Title: Heterogeneous Risk Preferences in Community-based Electricity Markets

Authors: Fabio Moret[1], Pierre Pinson[1], Athanasios Papakonstantinou[2]

Corresponding Authors: Fabio Moret (*fmoret@elektro.dtu.dk*), Pierre Pinson (*ppin@elektro.dtu.dk*)

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Keywords: OR in Energy, Decentralized Electricity Markets, Decision-making Under Uncertainties, Risk Hedging, Fairness.

[1]Technical University of Denmark, Department of Electrical Engineering, Elektrovej 325, DK-2800 Kgs. Lyngby, Denmark

[2]Technical University of Denmark, Department of Management Engineering, Produktionstorvet 426, DK-2800 Kgs. Lyngby, Denmark

Heterogeneous Risk Preferences in Community-based Electricity Markets

Fabio Moret[a,*], Pierre Pinson[a,*], Athanasios Papakonstantinou[b]

[a] *Technical University of Denmark, Department of Electrical Engineering, Elektrovej 325, DK-2800 Kgs. Lyngby, Denmark*

[b] *Technical University of Denmark, Department of Management Engineering, Produktionstorvet 426, DK-2800 Kgs. Lyngby, Denmark*

Abstract

Organization in electricity markets is evolving from centralized pool-based to decentralized peer-to-peer structures. Within this decentralized framework, agents are expected to negotiate their energy procurement individually while preserving their privacy. Since distributed power generation is mostly based on non-dispatchable energy resources with zero marginal cost, any proposed decentralized negotiation mechanism needs to account for uncertainties. When operating uncertain assets, decision makers are affected by subjective attitudes towards uncertain payoffs, impacting not only their energy procurement but also the whole market equilibrium. We propose a new definition of fairness in risky environments and show that, in decentralized electricity markets, heterogeneous risk aversion of participants compromises fairness of the resulting market payments. Consequently, we introduce financial contracts as risk hedging mechanisms and evaluate their impact on market equilibrium and payments. We show that by trading financial products, fairness is restored.

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1. Introduction

The ongoing decentralization of electricity generation and the increasing engagement of end-users in their energy procurement calls for a redesign of the electricity market structure. Currently, the hierarchical structure of electricity markets limits small-size producers or flexible consumers

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[*]Corresponding author

Email addresses: fmoret@elektro.dtu.dk (Fabio Moret), ppin@elektro.dtu.dk (Pierre Pinson)

to further invest in Distributed Energy Resources (DERs) and optimize their energy procurement. To accommodate for the ongoing transformation of the energy sector, a new branch of literature has recently proposed innovative designs for decentralized electricity markets [25]. Some of these market constructs do not largely differ from existing concepts, such as virtual power plants and aggregators. However, the role of prosumers in decentralized electricity markets is fundamentally different. By actively negotiating their energy procurement, end-users can trade energy according to their preferences, for instance on type of energy or on trading partners, instead of being scheduled or controlled by a profit-seeking external entity.

Large uncertainties on electricity prices and available renewable generation make risk management a fundamental decision-making problem. A large branch of literature exists addressing this challenge in electricity markets, for instance by means of multistage stochastic bidding strategies [27], flexibility activation [7] and purchase of forward contracts [9]. In current market structures, uncertainties of end-users are first internalized by bigger players, such as retailers and Distribution System Operators (DSOs), who, by managing a large portfolio of users, can benefit from reduced uncertainties of the aggregated energy profile. The costs of purchasing real-time adjustments to compensate for these uncertainties are then reflected by higher retail prices for the end-users. In the case of decentralized electricity markets, prosumers need to actively account for the uncertainty of their non-dispatchable generation and inflexible demand. Whenever facing future uncertain losses or payoffs, rational agents make informed decisions according to their attitudes towards risk. Several approaches are used to tackle the coordination of energy procurement of market participants in a decentralized way: among all, cooperative game theory [16], variational inequalities [13], matching algorithms [20], control theory [11] and distributed optimization [2]. To the best of our knowledge, the existing literature does not investigate neither the impact of stochastic processes on decentralized electricity markets, nor how individual agents internalize their risk in such context. Only authors in [22] propose a matching algorithm among prosumers while accounting for uncertainties of their assets, however no explicit model of risk is included.

