Asynchronous Real-Time Peer-to-Peer Electricity Market

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***Abstract*—Participants in electricity markets are becoming more proactive because of the rapid development of distributed energy resources and demand-side response management, which also boosts the evolution of peer-to-peer (P2P) market mechanisms. Moreover, the market also prefers operating in a real-time scheme in response to the fluctuant and uncertain wind or solar power generation. Therefore, a practicable real-time P2P market mechanism is in urgent need. However, it is technically challenging to combine real-time and P2P mechanisms together since the P2P mechanism brings a heavy communication burden, while the real-time scheme demands rapid computation. To overcome this challenge, we first propose a synchronous real-time P2P electricity market, which can minimize the social cost in the long run instead of each time slot, and effectively reduce the computational burden. However, the efficiency may be highly limited by the slow and debased agents. Thus, we further propose an innovative asynchronous real-time P2P market mechanism where all agents can freely trade without waiting for idle or inactive neighbor agents. The sublinear regret upper bound is proved, which also indicates the social cost will be minimized in the long run. Simulations show that our algorithm enjoys good convergence performance and fairness compared with the synchronous mechanism.**

***Index Terms*—Real-time P2P electricity market, online consensus ADMM, asynchronous market mechanism, forgetting factor, non-stationary regret**

I. Introduction

The ever-increasing distributed energy resources (DER) and demand-side response management (DRM) characterize the future of electrical power systems and market mechanisms. Market participants undertake a proactive behavior by managing their consumption, production, and energy storage. Therefore, electricity markets are evolving towards more decentralized mechanisms. However, current electricity markets still complete resource allocation and pricing based on the conventional hierarchical and top-down approach [1], which causes participants to be passive receivers. Recently, a novel idea of electricity markets has emerged: these so-called peer-to-peer (P2P) electricity markets rely on multi-bilateral trades among participants [2]–[9]. Employing a P2P market mechanism can yield a number of advantages, e.g., empowerment of participants, increasing the reliability of power system and protection of privacy [2], [5]. Existing works about P2P markets mainly focus on the following issues: reallocation of the costs [3], product differences [4], [9], dispatch fairness [8], communication properties [6] and costs [7].

In the meantime, renewable power generation, e.g., wind and solar power, is variable and uncertain. The characteristics of the underlying stochastic processes change with time. Besides, the cost or utility functions of the agents are also mostly time-varying. Consequently, it is required to continuously reschedule the trades among agents in a real-time mode to keep energy balance in response to these changes – hence calling for real-time P2P markets. It is noted that some other existing works already focused on deploying P2P mechanisms into real-time electricity markets [10]–[13]. For instance, [10] proposed bilateral contract networks for P2P energy trading in real-time markets, but it did not consider the changes and uncertainties of agents in real-time. [11] proposed a P2P local electricity market incorporating both energy and uncertainty tradings simultaneously, but only at the day-ahead stage. Other works [12], [13] employed blockchain-based approaches in real-time P2P market, however, the prices and payments are still centrally determined by the system coordinator. Eventually, the main challenge to combine P2P and real-time mechanisms is that P2P negotiation approaches rely on a heavy computational and communication burden [7], while real-time trading requires very fast calculation. As it may be too expensive to solve a full P2P market at each and every time slot, one needs to consider computationally cheaper approaches. To the best of our knowledge, none of the existing works can tackle this computational efficiency challenge well. This is what we do here, designing a real-time P2P market in an online optimization framework.

Specifically, this work turns into an alternative of minimizing the social cost for the P2P market in the long run, instead of performing optimization in every short real-time time slot, which is computationally lighter and more practicable. Online optimization is an efficient tool to solve this kind of problem where players' characteristics are time-varying [14]–[16], and the goal of this work is to design an online decentralized algorithm with high computational efficiency for the real-time P2P electricity market. To this end, we first propose an online consensus ADMM (OC-ADMM) algorithm, which blends the decentralization property of the consensus method with the superior convergence property of online ADMM. However, the OC-ADMM is performing in a synchronous mode, and the computational efficiency will be highly restricted by debased agents. To tackle these issues, we improve the OC-ADMM into an asynchronous algorithm where none of the agents are required to be synchronized with neighbor agents. Finally, the sublinear regret upper bound is proved, which indicates the social cost will be minimized in the long run. In simulations, real wind power data is employed for the comparisons between the synchronous mechanism, which demonstrates the good convergence performance and fairness of our asynchronous mechanism. Our contributions mainly lie as follows.

- We propose a novel idea of a real-time P2P electricity market using an online optimization approach, which highly increases the computational efficiency.
- We first propose the OC-ADMM algorithm, which is a synchronous algorithm but highly restricted by slow agents. To overcome this drawback, we further improve it into asynchronous OC-ADMM (AOC-ADMM) algorithm for real-time P2P market, where agents can freely trade without waiting for slow neighbor agents.
- The sublinear non-stationary regret upper bound for our AOC-ADMM algorithm is proved, which implies the social cost will be minimized in the long run.

The rest of the paper is organized as follows: Section II presents the real-time P2P electricity market model. Section III proposes the synchronous and asynchronous market mechanisms, followed by the regret and market properties analysis in Section IV. Numerical results and comparisons are presented in Section V. Finally, conclusions are drawn in Section VI.

II. Real-Time P2P Electricity Market Model

We consider a P2P electricity market with a set Ω of N agents who can be producers, consumers or prosumers over a period of time T. The duration of each single time slot is set to 5 minutes. We suppose all agents are rational and truthful as in [17], which means they always make strategic decisions to maximize individual profit.

