A Stochastic Market Design With Revenue Adequacy and Cost Recovery by Scenario: Benefits and Costs

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***Abstract*—Two desirable properties of electricity market mechanisms include: i) *revenue adequacy* for the market, and ii) *cost recovery* for all generators. Previously proposed *stochastic* market-clearing mechanisms satisfy both properties in expectation only, or satisfy one property by scenario and another in expectation. Consequently, market parties may perceive significant risks to participating in the market since they may lose money in one or more scenarios, and therefore be discouraged from offering in the market or perhaps even from investing. We develop a stochastic two-stage market-clearing model including day-ahead and real-time settlements with an energy-only pricing scheme that ensures both properties by scenario. However, this approach is cost-inefficient in general and may sacrifice other desirable market attributes. Undesirable consequences include: one group of participants will have to pay more to ensure that all other participants have their costs covered, and thus their prices will not be equilibrium supporting; and day-ahead and real-time prices are not arbitraged in expectation, although this can be fixed by allowing virtual bidders to arbitrage but at the potential cost of increased market inefficiency. Considering these pros and cons, we propose our model as an appropriate tool for market analysis, and not for clearing actual markets. Numerical results from case studies illustrate the benefits and costs of the proposed stochastic market design.**

Keywords: Two-stage stochastic market clearing, revenue adequacy, cost recovery, equilibrium

NOTATION

Indices and Sets:

d	Index for loads
i	Index for conventional generators
k	Index for wind power generators
n, m	Indices for nodes
s	Index for wind power scenarios
v	Index for virtual bidders
Φ_n	Set of nodes connected to node n
Ψ_n	Set of generators and loads located at node n

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Constants:

$B_{n,m}$	Susceptance of transmission line (n,m) [S]
C_i	Offer price of generator i [$/MWh], equal to its marginal cost
$F_{n,m}$	Capacity of transmission line (n,m) [MW]
L_d	Power consumption of load d [MW]
$P_i^{\max}$	Capacity of generator i [MW]
P_i^{adj}	Maximum power adjustment limit of generator i in real-time market [MW]
V_d	Value of lost load for load d [$/MWh]
$W_{k,s}$	Wind power realization of generator k in real-time market under scenario s [MW]
$W_k^{\max}$	Installed capacity of wind power generator k [MW]
ϕ_s	Probability of scenario s

Day-ahead scheduling variables (first-stage):

b_v^{DA}	Trading quantity of virtual bidder v [MW]
$f_{n,m}^{\mathrm{DA}}$	Power flow from node n to node m [MW]
l_d^{DA}	Power consumption of load d [MW]
p_i^{DA}	Power output of generator i [MW]
w_k^{DA}	Power output of wind generator k [MW]
θ_n^{DA}	Voltage angle of node n [rad]

Real-time operation variables (second-stage):

b_v^{RT}	Trading quantity of virtual bidder v [MW]
$f_{n,m,s}^{\mathrm{RT}}$	Power flow from node n to m under scenario s [MW]
l_d^{RT}	Incremental power consumption of load d [MW]
$l_{d,s}^{\mathrm{shed}}$	Involuntarily shedding of load d under scenario s [MW]
$p_{i,s}^{\mathrm{RT}}$	Power adjustment of generator i under scenario s [MW]
$w_{k,s}^{\mathrm{RT}}$	Deviation of wind generator k under scenario s [MW]
$\theta_{n,s}^{\mathrm{RT}}$	Voltage angle of node n under scenario s [rad]

Dual variables:

λ_n^{DA}	Day-ahead locational marginal price at node n [$/MWh]
$\lambda_{n,s}^{\mathrm{RT}}$	Probability-weighted real-time locational marginal price at node n under scenario s [$/MWh]
μ,ρ	Set of dual variables corresponding to day-ahead and real-time constraints, respectively

I. INTRODUCTION

DUE to an increasing contribution of renewable energy sources to electricity markets, new mechanisms are needed to cope with their production uncertainty. *Stochastic market clearing* has been proposed by many researchers, and could have a number of benefits in terms of managing variability and uncertainty more efficiently. In stochastic market-clearing models, uncertain parameters, e.g., wind power production, are characterized through a finite set of plausible scenarios and their corresponding probabilities. Compared to a deterministic model with a certain wind forecast, the stochastic one, in theory, leads to a lower expected system cost, assuming that a realistic range and probability distribution of scenarios are considered. The reason for this is that in the deterministic model, the operational reserve requirements are enforced via exogenous values, while those requirements are endogenously optimized within the stochastic market-clearing model.

In general, it is desirable that any market-clearing model, either deterministic or stochastic, has a pricing scheme that has the following two short-run properties: i) *revenue adequacy* for the market, and ii) *cost recovery* for each generator and for transmission operator, in which market revenues cover short-run (but not necessarily capital) costs. The first property, i.e., revenue adequacy, refers to a condition in which the market operator never incurs a financial deficit. In other words, the payments that the market operator receives from consumers is higher than or equal to its payment to the generators, curtailed loads and transmission operator.[1] The second property, i.e., cost recovery, corresponds to a condition in which the short-run profit (or "gross margin") of each generator, either conventional or non-dispatchable renewable, and transmission operator is non-negative, i.e., the revenue of that player is higher than or equal to its operating costs.

A barrier for stochastic market clearing is that heretofore no stochastic market design has been proposed that is simultaneously revenue adequate for the market and allows for cost recovery for all generators through market prices. Standard US practice (uplifts to cover losses) is not revenue adequate [1]-[2], and market parties will be distrustful of a stochastic system with probabilities they do not control and that could expose them to losses in some scenarios.

In this paper, we are interested in answering the following technical questions: is it possible to design a stochastic market-clearing mechanism that would satisfy revenue adequacy and cost recovery for each individual scenario? And if so, what is the "price" of doing so in terms of sacrificing other desirable market attributes? To answer these questions, we consider a two-settlement electricity market, including day-ahead (DA) and real-time (RT) settlements, and propose a stochastic clearing model. This proposed mechanism is in fact a stochastic equilibrium problem that can be recast as a mixed-integer linear programming (MILP) problem.

A. Literature Review and Contributions

There are several strands in the literature that have revisited conventional deterministic market designs under renewable uncertainty. The first strand maintains the deterministic and sequential structure of real-world electricity markets [3], but introduces new market products, e.g., flexible ramp [4]-[5]. These new products help deterministic mechanism to become more flexible against wind power uncertainty. The second strand explores a "robust" design for market clearing [6]-[9]. This mechanism considers an uncertainty set for the deviation of wind power production from the conditional mean forecast in DA, and then clears market optimally against the worst-case realization while ensuring that the outcomes are feasible for any potential wind realization within the uncertainty set.

The third strand, which is the focus of our paper, defines and analyzes stochastic market-clearing mechanisms [10]-[12], which consider a set of scenarios based on possible DA wind and load forecast errors. This stochastic clearing mechanism makes the DA decisions while explicitly recognizing what adjustments are required in RT for each of all foreseen scenarios. For instance, reference [13] proposes a stochastic equilibrium model for clearing a two-settlement DA-RT market while considering renewable premiums and risk aversion of producers. A distributed form of stochastic market-clearing mechanism is developed in [14]. Reference [15] proposes a stochastic two-settlement DA-RT market-clearing model that ensures incentive compatibility, but the market might not be revenue adequate in expectation.

One important observation is that the available stochastic market-clearing models in literature fulfill cost recovery and revenue adequacy in expectation only, e.g., [10] and [11], or satisfy one property by scenario and another in expectation, e.g., [16] and [17]. We now explain why this might be a disadvantage for the available stochastic market designs. The flexibility providers (e.g., fast-start generators and fast demand response resources) are the main market parties that participate in both DA and RT markets. The participation of these flexibility providers is essential for well-functioning of electricity markets with significant renewables. However, they may lose money in one or more scenarios under the available stochastic designs, though their expected profit is non-negative. This might discourage the flexible producers from making offers in short run or perhaps even investing in long run, especially if they perceive significant risks from market participation under a stochastic clearing mechanism. Therefore, any stochastic market-clearing mechanism that ensures cost recovery by scenario is more appealing for those producers. To this purpose, one potential alternative that is compatible with current US practice is to consider uplift payments to cover the potential financial losses of producers, but at the cost of sub-optimality since the uplift system is indeed an ex-post procedure. There are also a few papers in the literature that explicitly impose the cost recovery condition for all producers as part of market-clearing constraints. For example, [18] proposes an uplift-free market-clearing model with non-convexities (binary 0/1

[1]We note that the issue of revenue inadequacy is also frequently discussed in the context of financial transmission rights (FTR); revenue adequacy for FTRs is defined as occurring when the market operators congestion revenues are assured to be at least as much as the payouts to FTR holders. This issue is distinct from the issue of bid cost recovery and subsequent uplifts to consumers that we focus on in this paper, since market operators do not consider FTR revenue adequacy when determining cost recovery payments to generators or uplifts charged to consumers. Therefore, we do not consider revenue adequacy issues associated with FTRs in this paper.

variables indicating the commitment and start-up status of thermal units), but under deterministic conditions. A similar model but augmented for a two-settlement DA-RT stochastic system with renewables is proposed in [19]. Both references [18] and [19] include *explicit* constraints within their proposed market-clearing models to enforce the cost recovery condition per generator (and per scenario in [19]). These constraints are nonlinear (due to a revenue term including a product of price and quantity variables), and may need a considerable number of auxiliary binary variables to be approximately linearized. In addition, [18] and [19] do not address the market's efficiency and revenue adequacy problems.

To the best of our knowledge, there is no stochastic market-clearing mechanism in the literature that *implicitly* guarantees both revenue adequacy and cost recovery by scenario, which is in fact the novelty of the current paper. In other words, our proposed market design guarantees those two desirable properties by scenario without enforcing any explicit constraint for cost recovery and/or revenue adequacy.

As the main contribution of this paper, we develop a stochastic market-clearing mechanism with mathematical proofs that it implicitly satisfies revenue adequacy and cost recovery for each individual scenario. However, this appealing characteristic is achieved at the cost of potentially violating some or all three of the following desirable market properties:

(i) DA and RT prices are arbitraged in expectation,
(ii) prices are supporting of schedules for all market parties, and
(iii) system cost is minimized.

The first desirable market property lost, i.e., arbitraging DA and RT prices in expectation, can be restored by allowing virtual bidders[2] to arbitrage between the two markets, but at the cost of increased market inefficiency for some other participants. Another drawback of the proposed stochastic market design, compared to those in [10]-[12], is that it is formulated as an *equilibrium* model (similar to [13]) instead of an *optimization* problem, and eventually results in a MILP problem (similar to [17]-[19]) rather than a linear programming (LP) one.

The main insight provided by our proposed stochastic market-clearing model is that the satisfaction of revenue adequacy and cost recovery for each individual scenario has a price, in that the cost of serving load may increase. This requires making a *trade-off* between the desirable properties gained and those lost. We propose to view this stochastic clearing mechanism as an appropriate tool for market analysis and policy discussions of trade-offs, but not for use in practice to clear a market.