It is expected that in decentralized electricity markets driven by renewable generation, the marginal cost of all generation units and consequently the energy price, will always be zero. However, this is not the case, as agents decide their power set-point according to how they perceive the risk of future payments. For instance, while a risk neutral agent considers the mean, i.e. the expected value of a future loss function, a risk averse agent will consider the worse cases among all the possible scenario realizations as more likely to happen. Hence, she will procure energy in a more cautious way, so as to limit future losses. These risk attitudes can largely differ as prosumers affected by uncertainties are considerably different entities, e.g. households, industries and services. Therefore, an additional challenge is to design market mechanisms that ensure a certain level of fairness in how the risk is internalized and shared among different agents. In a fair market design risk averse agents will face higher costs that compensate only for their risk preferences and not the total welfare loss. Large literature exists on the impact of risk on agent decisions in the power system. In particular, expansion or investment planning has been largely investigated, where the installed capacity is defined also by the risk aversion towards future cash flows [28]. In the same way, risk affects market operations since the aversion towards real-time adjustment costs modifies the power dispatched in the period ahead, as in the case of the day-ahead market or future and forward procurement, [15]. In the existing literature it is not always clear how payments of each market agent are impacted not only by her risk preference but also by those of the other agents. This case is particularly relevant for local decentralized electricity markets applications, where the limited number and size of agents challenge fairness of payments. Additionally, heterogeneous description of uncertainties on agent assets make the market incomplete. As argued by Philpott et al. in [26], if agents do not share the same uncertainty description, in the form of a single stochastic process, then the social welfare optimization might not correspond to the competitive equilibrium solution. In order to hedge their risk towards uncertainties and complete the market, electricity market participants can choose among several financial products and derivatives as reviewed in [24] and [5]. In view of decentralized electricity markets, a further challenge is to adjust and include similar risk hedging mechanisms in the market clearing process.

Despite the fact that, as the literature suggests [30], the technological enablers for fully peer-to-peer electricity markets are currently in place, a transformation of the current market framework to a total decentralized one will be too disruptive. Therefore, market layouts in which agents gather as a community and co-optimize their excess/lack of energy before interfacing with the market and/or system operator can be seen as a first step towards the integration of peer-to-peer electricity markets in the current system [19]. In this work we focus on community-based markets, for which existing literature provides different methodological approaches but none of them considers either uncertainties or risk models [23, 14]. In order to address the challenges discussed above, we propose a formulation of decentralized electricity markets that includes both uncertainties description at agent level and that considers risk aversion in the decision making process. This work addresses the market clearing mechanism as an optimization problem, which can be solved via decomposition techniques, including a stochastic description of uncertainties in the negotiation mechanism and adding heterogeneous risk attitudes of agents by means of a Conditional Value at Risk (cVaR) measure. We then introduce a definition for fairness in risky environments that captures the dependency of the allocation of risk costs on risk preferences. That is, in addition to the increase of the energy cost a single agent may be exposed to as a result of changing her risk preference, there are costs that even agents that do not change their preferences have to face as a result of the actions of that agent. This relates to the notion of moral hazard, a well-known concept in economics that captures the option a market agent may have to exploit the risk of others and profit by doing so [21]. We extend our definition of fairness to the concept of individual fairness, inherited from the machine learning community, [8], and propose a fairness metric, to quantify the deviation from an ideal fair payment allocation. We then derive analytical expressions of agent payments, to assess the impact of considering risk when market agents, which we will equally refer to as market participants, negotiate their energy procurement. We use this fundamental understanding of risk impact on market equilibria to argue that there exist conditions of moral hazard and situations where agents are not incentivized to reveal their true risk preferences. Finally, this work addresses the modelling of risk hedging mechanisms, in the form of financial contracts, as an ideal benchmark for financial products to trade risk among agents. We investigate how risk hedging mechanisms impact market dynamics and in particular whether, or to what extent, fairness among agents can be restored.

The paper is laid out as follows. In Section 2 we describe the agent setup and market framework and propose a decentralized electricity market clearing formulation, including agent risk attitudes. Section 3 addresses the definition of fairness and the impact of risk in decentralized electricity markets in the form of market properties and analytical derivations. Section 4 provides a model of risk hedging mechanisms via financial contracts and analyzes their impact on fairness and agent payments. In Section 5, we present an illustrative example and a larger case study to visualize and verify our findings. Finally, Section 6 gathers a set of conclusions and hints at perspectives for future research.

2. Community-based Market Model with Risk towards Uncertainties

The impact of risk attitudes of market agents towards uncertain generation and consumption on energy procurement, in power systems heavy on non-dispatchable renewable sources, has been largely analyzed in the literature [3]. In this section, we first address the description of the market framework and of agent uncertainties. We then investigate an appropriate risk measure and propose a community-based market clearing formulation where agent risk preferences are included. We close this section by giving a definition of risk cost allocation.

2.1. Framework of community-based electricity markets

Community-based electricity markets can be seen as a specific form of fully decentralized electricity markets as argued in [2], and are more suitable for being integrated in the current market structure. As agents jointly optimize their assets within a community, a virtual agent is used as the interface to the existing markets, for instance day-ahead and balancing markets or retail market. For the sake of this study, we assume the presence of a backup retailer from whom the community can purchase energy both in day-ahead (hereafter referred to as first stage) and in real-time (hereafter referred to as second stage). Therefore we fix the buying (b) and selling (s) prices for first (γ) and second (λ) stages, such that $\lambda_b > \gamma_b > \gamma_s > \lambda_s$. However, uncertain prices could be easily modelled by adding a set of price scenarios to the loss function described below.