A. Peer-to-Peer Trading

A P2P electricity market mechanism is much more decentralized compared with centralized markets, which consists of negotiations over the price and quantity among agents. To model the trading process, the net power injection $E_{n,t}$ of agent $n \in \Omega$ at time slot t is split into a summation of bilaterally traded quantities with a set of neighbor agents $m \in \omega_n$ as

$$E_{n,t} = \sum_{m \in \omega_n} E_{nm,t}, \quad \forall n \in \Omega,\ t = 1, ..., T \tag{1}$$

A positive value of $E_{nm,t}$ corresponds to a sale/production and a negative value to a purchase/consumption. To lighten notations, $\mathbf{E}_{n,t} = \{E_{n1,t}, ..., E_{nm,t}\}$ is used to denote the whole transactions of agent n at time slot t. The power set-point of agent n at time slot t is constrained as

$$\underline{E}_{n,t} \leq E_{n,t} \leq \overline{E}_{n,t}, \quad \forall n \in \Omega,\ t = 1, ..., T \tag{2}$$

Here, the upper bound $\overline{E}_{n,t}$ is set to the actual power generation for renewable generators. Each agent can be either producer/consumer ($\underline{E}_{n,t}\overline{E}_{n,t} \geq 0$) or prosumer ($\underline{E}_{n,t}\overline{E}_{n,t} < 0$). Hence, $E_{nm,t}$ are constrained as

$$\begin{cases} E_{n,t} \geq E_{nm,t} \geq 0, \quad \forall (n,m) \in (\Omega_p, \omega_n),\ t = 1, ..., T \\ E_{n,t} \leq E_{nm,t} \leq 0, \quad \forall (n,m) \in (\Omega_c, \omega_n),\ t = 1, ..., T, \\ \underline{E}_{n,t} \leq E_{nm,t} \leq \overline{E}_{n,t}, \forall (n,m) \in (\Omega_{ps}, \omega_n),\ t = 1, ..., T, \end{cases} \tag{3}$$

where Ω_p, Ω_c and Ω_{ps} are the sets of producers, consumers and prosumers, respectively. Finally, the market equilibrium between production and consumption is represented by a set of reciprocity balance constraints as

$$E_{nm,t} + E_{mn,t} = 0, \quad \forall (n,m) \in (\Omega, \omega_n),\ t = 1, ..., T. \tag{4}$$

B. Social Cost Minimization Problem

The goal of the real-time P2P market is to minimize the social cost over a period of time T, which can be formulated as

$$\begin{aligned} \min_{\{E_{n \in \Omega}\}} \quad & \sum_{t=1}^{T} \left(\sum_{n \in \Omega} C_{n,t}(E_{n,t}) \right) \\ \text{s.t.} \quad & (1) - (4). \end{aligned} \tag{5}$$

In the real-time P2P market, each agent takes action in an online fashion since they are unaware of future information. To be specific, at the beginning of time slot t, each agent reveals the cost or utility function $C_{n,t}$, while the renewable agent updates the actual power generation. Then, based on the updated information, each agent will negotiate with neighbor agents again and again until satisfy the balance constraints (4) for time slot t. Thus, a double-loop algorithm is required, where the function $C_{n,t}$ changes in the outer loop, and the inner loop runs iteratively until reaching balance. However, in a P2P market, the number of communication times will increase with the square of the number of agents [7], which means if we want to obtain the optimal solution at each time slot, all the agents have to complete a large number of iterations and cause very heavy communication and computational burden. Thus, one needs to devise appropriate and computationally cheaper approaches. To this end, we propose an online optimization framework for the real-time P2P market, which is more practicable and applicable.

III. Asynchronous Real-Time P2P Electricity Market

In this section, we first propose the real-time P2P market mechanism using online consensus ADMM (OC-ADMM). However, the synchronous mechanism is highly restricted by slow and debased agents, which is not efficient for the P2P market. To overcome this problem, it is further improved to an asynchronous market mechanism.

A. Synchronous Market Mechanism

Since the market is running in an online framework, problem (5) is decomposed into each single time slot. The social cost minimization problem for time slot t is

$$\begin{aligned} \min_{\{\mathbf{E}_{n \in \Omega}\}} \quad & \sum_{n \in \Omega} \left(C_{n,t}(E_n) + \sum_{m \in \omega_n} \frac{\eta}{2} (E_{nm,t-1} - E_{nm})^2 \right) \\ \text{s.t.} \quad & (1) - (4), \end{aligned} \tag{6}$$

where η is the penalty factor and the term $\frac{\eta}{2}(E_{nm,t-1} - E_{nm})^2$ is appended to make the results close to previous value $E_{nm,t-1}$ in order to speed up the convergence process [18], [19]. ADMM is an algorithm that blends the decomposability of dual ascent with the superior convergence properties of the method of multipliers, but there is a central coordinator for updating the dual variable [20]. To get rid of the central coordinator, [20] proposed the consensus ADMM method, where all agents can reach a consensus value by only communicating with neighbor agents. For the plain ADMM, the objective functions are fixed and not time-varying. However, in practical applications, the objective functions usually change with time. To improve the practicability, [18] proposed the online ADMM, which is an efficient technique that combines plain ADMM with online learning theory, and the target is to minimize the objective value in the long run. Combining the above approaches, we propose the novel OC-ADMM algorithm to implement a real-time P2P market, which produces the following updates:

- **Energy Updates**: Each agent n updates their transactions with neighbor agents by solving individual optimization problem with constraints (1)-(3) as below:

$$\mathbf{E}_{n,t} = \underset{\{\mathbf{E}_n\}}{\operatorname{argmin}}\, C_{n,t}(E_n) + \sum_{m\in\omega_n} \lambda_{nm,t-1}(F_{nm,t-1} - E_{nm}) + \frac{\rho}{2}(F_{nm,t-1} - E_{nm})^2 + \frac{\eta}{2}(E_{nm} - E_{nm,t-1})^2, \tag{7}$$

  where ρ is the penalty factor and $\lambda_{nm,t-1}$ is the dual variable of the reciprocity constraint (4), which also defines the price for the traded energy $E_{nm,t-1}$. $\boldsymbol{\lambda}_{n,t-1} = \{\lambda_{n1,t-1}, ..., \lambda_{nm,t-1}\}$ is used to represent the whole energy prices of agent n to neighbor agents $m \in \omega_n$ at time slot $t-1$. $F_{nm,t-1}$ is the consensus variable defined as $\frac{E_{nm,t-1} - E_{mn,t-1}}{2}$ and $F_{n,t-1} = \sum_{m\in\omega_n} F_{nm,t-1}$. Then, each agent broadcasts $\mathbf{E}_{n,t}$ to neighboring agents.
- **Price Updates**: All agents update their energy prices to neighboring agents $m \in \omega_n$ as:

$$\lambda_{nm,t} = \lambda_{nm,t-1} - \rho(E_{nm,t} + E_{mn,t})/2. \tag{8}$$

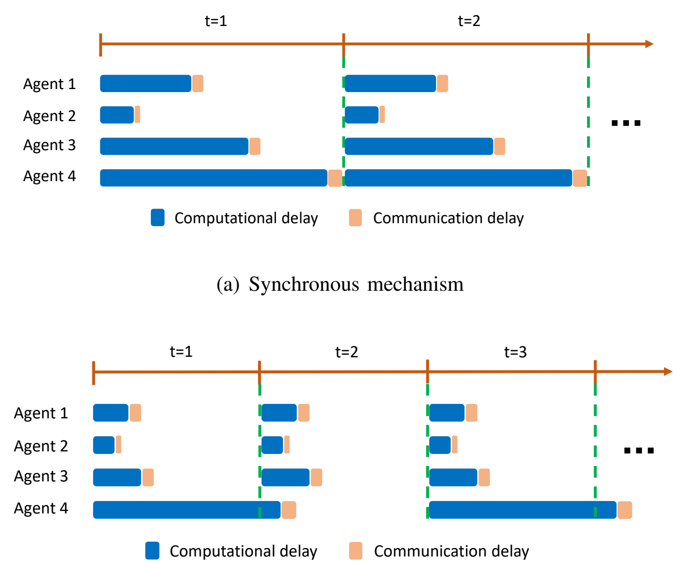
However, the OC-ADMM is running in a synchronous manner, and each agent has to wait to receive all bidding prices and quantities from neighbor agents, which results in the market efficiency being highly restricted by slow agents.

B. Asynchronous Market Mechanism

To overcome the drawback of the synchronous mechanism, we propose a novel asynchronous mechanism for the real-time P2P market. Although there are also other methods to accelerate the market, e.g., adaptive penalty factor [21] and alternative stopping criterion [7], this paper focuses on using the asynchronous mechanism.

As shown in Fig. 1(a), in the synchronous P2P market, agents have to wait to receive all bidding prices and quantities from neighbor agents, and a lot of time is wasted on waiting for the slowest agent. Since the scale of the P2P market is quite large and the real-time mechanism demands very fast trading, if there exists one invalid or damaged agent, the market efficiency will be significantly reduced. Thus, the synchronous mechanism is not suitable and practicable for the real-time large-scale P2P market. To overcome this issue, we propose a novel asynchronous real-time P2P electricity market, where the duration of each time slot Δt is fixed and shorter, thus the market can be running and trading more frequently. There are two reasons why we choose fixed time slots. First, each pair of agents prefer negotiating and trading for a known period of future time; second, it is helpful for us to analyze the optimality and convergence performance of the market.

A simple example is provided to show the asynchronous market mechanism. In Fig. 1(b), at time slot $t = 1$, the agents 1-3 are active and they will negotiate with each other to decide the energy prices and quantities without waiting for agent 4, whose energy prices and quantities between neighboring agents will remain unchanged. Then, at time slot $t = 2$, four agents are all active, and they will trade with each other. This market mechanism implies that none of the agents has to be synchronized with all neighboring agents and does not need to wait for the slowest agent either. Compared with the synchronous market in Fig. 1(a), Δt is shorter, and the market can be running more frequently.



(a) Synchronous mechanism

(b) Asynchronous mechanism

Fig. 1: Illustration of synchronous and asynchronous mechanisms.

Let $\mathcal{A}_{n,t} \subseteq \omega_n$ denote the subset of active neighbor agents of agent n at time slot t. For example, in Fig. 1, $\mathcal{A}_{1,1} = \{2, 3\}$ and $\mathcal{A}_{2,2} = \{1, 3, 4\}$. We use $\mathcal{A}^c_{n,t}$ to denote the complementary set of $\mathcal{A}_{n,t}$, i.e., $\mathcal{A}_{n,t} \cap \mathcal{A}^c_{n,t} = \varnothing$ and $\mathcal{A}_{n,t} \cup \mathcal{A}^c_{n,t} = \omega_n$. We define $\mathbf{E}^{\mathcal{A}}_n$ to be the energy transactions of agent n for active neighboring agents $m \in \mathcal{A}_{n,t}$.

Based on the OC-ADMM, we further propose the AOC-ADMM, and the algorithm produces the following updates.

- **Energy Updates**: The active agent n will update the energy transactions $\mathbf{E}^{\mathcal{A}}_{n,t+1}$ for active neighboring set $\mathcal{A}_{n,t}$, which is updated by solving below optimization problem:

$$\mathbf{E}_{n,t}^{\mathcal{A}}= \underset{\mathbf{E}_n^{\mathcal{A}}}{\operatorname{argmin}} \sum_{l=\bar{t}_n+1}^{t} \upsilon^{t-l} C_{n,l}(E_n) + \sum_{m\in\mathcal{A}_{n,t}} \lambda_{nm,t}(F_{nm,t-1}-E_{nm}) + \frac{\rho}{2}(F_{nm,t-1}-E_{nm})^2 + \frac{\eta}{2}(E_{nm}-E_{nm,t-1})^2 \tag{9}$$

while for the idle neighbor agents $m \in \mathcal{A}_{n,t}^c$, the energy transaction will remain unchanged, i.e., $E_{nm,t} = E_{nm,t-1},\ m \in \mathcal{A}_{n,t}^c$. $\upsilon \in (0,1]$ is the forgetting factor. A value of 1 results in no forgetting while decreasing values increase the amount of forgetting. Values slightly less than 1 are generally preferred. It should be noticed that, the energy upper bound $\overline{E}_{n,t}$ should be reduced by $\sum_{m\in\mathcal{A}_{n,t}^c} E_{nm,t}$. To ease the notation, we first define $G_{n,t}(E_n) = \sum_{l=\bar{t}_n+1}^{t} \upsilon^{t-l} C_{n,l}(E_n)$. After that, each agent broadcasts $\mathbf{E}_{n,t+1}$ to neighbor agents.