Our extensive numerical results (Section IV.B) demonstrate that for the case study considered, the proposed model successfully achieves cost recovery for generators and revenue adequacy for market not only by foreseen (in-sample) scenario but also by unseen (out-of-sample) scenario. A key point is that the in-sample scenarios, i.e., those scenarios which are included in the stochastic optimization, should be a good approximation of the distribution of out-of-sample scenarios; then our numerical results indicate that there is a very high probability that revenue adequacy and cost recovery will be achieved under any given out-of-sample scenario. Note that it is a numerical observation only, and it is not straightforward to mathematically prove that the proposed market design necessarily ensures cost recovery and revenue adequacy for any out-of-sample scenario.

It is worth mentioning that all available stochastic market-clearing mechanisms in literature (as well as our proposed stochastic market design) are theoretical models and none have been implemented in actual electricity markets. The reason is that the stochastic clearing models have difficulties for implementation in practice. For example, they place a large burden on the market operator to acquire and process probabilistic data needed for stochastic clearing (e.g., distribution of wind power across scenarios and their probabilities). However, stochastic clearing models (including our proposed model) can be viewed as benchmarks since they provide a lower bound for the system cost.[3] This benchmark can be used for assessing the performance of clearing models in actual markets (e.g., deterministic designs), and for understanding the efficiency loss that can occur if cost recovery by scenario is to be guaranteed through energy prices alone.

B. Model Assumptions and Paper Organization

We now review some general assumptions of this paper about the market parties. First, we assume that wind power production is the only source of uncertainty. A two-stage electricity pool (DA-RT) is assumed, being perfectly competitive, energy-only, and all players have same information in DA about the distribution of wind power scenarios in RT. The loads are assumed to be inelastic with respect to price. For simplicity, we consider a single-hour electricity pool since no inter-temporal constraints are enforced. To avoid pricing non-convexities, binary variables indicating the commitment status of conventional generators are not considered; the assumption of convexity is necessary for the proofs of this paper. A linearized lossless DC representation of the network is used in both DA and RT, yielding locational marginal prices (LMPs). Wind power production cost is assumed to be zero.

The remainder of this paper is organized as follows. Section II presents a general stochastic market-clearing model based on ones in the literature. Section III first presents the proposed model in the form of an equilibrium problem, and then describes its solution technique. Section IV provides and discusses the numerical results from a simple

[2] The virtual bidders are financial players who own no physical assets and buy/sell in the day-ahead market and then sell/buy the same amount back in the real-time market [20]-[24]. They are a part of market players in some US electricity markets, e.g., CAISO and PJM.

[3] In case the actual electricity markets decide to use a market-clearing model similar to the one proposed in this paper, the method used to solve DA unit commitment problem should also be modified. One potential approach could be the use of a Walrasian auction. In this iterative mechanism, the market operator specifies a set of prices, and then each market participant decides its own commitment and dispatch decisions. Then, based on the participants' dispatch decisions, the market operator checks whether nodal power balance conditions hold or not. If not, the operator systematically adjusts the prices and generates a new set to be disseminated among participants. Similar (but non-stochastic) market designs based on a tâtonnement process are available in [25] and [26].

test system and the IEEE two-area reliability test system (RTS), to illustrate the properties of our model. Section V concludes the paper. Appendix A derives the Karush-Kuhn-Tucker (KKT) optimality conditions of the stochastic market-clearing model presented in Section II. Appendix B obtains the KKT optimality conditions of the proposed market-clearing model. Appendix C mathematically proves that the proposed model is revenue adequate by scenario. Appendix D provides a mathematical proof for cost recovery of all generators and transmission operator by scenario. Finally, Appendix E derives a linear expression to be used in the proposed model.

II. A Generic Stochastic Market-Clearing Model

Most of the stochastic market-clearing models in the literature can be stated concisely as a two-stage LP problem as given by optimization problem (1) below. The first-stage provides the DA schedules (here-and-now decisions), whereas the second-stage adjusts the energy imbalances due to wind power deviations in RT (wait-and-see decisions). Objective function (1a) minimizes the *expected* system cost that includes energy dispatch costs in DA, expected adjustment costs in RT, and expected load shedding costs in RT. This objective function is subject to scenario-independent DA constraints (1b)-(1g) and scenario-dependent RT constraints (1h)-(1o). Note that the dual variables are listed alongside each constraint:

$$\underset{p_i^{\mathrm{DA}}, w_k^{\mathrm{DA}}, f_{n,m}^{\mathrm{DA}}, \theta_n^{\mathrm{DA}}, p_{i,s}^{\mathrm{RT}}, w_{k,s}^{\mathrm{RT}}, l_{d,s}^{\mathrm{shed}}, f_{n,m,s}^{\mathrm{RT}}, \theta_{n,s}^{\mathrm{RT}}}{\text{Minimize}} \quad \sum_i C_i \; p_i^{\mathrm{DA}} + \sum_s \phi_s \left\{ \sum_i C_i \; p_{i,s}^{\mathrm{RT}} + \sum_d V_d \; l_{d,s}^{\mathrm{shed}} \right\} \tag{1a}$$

subject to:

$$\sum_{d \in \Psi_n} L_d + \sum_{m \in \Phi_n} f_{n,m}^{\mathrm{DA}} - \sum_{i \in \Psi_n} p_i^{\mathrm{DA}} - \sum_{k \in \Psi_n} w_k^{\mathrm{DA}} = 0 : \; \lambda_n^{\mathrm{DA}} \quad \forall n \tag{1b}$$

$$0 \le p_i^{\mathrm{DA}} \le P_i^{\max} \quad : \underline{\mu}_i^{\mathrm{P}}, \overline{\mu}_i^{\mathrm{P}} \quad \forall i \tag{1c}$$

$$0 \le w_k^{\mathrm{DA}} \le W_k^{\max} \quad : \underline{\mu}_k^{\mathrm{W}}, \overline{\mu}_k^{\mathrm{W}} \quad \forall k \tag{1d}$$

$$B_{n,m} \left(\theta_n^{\mathrm{DA}} - \theta_m^{\mathrm{DA}} \right) = f_{n,m}^{\mathrm{DA}} \quad : \mu_{n,m}^{\theta} \quad \forall n, \forall m \in \Phi_n \tag{1e}$$

$$f_{n,m}^{\mathrm{DA}} \le F_{n,m}^{\max} \quad : \mu_{n,m}^{\mathrm{F}} \quad \forall n, \forall m \in \Phi_n \tag{1f}$$

$$\theta_{(n=1)}^{\mathrm{DA}} = 0 \quad : \mu^1 \tag{1g}$$

$$\sum_{m \in \Phi_n} \left(f_{n,m,s}^{\mathrm{RT}} - f_{n,m}^{\mathrm{DA}} \right) - \sum_{i \in \Psi_n} p_{i,s}^{\mathrm{RT}} - \sum_{k \in \Psi_n} w_{k,s}^{\mathrm{RT}} - \sum_{d \in \Psi_n} l_{d,s}^{\mathrm{shed}} = 0 \quad : \lambda_{n,s}^{\mathrm{RT}} \quad \forall n, \forall s \tag{1h}$$

$$0 \le \left(p_i^{\mathrm{DA}} + p_{i,s}^{\mathrm{RT}} \right) \le P_i^{\max} \quad : \underline{\rho}_{i,s}^{\mathrm{P}}, \overline{\rho}_{i,s}^{\mathrm{P}} \; \forall i, \forall s \tag{1i}$$

$$-P_i^{\mathrm{adj}} \le p_{i,s}^{\mathrm{RT}} \le P_i^{\mathrm{adj}} \quad : \underline{\rho}_{i,s}^{\mathrm{adj}}, \overline{\rho}_{i,s}^{\mathrm{adj}} \quad \forall i, \forall s \tag{1j}$$

$$0 \le \left(w_k^{\mathrm{DA}} + w_{k,s}^{\mathrm{RT}} \right) \le W_{k,s} \quad : \underline{\rho}_{k,s}^{\mathrm{W}}, \overline{\rho}_{k,s}^{\mathrm{W}} \quad \forall k, \forall s \tag{1k}$$

$$0 \le l_{d,s}^{\mathrm{shed}} \le L_d \quad : \underline{\rho}_{d,s}^{\mathrm{shed}}, \overline{\rho}_{d,s}^{\mathrm{shed}} \quad \forall d, \forall s \tag{1l}$$

$$B_{nm} \left(\theta_{n,s}^{\mathrm{RT}} - \theta_{m,s}^{\mathrm{RT}} \right) = f_{n,m,s}^{\mathrm{RT}} \quad : \rho_{n,m,s}^{\theta} \quad \forall n, \forall m \in \Phi_n, \forall s \tag{1m}$$

$$f_{n,m,s}^{\mathrm{RT}} \le F_{n,m}^{\max} \quad : \rho_{n,m,s}^{\mathrm{F}} \quad \forall n, \forall m \in \Phi_n, \forall s \tag{1n}$$

$$\theta_{(n=1),s}^{\mathrm{RT}} = 0 \quad : \rho_s^1 \quad \forall s. \tag{1o}$$

Constraint (1b) represents the DA power balance at node n, whose dual variable (λ_n^{DA}) provides the day-ahead LMP at that node. Constraints (1c) and (1d) enforce the lower and upper bounds for production schedules of conventional and wind power generators, respectively. Constraint (1e) obtains the power flow schedule across transmission lines as functions of nodal voltage angles. The capacity of each transmission line is enforced through (1f), and constraint (1g) sets node $n = 1$ as the reference node. Regarding operating conditions in RT, constraint (1h) represents the power balance in an incremental form at node n and scenario s, whose dual variable ($\lambda_{n,s}^{\mathrm{RT}}$) provides the corresponding probability-weighted real-time LMP. According to (1h), wind power deviation in RT is met by power adjustments of flexible conventional generators and/or load curtailments. Constraints (1i) and (1j) limit the power adjustment of each conventional generator. Constraint (1k) restricts the total wind power production of each generator for each scenario, i.e., the DA wind schedule plus its deviation in RT, to lie within zero and wind power realization (i.e., uncertain parameter $W_{k,s}$). Note that this constraint allows excess wind power to be spilled. Constraint (1l) limits the level of unserved load. Finally, constraints (1m)-(1o) are similar to (1e)-(1g) but for RT operation.

As mathematically proven in [11], the stochastic market-clearing model (1) ensures revenue adequacy and cost recovery *in expectation*, providing that an energy-only pricing scheme is considered based on day-ahead LMPs, i.e., $\lambda_n^{\mathrm{DA}} \; \forall n$, and probability-adjusted real-time LMPs, i.e., $\frac{\lambda_{n,s}^{\mathrm{RT}}}{\phi_s} \; \forall n, \forall s$. This result necessarily assumes convex costs, e.g., no binary unit commitment variables. Hereafter, model (1), which represents a typical stochastic market-clearing setup in the literature, is called model $\mathcal{M}1$.

