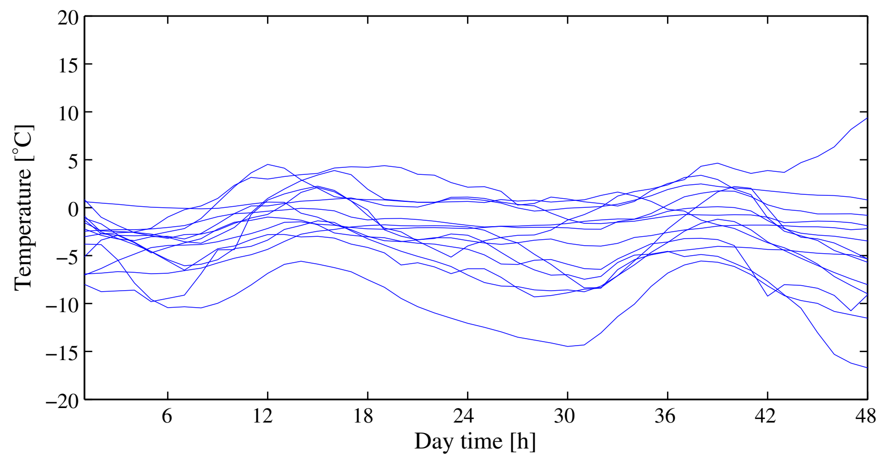

Fig. 3. Scenarios for temperature T^{a}_{t,ω_2} obtained from measurements during March 2011 available at Iowa Environmental Mesonet website (2011).

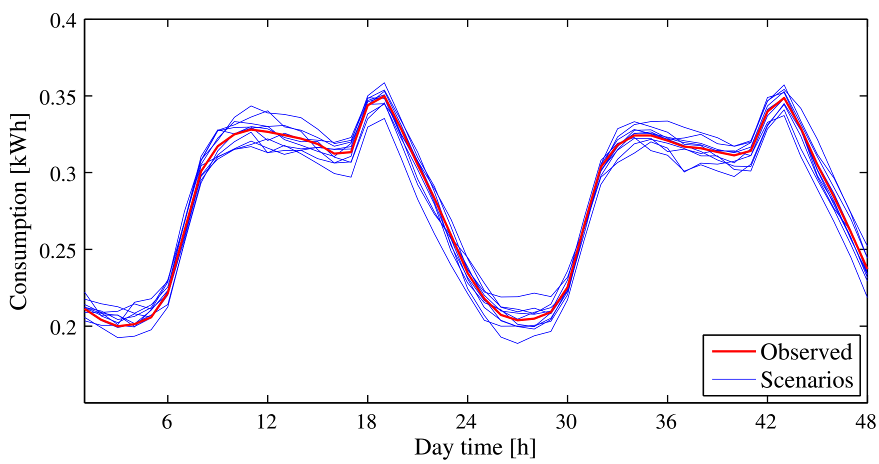

Fig. 4. Scenarios for the inflexible part of the load l^{i}_{t,ω_3} for the period 15–16th March 2011.

takes place more likely in time periods where the price tends to be low. This appears to hold rather generally across all the consumer groups considered.

Analysing the results further, one notices scenarios where the price chosen by the retailer implies multiple solutions for the consumer. In this case, the consumer is indifferent with respect to choosing any of these solutions, and might therefore pick randomly or decide according to a secondary criterion (e.g. choosing, among the solutions delivering the minimum cost, the one minimising the consumption). The results presented here refer to the case where the consumer selects, among the optimal solutions, the one that yields the best profit for the retailer. In mathematical terms, this is the *optimistic* or *strong Stackelberg solution* (Loridan and Morgan, 1996). Notice that by definition every solution to the MPEC in the general form (Eq. (1)) is a strong Stackelberg solution. In practice this means that the results of the bilevel model could be too optimistic, unless there is a reason why the consumer would choose the strong Stackelberg solution instead of any other element in his/her optimal set $\mathbf{y} \in \mathbf{S}(\mathbf{x})$. For example, the retailer could communicate, along with the price signal, a suggested consumption level to choose in case multiple solutions are found. As an alternative, one could modify the setup of the lower level problem so that it always has a unique solution. In other words, one must define a lower level problem as a variational inequality where the functional is *strongly monotone* on the feasible set, see Luo et al. (1996). Future research in this direction is needed.