- **Price Updates**: All agents update their energy prices to neighbor agents $m \in \omega_n$ as:

$$\lambda_{nm,t} = \begin{cases} \lambda_{nm,t-1} - \rho\frac{E_{nm,t}+E_{mn,t}}{2},\ \forall m \in \mathcal{A}_{n,t} \\ \lambda_{nm,t-1},\ \forall m \in \mathcal{A}_{n,t}^c \end{cases} \tag{10}$$

However, since we only run one iteration at each time slot, the energy balance between two agents will probably not be balanced, i.e., $E_{nm,t}+E_{mn,t} \neq 0$. To address this problem, we design a projection-based energy update process to guarantee the energy balance.

C. Projection-based Energy Update

In the electricity market, it is necessary to guarantee the balance for trade among agents. Thus, a projection-based energy update process is designed to determine the final quantities. In this process, iterations are indexed with l.

First, $\hat{E}_{nm,t}^{l+1}$ is set to be the consensus value of the trade between agents n and m from last iteration as

$$\hat{E}_{nm,t}^{l+1} = \hat{F}_{nm,t}^{l} = (\hat{E}_{nm,t}^{l} - \hat{E}_{mn,t}^{l})/2. \tag{11}$$

Here, the initial value $\hat{E}_{nm,t}^{1}$ is obtained after the primal-dual alternate update process, i.e., $\hat{E}_{nm,t+1}^{1} = E_{nm,t+1}^{k}$. Then the total energy power is obtained as $\hat{E}_{n,t}^{l+1} = \sum_{m\in\omega_n} \hat{E}_{nm,t}^{l+1}$. After projecting into the feasible region (2), the projected power $\hat{E}_{n,t}^{proj,l+1}$ is updated as

$$\hat{E}_{n,t}^{proj,l+1} = \max\left\{\min\left\{\hat{E}_{n,t}^{l+1}, \overline{E}_{n,t}\right\}, \underline{E}_{n,t}\right\}. \tag{12}$$

Then the trade $\hat{E}_{nm,t}^{l+1}$ to neighbor agent m is updated as

$$\hat{E}_{nm,t}^{l+1} = \hat{E}_{nm,t}^{l+1} \frac{\hat{E}_{n,t}^{proj,l+1}}{\hat{E}_{n,t}^{l+1}}. \tag{13}$$

Finally, all agents send the updated trade quantities $\hat{E}_{nm,t}^{l+1}$ to its neighbor agents, and check if the balance equation $\hat{E}_{nm,t}^{l+1}+\hat{E}_{mn,t}^{l+1} < \epsilon$ is satisfied, where ϵ is the allowed maximal violation; if not, repeat the processes (11)-(13) until balance.

IV. Market Analysis

In this section, we first analyze the convergence performance for our online algorithm, which is usually measured by the regret, i.e., the accumulated gap between the online solution and the best solution in hindsight [18], [22]–[24]. Under some standard assumptions, the sublinear regret upper bound for our algorithm is proved, which indicates the social cost will be minimized in the long run. Finally, the desirable four properties of market mechanisms are analyzed.

A. Regret Analysis

Since the renewable power generation is changing with time, the non-stationary regret [24] is adopted, which compares the accumulated gap between the online solution and the best solution at each time slot as defined below for our problem

$$\tilde{R}(T) = \sum_{t=1}^{T}\left(\sum_{n\in\Omega} C_{n,t}(\hat{E}_{n,t})\right) - \sum_{t=1}^{T}\left(\sum_{n\in\Omega} C_{n,t}(E_{n,t}^{*})\right) \tag{14}$$

where $E_{n,t}^{*}$ is the optimal solution of agent n at time slot t.

Before presenting the results, some needed definition, assumptions and lemma are introduced to derive the sublinear regret upper bound to be presented later in Theorem 1.

Definition 1. *Path variation is defined as the accumulated absolute value of variations of the optimums sequence:*

$$P_{nm,T} = \sum_{t=1}^{T} \left| E_{nm,t}^{*} - E_{nm,t+1}^{*} \right|. \tag{15}$$

Assumption 1.

(a) $C_{n,t}$ are convex with bounded subgradients for $m \in \omega_n$, i.e., $\frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}} \leq G,\ \forall (n,m) \in (\Omega, \omega_n)$, with G being a positive constant.

(b) The initial values are set to zero, i.e., $\lambda_{nm,1} = 0$ and $E_{nm,1} = 0,\ \forall (n,m) \in (\Omega, \omega_n)$.

(c) We assume the gap between the optimal solution and initial one are bounded, i.e., $(E_{nm,t}^{*} - E_{nm,1})^2 \leq D_1$ and $(F_{nm,t}^{*} - F_{nm,1})^2 \leq D_2,\ \forall (n,m) \in (\Omega, \omega_n)$, with D_1 and D_2 being positive constants.

(d) We assume the gap between the online solution and optimal one are bounded, i.e., $C_{n,t}(E_{n,t}) - C_{n,t}(E_{n,t}^{*}) \leq J,\ \forall n \in \Omega$, with J being a positive constant.

(e) The optimal solutions do not change dramatically, or in mathematical sense, the path variation is bounded as $P_{nm,T} \leq P,\ \forall (n,m) \in (\Omega, \omega_n)$, with P being a positive constant called variation budget.

(f) The electricity prices do not change dramatically, or in mathematical sense, the price variation between two time slots is bounded as $|\lambda_{nm,t+1} - \lambda_{nm,t}| \leq \Lambda,\ \forall (n,m) \in (\Omega, \omega_n)$, with Λ being a positive constant.