Inspired by [27] that refers to a deterministic but oligopolistic market, we mathematically prove that optimization model $\mathcal{M}1$ is *equivalent* to an equilibrium model given by (2)-(6) below. We refer to this equivalent equilibrium model as $\mathcal{M}2$. The basis of this proof is that the KKT conditions of model $\mathcal{M}1$ are identical to the equilibrium conditions of model $\mathcal{M}2$, as shown in Appendix A. To define the equilibrium problem $\mathcal{M}2$, it is necessary to define a profit-maximization problem for each market player, obtain the KKT conditions for each, and finally concatenate them with market-clearing conditions (power balance). Within the equivalent equilibrium model $\mathcal{M}2$, optimization problem (2) presents the expected profit-maximization problem for each conventional generator i as given below:

$$\left\{ \begin{aligned} & \underset{p_i^{\mathrm{DA}}, p_{i,s}^{\mathrm{RT}}}{\text{Maximize}} \quad p_i^{\mathrm{DA}} \left(\lambda_{n:i \in \Psi_n}^{\mathrm{DA}} - C_i \right) + \sum_s p_{i,s}^{\mathrm{RT}} \left[\lambda_{(n:i \in \Psi_n),s}^{\mathrm{RT}} - \phi_s \; C_i \right] && \text{(2a)} \\ & \text{subject to: (1c), (1i), (1j)} && \text{(2b)} \end{aligned} \right\} \forall i.$$

The first row of objective function (2a) refers to the DA profit of generator i, whereas the second row is associated with its expected profit in RT. Similarly, optimization problem (3) maximizes the expected profit of each wind power generator k:

$$\left\{\begin{array}{l}\underset{w_k^{\mathrm{DA}}, w_{k,s}^{\mathrm{RT}}}{\text{Maximize}} \quad w_k^{\mathrm{DA}} \lambda_{n:k\in\Psi_n}^{\mathrm{DA}} + \sum_s w_{k,s}^{\mathrm{RT}} \ \lambda_{(n:k\in\Psi_n),s}^{\mathrm{RT}} \qquad (3a) \\ \text{subject to: (1d), (1k)} \end{array}\right\} \ \forall k. \qquad (3b)$$

Likewise, optimization problem (4) maximizes the expected profit of transmission operator obtained from energy transactions across lines. In the DA market, the transmission operator buys power $f_{n,m}^{\mathrm{DA}}$ at node n at price λ_n^{DA}, and then sells it at node m at price λ_m^{DA}. Similarly, it trades in RT based on the incremental power flow:

$$\underset{f_{n,m}^{\mathrm{DA}}, \theta_n^{\mathrm{DA}}, f_{n,m,s}^{\mathrm{RT}}, \theta_{n,s}^{\mathrm{RT}}}{\text{Maximize}} \sum_{n,(m\in\Phi_n)} \Big[f_{m,n}^{\mathrm{DA}} \ \lambda_n^{\mathrm{DA}} + \sum_s \left(f_{m,n,s}^{\mathrm{RT}} - f_{m,n}^{\mathrm{DA}}\right) \lambda_{n,s}^{\mathrm{RT}} \Big] \qquad (4a)$$

$$\text{subject to: (1e)} - \text{(1g), (1m)} - \text{(1o)}. \qquad (4b)$$

In addition, optimization problem (5) minimizes the expected load shedding cost for each inelastic load d, which represents the consumer's problem:

$$\left\{\begin{array}{l}\underset{l_{d,s}^{\mathrm{shed}}}{\text{Minimize}} \quad \sum_s l_{d,s}^{\mathrm{shed}} \left(\phi_s V_d - \lambda_{(n:d\in\Psi_n),s}^{\mathrm{RT}}\right) \qquad (5a) \\ \text{subject to: (1l)} \end{array}\right\} \ \forall d. \qquad (5b)$$

Finally, (6) includes the nodal power balance equalities as market constraints, i.e.,

$$\text{(1b), (1h)}. \qquad (6)$$

Similar to model $\mathcal{M}1$, the dual variables of (1b) and (1h) in (6) provide DA and probability-weighted RT LMPs, respectively. These prices are variables within equilibrium model $\mathcal{M}2$, but treated as exogenous parameters within the optimization problems (2)-(5).

In models $\mathcal{M}1$ and $\mathcal{M}2$, it is straightforward to mathematically prove that the DA and expected RT prices at each node are equal, providing that there is at least one market party at that node who acts as an unrestrained arbitrager between DA and RT markets. The equality of DA and expected RT prices is a desirable property, as discussed in [28].

III. Proposed Stochastic market-clearing Model

In this section, we first propose a stochastic market-clearing model as an equilibrium problem that ensures revenue adequacy for the market and cost recovery for all generators and for transmission operator *by scenario*. Then, we propose a solution technique.

A. Proposed Model:

The proposed model in this paper is an equilibrium problem that includes problems (7) to (10). Hereafter, we refer to (7)-(10) as model $\mathcal{M}3$. Note that the augmented version of model $\mathcal{M}3$, i.e., model $\mathcal{M}3$ with virtual bidders, includes problem (11) as well. We compare the proposed model $\mathcal{M}3$ with model $\mathcal{M}2$ since both are equilibrium models, while $\mathcal{M}1$ is a single optimization model. However, recall that models $\mathcal{M}1$ and $\mathcal{M}2$ are equivalent. Compared to model $\mathcal{M}2$, the proposed equilibrium model $\mathcal{M}3$ embodies three main differences, as follows:

First, problems (2), (3), and (4) in model $\mathcal{M}2$ maximize the *expected profit* of conventional generator i, wind power generator k, and transmission operator, respectively. However, problems (7), (8), and (9) within model $\mathcal{M}3$ maximize their *probability-weighted profit for each individual scenario*.

Secondly, model $\mathcal{M}3$ omits the cost-minimization (or profit-maximization) of one pre-selected party or set of parties within the equilibrium problem, and thereby, that party cannot affect the market price formation, and their decisions are unsupported by market prices. This results in the cost of uncertainty (i.e., the cost of augmenting market to ensure revenue adequacy and cost recovery by scenario) being assigned to that party, whose optimization problem is excluded. In our proposed model, we choose "loads" as the party whose cost-minimization problems are excluded from the equilibrium model $\mathcal{M}3$. This selection is consistent with the current US practice, since the loads pay the uplifts to cover losses. Because the load's cost-minimization problem is excluded from the equilibrium, this is equivalent to the operator deciding which market loads will be served day-ahead as opposed to real-time (load will not be allowed to arbitrage) and the total amount that load will pay by scenario. However, this does not mean that the total payments by load in model $\mathcal{M}3$ are necessarily higher than in models $\mathcal{M}1$ and $\mathcal{M}2$; in fact, as the first example shows later, consumer expenditures can be lower under model $\mathcal{M}3$.

Although we select loads to pay the cost of uncertainty, the structure of the proposed equilibrium model $\mathcal{M}3$ is flexible and can allow the cost of uncertainty to be assigned to other party. For example, wind power generators would pay the cost of their own uncertainty if their profit-maximization problems are excluded from the equilibrium model, while the optimization problems of conventional generators, loads, and transmission operator are included.

Thirdly, the proposed model $\mathcal{M}3$ allows the market operator to settle loads in both DA and RT markets. In contrast, the loads in models $\mathcal{M}1$ and $\mathcal{M}2$ are fully settled in DA market. Within the proposed model, two scenario-independent non-negative variables l_d^{DA} and l_d^{RT} are defined for each inelastic load d referring to its consumption level in DA and RT markets, respectively. However, the summation of l_d^{DA} and l_d^{RT} is fixed to the total load, i.e., parameter L_d.

Similar to models $\mathcal{M}1$ and $\mathcal{M}2$, we use an energy-only pricing scheme in model $\mathcal{M}3$ based on day-ahead LMPs, i.e, λ_n^{DA} $\forall n$, and probability-adjusted real-time LMPs, i.e., $\frac{\lambda_{n,s}^{\mathrm{RT}}}{\phi_s}$ $\forall n, \forall s$. We now describe each market party's profit-maximization problem. Within the proposed model $\mathcal{M}3$, optimization problem (7) maximizes the probability-weighted profit for each conventional generator i under each scenario s:

$$\left\{\underset{p_i^{\mathrm{DA}},p_{i,s}^{\mathrm{RT}}}{\text{Maximize}} \quad \phi_s \Bigg[p_i^{\mathrm{DA}} \left(\lambda_{n:i\in\Psi_n}^{\mathrm{DA}} - C_i\right) + p_{i,s}^{\mathrm{RT}} \left(\frac{\lambda_{(n:i\in\Psi_n),s}^{\mathrm{RT}}}{\phi_s} - C_i\right)\Bigg] \right. \tag{7a}$$

subject to:

$$0 \le p_i^{\mathrm{DA}} \le P_i^{\max} \qquad : \underline{\mu}_{i,s}^{\mathrm{P}}, \overline{\mu}_{i,s}^{\mathrm{P}} \tag{7b}$$

$$\left.(1\mathrm{i}),(1\mathrm{j}) \right\} \quad \forall i, \forall s. \tag{7c}$$

The objective function (7a) is multiplied by ϕ_s to weight problem (7) within the proposed equilibrium model $\mathcal{M}3$. Similar to models $\mathcal{M}1$ and $\mathcal{M}2$, the DA schedules, i.e., p_i^{DA} are scenario-independent (enforcing non-anticipativity); however, the dual variables associated with DA constraints, i.e., $\underline{\mu}_{i,s}^{\mathrm{P}}$ and $\overline{\mu}_{i,s}^{\mathrm{P}}$ in (7b), are scenario-dependent (indexed by s) since problem (7) corresponds to scenario s. The KKT conditions associated with (7) are given in Appendix B. A comparison between the KKT conditions of conventional generator's problem in models $\mathcal{M}2$ and $\mathcal{M}3$, i.e., (2) and (7), further clarifies the mathematical differences. For example, the KKT equality (14ab) in model $\mathcal{M}2$ provides a single condition across all scenarios, while the analogous equality in model $\mathcal{M}3$, i.e., (15b), provides a set of conditions by scenario. The KKT conditions (14ab) and (15b) would be equivalent if the values obtained for dual variables $\underline{\mu}_{i,s}^{\mathrm{P}}$, $\overline{\mu}_{i,s}^{\mathrm{P}}$, $\underline{\rho}_{i,s}^{\mathrm{P}}$, and $\overline{\rho}_{i,s}^{\mathrm{P}}$ in model $\mathcal{M}3$ are identical to values obtained for $\phi_s\underline{\mu}_i^{\mathrm{P}}$, $\phi_s\overline{\mu}_i^{\mathrm{P}}$, $\phi_s \sum_s \underline{\rho}_{i,s}^{\mathrm{P}}$, and $\phi_s \sum_s \overline{\rho}_{i,s}^{\mathrm{P}}$ in model $\mathcal{M}2$, respectively.