Let us now consider the relationship between the total consumption $l_{t,\omega_2} + l^{i}_{t,\omega_3}$ and the price π^{s}_{t,ω_2} paid by the retailer at the spot market under the dynamic-pricing scheme. As one can see in Fig. 8, total consumption peaks when the spot price is at the lowest point, i.e. during the night in both the first and the second day included in the horizon. Furthermore, another peak of smaller intensity and shorter duration is observed around the 13th hour of the simulation, where the spot price appears on average to have a local valley. An intuitive explanation for this is that part of the load cannot be postponed to the following night (or shifted to the previous one) without violating the comfort band; therefore accepting a locally minimum price is a good compromise. Observing this type of behaviour was one of the reasons behind the choice of a model capturing the dynamics in the consumer flexibility. Finally, the energy E^{s} purchased by the retailer at the spot market resembles the pattern of the average consumption, though shifted somewhat up due to the lower expected costs for down-regulation compared to up-regulation.

Finally, it is of interest to analyse the impact of the introduction of dynamic prices on the retailer's energy imbalance. Indeed demand

Table 2
Details of the Time-Of-Use (TOU) pricing scheme employed. Prices are in €/kWh.

Day time	1–7	8–10	11–14	15–16	17–20	21–23	24
Type	Valley	Flat	Peak	Flat	Peak	Flat	Valley
Price	0.1	0.2	0.3	0.2	0.3	0.2	0.1

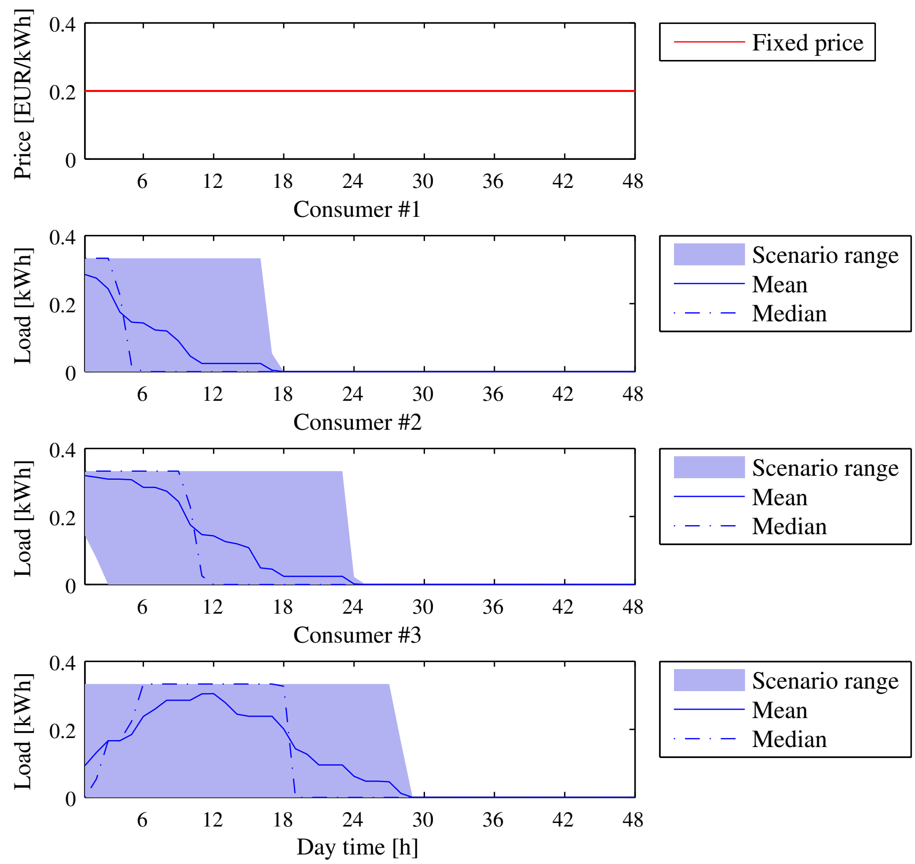

Fig. 5. Flexible consumption (l) patterns for the consumer types with fixed price $\tilde{\pi}$.