2.2. Agent uncertainties and associated loss functions

Decentralized electricity market structures are meant to accommodate a large diversity of agents both in terms of assets availability and of individual objectives and preferences. We model agents as prosumers, namely agents that can behave as producers or consumers depending on the available assets and on the market equilibrium. We then distinguish among all the assets $a \in \mathcal{A}_i$ available to each agent $i \in \mathcal{I}$, a subset $\mathcal{U}_i \subseteq \mathcal{A}_i$ of assets affected by uncertainties. We consider a single-settlement market where each asset dispatch $x_{i,a}$ (positive for generation and negative for consumption) is chosen according to its cost (or utility) function $f_{i,a}$. In order to account for uncertainties, the stochastic process for each asset $u \in \mathcal{U}_i$ is described by means of a set of scenarios $s \in \mathcal{S}_i$ of the realized energy production or consumption with its cardinality denoted by N_{S_i}.

Considering current balancing markets, imbalances are settled after operation, using the measured power dispatch of each asset. In this framework, we can write a second-stage loss function as

$$\begin{aligned} L_{i,u}^s &= \lambda_b |x_{i,u} - \tilde{x}_{i,u}^s|^+ + \lambda_s |x_{i,u} - \tilde{x}_{i,u}^s|^- \\ &= \omega |x_{i,u} - \tilde{x}_{i,u}^s| + \vartheta (x_{i,u} - \tilde{x}_{i,u}^s) \end{aligned} \tag{1}$$

where

$$\omega = \frac{\lambda_b - \lambda_s}{2}, \quad \vartheta = \frac{\lambda_b + \lambda_s}{2}$$

The loss function $L_{i,u}^s$ describes the cost (or revenue if negative) per scenario s associated to asset u of agent i, caused by the difference from the dispatched energy $x_{i,u}$ and the correspondent realization $\tilde{x}_{i,u}^s$ in scenario s. For simplicity of notation, in this work we consider a simplified model of prosumers, i.e. equipped with one uncertain renewable generator (with zero marginal cost) and a fixed load (henceforth we drop the asset subscript for all variables and parameters under the assumption of $|\mathcal{U}_i| = |\mathcal{A}_i| = 1$). In this framework, first-stage decisions of each agent are made only as function of her uncertain second-stage loss function.

2.3. Risk measures with heterogeneous risk preferences

Different risk attitudes imply different perceptions of the loss function in (1) when optimizing the energy procurement. We describe the risk attitude of market participants by means of a risk measure

$$\mathcal{M}_i \; : \; \mathbb{R}_i^S \mapsto \mathbb{R}$$

defined as a functional that maps the stochastic loss function to a deterministic one. Especially in case of large heterogeneity of agents, a coherent risk measure, i.e., a measure satisfying the properties of translational invariance, sub-additivity, homogeneity and monotonicity [1] is preferred, since the sub-addivity property guarantees action diversification. Several coherent risk measures exist, each with their advantages and caveats, as reviewed in [6]. For the sake of this work, we employ the cVaR as a coherent risk measure for two main reasons highlighted by the authors of [29]: it is easy to integrate in an optimization problem and it better handles decision making of agents under "not-normally" distributed uncertainty descriptions than other metrics, e.g. Value at Risk (VaR). Given $\psi_i(l)$ the probability distribution function, of which L_i^s in (1) are equiprobable realizations, the cVaR, parametric on the risk attitude χ_i, is defined as the average of the $1 - \chi_i$ worse realizations. In other words, the cVaR is the average of all realizations larger than the VaR, where $\text{VaR}_i = \{\eta_i \mid \Pr[L_i^s \leq \eta_i] = \chi_i\}$. As displayed in Figure 1 and following [31], one can write a convex sample average approximation of the cVaR as the sum of the VaR (η_i) and the average of the positive residuals (u_i^s). This writes

$$\text{cVaR}_i = \mathcal{M}_i(L_i^s) = \eta_i + \sum_{s \in S_i} \frac{u_i^s}{N_{S_i}(1-\chi_i)} \tag{2}$$

with $u_i^s \geq 0$ and

$$L_i^s \leq \eta_i + u_i^s\,, \qquad \forall s \in \mathcal{S}_i \tag{3}$$

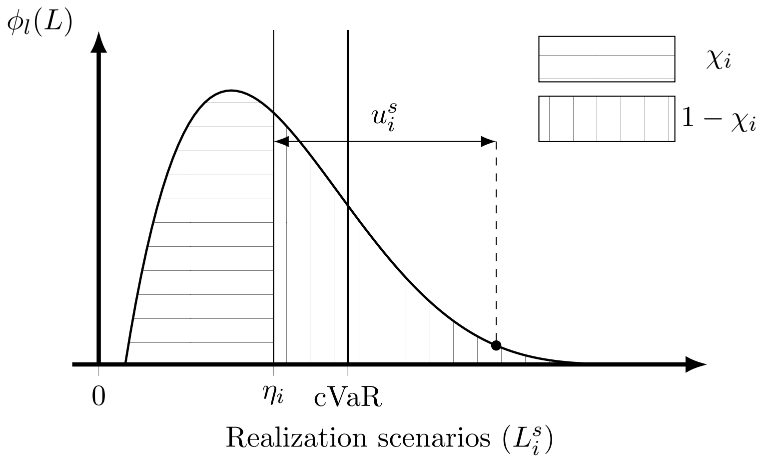

Figure 1: Conditional Value at Risk (cVaR) of the loss scenarios L_i^s.