Similarly, the probability-weighted profit-maximization problem for each wind power generator k under each scenario s is given by (8) below:

$$\left\{\underset{w_k^{\mathrm{DA}},w_{k,s}^{\mathrm{RT}}}{\text{Maximize}} \quad \phi_s \left[w_k^{\mathrm{DA}} \lambda_{n:k\in\Psi_n}^{\mathrm{DA}} + w_{k,s}^{\mathrm{RT}} \frac{\lambda_{(n:k\in\Psi_n),s}^{\mathrm{RT}}}{\phi_s}\right] \right. \tag{8a}$$

subject to:

$$0 \le w_k^{\mathrm{DA}} \le W_k^{\max} \qquad : \underline{\mu}_{k,s}^{\mathrm{W}}, \overline{\mu}_{k,s}^{\mathrm{W}} \tag{8b}$$

$$\left.(1\mathrm{k}) \right\} \quad \forall k, \forall s. \tag{8c}$$

Likewise, the probability-weighted profit-maximization problem for transmission operator under each scenario s is given by (9) below:

$$\left\{\underset{f_{n,m}^{\mathrm{DA}},\theta_n^{\mathrm{DA}},f_{n,m,s}^{\mathrm{RT}},\theta_{n,s}^{\mathrm{RT}}}{\text{Maximize}} \quad \phi_s \sum_{n,(m\in\Phi_n)} \Bigg[f_{m,n}^{\mathrm{DA}}\ \lambda_n^{\mathrm{DA}} + \left(f_{m,n,s}^{\mathrm{RT}} - f_{m,n}^{\mathrm{DA}}\right) \frac{\lambda_{n,s}^{\mathrm{RT}}}{\phi_s}\Bigg] \right. \tag{9a}$$

subject to:

$$B_{n,m}\left(\theta_n^{\mathrm{DA}} - \theta_m^{\mathrm{DA}}\right) = f_{n,m}^{\mathrm{DA}} \qquad : \mu_{n,m,s}^{\theta} \qquad \forall n, \forall m \in \Phi_n \tag{9b}$$

$$f_{n,m}^{\mathrm{DA}} \le F_{n,m}^{\max} \qquad : \mu_{n,m,s}^{\mathrm{F}} \qquad \forall n, \forall m \in \Phi_n \tag{9c}$$

$$\theta_{(n=1)}^{\mathrm{DA}} = 0 \qquad : \mu_s^1 \tag{9d}$$

$$\left.(1\mathrm{m}) - (1\mathrm{o}) \right\} \quad \forall s. \tag{9e}$$

Finally, conditions (10) include the nodal power balance equalities in DA and RT as well as load constraints:

$$\sum_{d\in\Psi_n} l_d^{\mathrm{DA}} + \sum_{m\in\Phi_n} f_{n,m}^{\mathrm{DA}} - \sum_{i\in\Psi_n} p_i^{\mathrm{DA}} - \sum_{k\in\Psi_n} w_k^{\mathrm{DA}} = 0 \quad : \ \lambda_n^{\mathrm{DA}} \quad \forall n \tag{10a}$$

$$\sum_{d\in\Psi_n} \left(l_d^{\mathrm{RT}} - l_{d,s}^{\mathrm{shed}}\right) + \sum_{m\in\Phi_n} \left(f_{n,m,s}^{\mathrm{RT}} - f_{n,m}^{\mathrm{DA}}\right) - \sum_{k\in\Psi_n} w_{k,s}^{\mathrm{RT}} - \sum_{i\in\Psi_n} p_{i,s}^{\mathrm{RT}} = 0 \quad : \lambda_{n,s}^{\mathrm{RT}} \quad \forall n, \forall s \tag{10b}$$

$$l_d^{\mathrm{DA}} \ge 0; \quad l_d^{\mathrm{RT}} \ge 0; \quad l_d^{\mathrm{DA}} + l_d^{\mathrm{RT}} = L_d \quad \forall d \tag{10c}$$

$$0 \le l_{d,s}^{\mathrm{shed}} \le L_d \quad \forall d, \forall s. \tag{10d}$$

The dual variables of (10a) and (10b) present DA and probability-weighted RT LMPs, respectively. Similar to equilibrium model $\mathcal{M}2$, the DA and RT prices are variables within equilibrium model $\mathcal{M}3$, but treated as exogenous parameters within the optimization problems (7)-(9), and within optimization problem (11) that is presented later.

The KKT optimality conditions associated with the proposed model $\mathcal{M}3$ are given by (15) in Appendix B. We now list four properties of model $\mathcal{M}3$:

First, as mathematically proven in Appendix C, the cost recovery by scenario is achieved, i.e., the profit of each conventional generator i, each wind power generator k, and transmission operator is non-negative for each individual scenario. The reason is that each party (excluding load) maximizes its profit for each scenario individually, and therefore, it will never take a position resulting a negative profit in that scenario.

Second, as mathematically proven in Appendix D, model $\mathcal{M}3$ ensures the revenue adequacy for the market by scenario. Intuitively speaking, loads' cost-minimization problems are excluded within the equilibrium problem. This brings a flexibility to market operator to decide which market the loads are settled (without allowing them to do arbitrage), and what the total amount that loads will pay by scenario. In addition, the RT market price will never be formed at the value of lost load (VOLL), even though load may be curtailed - note that there is no KKT equality in (15) linking VOLL and RT market price.

Third, the exclusion of cost-minimization problem of loads in model $\mathcal{M}3$ makes the KKT conditions (15) *non-square* in the sense that the number of variables is more than the number of conditions. Therefore, the proposed equilibrium model $\mathcal{M}3$ may have *multiple* solutions.

Fourth, unlike models $\mathcal{M}1$ and $\mathcal{M}2$, the DA and expected RT prices are not necessarily arbitraged in model $\mathcal{M}3$, which is an undesirable property. This price distortion in model $\mathcal{M}3$

can be corrected by virtual bidders. However, as we show later, this may further increase costs to loads. In model $\mathcal{M}3$ with virtual bidders, the optimization problem (11) below for each virtual bidder v should also be included within the equilibrium model:

$$\left\{\begin{aligned} &\underset{b_v^{\mathrm{DA}},b_v^{\mathrm{RT}}}{\text{Maximize}} \quad b_v^{\mathrm{DA}}\ \lambda^{\mathrm{DA}}_{n:v\in\Psi_n} + \sum_s\ b_v^{\mathrm{RT}}\ \lambda^{\mathrm{RT}}_{(n:v\in\Psi_n),s} && \text{(11a)}\\ &\text{subject to:} \quad b_v^{\mathrm{DA}} + b_v^{\mathrm{RT}} = 0 \quad : \rho_v \end{aligned}\right\} \ \forall v \qquad \text{(11b)}$$

where the objective function (11a) maximizes the expected profit of virtual bidder v, subject to constraint (11b) that forces its total production in DA and RT is zero. Note that both variables b_v^{DA} and b_v^{RT} are scenario-independent to ensure that the total production of virtual bidder is zero irrespective of the scenario realized. Note also that in model $\mathcal{M}3$ with virtual bidders, $\sum_{v\in\Psi_n} b_v^{\mathrm{DA}}$ and $\sum_{v\in\Psi_n} b_v^{\mathrm{RT}}$ should be added to the left-hand side of power balance equalities (10a) and (10b), respectively. One important observation is that the inclusion of (11) within the equilibrium model $\mathcal{M}3$ implicitly enforces the equality of DA and expected RT prices at bus n [23]-[24]. This price equality condition can be readily derived from the KKT conditions of (11).

B. Solution Technique:

In order to choose one solution from the multiple possible equilibria of model $\mathcal{M}3$, we formulate an auxiliary optimization problem, whose objective function could be arbitrarily selected, however, it is constrained by optimality conditions (15). Note that different objective functions may lead to different solutions. In order to choose from among alternative solutions, we consider the minimization of total expected cost paid by all loads as objective function. This means that among all possible market-clearing solutions, we select a solution which is the best for the loads in expectation. The reason for this selection is that the loads in model $\mathcal{M}3$ have been already forced to pay the cost of uncertainty by excluding their cost-minimization problems from the equilibrium problem. Accordingly, the following auxiliary problem is formulated:

$$\textit{Minimize } \Pi, \text{ subject to (15)} \qquad (12)$$

where Π is the total expected cost of all loads including their expected payments and shedding costs. Note that the auxiliary problem (12) is in fact a mathematical program with equilibrium constraints (MPEC) as it is constrained by market-clearing conditions. This MPEC can be then recast as a MILP as follows:

$$\begin{aligned} &\textit{Minimize linear equivalent of } \Pi && \text{(13a)}\\ &\text{subject to } \textit{mixed-integer linear form of } \text{(15)} && \text{(13b)} \end{aligned}$$

where the linear equivalent of Π is provided in Appendix E. In addition, conditions (15) are linearized through replacing complementarity conditions (15g)-(15l) by their mixed-integer linear equivalent. More specifically, each complementarity condition of the form $0 \le a \perp b \ge 0$ is replaced by $a \ge 0$, $b \ge 0$, $a \le M(1-z)$ and $b \le Mz$, where z is an auxiliary binary variable and M is a large enough positive constant [29]-[30]. Another alternative for complementarity linearization is to use auxiliary SOS1 variables as proposed in [31]. This SOS1-based technique replaces each complementarity condition of the form $0 \le a \perp b \ge 0$ by the following set of equations: $a \ge 0$, $b \ge 0$, $a + b = c + d$ and $a - b = c - d$. Note that c and d are SOS1 variables, i.e., at most one of them can take a strictly positive (non-zero) value. We use both complementarity linearization techniques above in our large case study.

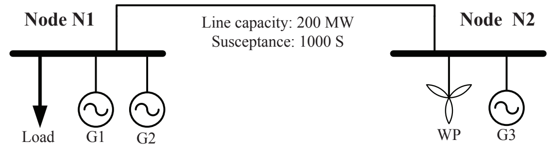

Fig. 1. Network of the illustrative example.

TABLE I
ILLUSTRATIVE EXAMPLE: DATA FOR CONVENTIONAL GENERATORS

Conventional generator	$P_i^{\max}$ [MW]	P_i^{adj} [MW]	C_i [$/MWh]
G1	50	0	10
G2	110	0	25
G3	100	45	35

IV. NUMERICAL RESULTS

This section provides the numerical results from a small-scale illustrative example (Section IV.A) and a large-scale case study based on IEEE two-area RTS (Section IV.B). The computational performance of different models is discussed in Section IV.C.

A. Simple Illustrative Example

We consider a two-node (N1 and N2) system as illustrated in Fig. 1. This system includes three conventional generators (G1, G2 and G3), whose technical data are provided in Table I. A wind power generator (WP) with an installed capacity of 50 MW is considered, and its production uncertainty is modeled through three scenarios: 50 MW, 22 MW and 10 MW with probabilities 0.2, 0.5 and 0.3, respectively. The load is 200 MW, and its VOLL is $200/MWh.

Table II gives the market outcomes obtained from models $\mathcal{M}1$, $\mathcal{M}2$, $\mathcal{M}3$, and $\mathcal{M}3$ with virtual bidders (VB). The transmission line is not congested. As proven in Appendix A, models $\mathcal{M}1$ and $\mathcal{M}2$ are equivalent, and therefore, they result in identical outcomes. In these two equivalent models, load is fully settled DA, and DA and expected RT prices are equal ($28/MWh). Model $\mathcal{M}3$ yields different outcomes; the market operator settles 150 MW of load in DA and remaining 50 MW in RT. Also, model $\mathcal{M}3$ results in different values for DA and expected RT prices ($25/MWh and $33/MWh), which is undesirable. Virtual bidding could fix this price difference

TABLE II
ILLUSTRATIVE EXAMPLE: MARKET-CLEARING OUTCOMES

Model	Market outcome	DA schedule	RT (scenario 1)	RT (scenario 2)	RT (scenario 3)
$\mathcal{M}1$ and $\mathcal{M}2$	G1 [MW]	50	0	0	0
	G2 [MW]	110	0	0	0
	G3 [MW]	40	-40	-22	-10
	WP [MW]	0	+40 (10 spilled)	+22	+10
	Load [MW]	200	0	0	0
	LMP [$/MWh]	28	0	35	35
$\mathcal{M}3$	G1 [MW]	50	0	0	0
	G2 [MW]	100	0	0	0
	G3 [MW]	0	0	+28	+40
	WP [MW]	0	+50	+22	+10
	Load [MW]	150	50	50	50
	LMP [$/MWh]	25	25	35	35
$\mathcal{M}3$ with VB	G1 [MW]	50	0	0	0
	G2 [MW]	100	0	0	0
	G3 [MW]	0	0	0	0
	WP [MW]	0	+50	+22	+10
	VB [MW]	+50	-50	-50	-50
	Load [MW]	200	0	0 (28 shed)	0 (40 shed)
	LMP [$/MWh]	25	25	25	25

TABLE III
ILLUSTRATIVE EXAMPLE: TOTAL EXPECTED SYSTEM COST AND TOTAL EXPECTED COST OF LOAD [$]

Model	Total expected system cost*	Total expected cost of load†
$\mathcal{M}1$ and $\mathcal{M}2$	3,880	5,600
$\mathcal{M}3$	3,910	5,400
$\mathcal{M}3$ with VB	8,200	9,550

* This value includes generation-side costs and load shedding costs.
† This value consists of demand-side payments and load shedding costs.