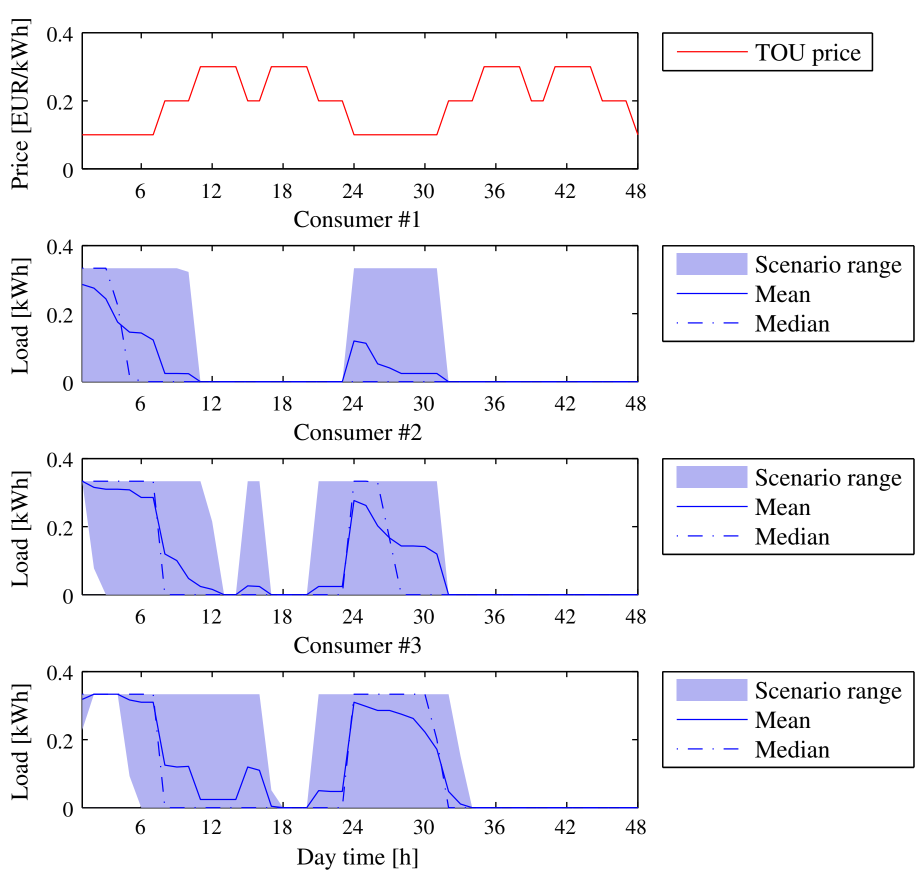

Fig. 6. Flexible consumption (l) patterns for the consumer types with Time-Of-Use (TOU) based price $\tilde{\pi}$.

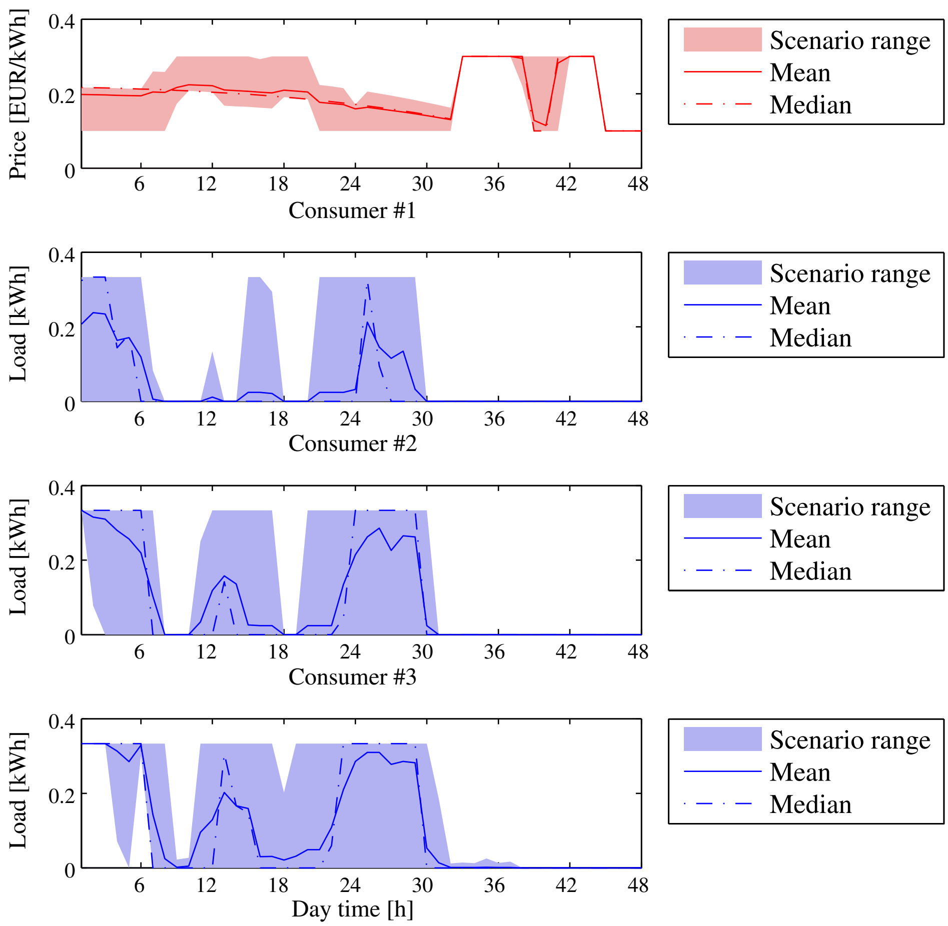

Fig. 7. Flexible consumption (l) patterns for the consumer types with dynamic price $\tilde{\pi}$.