It should be noted that for $\chi_i = 0$ the cVaR coincides with the expected value and for $\chi_i \to 1$ the market equilibrium is optimized in view of the worst scenario realization for each agent, as in robust optimization. Participants of decentralized electricity markets are likely to have largely different perceptions of risk towards uncertainty, as they include diverse types of agents. We refer to this market participants as agents with heterogeneous risk attitudes, in contrast to homogeneous preferences, where every agent perceive her risk in the same way. Note that in this work we refer to risk preferences and attitudes without distinction.

Definition 1. *Give a set of risk attitudes $\boldsymbol{\chi} = \{\chi_i\}_{i \in \mathcal{I}}$, the set is said to be homogeneous if*

$$\chi_i = \chi_j \qquad \forall i, j \in \mathcal{I}$$

and heterogeneous otherwise.

According to Definition 1, a risk neutral market equilibrium is a specific case of homogeneous risk attitudes, where $\chi_i = 0 \ \forall i \in \mathcal{I}$.

2.4. Community-based market clearing

We extend the community-based market clearing formulation originally proposed in [18] by considering uncertainty description of renewable assets and by introducing risk attitudes in the form of cVaR for each market participant. The optimization problem becomes

$$\begin{aligned}
\min_{\Gamma} \quad & \sum_{i \in \mathcal{I}} \left[\eta_i + \sum_{s \in S_i} \frac{u_i^s}{N_{S_i}(1-\chi_i)} \right] + \gamma_b q_b - \gamma_s q_s && && \text{(4a)} \\
\text{s.t.} \quad & -x_i - q_i + D_i = 0 && \forall i \quad [\pi_i] && \text{(4b)} \\
& \sum_{i \in \mathcal{I}} q_i = q_b - q_s && [\lambda_c] && \text{(4c)} \\
& x_i \leq \overline{X}_i && \forall i \quad [\overline{\mu}_i] && \text{(4d)} \\
& x_i \geq \underline{X}_i && \forall i \quad [\underline{\mu}_i] && \text{(4e)} \\
& \omega |(x_i - \tilde{x}_i^s)| + \vartheta (x_i - \tilde{x}_i^s) \leq u_i^s + \eta_i && \forall i, s \quad [\nu_i^s] && \text{(4f)} \\
& u_i^s \geq 0 && \forall i, s \quad [\tau_i^s] && \text{(4g)} \\
& q_b \geq 0 && [\mu_b] && \text{(4h)} \\
& q_s \geq 0 && [\mu_s] && \text{(4i)} \\
& q_i, \eta_i \in \mathbb{R}
\end{aligned}$$

where $\Gamma = [\boldsymbol{x}, \boldsymbol{q}, q_b, q_s, \boldsymbol{u}, \boldsymbol{\eta}]$ is the set of decision variables. Bold letters are used to group variables over agents and scenarios and dual variables of each constraint are represented within square brackets. The market is modelled as a minimization of procurement costs (4a). Under the assumption that the community operates only renewable generators and fixed loads, the objective function simplifies to only first-stage procurement costs, $\gamma_b q_b - \gamma_s q_s$ respectively the cost of buying energy from and the revenues of selling energy to the retailer, and second-stage adjustment costs, in the form of cVaR as in (2). Each agent i is subjected to generation boundaries (4d) and (4e) and to an energy balance (4b) of the dispatched generation x_i, the fixed load D_i and the energy traded with the community q_i. Additionally, (4f) is used for the definition of the cVaR as in (3) with the loss function defined as in (1). Finally, (4c) ensures energy balance at community level between the community members and the exchanges with the retailer. Hereafter, we refer to this decentralized electricity market with risky agents as a *risk-augmented electricity market.*

The market clearing (4) can be solved as a centralized optimization problem, requiring a central agent to have full information on the uncertain description of each participant, or by means of decomposition techniques. Several methods can be used to decompose the problem as (4c) is the only shared constraint among agents or, alternatively, the set of traded energy $\boldsymbol{q}$ is the complicating variable. Hence, both methods for exchange problems, with sharing constraints, and methods for consensus problems, with complicating variables, can be employed. Among all, algorithms such as the Alternating Direction Method of Multipliers (ADMM), both in its exchange and consensus form, Dantzig-Wolfe, gradient descent, Benders decomposition and gossip algorithms can be readily applied to clear the market, since the description of uncertainties and the risk measure are specific of each agent and they do not add any complicating constraints nor variables. Since (4) can be intended as an optimal power flow without considering network constraints, all decomposition methods reviewed in [17] can be applied.