TABLE IV
ILLUSTRATIVE EXAMPLE: PROFIT/COST OF MARKET PLAYERS IN EXPECTATION AND BY SCENARIO

Model	Market player	Scenario 1 realization	Scenario 2 realization	Scenario 3 realization	Expected
$\mathcal{M}1$ and $\mathcal{M}2$	G1 [$]	900	900	900	900
	G2 [$]	330	330	330	330
	G3 [$]	1,120	-280	-280	0
	WP [$]	0	770	350	490
	Load [$]	5,600	5,600	5,600	5,600
$\mathcal{M}3$	G1 [$]	750	750	750	750
	G2 [$]	0	0	0	0
	G3 [$]	0	0	0	0
	WP [$]	1,250	770	350	740
	Load [$]	5,000	5,500	5,500	5,400
$\mathcal{M}3$ with VB	G1 [$]	750	750	750	750
	G2 [$]	0	0	0	0
	G3 [$]	0	0	0	0
	WP [$]	1,250	550	250	600
	VB [$]	0	0	0	0
	Load [$]	5,000	9,900	12,000	9,550

TABLE V
IEEE TWO-AREA RTS CASE STUDY: DATA FOR CONVENTIONAL GENERATORS

Generator	Location [node]	$P_i^{\max}$ [MW]	P_i^{adj} [MW]	C_i [$/MWh]
GA1, GA2, GB1, GB2	A1, A2, B1, B2	40	0	11.09
GA3, GA4, GB3, GB4	A1, A2, B1, B2	152	80	16.60
GA5, GB5	A7, B7	300	160	18.52
GA6, GB6	A13, B13	591	280	19.10
GA7, GB7	A15, B15	60	60	22.41
GA8, GA9, GB8, GB9	A15, A16, B15, B16	155	60	14.08
GA10, GA11, GB10, GB11	A18, A21, B18, B21	400	0	10.17
GA12, GB12	A22, B22	300	0	6.10
GA13, GB13	A23, B23	310	80	14.08
GA14, GB14	A23, B23	350	75	12.46

in model $\mathcal{M}3$ and result in identical DA and expected RT prices ($25/MWh), but at the cost of load curtailment under two scenarios. Note that the RT prices are not equal to VOLL, though the load is curtailed. The reason is that the cost-minimization problem of load is not included in equilibrium model $\mathcal{M}3$, and thereby, load's cost function cannot affect the RT market price formation.

Table III gives the values obtained for total expected system cost and total expected cost of load. The total expected system cost in models $\mathcal{M}1$ and $\mathcal{M}2$ is comparatively lower than that in model $\mathcal{M}3$, though wind power is spilled under one scenario in the cost-minimization models. The reason for this lower expected cost is that the costly generator G3 is operated more in model $\mathcal{M}3$ compared to other two models. This cost is significantly higher in model $\mathcal{M}3$ with VB due to load shedding. The total expected cost of load in models $\mathcal{M}1$ and $\mathcal{M}2$ is comparatively higher than that in model $\mathcal{M}3$. However, this may change in different cases, since the cost-minimization problem of load is excluded from the market equilibrium problem in model $\mathcal{M}3$. The cost of load is considerably high in model $\mathcal{M}3$ with VB due to curtailed load.

Table IV gives profits and cost for the different market parties in expectation and by scenario. One important observation is that models $\mathcal{M}1$ and $\mathcal{M}2$ do not ensure cost recovery for all generators by scenario; for example, the profit of generator G3 under scenarios 2 and 3 is negative (-$280), while its expected profit is non-negative (zero). In contrast, model $\mathcal{M}3$ (with or without VB) results in non-negative profit for all generators not only in expectation but also by scenario, which is its advantage over models $\mathcal{M}1$ and $\mathcal{M}2$. This is true even though consumers pay less under model $\mathcal{M}3$ in this case. Another observation is that the conventional generators earn higher profit in expectation in models $\mathcal{M}1$ and $\mathcal{M}2$, whereas the wind power generator's expected profit is comparatively higher in model $\mathcal{M}3$ (with or without VB). Regarding revenue adequacy for the market, it is satisfied in all models by scenario, and the profit of the system operator is zero since the line is never congested.

B. IEEE Two-Area RTS Case Study

We consider the IEEE RTS [32] including two areas (A and B), 48 nodes (A1 to A24 and B1 to B24), 34 loads and 28 conventional generators (i.e., GA1 to GA14 located in area A, and GB1 to GB14 located in area B). The loads are identical to that in [32] raised by 5%, yielding a total load of 5,985 MW. Technical data for conventional generators are given in Table V. In addition, two wind power generators (WP1 and WP2) are considered that are located at nodes A11 and B16, respectively. The per-unit power production of wind generators WP1 and WP2 is modeled using a Beta distribution with shape parameters, (α, β), equal to (1.89, 4.48) and (2.09, 3.12), respectively. We generate 300 samples; each one includes the production of both wind generators. According to these 300

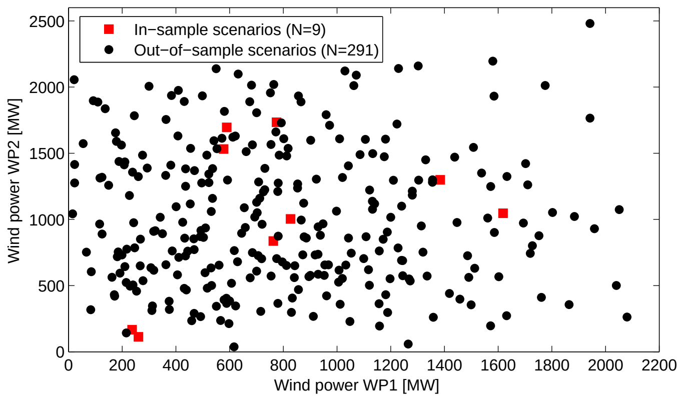

Fig. 2. IEEE two-area RTS case study: In-sample and out-of-sample scenarios

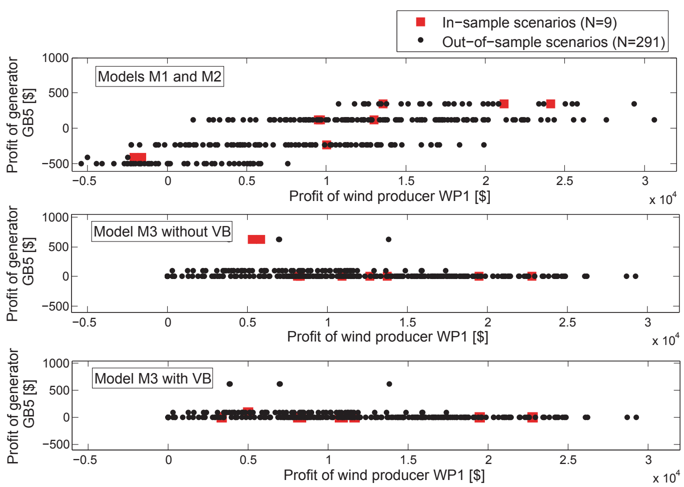

Fig. 3. IEEE two-area RTS case study: Profit of generators GB5 and WP1 in models $\mathcal{M}1$ and $\mathcal{M}2$ (upper plot), model $\mathcal{M}3$ without VB (intermediate plot) and model $\mathcal{M}3$ with VB (lower plot) under each in-sample and out-of-sample scenario

samples, wind power penetration, i.e., total expected wind power divided by total load, is 30.4%. Wind power uncertainty, i.e., standard deviation of wind production across scenarios divided by expected wind, is 55.1%. We then select nine of these samples as in-sample equiprobable scenarios (s_1 to s_9) for use within the stochastic optimization, and the remaining 291 samples are used for an out-of-sample simulation. The reason for selecting these specific nine scenarios is that they give nearly identical values for wind power penetration and wind uncertainty as the full original set of 300 samples. Both sets of in-sample and out-of-sample scenarios are illustrated in Fig. 2. The in-sample simulation considers the nine scenarios s_1 to s_9, and treats them as the only potential realizations in RT within the stochastic market model. In the out-of-sample simulation, the DA schedules are fixed to those obtained in the in-sample simulation, and then the RT market is cleared deterministically for each of 291 out-of-sample scenarios. The VOLL for all loads is assumed to be identical, i.e., \$200/MWh. The capacity of transmission lines is raised by 30% to facilitate wind integration.

As examples of generators' profits in different models, Fig. 3 illustrates the profits of conventional generator GB5 and wind power generator WP1 under each in-sample and out-of-sample scenario. We first investigate their profits achieved in the in-sample simulations (scenarios s_1-s_9). Similar to the results of illustrative example in Section IV.A, models $\mathcal{M}1$ and $\mathcal{M}2$ do not guarantee cost recovery of generators by in-sample scenario (upper plot of Fig. 3). For example, the profit of generator GB5 in three in-sample scenarios and that of WP1 in two scenarios are negative, though their expected profits are non-negative (\$39 for GB5, and \$10,809 for WP1). In contrast, model $\mathcal{M}3$ (with or without VB) yields non-negative profits for *all* generators in each and every in-sample scenario, as well as in expectation. We now analyze their profits in the 291 out-of-sample simulations. Similar to the in-sample simulation, the profit of at least one of generators GB5 and WP1 in models $\mathcal{M}1$ and $\mathcal{M}2$ is negative under about half of out-of-sample scenarios. Remarkably, such profits are still non-negative for *every* out-of-sample scenario in model $\mathcal{M}3$ (with or without VB), see intermediate and lower plots of Fig. 3. We had expected, in contrast, that sampling error would produce at least a few out-of-sample scenarios with negative profits.

Table VI gives the market-clearing outcomes of the three different models obtained from in-sample and out-of-sample simulations. We first analyze the results of the in-sample simulations. As expected, total expected system cost and its standard deviation are comparatively lower in models $\mathcal{M}1$ and $\mathcal{M}2$ compared to those in model $\mathcal{M}3$. The profits of all generators in model $\mathcal{M}3$ (with or without VB) are non-negative for each individual in-sample scenario (by construction), while they could be negative in models $\mathcal{M}1$ and $\mathcal{M}2$ as already shown in Fig. 3. We assume that such negative profits of generators across scenarios are compensated by loads, as in the uplift system in existing markets. Unlike the simple illustrative example in Section IV.A, total cost of loads is also lower in models $\mathcal{M}1$ and $\mathcal{M}2$ compared to that in model $\mathcal{M}3$, even though this cost includes the uplift payments. This demonstrates the cost-inefficiency of model $\mathcal{M}3$ as a potentially undesirable consequence of a stochastic market design that ensures cost recovery and revenue adequacy by scenario. Adding VB to model $\mathcal{M}3$ results in a considerable increase in system cost and the cost to load due to significant load shedding. The DA and RT prices are arbitraged in expectation in models $\mathcal{M}1$, $\mathcal{M}2$ and $\mathcal{M}3$ with VB, but not in model $\mathcal{M}3$ without VB.