response, if managed with correct policies, has the potential to reduce both the magnitude and the total cost of regulation. Deviations from the day-ahead schedule and imbalance penalties are shown in Figs. 9, 10 and 11, in the cases of fixed, TOU and dynamic price, respectively. It is worth pointing out that generally the retailer prefers being long, i.e. contracting more energy at the spot market than needed on average.

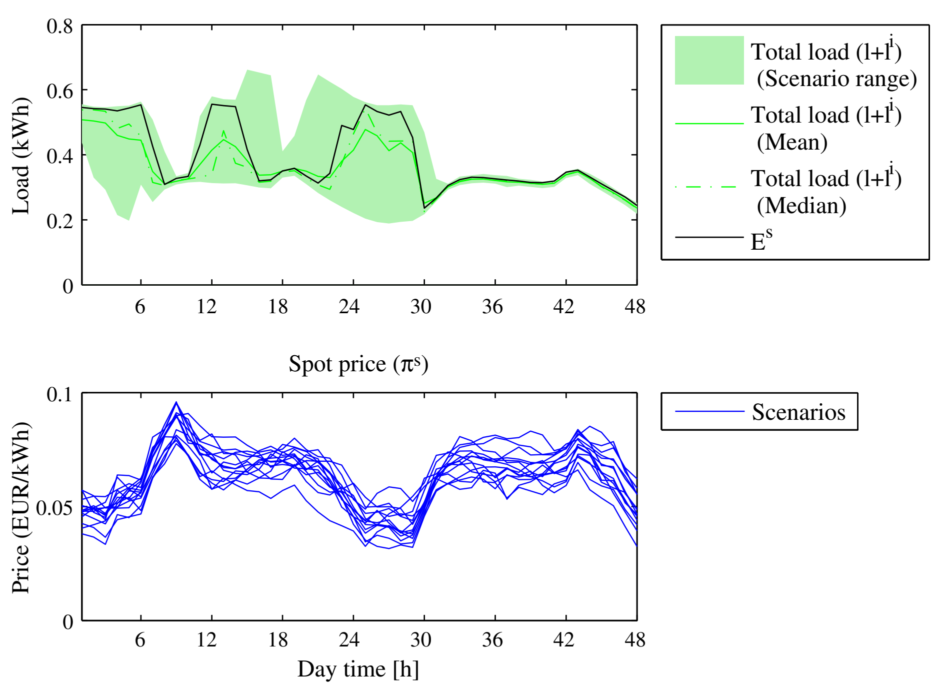

Fig. 8. Total consumption (aggregating flexible and inflexible load $l+l^i$) and spot market offer E^s versus spot market price π^s.