2.5. Payments and risk cost allocation

In decentralized electricity markets, small-size agents are actively and directly exposed to market dynamics, thus it is fundamental to design mechanisms that ensure fair payments for each market participant. Before addressing the concept of fairness, especially in risky environments, we proceed by defining payments of each market agent. First-stage energy procurement costs can be calculated as

$$C_i^{\mathrm{I}} = \lambda_c \, q_i \tag{5}$$

with λ_c the community energy price computed as the dual variable of (4c). One should note that these are purely procurement costs and, as we only consider renewable generators in this work, first-stage net costs of agents coincide with their procurement costs. Computing second-stage costs requires making a distinction between expected costs

$$\mathbb{E}\left[C_i^{\mathrm{II}}\right] = \int l \, \psi_i(l) dl \tag{6}$$

and risk-adjusted costs

$$\mathbb{R}\left[C_i^{\mathrm{II}}\right] = \frac{1}{1-\chi_i} \int_{\eta_i}^{+\infty} l \, \psi_i(l) dl \tag{7}$$

with $\psi_i(l)$ the probability distribution of second-stage adjustment costs as in (1).

When describing the uncertain process by means of scenarios, the cost definitions (6) and (7) are to be discretized. Since the market clearing is optimized according to a cVaR measure of second-stage costs, market participants procure their energy according to their risk-adjusted perception. From the definition of cVaR in (2) and (3), one can derive that the set of risk adjusted probabilities implies zero probability on the scenarios with $L_i^s < \eta_i$ and probability $\frac{1}{N_{S_i}(1-\chi_i)}$ for scenarios with $L_i^s \geq \eta_i$. These risk adjusted probabilities, as consequence of complementarity of the KKT conditions of (4), coincide with the dual variables ν_i^s of (4f). Hence, we define the total risk-adjusted costs (first plus second stage) of agent i as

$$\hat{C}_i = \lambda_c \; q_i + \sum_{s \in \mathcal{S}_i} \nu_i^s \left[\omega |x_i - \tilde{x}_i^s| + \theta(x_i - \tilde{x}_i^s) \right] \tag{8}$$

However, in reality, all the scenarios describing the uncertain assets are equiprobable with probability $1/N_{S_i}$. Hence, each agent will effectively not face the risk-adjusted costs, that drive her first-stage decision, but the expected costs instead. Therefore, we define the total expected costs of agent i as

$$C_i = \lambda_c \; q_i + \frac{1}{N_{S_i}} \sum_{s \in \mathcal{S}_i} \left[\omega |x_i - \tilde{x}_i^s| + \theta(x_i - \tilde{x}_i^s) \right] \tag{9}$$

Each cost allocation, i.e., a collection of payments of each market participant $\boldsymbol{C} = \{C_i\}_{i=1 \dots N}$, is a function of the risk preferences $\boldsymbol{\chi}$, as these preferences impact the dispatched power of each agent and, hence, the market equilibrium. We can therefore associate a risk cost to the difference between two market equilibria under different risk preferences.

Definition 2. *Let $\boldsymbol{\chi}', \boldsymbol{\chi}'' \in [0,1)^N$ be two sets of risk preferences for the N market participants and $\boldsymbol{C}(\boldsymbol{\chi}')$ and $\boldsymbol{C}(\boldsymbol{\chi}'')$ the respective cost allocations. We define the risk cost allocation as*

$$\begin{aligned} \boldsymbol{R}(\boldsymbol{\chi}', \boldsymbol{\chi}'') &= \boldsymbol{C}(\boldsymbol{\chi}'') - \boldsymbol{C}(\boldsymbol{\chi}') \\ &= \left\{ R_i \in \mathbb{R}, \;\; \forall i \in \mathcal{I} \;\middle|\; R_i(\boldsymbol{\chi}', \boldsymbol{\chi}'') = C_i(\boldsymbol{\chi}'') - C_i(\boldsymbol{\chi}') \right\} \end{aligned} \tag{10}$$

Whenever agents increase their risk aversion, for instance from risk neutral to risk averse, system costs always increase, both in expectation (across scenarios) and with risk-adjusted probabilities. In decentralized markets, it is fundamental to investigate not only the system cost of including risk preferences ($\tilde{R}$), but also how this cost is allocated among agents (R_i). Our objective is to characterize how the system cost of risk is allocated to each agent

$$\boldsymbol{R} = \left\{ R_i \in \mathbb{R},\ \ \forall i \in \mathcal{I} \;\middle|\; \sum_{i \in \mathcal{I}} R_i = \tilde{R} \right\} \tag{11}$$

as a function of the risk preferences. In particular, we investigate whether risk cost allocations under this market design are fair among market participants.