We now analyze the results in Table VI obtained from the out-of-sample simulation. The market outcomes (profits and costs) of models $\mathcal{M}1$ and $\mathcal{M}2$ in the out-of-sample simulations are not significantly changed compared to those obtained from in-sample simulation, although these models result in a negative profit for at least one generator in 130 of the 291 out-of-sample scenarios.

Fig. 4 illustrates the distribution of system cost versus total cost of loads in models $\mathcal{M}1$ and $\mathcal{M}2$ (upper plot), model $\mathcal{M}3$ without VB (intermediate plot), and model $\mathcal{M}3$ with VB (lower plot) for both in-sample and out-of-sample simulations. According to the upper plot and the results of Table VI, the expected value and standard deviation of total

TABLE VI
IEEE TWO-AREA RTS CASE STUDY: MARKET-CLEARING OUTCOMES IN DIFFERENT MODELS

		In-sample simulations (N=9)			Out-of-sample simulations (N=291)		
		$\mathcal{M}1$ and $\mathcal{M}2$	$\mathcal{M}3$	$\mathcal{M}3$ with VB	$\mathcal{M}1$ and $\mathcal{M}2$	$\mathcal{M}3$	$\mathcal{M}3$ with VB
Total system cost[1] [\$]	Expected	46,997	54,554	90,404	47,156	47,717	47,717
	Standard deviation	13,580	27,355	94,255	9,624	11,439	11,439
Number of scenarios with a negative profit[2]		3 out of 9	0 out of 9	0 out of 9	130 out of 291	0 out of 291	0 out of 291
Total negative profit of generators[3] [\$]	Expected	768	0	0	714	0	0
	Standard deviation	1,200	0	0	1,079	0	0
Total cost of loads including uplifts[4] [\$]	Expected	90,158	97,060	128,388	90,103	91,153	85,878
	Standard deviation	1,200	20,836	84,726	1,079	38,401	8,980
Total expected wind power spillage [MW]		0	0	0	33.5	20.4	20.4
Total expected load shedding [MW]		0	39.0	236.1	0	1.7	1.3
DA price[5] [\$/MWh]		14.93	14.08	14.08	14.93	14.08	14.08
Expected RT price[5] [\$/MWh]		14.93	16.49	14.08	14.15	16.91	16.91

1. This value includes generation-side costs and load shedding costs.
2. This number considers the scenarios with a negative profit for at least one conventional or wind power generator (but not virtual bidders).
3. This value considers the negative profits of generators across scenarios, i.e., the non-negative profits are excluded. These losses are compensated by uplift payments of loads.
4. This value consists of demand-side payment, load shedding cost, and uplifts for compensating the negative profits of generators (but not virtual bidders).
5. All nodal LMPs are identical since there is no transmission congestion.

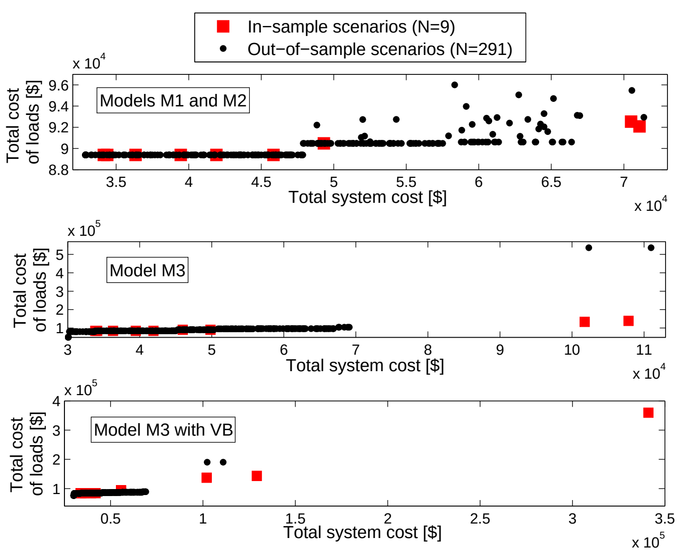

Fig. 4. IEEE two-area RTS case study: Total system cost and total cost of loads in models $\mathcal{M}1$ and $\mathcal{M}2$ (upper plot), model $\mathcal{M}3$ without VB (intermediate plot) and model $\mathcal{M}3$ with VB (lower plot) under each in-sample and out-of-sample scenario

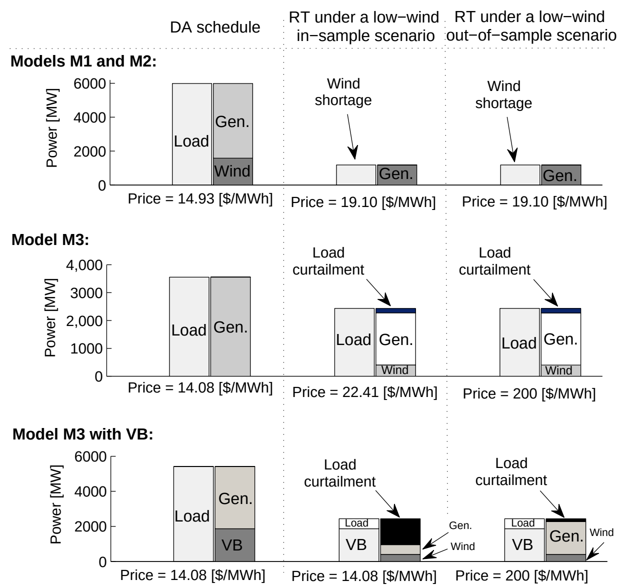

Fig. 5. IEEE two-area RTS case study: DA dispatch, RT re-dispatch under a low-wind (403.2 MW) in-sample scenario, and RT re-dispatch under a similar low-wind (400.4 MW) but out-of-sample scenario in models $\mathcal{M}1$ and $\mathcal{M}2$ (upper plot), model $\mathcal{M}3$ without VB (intermediate plot) and model $\mathcal{M}3$ with VB (lower plot)

system cost and total cost of loads in models $\mathcal{M}1$ and $\mathcal{M}2$ in the out-of-sample simulations are close to their values in the in-sample simulation (i.e., the scenarios considered in the original market model). This shows the robustness of models $\mathcal{M}1$ and $\mathcal{M}2$ against scenarios not anticipated by the market parties. In addition, the upper plot of Fig. 5 shows that wind power is scheduled in DA, and the wind shortage in low-wind scenarios (either in-sample or out-of-sample) is compensated by re-dispatching flexible conventional generators in RT. The average RT market price in both low-wind in-sample and out-of-sample scenarios is identical, i.e., \$19.10/MWh. Unlike models $\mathcal{M}1$ and $\mathcal{M}2$, the system cost and cost of loads are more widely dispersed in model $\mathcal{M}3$ (especially with VB). For example, as given in the third and fourth rows of Table VI, standard deviation of total system cost across in-sample scenarios divided by expected total system cost in models $\mathcal{M}1$ and $\mathcal{M}2$ is 29%, while it is 50% and 104% in model $\mathcal{M}3$ without and with VB, respectively. In addition, out-of-sample costs are very different than the in-sample ones in mean and standard deviation (Table VI), illustrating that the in-sample costs in model $\mathcal{M}3$ are not highly robust against non-modeled scenarios.

In model $\mathcal{M}3$ (without VB), unlike models $\mathcal{M}1$ and $\mathcal{M}2$, market outcomes with out-of-sample scenarios are moderately different than those in in-sample simulation. For example, the expected cost to load in the out-of-sample simulation is 6.4% lower than that in the in-sample simulations, while the standard deviation of that cost is significantly higher. As another example, the intermediate plot of Fig. 4 illustrates that the total system cost under the two low-wind in-sample scenarios considered is significantly higher than that in the rest of in-sample scenarios, yielding a comparatively high standard deviation for the total system cost in in-sample simulations ($27,355). However, that value is significantly lower in the out-of-sample simulation ($11,439). The difference between in-sample and out-of-sample market outcomes in model $\mathcal{M}3$ is further emphasized in the second plot of Fig. 5. Although the RT re-dispatch under both low-wind in-sample and out-of-sample scenarios is very similar, their resulting market prices are significantly different. As already discussed in Section III.A, the loads in model $\mathcal{M}3$ cannot affect market price formation. Therefore, the RT price under in-sample scenario is not equal to VOLL (i.e., $200/MWh), although a portion of the load is curtailed. However, we solve a deterministic RT market-clearing model for the low-wind out-of-sample scenario, in which the curtailed load sets the RT market price to $200/MWh. Note that a considerable part of the unserved loads in model $\mathcal{M}3$ winds up being supplied in the RT stage. It is also worth mentioning that in all 291 out-of-sample scenarios, model $\mathcal{M}3$ ensures cost recovery for all generators and revenue adequacy for the market. Whether this a general result for model $\mathcal{M}3$ would require additional analysis for a wider variety of systems.

Finally, the out-of-sample simulation shows that model $\mathcal{M}3$ with VB is not robust against the unseen scenarios, since the market outcomes (e.g., total system cost and total cost of loads) with foreseen and unseen scenarios are significantly different, although in this example cost recovery for all conventional and wind power generators (but not necessarily for virtual bidders) is successfully achieved under all out-of-sample scenarios.

This important difference in market outcomes of model $\mathcal{M}3$ with VB is highlighted in the lower plot of Fig. 4, where the total system cost and total cost of loads are large under one of the in-sample scenarios, representing a low-wind condition. The reason for these large costs is revealed if we examine the lower plot of Fig. 5. In this example, the aggregation of virtual bidders behaves as a generator in the DA stage, while in RT they buy back the same amount of energy that they already sold in DA. In this way, the DA and (in-sample) RT prices are arbitraged in expectation. In this specific low-wind in-sample scenario, the RT price is identical to the DA price, which is $14.08/MWh. This price is lower than the marginal cost of most of flexible conventional generators; therefore, the wind shortage in this scenario is mostly met by load curtailment (1480 MW) – recall that loads cannot contribute to market price formation in model $\mathcal{M}3$, and therefore, the RT market price under this in-sample scenario is not $200/MWh. This major load curtailment greatly increases the system cost and the cost of loads, even in expectation. However, the deterministic RT market-clearing model used for the out-of-sample scenario yields different re-dispatch outcomes, since the curtailed load sets the market price to $200/MWh, while the flexible conventional generators offset a large portion of wind shortage. Since a single low-wind scenario out of nine in-sample scenarios drastically changes the in-sample market outcomes of model $\mathcal{M}3$ with VB, we hypothesize that including a higher number of in-sample scenarios (while including more low-wind scenarios) in this model will not decrease the gap between the market outcomes of model $\mathcal{M}3$ with VB between the in-sample and out-of-sample simulations.