In Assumption 1, (a)-(d) are usually required in the online optimization settings. For (e), [24] proved that the non-stationary regret is linear in T if there is no restriction on the path variation. Thus we place some restrictions on the possible variation of the optimums, which is also proposed in some previous works [24]–[26]. (f) is reasonable in the electricity market background since the electricity price cannot be infinite.

Lemma 1. *Let* $\frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}}$ *be the gradient of* $G_{n,t}(E_n)$ *at* $E_{nm,t}$*. Since* $\mathbf{E}_{n,t}$ *minimizes (9), combining (10), we have*

$$\begin{aligned}\frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}} &= \lambda_{nm,t} + \rho(F_{nm,t-1} - F_{nm,t}) \\ &+ \eta(E_{nm,t-1} - E_{nm,t}),\ \forall m \in \mathcal{A}_{n,t}\end{aligned} \tag{16}$$

Bearing all above in mind, the following theorem establishes the regret bound for the AOC-ADMM.

Theorem 1. *The AOC-ADMM algorithm has the following sublinear non-stationary regret upper bound by setting* $\rho = \sqrt{T}$ *and* $\eta = \sqrt{T}$

$$\tilde{R}(T) \leq N(N-1)\left(\frac{D_1 + D_2 + \Lambda^2 + G^2}{2} + LP\right)\sqrt{T} + \frac{NJ}{1-\upsilon} \tag{17}$$

where $L = \max_{n\in\Omega} 2 * (\overline{E}_n - \underline{E}_n)$.

Proof. See Appendix A. □

Since the regret has $\mathcal{O}(\sqrt{T})$ upper bound, we have $\lim_{T\to\infty} \frac{\tilde{R}(T)}{T} = 0$, which implies the accumulated social cost between our online solution and optimal one are approaching to zero in the long run on time average, which also means the social cost is minimized at each single time slot in a long period of time. This is the main target for designing an online optimization algorithm.

B. Desirable Properties of Market Mechanism

It is important and necessary to evaluate a market mechanism by checking the four desirable properties, which are market efficiency[1], incentive compatibility[2], cost recovery[3] and revenue adequacy[4]. Based on the Hurwicz theorem [27], no mechanism is capable of achieving all those properties at the same time.

1) Market efficiency: From the Theorem 1, we have $\lim_{T\to\infty} \frac{\tilde{R}(T)}{T} = 0$, which implies the social cost is minimized or the market efficiency is maximized in the long run.

2) Incentive compatibility: A market agent can gain profit by not trustfully bidding price or quantity, but we assume that they are all truthful in this work.

3) Cost recovery: The individual profit for an agent at time slot t is

$$\sum_{m\in\omega_n} \lambda_{nm,t} E_{nm,t} - C_{n,t}(E_{n,t}) \tag{18}$$

Since the quadratic cost function is convex, monotonically increasing, and passing through the origin, the agent can always set $E_{n,t} = E_{nm,t} = 0$ to avoid a negative profit. Thus the cost recovery is satisfied.

[1]Market efficiency is maximized when outcomes maximize social welfare.

[2]A mechanism is called incentive-compatible if every participant can maximize its objective just by acting according to its true preferences.

[3]Cost recovery implies that individual profit is non-negative.

[4]Revenue adequacy implies that there is no financial deficit in the market.

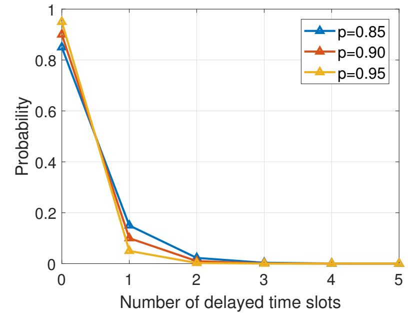


Fig. 2: The probability of delayed time slots with different active rate.

4) Revenue adequacy: From (10), we can conclude the prices between agents are identical, i.e., $\lambda_{nm,t} = \lambda_{mn,t}$, and after the projection, the trades among agents are balanced, i.e., $\hat{E}_{nm,t} + \hat{E}_{mn,t} = 0$. Thus, the revenue adequacy is satisfied.

V. Simulation Results

Our online algorithm is tested on a dataset of wind power generation at 20 wind farms in Australia [28] to show the convergence performance and comparisons with the synchronous market. To better show the performance, uniformly distributed stochastic parameter settings are applied. We perform simulations using Matlab R2017b on a PC with 1.6 GHz Intel Core 4 Duo CPU and 8 GB memory, and the convex optimization problem is solved by CVX Sedumi solver.

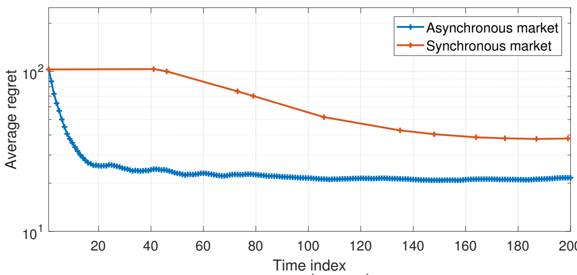
A. Convergence Performance

We build a market composed of 20 conventional generators, 20 users, and 20 wind generators with the real wind data. The convergence performance and comparisons between the synchronous mechanism are tested. We assume that the active rate of each agent at each time slot satisfies the Bernoulli distribution. The active rate is p_n and the idle rate is $1 - p_n$. The probability of the number of continuously delayed time slots is shown in Fig. 2 with different active rate. As seen from the profile, the agent is very unlikely to be infinitely delayed.