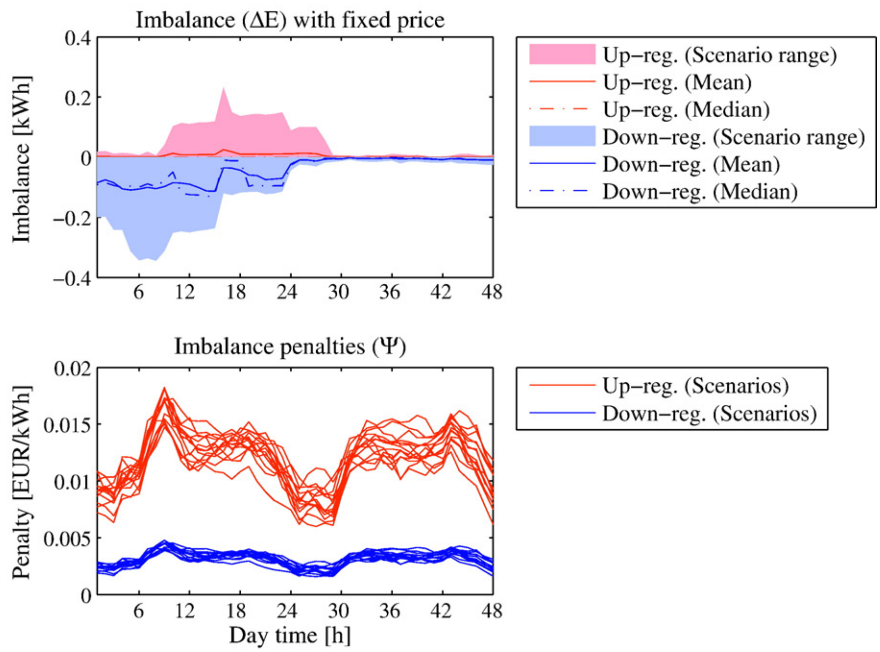

Fig. 9. Retailer energy imbalance $\Delta E^{\uparrow}_{t,\omega_2,\omega_3}$ (up-regulation), $\Delta E^{\downarrow}_{t,\omega_2,\omega_3}$ (down-regulation) and imbalance penalties $\psi^{\uparrow}_{t,\omega_2}$ (up-regulation), $\psi^{\downarrow}_{t,\omega_2}$ (down-regulation) with fixed consumer price $\tilde{\pi}$.

This is confirmed by the prevalence of down-regulation in the three figures. Furthermore, we remark that in the TOU-price case in Fig. 10, the largest imbalances are moved to the valley hours. In a similar fashion, the retailer manages to move the largest imbalances away from periods where regulation prices peak under dynamic pricing. This is illustrated in Fig. 11.

The main results for the retailer in the simulations with fixed, TOU and dynamic price are summarised in Table 3. It emerges from these results that the retailer improves its performance when it is allowed to send a dynamic price signal to its flexible consumers. The expected profit $\phi(\mathbf{x},\mathbf{y})$ rises by approximately 5% compared to the fixed-price case, both due to an increase in revenues and a cost reduction. On the contrary, the TOU-pricing scheme yields the lowest profits among the three cases considered.

The retailer revenues consist of returns of the sale of energy for flexible and inflexible consumption at the price imposed by the retailer, that is

$$\sum_{t=1}^{N_T} \tilde{\pi}_{t,\omega_2} l_{t,\omega_2} + \tilde{\pi}_{t,\omega_2} l^{i}_{t,\omega_3}. \quad (38)$$

The two components, averaged over the scenarios used in this example, are presented separately in the table. In the case with TOU-

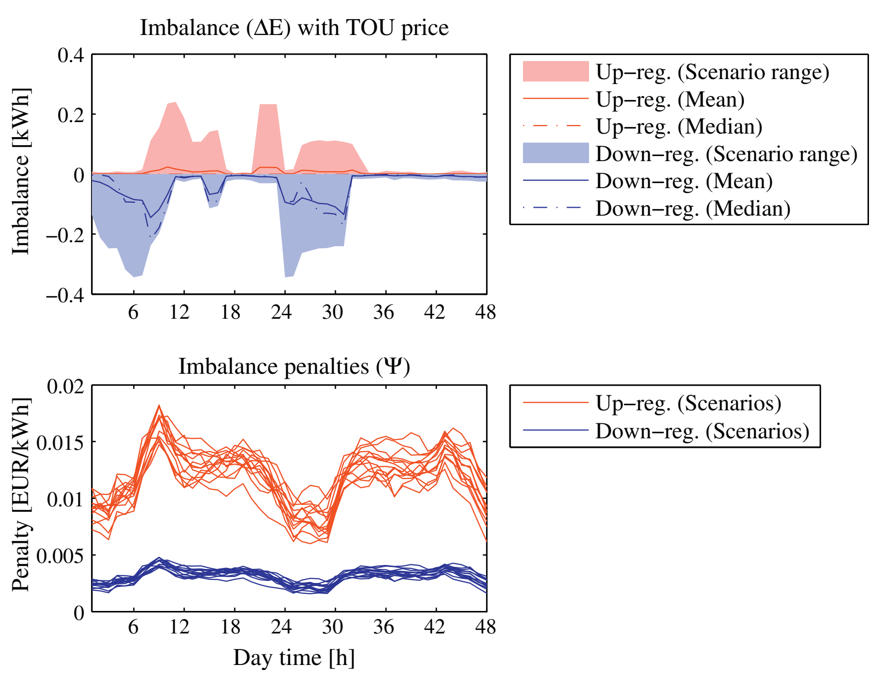

Fig. 10. Retailer energy imbalance $\Delta E^{\uparrow}_{t,\omega_2,\omega_3}$ (up-regulation), $\Delta E^{\downarrow}_{t,\omega_2,\omega_3}$ (down-regulation) and imbalance penalties $\psi^{\uparrow}_{t,\omega_2}$ (up-regulation), $\psi^{\downarrow}_{t,\omega_2}$ (down-regulation) with Time-Of-Use (TOU) based consumer price $\tilde{\pi}$.