As an additional test of models $\mathcal{M}1$ and $\mathcal{M}2$, and both versions of model $\mathcal{M}3$, we have also applied them to the IEEE 118-bus test system with 19 thermal generators [33]. This system is augmented with wind farms at buses 9 and 64 whose uncertain output is described by 15 scenarios. The results are consistent with the above two-area RTS system (details available from authors): day-ahead and real-time prices converge in expectation in models $\mathcal{M}1$ and $\mathcal{M}2$, and $\mathcal{M}3$ with VB; model $\mathcal{M}3$ is costlier than $\mathcal{M}1$ and $\mathcal{M}2$, with VB increasing cost further; and both versions of model $\mathcal{M}3$ ensure revenue adequacy and cost recovery for each and every scenario, unlike models $\mathcal{M}1$ and $\mathcal{M}2$.

C. Computational Performance

The LP problem in model $\mathcal{M}1$ and the MILP problems in models $\mathcal{M}2$ and $\mathcal{M}3$ (with and without VB) are solved using CPLEX under GAMS on an Intel(R) Xeon(R) E5-1650 with 12 processors clocking at 3.50 GHz and 32 GB of RAM. The CPU time for the LP problem of model $\mathcal{M}1$ in Section IV.B (IEEE two-area RTS case study with 9 in-sample scenarios) is 0.2 seconds, while it is 244 seconds for the MILP problem of model $\mathcal{M}3$ with a zero optimality gap. The maximum CPU time occurs in model $\mathcal{M}3$ with VB, which is 7 hours with an optimality gap of 1%. The CPU time increases drastically with higher number of scenarios, so that MILP models with a high number of scenarios become computationally intractable. In particular, we were unable to solve model $\mathcal{M}3$ using the computing system mentioned for the same large case study when there are either 10 and 15 in-sample scenarios for the with and without VB cases, respectively. The much longer computational times for model $\mathcal{M}3$ raise issues of scalability, and are consistent with our earlier point that a cost of implementing the revenue adequacy-by-scenario framework would be computational inefficiencies.

As a potential future work, we propose the application of decomposition and distributed optimization techniques to models $\mathcal{M}2$ and $\mathcal{M}3$ with high number of scenarios. This might diminish the computational disadvantages of the revenue adequacy-by-scenario model. One interesting observation is that the equilibrium models $\mathcal{M}2$ and $\mathcal{M}3$ are decomposable – the relaxation of nodal power balance conditions as shared market constraints decomposes them to several smaller sub-problems, one per market party in model $\mathcal{M}2$, and one per market party per scenario in model $\mathcal{M}3$.

V. Conclusions

This paper proposes a stochastic market design that ensures i) cost recovery for all generators and transmission operator, and ii) revenue adequacy for the market, not only in expectation but also by scenario. However, these properties have a price: generation and demand-side costs may increase; market prices will not be equilibrium supporting all parties;

and DA and RT prices are not arbitraged in expectation. The latter could be fixed by virtual bidders, but they may increase the demand-side costs. From mathematical point of view, the proposed model is an equilibrium problem, which is recast as an MPEC, and then linearized.

Appendix A: KKTs of Models $\mathcal{M}1$ and $\mathcal{M}2$

The KKT optimality conditions associated with model $\mathcal{M}1$ are given by (14) below. Note that $\mathcal{L}$ is the Lagrangian function with respect to problem (1). An identical set of KKT optimality conditions is derived from model $\mathcal{M}2$.

$$\text{(1b)}, \text{(1e)}, \text{(1g)}, \text{(1h)}, \text{(1m)}, \text{(1o)} \tag{14aa}$$

$$\frac{\partial \mathcal{L}}{\partial p_i^{\mathrm{DA}}} = C_i - \lambda_{n:i\in\Psi_n}^{\mathrm{DA}} + \overline{\mu}_i^{\mathrm{P}} - \underline{\mu}_i^{\mathrm{P}} + \sum_s \left(\overline{\rho}_{i,s}^{\mathrm{P}} - \underline{\rho}_{i,s}^{\mathrm{P}}\right) = 0 \quad \forall i \tag{14ab}$$

$$\frac{\partial \mathcal{L}}{\partial w_k^{\mathrm{DA}}} = -\lambda_{n:k\in\Psi_n}^{\mathrm{DA}} + \overline{\mu}_k^{\mathrm{W}} - \underline{\mu}_k^{\mathrm{W}} + \sum_s \left(\overline{\rho}_{k,s}^{\mathrm{W}} - \underline{\rho}_{k,s}^{\mathrm{W}}\right) = 0 \quad \forall k \tag{14ac}$$

$$\frac{\partial \mathcal{L}}{\partial f_{n,m}^{\mathrm{DA}}} = \lambda_n^{\mathrm{DA}} - \mu_{n,m}^{\theta} + \mu_{n,m}^{\mathrm{F}} - \sum_s \lambda_{n,s}^{\mathrm{RT}} = 0 \quad \forall n, \forall m \in \Phi_n \tag{14ad}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_n^{\mathrm{DA}}} = \sum_{m\in\Phi_n} B_{n,m}\left(\mu_{n,m}^{\theta} - \mu_{m,n}^{\theta}\right) + \left(\mu^1\right)_{n=1} = 0 \quad \forall n \tag{14ae}$$

$$\frac{\partial \mathcal{L}}{\partial p_{i,s}^{\mathrm{RT}}} = \phi_s\ C_i - \lambda_{(n:i\in\Psi_n),s}^{\mathrm{RT}} + \overline{\rho}_{i,s}^{\mathrm{P}} - \underline{\rho}_{i,s}^{\mathrm{P}} + \overline{\rho}_{i,s}^{\mathrm{adj}} - \underline{\rho}_{i,s}^{\mathrm{adj}} = 0 \quad \forall i, \forall s \tag{14af}$$

$$\frac{\partial \mathcal{L}}{\partial w_{k,s}^{\mathrm{RT}}} = -\lambda_{(n:k\in\Psi_n),s}^{\mathrm{RT}} + \overline{\rho}_{k,s}^{\mathrm{W}} - \underline{\rho}_{k,s}^{\mathrm{W}} = 0 \quad \forall k, \forall s \tag{14ag}$$

$$\frac{\partial \mathcal{L}}{\partial l_{d,s}^{\mathrm{shed}}} = \phi_s\ V_d - \lambda_{(n:d\in\Psi_n),s}^{\mathrm{RT}} + \overline{\rho}_{d,s}^{\mathrm{shed}} - \underline{\rho}_{d,s}^{\mathrm{shed}} = 0 \quad \forall d, \forall s \tag{14ah}$$

$$\frac{\partial \mathcal{L}}{\partial f_{n,m,s}^{\mathrm{RT}}} = \lambda_{n,s}^{\mathrm{RT}} - \rho_{n,m,s}^{\theta} + \rho_{n,m,s}^{\mathrm{F}} = 0 \quad \forall n, \forall m \in \Phi_n, \forall s \tag{14ai}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_{n,s}^{\mathrm{RT}}} = \sum_{m\in\Phi_n} B_{n,m}\left(\rho_{n,m,s}^{\theta} - \rho_{m,n,s}^{\theta}\right) + \left(\rho_s^1\right)_{n=1} = 0 \quad \forall n, \forall s \tag{14aj}$$

$$0 \le p_i^{\mathrm{DA}} \perp \underline{\mu}_i^{\mathrm{P}} \ge 0 \quad \forall i \tag{14ak}$$

$$0 \le \left(P_i^{\max} - p_i^{\mathrm{DA}}\right) \perp \overline{\mu}_i^{\mathrm{P}} \ge 0 \quad \forall i \tag{14al}$$

$$0 \le w_k^{\mathrm{DA}} \perp \underline{\mu}_k^{\mathrm{W}} \ge 0 \quad \forall k \tag{14am}$$

$$0 \le \left(W_k^{\max} - w_k^{\mathrm{DA}}\right) \perp \overline{\mu}_k^{\mathrm{W}} \ge 0 \quad \forall k \tag{14an}$$

$$0 \le \left(F_{n,m}^{\max} - f_{n,m}^{\mathrm{DA}}\right) \perp \mu_{n,m}^{\mathrm{F}} \ge 0 \quad \forall n, \forall m \in \Phi_n \tag{14ao}$$

$$0 \le \left(p_i^{\mathrm{DA}} + p_{i,s}^{\mathrm{RT}}\right) \perp \underline{\rho}_{i,s}^{\mathrm{P}} \ge 0 \quad \forall i, \forall s \tag{14ap}$$

$$0 \le \left(P_i^{\max} - p_i^{\mathrm{DA}} - p_{i,s}^{\mathrm{RT}}\right) \perp \overline{\rho}_{i,s}^{\mathrm{P}} \ge 0 \quad \forall i, \forall s \tag{14aq}$$

$$0 \le \left(P_i^{\mathrm{adj}} + p_{i,s}^{\mathrm{RT}}\right) \perp \underline{\rho}_{i,s}^{\mathrm{adj}} \ge 0 \quad \forall i, \forall s \tag{14ar}$$

$$0 \le \left(P_i^{\mathrm{adj}} - p_{i,s}^{\mathrm{RT}}\right) \perp \overline{\rho}_{i,s}^{\mathrm{adj}} \ge 0 \quad \forall i, \forall s \tag{14as}$$

$$0 \le \left(w_k^{\mathrm{DA}} + w_{k,s}^{\mathrm{RT}}\right) \perp \underline{\rho}_{k,s}^{\mathrm{W}} \ge 0 \quad \forall k, \forall s \tag{14at}$$

$$0 \le \left(W_{k,s} - w_k^{\mathrm{DA}} - w_{k,s}^{\mathrm{RT}}\right) \perp \overline{\rho}_{k,s}^{\mathrm{W}} \ge 0 \quad \forall k, \forall s \tag{14au}$$

$$0 \le l_{d,s}^{\mathrm{shed}} \perp \underline{\rho}_{d,s}^{\mathrm{shed}} \ge 0 \quad \forall d, \forall s \tag{14av}$$

$$0 \le \left(L_d - l_{d,s}^{\mathrm{shed}}\right) \perp \overline{\rho}_{d,s}^{\mathrm{shed}} \ge 0 \quad \forall d, \forall s \tag{14aw}$$

$$0 \le \left(F_{n,m}^{\max} - f_{n,m,s}^{\mathrm{RT}}\right) \perp \rho_{n,m,s}^{\mathrm{F}} \ge 0 \quad \forall n, \forall m \in \Phi_n, \forall s. \tag{14ba}$$

Appendix B: KKTs of Model $\mathcal{M}3$

The KKT optimality conditions associated with the proposed model $\mathcal{M}3$, i.e., problem (7)-(10), are given by (15) below. Note that $\mathcal{L}^{(7)}$, $\mathcal{L}^{(8)}$ and $\mathcal{L}^{(9)}$ are the Lagrangian functions with respect to problems (7), (8) and (9), respectively.