3. Fairness of Risk Cost Allocation

Investigating the impact of risk preferences on cost allocation in decentralized electricity markets becomes fundamental in order to design mechanisms that guarantee fairness among prosumers. However, it is a challenging task to define the concept of fairness in the context of risky market environments and it only marginally relates to previous definitions of fairness in the literature, e.g., in communication networks. Therefore, in this section, we introduce an intuitive description of fairness based on risk cost allocation and provide formal definitions which we then use to assess whether a risk-augmented electricity market respects fairness.

3.1. Basis of risk cost allocation

The existence of system and agent costs brings an additional dimension into out concept of fairness. It may be considered unfair for the agent that changes its risk preference to face costs higher than its expected gains, while also other agents should not be burdened with additional costs. In this context, fairness is regarded as a property of the risk cost allocation given in Definition 2. For a risk cost allocation to be considered fair, each agent has to face the same cost that corresponds to the system cost increase that specific agent is responsible for, when changing her risk preference. If an agent faces higher costs the allocation is not fair towards herself, while facing lower costs implies that someone else has to compensate for this missing money, hence the allocation is not fair towards these individuals.

Definition 3. *Let $\boldsymbol{\chi}'$ and $\boldsymbol{\chi}'' \in [0,1)^N$ be two sets of risk preferences for the N market participants. For each agent i, let $\boldsymbol{\chi}^{(i)}$ be a third set of risk preferences, such that $\chi_i^{(i)} = \chi_i''$ and $\chi_j^{(i)} = \chi_j'$ $\forall j \neq i$. The risk cost allocation $\boldsymbol{R}(\boldsymbol{\chi}', \boldsymbol{\chi}'')$ is fair if*

$$R_i(\boldsymbol{\chi}', \boldsymbol{\chi}'') = R_i(\boldsymbol{\chi}', \boldsymbol{\chi}^{(i)}) \tag{12}$$

holds for each agent $i \in \mathcal{I}$.

Definition 3 addresses fairness in terms of how agents internalize their risk costs. Specifically, we argue that a market design grants fairness of risk cost allocation, if participants are allocated only the extra costs they individually cause to the system. In (12) we distinguish between the cost of including the risk preference of agent i in the system (right-hand side) and the cost actually paid (left-hand side) when each market participant reveals her risk preference. Under this definition, when only one agent is risk averse, she should be the only one facing an extra cost, while the payments of the other market participants do not change, as summarized in the following remark.

Remark 1. *In case only agent i is risk averse, hence $\boldsymbol{\chi}'' = \boldsymbol{\chi}^{(i)}$, then the risk cost allocation $\boldsymbol{R}(\boldsymbol{\chi}', \boldsymbol{\chi}'')$ is fair if*

$$R_k = \begin{cases} R_k(\boldsymbol{\chi}', \boldsymbol{\chi}'') = \tilde{R} & \text{if } k = i \\ 0 & \text{if } k \neq i \end{cases} \tag{13}$$

which implies allocating the entire system risk costs $\tilde{R}$ to agent i.

Definition 3 implies that a cost allocation is fair if the risk cost of each agent does not increase to compensate costs of other agents. In other words, in a fair market design, each agent will pay for her risk independently of the risk preferences of other agents. To verify whether a risk-augmented electricity market is fair according to Definition 3, we derive an analytical representation of the costs of each agent as function of their risk parameters.

Lemma 1. *Let $\boldsymbol{\chi}$ be the set of risk preferences of market participants, $\boldsymbol{\delta}$ and $\boldsymbol{\phi}(\boldsymbol{x})$ respectively the sets of quantiles and probability distribution functions corresponding to the set-point $\boldsymbol{x}$ of the uncertain assets; then the derivative of the total expected costs of each market participant as function of the risk preference of agent $i \in \mathcal{I}$ is*

$$\frac{\partial C_i}{\partial \chi_i} = (\lambda_b - \lambda_s)\left(K_i\, q_i - \frac{\partial \delta_i}{\partial \chi_i} \frac{\delta_i \chi_i}{\phi(x_i)(1-\chi_i)}\right) \tag{14a}$$

$$\frac{\partial C_j}{\partial \chi_i} = K_i(\lambda_b - \lambda_s)\left(q_j - \frac{\delta_j \chi_j}{\phi(x_j)}\right) \qquad \forall j \in \mathcal{I} \setminus i \tag{14b}$$

with $K_i = \left(\frac{\partial \delta_i}{\partial \chi_i}(1-\chi_i) + \delta_i\right)\frac{1}{(1-\chi_i)^2}$. The derivative of the total risk-adjusted costs of each market participant as function of the risk preference of agent $i \in \mathcal{I}$ is

$$\frac{\partial \hat{C}_i}{\partial \chi_i} = (\lambda_b - \lambda_s)K_i\, q_i + \frac{cVaR_i - \eta_i}{1-\chi_i} \tag{15a}$$

$$\frac{\partial \hat{C}_j}{\partial \chi_i} = (\lambda_b - \lambda_s)K_i\, q_j \qquad \forall j \in \mathcal{I} \setminus i \tag{15b}$$

Proof. [Sketch] We calculate the KKT conditions of (4) and manipulate them to achieve an expression of total costs of agents, both in expectation (9) and with risk-adjusted probabilities (8), as a function of χ_i. We then take the derivative over χ_i to achieve expressions (14) and (15). For the full proof, see Appendix 7.1. □

Lemma 1 gives an analytical representation of the derivative of the costs an agent i and of any other agent j as function of the risk attitude of agent i. This allows us to evaluate the fairness of the market structure as in Definition 3.