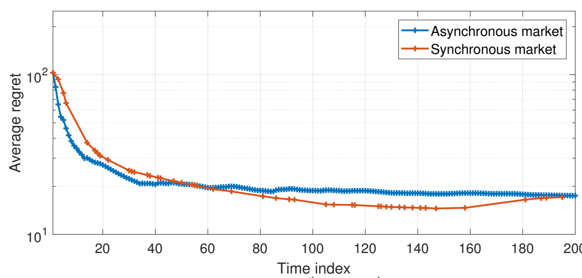
Fig. 3 shows the convergence performance of our asynchronous market mechanism. It can be seen that $\tilde{R}(T)/T$ decreases quite fast at the first 20 time slots, while after that, the average regret decreases much slower but still keeps going down with a little oscillation, which is mainly due to the large uncertainty of renewable generation.

B. Comparisons with Synchronous Mechanism

We run the synchronous and asynchronous market mechanisms under two different active rate ranges, i.e., $p_n \in (0.9, 1)$ and $p_n \in (0.95, 1)$. As seen in Fig. 3, for our asynchronous mechanism, the change of active rate only has a slight impact on the convergence performance, which shows the robustness of the asynchronous mechanism. While for the synchronous mechanism, with the increase of active rate, the probability of all agents being active will increase. Therefore, the synchronous mechanism can be running more frequently, which results in boosting the average regret convergence speed, even better than the asynchronous mechanism. However, it should be noticed that in the real world P2P market, the number of agents is usually substantial, much more than 60, which means as long as there exists one agent who has a very low active rate, the synchronous mechanism could be very inefficient.



(a) $p_n \in (0.9, 1)$



(b) $p_n \in (0.95, 1)$

Fig. 3: Average regret $\tilde{R}(T)/T$ of synchronous and asynchronous markets with different active rate ranges.

We also compare the average number of time slots between two iterations, i.e., market transactions, of synchronous and asynchronous mechanisms under different active rate ranges. As seen in Fig. 4, with the decrease of active rate range, the average number of time slots between two iterations for synchronous market mechanism will be exponentially increasing. While for the asynchronous market, since it can be running and trading for any time slot, the average value keeps at 1. Thus, our asynchronous market mechanism can be running more frequently and better catch up with the uncertainty of renewable power generation.

We also study individual profits under synchronous and asynchronous mechanisms. We build a market with 5 conventional generators, 5 users, and 5 renewable generators. The characteristics of the same type of agents are identical. The active rates of the five conventional generators are set to 0.6, 0.7, 0.8, 0.9, 1, respectively, while the active rates of other agents are 1. Then, the accumulative individual profit of conventional generators and users over 200 time slots are shown in Fig. 5. The results indicate the profit of the same type of agents are identical in the synchronous market since the agents have to trade together. While in the asynchronous market, the agent whose active rate is higher can trade more frequently and gain more profit. Thus, our asynchronous mechanism is fairer, which can encourage agents to improve

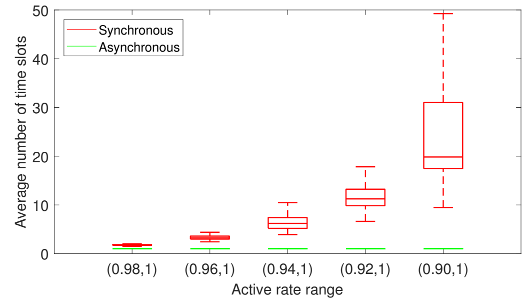


Fig. 4: The average number of time slots between two iterations of two mechanisms under different active rate ranges.

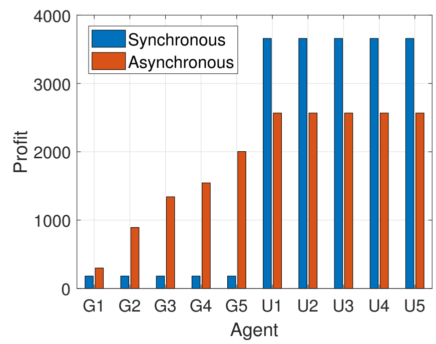


Fig. 5: The profit of agents under two mechanisms

quality and increase the active rate.

C. The Impact of Forgetting Factor

The impact of the forgetting factor on the convergence performance is tested. Fig. 6 shows that the decrease of forgetting factor values will increase the amount of forgetting, which means the agent focuses more on the updated new information, and the convergence performance will be improved.

VI. Conclusion

P2P markets are considered as an evolution of the future electricity markets driven by distributed energy resources and demand response management development. How to design a proper P2P electricity market mechanism to satisfy the fast calculation requirement of the real-time market remains a challenge. To this end, we propose a novel asynchronous real-time P2P market mechanism, where agents can freely trade and

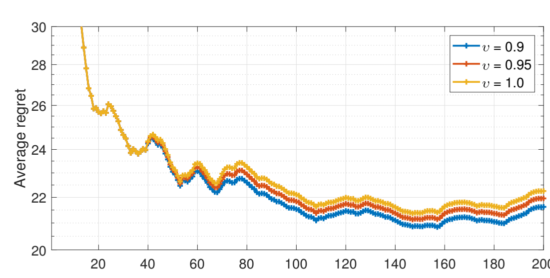


Fig. 6: The impact of forgetting factor value.

negotiate with each other without waiting to receive all bidding information from neighbor agents. We give proof of the sublinear non-stationary regret upper bound for our algorithm, which indicates the social welfare will be maximized in the long run. Simulation results show that our asynchronous market mechanism has good convergence performance, robustness, and fairness compared with the synchronous mechanism.