$$\text{(9b)}, \text{(9d)}, \text{(1m)}, \text{(1o)}, \text{(10)} \tag{15a}$$

$$\frac{\partial \mathcal{L}^{(7)}}{\partial p_i^{\mathrm{DA}}} = \phi_s\ C_i - \phi_s\ \lambda_{n:i\in\Psi_n}^{\mathrm{DA}} + \overline{\mu}_{i,s}^{\mathrm{P}} - \underline{\mu}_{i,s}^{\mathrm{P}} + \overline{\rho}_{i,s}^{\mathrm{P}} - \underline{\rho}_{i,s}^{\mathrm{P}} = 0 \quad \forall i, \forall s \tag{15b}$$

$$\frac{\partial \mathcal{L}^{(8)}}{\partial w_k^{\mathrm{DA}}} = -\phi_s\ \lambda_{n:k\in\Psi_n}^{\mathrm{DA}} + \overline{\mu}_{k,s}^{\mathrm{W}} - \underline{\mu}_{k,s}^{\mathrm{W}} + \overline{\rho}_{k,s}^{\mathrm{W}} - \underline{\rho}_{k,s}^{\mathrm{W}} = 0 \quad \forall k, \forall s \tag{15c}$$

$$\frac{\partial \mathcal{L}^{(9)}}{\partial f_{n,m}^{\mathrm{DA}}} = \phi_s\ \lambda_n^{\mathrm{DA}} - \mu_{n,m,s}^{\theta} + \mu_{n,m,s}^{\mathrm{F}} - \lambda_{n,s}^{\mathrm{RT}} = 0 \quad \forall n, \forall m \in \Phi_n, \forall s \tag{15d}$$

$$\frac{\partial \mathcal{L}^{(9)}}{\partial \theta_n^{\mathrm{DA}}} = \sum_{m\in\Phi_n} B_{n,m}\left(\mu_{n,m,s}^{\theta} - \mu_{m,n,s}^{\theta}\right) + \left(\mu_s^1\right)_{n=1} = 0 \quad \forall n, \forall s \tag{15e}$$

$$\text{(14af)} - \text{(14ag)}, \text{(14ai)}, \text{(14aj)} \tag{15f}$$

$$0 \le p_i^{\mathrm{DA}} \perp \underline{\mu}_{i,s}^{\mathrm{P}} \ge 0 \quad \forall i, \forall s \tag{15g}$$

$$0 \le \left(P_i^{\max} - p_i^{\mathrm{DA}}\right) \perp \overline{\mu}_{i,s}^{\mathrm{P}} \ge 0 \quad \forall i, \forall s \tag{15h}$$

$$0 \le w_k^{\mathrm{DA}} \perp \underline{\mu}_{k,s}^{\mathrm{W}} \ge 0 \quad \forall k, \forall s \tag{15i}$$

$$0 \le \left(W_k^{\max} - w_k^{\mathrm{DA}}\right) \perp \overline{\mu}_{k,s}^{\mathrm{W}} \ge 0 \quad \forall k, \forall s \tag{15j}$$

$$0 \le \left(F_{n,m}^{\max} - f_{n,m}^{\mathrm{DA}}\right) \perp \mu_{n,m,s}^{\mathrm{F}} \ge 0\ \forall n, \forall m \in \Phi_n, \forall s \tag{15k}$$

$$\text{(14ap)} - \text{(14au)}, \text{(14ba)}. \tag{15l}$$

Appendix C: Cost recovery by scenario

Mathematically, the profit of all generators, either conventional or renewable, under any scenario realization are non-negative if, at the optimal solution, they hold that

$$\left[p_i^{\mathrm{DA}*}\left(\lambda_{n:i\in\Psi_n}^{\mathrm{DA}*} - C_i\right) + p_{i,s}^{\mathrm{RT}*}\left(\frac{\lambda_{(n:i\in\Psi_n),s}^{\mathrm{RT}*}}{\phi_s} - C_i\right)\right] \ge 0 \quad \forall i, \forall s \tag{16a}$$

$$\left[w_k^{\mathrm{DA}*}\ \lambda_{n:k\in\Psi_n}^{\mathrm{DA}*} + w_{k,s}^{\mathrm{RT}*}\ \frac{\lambda_{(n:k\in\Psi_n),s}^{\mathrm{RT}*}}{\phi_s}\right] \geq 0 \quad \forall k, \forall s \tag{16b}$$

where superscript * stands for the optimal values.

In addition, the profit of transmission operator for any scenario realization is non-negative if, at the optimal solution, it holds that

$$\sum_{n,(m\in\Phi_n)} \left[f_{m,n}^{\mathrm{DA}*}\ \lambda_n^{\mathrm{DA}*} + \left(f_{m,n,s}^{\mathrm{RT}*} - f_{m,n}^{\mathrm{DA}*}\right) \frac{\lambda_{n,s}^{\mathrm{RT}*}}{\phi_s}\right] \geq 0 \quad \forall s. \tag{16c}$$

For notational clarity, we denote the left-hand side of equations (16a), (16b) and (16c) as $\Gamma_{i,s}^{\text{(16a)}}$, $\Gamma_{k,s}^{\text{(16b)}}$, and $\Gamma_s^{\text{(16c)}}$, respectively.

To prove that conditions (16a)-(16c) hold, we derive the strong duality equality corresponding to problems (7), (8), and (9) within the proposed model $\mathcal{M}3$. Note that for each optimization problem the strong duality equality enforces that the values of primal and dual objective functions at the optimal solution are identical. Thus, we get

$$\Gamma_{i,s}^{\text{(16a)}} = \frac{1}{\phi_s}\left[P_i^{\max}\left(\overline{\mu}_{i,s}^{\mathrm{P}*} + \overline{\rho}_{i,s}^{\mathrm{P}*}\right) + P_i^{\mathrm{adj}}\left(\underline{\rho}_{i,s}^{\mathrm{adj}*} + \overline{\rho}_{i,s}^{\mathrm{adj}*}\right)\right] \quad \forall i, \forall s \tag{16d}$$

$$\Gamma_{k,s}^{\text{(16b)}} = \frac{1}{\phi_s}\left[W_k^{\max}\ \overline{\mu}_{k,s}^{\mathrm{W}*} + W_{k,s}\ \overline{\rho}_{k,s}^{\mathrm{W}*}\right] \quad \forall k, \forall s \tag{16e}$$

$$\Gamma_s^{\text{(16c)}} = \frac{1}{\phi_s}\sum_{n,(m\in\Phi_n)} F_{n,m}^{\max}\left(\mu_{n,m,s}^{\mathrm{F}*} + \rho_{n,m,s}^{\mathrm{F}*}\right) \forall s. \tag{16f}$$

The right-hand side of equations (16d)-(16f) include the summation of several expressions, each one is a product of a parameter and a dual variable. Observe that all those parameters and dual variables are non-negative. Therefore, the right-hand side of each equation (16d), (16e), and (16f) is necessarily non-negative. This concludes $\Gamma_{i,s}^{\text{(16a)}} \geq 0$, $\Gamma_{k,s}^{\text{(16b)}} \geq 0$, and $\Gamma_s^{\text{(16c)}} \geq 0$.

Appendix D: Revenue Adequacy by Scenario

This appendix proves that the proposed model $\mathcal{M}3$ is revenue-adequate by scenario. To this purpose, at the optimal solution, we multiply each expression within the DA nodal equalities (10a) by $\lambda_n^{\mathrm{DA}*}$. Similarly, all expressions within the RT nodal equalities (10b) are multiplied by $\frac{\lambda_{n,s}^{\mathrm{RT}*}}{\phi_s}$ at the optimal solution. Then, we sum all equalities obtained, i.e.,

$$\sum_{n,(d\in\Psi_n)} \left[l_d^{\mathrm{DA}*}\ \lambda_n^{\mathrm{DA}*} + \left(l_d^{\mathrm{RT}*} - l_{d,s}^{\mathrm{shed}*}\right)\ \frac{\lambda_{n,s}^{\mathrm{RT}*}}{\phi_s}\right] = \sum_{n,(i\in\Psi_n)} \left[p_i^{\mathrm{DA}*}\ \lambda_n^{\mathrm{DA}*} + p_{i,s}^{\mathrm{RT}*} \frac{\lambda_{n,s}^{\mathrm{RT}*}}{\phi_s}\right] + \sum_{n,(k\in\Psi_n)} \left[w_k^{\mathrm{DA}*}\ \lambda_n^{\mathrm{DA}*} + w_{k,s}^{\mathrm{RT}*}\ \frac{\lambda_{n,s}^{\mathrm{RT}*}}{\phi_s}\right] + \sum_{n,(m\in\Phi_n)} \left[f_{m,n}^{\mathrm{DA}*}\ \lambda_n^{\mathrm{DA}*} + \left(f_{m,n,s}^{\mathrm{RT}*} - f_{m,n}^{\mathrm{DA}*}\right) \frac{\lambda_{n,s}^{\mathrm{RT}*}}{\phi_s}\right] \forall s. \tag{17}$$

According to (17), under any scenario, the total payment of demand-side to market operator, i.e., the left-hand side, equals to the total payment of market operator to all conventional generators, wind power generators, and transmission operator. Therefore, the market operator never incurs a financial deficit under any scenario, i.e., the market is revenue-adequate by scenario. Note that in cases in which the transmission system belongs to the market operator, the market is still revenue-adequate by scenario because the transmission operator's profit for each scenario, i.e., the expression in the last row of (17), is non-negative as proven in Appendix C.

Appendix E: A Linear Expression for Π

Total expected cost of all loads (Π) to be included in objective function (13a) contains the expected payment and shedding cost of all loads, i.e.,

$$\Pi = \sum_{n,(d\in\Psi_n),s} \phi_s\left[l_d^{\mathrm{DA}}\ \lambda_n^{\mathrm{DA}} + \left(l_d^{\mathrm{RT}} - l_{d,s}^{\mathrm{shed}}\right)\ \frac{\lambda_{n,s}^{\mathrm{RT}}}{\phi_s} + V_d\ l_{d,s}^{\mathrm{shed}}\right] \tag{18a}$$

Note that (18a) is non-linear due to bilinear terms. This appendix provides a linear expression for Π.

As proven in Appendix D, for each scenario s, the total demand-side payment equals to total payment of the market operator to conventional generators, wind power generators, and transmission operator. Observe that the expressions in the second, third, and fourth rows of (17) are included in objective functions (7a), (8a), and (9a), respectively. All those expressions are non-linear. However, their linear equivalents can be derived through the strong duality equalities corresponding to problems (7), (8) and (9). Accordingly, a linear equivalent for Π is obtained as follows:

$$\Pi = \sum_s \phi_s\left(\Pi_s + \sum_d V_d\ l_{d,s}^{\mathrm{shed}}\right) \tag{18b}$$

where

$$\begin{aligned}\Pi_s = &\sum_i C_i\left(p_i^{\mathrm{DA}} + p_{i,s}^{\mathrm{RT}}\right)\\ &+ \frac{1}{\phi_s}\Bigg\{\sum_i\left[P_i^{\max}\left(\overline{\mu}_{i,s}^{\mathrm{P}} + \overline{\rho}_{i,s}^{\mathrm{P}}\right) + P_i^{\mathrm{adj}}\left(\underline{\rho}_{i,s}^{\mathrm{adj}} + \overline{\rho}_{i,s}^{\mathrm{adj}}\right)\right]\\ &+ \sum_k\left(W_k^{\max}\ \overline{\mu}_{k,s}^{\mathrm{W}} + W_{k,s}\ \overline{\rho}_{k,s}^{\mathrm{W}}\right)\\ &+ \sum_{n,(m\in\Phi_n)} F_{n,m}^{\max}\left(\mu_{n,m,s}^{\mathrm{F}} + \rho_{n,m,s}^{\mathrm{F}}\right)\Bigg\} \quad \forall s.\end{aligned} \tag{18c}$$

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