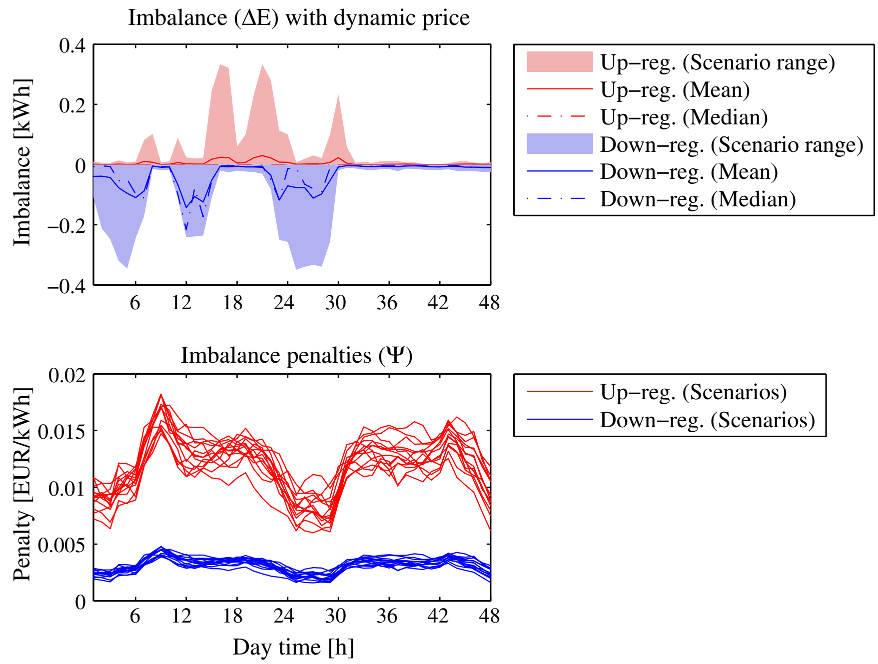

Fig. 11. Retailer energy imbalance $\Delta E^{\uparrow}_{t,\omega_2,\omega_3}$ (up-regulation), $\Delta E^{\downarrow}_{t,\omega_2,\omega_3}$ (down-regulation) and imbalance penalties $\psi^{\uparrow}_{t,\omega_2}$ (up-regulation), $\psi^{\downarrow}_{t,\omega_2}$ (down-regulation) with dynamic consumer price $\tilde{\pi}$.

price, revenues from the flexible part of the load are dramatically lower if compared to the corresponding quantities with fixed- and dynamic-price. This clearly indicates that this pricing scheme is the most favourable to the consumers. With dynamic pricing, the total revenues are maximised, indicating that the retailer is fully exploiting its market power over the consumers.

The costs for the retailer are presented in two different formulations. In the first formulation they are calculated by summing the payment for purchasing power at the spot market and the cost (revenue) of buying (selling) energy at the real-time market

$$\sum_{t=1}^{N_T} \pi^s_{t,\omega_2} E^s_t + \pi^{\uparrow}_{t,\omega_2} \Delta E^{\uparrow}_{t,\omega_2,\omega_3} - \pi^{\downarrow}_{t,\omega_2} \Delta E^{\downarrow}_{t,\omega_2,\omega_3}. \quad (39)$$

The payment at the spot market is lowest with dynamic-pricing, i.e. when the retailer can indirectly shift the load by communicating a price signal to the consumer, while the TOU pricing scheme ranks second. The results in the real-time market seem, at a superficial analysis, counterintuitive, since the revenues are lower in the dynamic-price case than in the fixed-price one. However, it is not straightforward from this formulation to understand whether dynamic prices can help to achieve better results in terms of imbalance costs. For this reason we consider reformulation (30) and break down the retailer costs in the following way

$$\sum_{t=1}^{N_T} \pi^s_{t,\omega_2} \left(l_{t,\omega_2} + l^i_{t,\omega_3} \right) + \psi^{\uparrow}_{t,\omega_2} \Delta E^{\uparrow}_{t,\omega_2,\omega_3} + \psi^{\downarrow}_{t,\omega_2} \Delta E^{\downarrow}_{t,\omega_2,\omega_3} \quad (40)$$

where the first term can be considered as the spot market costs if the retailer had perfect information on future consumption, and the last two terms are the imbalance penalties, i.e. the cost of imperfect information. As shown in Table 3, not only the spot market "virtual" payment under perfect information, but also the imbalance costs behave according to intuition. Indeed, the dynamic pricing scheme (which is optimal) performs best, followed by the case with TOU price (which is suboptimal, but designed to reduce costs on average), while the fixed price case ranks last. This confirms that demand response can be employed both for reducing the cost of energy procurement (i.e. peak-shifting) and for cutting the regulation cost.