Proposition 1. *The risk cost allocation of a market participant in a risk-augmented electricity market, is dependent on the risk attitudes of other agents. Hence, the market is not fair.*

Proof. We consider the infinitesimal risk cost allocation, i.e. we look at the derivative of agent costs over their risk attitude rather than the difference. From Lemma 1, we analyze 14b and 15b. If $K_i \neq 0$, then the cost of an agent j changes as function of the risk attitude of another agent i, violating the definition of fairness.

The condition $K_i \neq 0$ is always satisfied if the cumulative distribution function δ_i is sub-linear on the risk attitude χ_i. In other words, an increasing risk aversion of agent i leads to a decrease of the quantile δ_i of her uncertain assets of at most the same magnitude. Under the assumption from the proof of Lemma 1 that $\delta_i < 1 - \chi_i$, $K_i > 0$ if

$$\frac{\partial \delta_i}{\partial \chi_i}(1-\chi_i) + \delta_i > \left(\frac{\partial \delta_i}{\partial \chi_i} + 1\right)(1-\chi_i) \geq 0 \tag{16}$$

it follows

$$\frac{\partial \delta_i}{\partial \chi_i} \geq -1 \tag{17}$$

verifying sub-linearity. □

Agents of decentralized electricity markets influence the clearing price as function of their risk preferences. The community energy price increases as a result of decreased generation (or increased consumption) whenever agent i increases her risk aversion and vice-versa.

Corollary 1. *Whenever one market participant i increases her risk aversion, the market price increases according to*

$$\frac{\partial \lambda_c}{\partial \chi_i} = (\lambda_b - \lambda_s)K_i \geq 0 \tag{18}$$

Proof. From the proof of Proposition 1, we can extrapolate

$$\lambda_c = \lambda_s + \frac{2\omega\delta_i}{1-\chi_i} \tag{19}$$

Then calculating the derivative over χ_i concludes the proof. □

Corollary 1 underlines the interdependence of agents, and their risk attitudes, in decentralized markets and their impact on the price formation. Especially in situations where the number of market agents is not large enough to adopt perfect competitive assumptions, market participants can act strategically to their advantages. When it comes to risk-augmented electricity markets, we show that large generators could act upon risk attitudes different from their real ones to increase their revenues.

Corollary 2. *In a risk-augmented electricity market, the risk attitude χ_i of each market participant determines a threshold T_i such that if the net generation of agent i exceeds T_i, then she can benefit from misrepresenting her risk aversion.*

Proof. Assuming that $-1 \leq \frac{\partial \delta_i}{\partial \chi_i} \leq 0$ as in the proof of Proposition 1, it follows that $K_i \geq 0$. Hence, from (14a) we verify that there exist conditions for which one agent can face lower total expected costs by becoming risk averse. In fact

$$\frac{\partial C_i}{\partial \chi_i} \leq 0 \qquad \text{for} \qquad q_i \leq -\frac{\delta_i \chi_i}{\phi(x_i)} \left(\frac{\delta_i}{K_i(1-\chi_i)^2} - 1 \right) = -T_i \tag{20}$$

with T_i the threshold for which agent i has market power. If she generates more than T_i, she can raise the first-stage price to compensate for a loss in the second stage and get better off in expectation. □

Corollary 2 not only opens new research questions on how to avoid strategic behaviour in decentralized electricity markets, but also reveals the necessity of reducing differentiation across groups of players, e.g. net producers and net consumers. Fairness as defined in Definition 3 can not be achieved, especially when the number of agents is not large enough to make the impact of a single market participant negligible. Therefore, we rethink the definition of fairness from the perspective of *heterogeneity* and *homogeneity* of market participant risk attitudes.

3.2. Fairness in risk-augmented electricity markets

Compared to the current electricity market structures, decentralized electricity markets have peculiar characteristics. Market participants manage uncertain assets (both generation and consumption) mostly with zero marginal costs and infinite marginal utility and have intrinsically heterogeneous risk attitudes towards uncertainties. In a zero marginal cost market, the set of risk preferences determines the price and quantity bid by agents, as a function of the future costs of imbalance. It follows that, under heterogeneous risk attitudes, resources with the same (zero) marginal costs will be dispatched differently. This contradicts the concept of individual fairness, a well-known paradigm in the machine learning community for which a fair model treats similar individuals in a similar way, [8]. Similarity is defined by means of two distance metrics, one in the input and one in the outcome space. We adapt this definition in the context of a risk-augmented electricity market, where the distance in the input space relates to the generation type, and more precisely to its marginal cost. As for the distance in the output space we consider the difference across quantiles ($\boldsymbol{\delta}$) of the uncertain assets.