APPENDIX A
PROOF OF THE SUBLINEAR REGRET UPPER BOUND

Since $G_{n,t}$ is a convex function and its subgradient at $E_{nm,t+1}$ is given in (16), for optimal solution $E^*_{n,t}$ we have

$$\begin{aligned}
&G_{n,t}(E_{n,t}) - G_{n,t}(E^*_{n,t}) \\
&\le \sum_{m\in\mathcal{A}_{n,t}} \frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}}(E_{nm,t} - E^*_{nm,t}) \\
&= \sum_{m\in\mathcal{A}_{n,t}} \lambda_{nm,t}(E_{nm,t}-F^*_{nm,t}) \\
&\quad + \sum_{m\in\mathcal{A}_{n,t}} \rho(F_{nm,t-1}-F_{nm,t})(E_{nm,t}-F^*_{nm,t}) \\
&\quad + \sum_{m\in\mathcal{A}_{n,t}} \eta\,(E_{nm,t-1} - E_{nm,t})\,(E_{nm,t}-E^*_{nm,t}) \\
&= \sum_{m\in\mathcal{A}_{n,t}} \lambda_{nm,t}(E_{nm,t}-F^*_{nm,t}) \\
&\quad + \sum_{m\in\mathcal{A}_{n,t}} \frac{\rho}{2}\big[(F_{nm,t-1}-F^*_{nm,t})^2-(F_{nm,t}-F^*_{nm,t})^2 \\
&\quad +(E_{nm,t}-F_{nm,t})^2-(E_{nm,t}-F_{nm,t-1})^2\big] \\
&\quad + \frac{\eta}{2}\big[(E^*_{nm,t} - E_{nm,t-1})^2-(E^*_{nm,t} - E_{nm,t})^2 \\
&\quad -(E_{nm,t} - E_{nm,t-1})^2\big]
\end{aligned} \tag{19}$$

According to the Fenchel-Young's inequality [29], we have

$$\begin{aligned}
&G_{n,t}(\hat{E}_{n,t}) - G_{n,t}(E_{n,t}) \\
&\le \sum_{m\in\mathcal{A}_{n,t}} \frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}}(\hat{E}_{nm,t} - E_{nm,t}) \\
&\le \sum_{m\in\mathcal{A}_{n,t}} \frac{1}{2\rho}\left(\frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}}\right)^2 + \frac{\rho}{2}(\hat{E}_{nm,t} - E_{nm,t})^2 \\
&\le \sum_{m\in\mathcal{A}_{n,t}} \frac{1}{2\rho}\left(\frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}}\right)^2 + \frac{\rho}{2}(F_{nm,t} - E_{nm,t})^2 \\
&\le \sum_{m\in\mathcal{A}_{n,t}} \frac{1}{2\rho}\left(\frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}}\right)^2 + \frac{(\lambda_{nm,t-1} - \lambda_{nm,t})^2}{2\rho}
\end{aligned} \tag{20}$$

For the penultimate inequality, because $\hat{E}_{nm,t}$ begins updating from $F_{nm,t}$, if the projection update finishes at the first time, which also means both agents do not touch the bound, the term $(\hat{E}_{nm,t} - E_{nm,t})^2$ reaches the maximal value $(F_{nm,t} - E_{nm,t})^2$; The worst case is that the final trade reaches at the initial value $E_{nm,t}$ or $-E_{mn,t}$, which means at the beginning, one of the agents n or m has reached the bound, then the term $(\hat{E}_{nm,t} - E_{nm,t})^2$ reaches the minimal value zero.

Combining (19)-(20), we have

$$\begin{aligned}
&G_{n,t}(\hat{E}_{n,t}) - G_{n,t}(E^*_{n,t}) \\
&\le \sum_{m\in\mathcal{A}_{n,t}} \lambda_{nm,t}(E_{nm,t}-F^*_{nm,t})+\frac{\rho}{2}(E_{nm,t}-F_{nm,t})^2 \\
&\quad + \sum_{m\in\mathcal{A}_{n,t}} \frac{\rho}{2}\left[(F_{nm,t-1}-F^*_{nm,t})^2-(F_{nm,t}-F^*_{nm,t})^2\right] \\
&\quad + \sum_{m\in\mathcal{A}_{n,t}} \frac{\eta}{2}\left[(E^*_{nm,t} - E_{nm,t-1})^2-(E^*_{nm,t} - E_{nm,t})^2\right] \\
&\quad + \sum_{m\in\mathcal{A}_{n,t}} \frac{1}{2\rho}\left(\frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}}\right)^2 + \frac{(\lambda_{nm,t-1} - \lambda_{nm,t})^2}{2\rho}
\end{aligned} \tag{21}$$

For the first term, using $E_{nm,t} - F_{nm,t} = \frac{\lambda_{nm,t-1}-\lambda_{nm,t}}{\rho}$, $F^*_{nm,t} + F^*_{mn,t} = 0$, $\lambda_{nm,t} = \lambda_{mn,t}$ and summing up for all $n \in \Omega$ yields

$$\begin{aligned}
&\sum_{n\in\mathcal{A}_t}\sum_{m\in\mathcal{A}_{n,t}} \lambda_{nm,t}(E_{nm,t}-F^*_{nm,t})+\frac{\rho}{2}(E_{nm,t}-F_{nm,t})^2 \\
&= \frac{1}{2}\sum_{\forall(n,m)\in(\Omega,\mathcal{A}_{n,t})} \lambda_{nm,t}\big[E_{nm,t}+E_{mn,t}-(F^*_{nm,t}+F^*_{mn,t})\big] \\
&\quad + \frac{\rho}{2}(E_{nm,t}-F_{nm,t})^2+\frac{\rho}{2}(E_{mn,t}-F_{mn,t})^2 \\
&= \frac{1}{2}\sum_{\forall(n,m)\in(\Omega,\mathcal{A}_{n,t})} 2\lambda_{nm,t}(E_{nm,t}-F_{nm,t}) \\
&\quad + \frac{1}{2\rho}(\lambda_{nm,t-1}-\lambda_{nm,t})^2+\frac{1}{2\rho}(\lambda_{mn,t-1}-\lambda_{mn,t})^2 \\
&= \sum_{\forall(n,m)\in(\Omega,\mathcal{A}_{n,t})} \frac{1}{2\rho}(\lambda^2_{nm,t-1}-\lambda^2_{nm,t})
\end{aligned} \tag{22}$$