The results of the simulations for the consumers are summarised in Table 4. Note that the consumer costs in this table are equal to the retailer revenues in Table 3. As already mentioned in the discussion above, the electricity procurement payments for the consumers are maximised under the dynamic pricing scheme, and lowest with TOU price. Especially the fact that the consumer payments for the flexible part of the load are highest with dynamic pricing tells us

Table 3
Market performance of the retailer in the simulations with fixed and dynamic price. All the values are averages for the considered scenarios expressed in €.

Retailer performance index		Pricing		
		Fixed	TOU	Dynamic
Profits		2.3139	2.2296	2.4286
Revenues	Flexible load	0.7050	0.4300	0.7180
	Inflexible load	2.7155	2.8865	2.7868
	Total	3.4205	3.3164	3.5049
Costs	Spot market	1.2146	1.1676	1.1578
	Regulation market	−0.1080	−0.0808	−0.0814
	Total	1.1067	1.0868	1.0763
Costs (reformulated)	Perfect information	1.0970	1.0781	1.0680
	Real-time penalties	0.0096	0.0088	0.0083
	Total	1.1067	1.0868	1.0763

Table 4
Consumer results in the simulations with fixed and dynamic price. All the values are averages for the considered scenarios.

Consumer result index		Unit	Pricing		
			Fixed	TOU	Dynamic
Costs	Flexible load	€	0.7050	0.4300	0.7180
	Inflexible load	€	2.7155	2.8865	2.7868
	Total	€	3.4205	3.3164	3.5049
Price	Flexible load	€/kWh	0.2000	0.1136	0.1885
	Inflexible load	€/kWh	0.2000	0.2126	0.2053

that dynamic prices alone do not necessarily result in higher benefit for the flexible consumers. In this example, the consumer is better off with TOU- or fixed-price contracts than with a dynamic-price one with equal average prices over the day. Reductions in the average real-time consumer price could be considered as an incentive for consumers to switch to dynamic-price contracts. Therefore, determining an average value making dynamic real-time prices beneficial also for consumers is an interesting problem for which models of this type could be employed. Finally we point out that, despite the electricity procurement costs for the consumers are higher in the dynamic-price case compare to the fixed pricing scheme, the average price paid by the flexible part of the load decreases quite sensibly. This implies that the total electricity consumption is higher in the dynamic-price case.

To conclude, we can interpret the reduction of the market costs for the retailer in the dynamic-price case as an increase of social welfare. This is because the transfer of money from consumers to retailer cancels out in a social welfare calculation. Since there were no deviations from the comfort band in any of the cases and scenarios considered in the example, we can conclude that the consumer benefit is constant. As a result, the social welfare is given by the generation costs changed in sign. These cannot be directly calculated, since this model does not include the supply side. Nevertheless, we can consider the reduction in retailer market costs as a proxy for the reduction of generation cost. On the other hand, the increase in consumer payments to the retailer implies that the redistribution of this additional welfare between the players might not be fair under this retailer-consumer configuration. Once again, though, we point out that these considerations hold for the considered example and with the considered setup. Different dynamic price contracts, i.e. different parameters in the constraints (5)–(7), could yield a fairer redistribution of the welfare.

4.3.2. Impact of consumer flexibility

We now consider how different levels of demand flexibility impact the results for both the retailer and the consumer. This is done by carrying out two additional simulations with different distributions into the consumer groups described in Section 1. In the first run of the model, aimed at simulating a situation of high demand flexibility, we consider the consumer group distribution [0.6, 0.3, 0.1]. The situation is reversed to the distribution [0.1, 0.3, 0.6] in the last run of the model, simulating low demand flexibility. Both cases are compared to the reference case in the previous section, where demand flexibility is medium due to the choice of the distribution [0.3, 0.4, 0.3].

Table 5 illustrates the retailer market performance in the three cases of demand flexibility, this time only with dynamic price. Observe that higher demand flexibility results in lower average profits for the retailer. This is the result of two contrasting trends. On the one side, total revenues for electricity sale diminish as demand flexibility increases. This is in line with the expectations that retailers have lower market power, i.e. ability to impose prices to the demand, as consumers get more flexible. On the other side, total market costs drop as well with higher consumer flexibility. This drop is due to cuts both in spot market costs for electricity procurement (perfect information row in Table 5) and in regulation penalty costs. Nevertheless the overall effect is still of decreasing retailer profits with increasing flexibility, because the cuts in market costs are not large enough to offset the reduction in revenues.

The results for the consumer are shown in Table 6. As already pointed out, demand experiences a cut in the electricity costs as it gets more and more flexible. This is due to a quite dramatic drop in the cost of flexible load and only a slight increase in the cost of must-serve load. Therefore, there is a clear economic signal suggesting the demand to adopt more flexible consumption preferences, and to increase the share of flexible demand. The decrease in the average price per kWh paid for flexible load confirms that a more cost-effective load pattern is adopted by the consumer.

Table 5
Market performance of the retailer in simulations with different demand flexibility. All the values are averages for the considered scenarios.

Retailer performance index		Unit	Flexibility		
			High	Medium	Low
Profits		€	2.3199	2.4286	2.5311
Revenues	Flexible load	€	0.5517	0.7180	0.8757
	Inflexible load	€	2.7969	2.7868	2.7767
	Total	€	3.3486	3.5049	3.6524
Costs	Spot market	€	1.1058	1.1578	1.2057
	Regulation market	€	−0.0771	−0.0814	−0.0845
	Total	€	1.0287	1.0763	1.1212
Costs (reformulated)	Perfect information	€	1.0211	1.0680	1.1114
	Real-time penalties	€	0.0076	0.0083	0.0099
	Total	€	1.0287	1.0763	1.1212

Table 6
Consumer results in simulations with different demand flexibility. All the values are averages for the considered scenarios.

Consumer result index		Unit	Flexibility		
			High	Medium	Low
Costs	Total	€	3.3486	3.5049	3.6524
	Flexible load	€	0.5517	0.7180	0.8757
	Inflexible load	€	2.7969	2.7868	2.7767
Price	Flexible load	€/kWh	0.1870	0.1885	0.1921
	Inflexible load	€/kWh	0.2060	0.2053	0.2045

5. Conclusions

This paper presents a game theoretical model for the participation of energy retailers in electricity markets with flexible demand and real-time consumer prices. The hierarchical structure in the relation between retailers and consumers, pertaining to the so-called Stackelberg (or leader–follower) games, is imposed by the formulation as a bilevel optimisation problem. The model has three-stages to reflect the fact that decisions are made day-ahead, real-time and ex-post with different information structure on the stochastic variables involved. Furthermore a dynamic model for the demand flexibility based on realistic consumer preferences is employed.

In an illustrative example, the model is simulated in a realistic setup, which allows the comparison of the results obtained using the optimal dynamic price with the ones under fixed and time-of-use pricing schemes. We show that, in the dynamic-price case, the retailer, while maximising its profits, sends the consumer a price-incentive to shift his/her demand to periods of the day characterised by low spot market prices. Similarly, a drop in the imbalance costs borne by the retailer, due to deviations of the actual consumption from the energy contracted day-ahead, is experienced when switching from a fixed or a time-of-use to a real-time consumer price regime. It turns out that the dynamic pricing scheme minimises the retailer net payments in the day-ahead and real-time markets. On the contrary, the fixed price yields the highest costs among the pricing schemes considered, while the time-of-use price has a middle performance.

We link the reduction of procurement and regulation costs, obtained by shifting the load, to an increase in social welfare. As simulations show, though, the redistribution of the additional welfare is not fair in the dynamic-price scheme considered, as the retailer absorbs entirely the added welfare. Indeed, the consumer payments to the retailer are highest under the dynamic-pricing scheme in the considered example. On the contrary, the time-of-use setup yields the lowest costs for the consumers among the pricing schemes considered. These results, however, do not account for the effect of competition among retailers. In any case, particular care should be taken in designing pricing schemes that can effectively motivate consumers' participation in real-time price programmes.

Finally, through a sensitivity analysis it is shown that, once real-time contracts are in place, there is an economic incentive for the consumers to increase their flexibility.

Future extensions of this research could move in several directions. Different utility functions to model the trade-off for the consumer between electricity price and comfort could be defined and simulated. For example, the lower and upper bounds of the comfort band could be linear functions of the price, or a quadratic penalty for deviations of the temperature from a reference could be used. Furthermore, different forms of consumer flexibility could be considered, for example by modelling the consumption of "intelligent appliances" such as price-responsive washing machines and electric vehicles. Besides, a different setup ensuring a unique solution to the lower-level optimisation problem could be proposed so as to improve the controllability of the load from the retailer perspective, i.e. to ensure that the strong Stackelberg solution is also unique. Furthermore, the effect of renewable power on market prices could be introduced in the model, thus paving the way for an assessment of the value of demand response programmes in the integration of renewable generation in the system. Additionally, the optimisation model for the retailer could be refined by considering a diversified portfolio including e.g. futures and options, and by including risk management. Finally, competition among retailers could be modelled in the framework of *Equilibrium Programs with Equilibrium Constraints* (EPECs).

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