For the second term

$$\begin{aligned}
&(F_{nm,t-1}-F^*_{nm,t})^2-(F_{nm,t}-F^*_{nm,t})^2 \\
&= (F_{nm,t-1}-F^*_{nm,t-1})^2-(F_{nm,t}-F^*_{nm,t})^2 \\
&\quad + (F_{nm,t-1}-F^*_{nm,t})^2-(F_{nm,t-1}-F^*_{nm,t-1})^2 \\
&\le (F_{nm,t-1}-F^*_{nm,t-1})^2-(F_{nm,t}-F^*_{nm,t})^2 \\
&\quad + (2F_{nm,t-1} - F^*_{nm,t-1} - F^*_{nm,t})(F^*_{nm,t-1} - F^*_{nm,t}) \\
&\le (F_{nm,t-1}-F^*_{nm,t-1})^2-(F_{nm,t}-F^*_{nm,t})^2 \\
&\quad + L\left|E^*_{nm,t-1} - E^*_{nm,t}\right|
\end{aligned} \tag{23}$$

Similarly, for the third term

$$\begin{aligned}
&(E^*_{nm,t} - E_{nm,t-1})^2-(E^*_{nm,t} - E_{nm,t})^2 \\
&\le (E_{nm,t-1}-E^*_{nm,t-1})^2-(E_{nm,t}-E^*_{nm,t})^2 \\
&\quad + L\left|E^*_{nm,t-1}-E^*_{nm,t}\right|
\end{aligned} \tag{24}$$

Combining (21)-(22) and based on Assumption 1, we have

$$\begin{aligned}
&\sum_{t=1}^{T}\left(\sum_{n\in\mathcal{A}_t} G_{n,t}(\hat{E}_{n,t})\right)-\sum_{t=1}^{T}\left(\sum_{n\in\mathcal{A}_t} G_{n,t}(E^*_{n,t})\right) \\
&\le \sum_{\forall(n,m)\in(\mathcal{A}_t,\mathcal{A}_{n,t})} \frac{1}{2\rho}(\lambda^2_{nm,1}-\lambda^2_{nm,T}) \\
&\quad + \sum_{n\in\mathcal{A}_t}\sum_{m\in\mathcal{A}_{n,t}} \frac{\rho}{2}\left[(F_{nm,1}-F^*_{nm,1})^2-(F_{nm,T}-F^*_{nm,T})^2\right]
\end{aligned}$$

$$+\sum_{n\in\mathcal{A}_t}\sum_{m\in\mathcal{A}_{n,t}}\frac{\eta}{2}\left[(E_{nm,1}-E^*_{nm,1})^2-(E_{nm,T}-E^*_{nm,T})^2\right]$$
$$+\sum_{t=1}^{T}\sum_{n\in\mathcal{A}_t}\sum_{m\in\mathcal{A}_{n,t}}\frac{(\rho+\eta)L}{2}\left|E^*_{nm,t}-E^*_{nm,t}\right|$$
$$+\sum_{t=1}^{T}\sum_{n\in\mathcal{A}_t}\sum_{m\in\mathcal{A}_{n,t}}\frac{1}{2\rho}\left(\frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{nm,t}}\right)^2$$
$$+\sum_{t=1}^{T}\sum_{n\in\mathcal{A}_t}\sum_{m\in\mathcal{A}_{n,t}}\frac{(\lambda_{nm,t-1}-\lambda_{nm,t})^2}{2\rho}$$
$$\leq N(N-1)\left(\frac{\rho D_2}{2}+\frac{\eta D_1}{2}+\frac{(\rho+\eta)LP}{2}+\frac{TG^2}{2\rho}+\frac{T\Lambda^2}{2\rho}\right) \tag{25}$$

Since we have

$$\sum_{t=1}^{T}\left(\sum_{n\in\mathcal{A}_t}G_{n,t}(\hat{E}_{n,t})\right)-\sum_{t=1}^{T}\left(\sum_{n\in\Omega}G_{n,t}(E^*_{n,t})\right)$$
$$=\sum_{t=1}^{T}\sum_{n\in\mathcal{A}_t}\sum_{l=\bar{t}_n+1}^{t}\upsilon^{t-l}\left(C_{n,l}(\hat{E}_{n,t})-C_{n,l}(E^*_{n,t})\right)$$
$$=\sum_{t=1}^{T}\sum_{n\in\Omega}\upsilon^{t-l}\left(C_{n,t}(\hat{E}_{n,t})-C_{n,t}(E^*_{n,t})\right)$$
$$-\sum_{n\in\mathcal{A}^c_t}\sum_{l=\bar{t}_n+1}^{T}\upsilon^{T-l}\left(C_{n,l}(\hat{E}_{n,t})-C_{n,l}(E^*_{n,t})\right) \tag{26}$$

Noting that $\hat{E}_{n,l}=\hat{E}_{n,t}$ for $l=\bar{t}_n+1,...,t$ and using Assumption 1(f), the last term in (26) can be bounded as follows:

$$\sum_{n\in\mathcal{A}^c_t}\sum_{l=\bar{t}_n+1}^{T}\upsilon^{T-l}\left(C_{n,l}(\hat{E}_{n,t})-C_{n,l}(E^*_{n,t})\right) \tag{27}$$
$$\leq NJ\frac{1-\upsilon^{T-\bar{t}_n-1}}{1-\upsilon}\leq\frac{NJ}{1-\upsilon} \tag{28}$$

where we have used $\mathcal{A}^c_t\in\Omega$. By using (27) and (26) in (25), we finally have

$$\tilde{R}(T)=\sum_{t=1}^{T}\sum_{n\in\Omega}\left(C_{n,l}(\hat{E}_{n,t})-C_{n,t}(E^*_{n,t})\right)$$
$$\leq N(N-1)\left(\frac{\rho D_2}{2}+\frac{\eta D_1}{2}+\frac{(\rho+\eta)LP}{2}+\frac{TG^2}{2\rho}+\frac{T\Lambda^2}{2\rho}\right)$$
$$+\frac{NJ}{1-\upsilon} \tag{29}$$

Setting $\rho=\sqrt{T}$ and $\eta=\sqrt{T}$ yields sublinear regret $\tilde{R}(T)$.

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