From this perspective, an ideal market equilibrium would be achieved when all market participants have the same risk attitudes, i.e. with an homogeneous set of risk preferences. Definition 3 describes an ideal situation, in which agent payments are not influenced by other agent risk attitudes, and cannot be achieved in a zero marginal cost market, where risk attitudes determines the bids of market participants. Therefore, we hereby propose an alternative definition of *risk-fairness* using as a focal point risk preferences, instead of payments.

Definition 4. *A decentralized electricity market is risk-fair if all market participants act upon an homogeneous set of risk attitudes.*

The wording "act upon" in Definition 4 is crucial. On the one hand, it is trivial that, if all market participants perceive risk in the same way, i.e. the set of risk preferences is homogeneous, then the market is risk-fair. However, with the right incentives, agents with heterogeneous risk attitudes could procure their energy as if they had homogeneous preferences. We discuss this in the following section, while analysing risk hedging mechanisms. Additionally, risk-fairness implies that the total system welfare attained is the best possible and coincides with the risk neutral case.

Proposition 2. *If a risk-augmented decentralized electricity market is risk-fair, then the expected* *total system costs do not depend on the risk aversion of market participants and they yield the best possible total social welfare.*

Proof. From the proof of Lemma 1, it follows

$$\frac{\delta_i}{1-\chi_i} = \frac{\delta_j}{1-\chi_j} \qquad \forall i,j \tag{21}$$

Therefore, homogeneous risk attitudes $\chi_i = \chi_j \;\; \forall i,j$ imply that agents will dispatch their assets at the same quantiles $\delta_i = \delta_j \;\; \forall i,j$.

Let us consider two market equilibria with two different sets of homogeneous risk attitudes but the same total demand $(\sum_i D_i)$, i.e., the same total generation to be supplied, and the same uncertainty scenarios. Since the dispatch quantiles are constrained to be equal across agents from (21), the quantiles are the same in the two market equilibria. The total first-stage costs sum to zero, while the expected second-stage costs do not change since the quantiles are the same.

Given that the total expected system costs are independent from the value of the risk attitudes of market participants as long as they are homogeneous, the total social welfare coincides with the one of a risk neutral market clearing, which is the best possible. □

Granting risk-fairness is fundamental for decentralized electricity markets. It erases discrimination of resources usage among agents as per Definition 4, while it also yields the best possible social welfare as proved in Proposition 2. Therefore, we investigate on products that recover risk-fairness in case of heterogeneous risk preferences of market participants.

4. Risk Hedging in Decentralized Electricity Markets

We include financial contracts in risk-augmented electricity markets to mitigate the effect of heterogeneous risk attitudes on fairness of risk cost allocation. We then provide a fairness indicator to assess whether these risk hedging mechanisms help to enhance risk-fairness of cost allocation and to reduce suboptimality of market equilibrium.

4.1. Risk hedging via financial contracts

Financial contracts are often employed in markets where the level of uncertainty on prices or volumes is considerable. As they are intended to hedge the risk on first-stage decisions in view of second-stage adjustments, these financial products are purchased by agents only if profitable for them. We model financial contracts in the form of Arrow-Debreu securities, i.e. financial contracts purchased per scenario s in the first-stage market at a certain price λ_w^s with value of 1 in the second-stage market [10]. We model this by including in (4f) the value of the contract w_i^s in the loss function of scenario s with the convention that $w_i^s \geq 0$ if agent i buys a financial contract to secure a revenue in case scenario s realizes. It follows

$$\omega\Delta_i^s + \vartheta(x_i - \tilde{x}_i^s) - w_i^s \leq u_i^s + \eta_i, \qquad \forall i, s \qquad [\nu_i^s] \tag{22}$$

We then add a risk trade balance per scenario

$$\sum_{i\in\mathcal{I}} w_i^s = 0, \quad \forall s \qquad [\lambda_w^s] \tag{23}$$

with dual variables λ_w^s being the prices of the financial contracts. A regularization term could be added to the objective function, in the form of a transaction cost, such as $\sum_{i,s}\gamma_w|w_i^s|$. Hereafter, we refer to a risk-augmented electricity markets with financial contracts as in (22) and (23) as a risk-adjusted electricity market.

Lemma 2. *A risk-adjusted electricity market is complete for risk and risk-adjusted probabilities are aligned across market participants. The risk-adjusted probabilities coincide with those of the least risk averse agent.*

Proof. By allowing each market participant to trade her risk for each scenario (assuming the same number of scenarios N_S across market participants), we complete the market for risk in accordance to the definition provided by the authors of [4].

From the KKT conditions of (4) with (22) and (23), it follows

$$\lambda_w^s = \nu_i^s = \frac{1}{N_S(1-\chi_i)} - \tau_i^s \qquad \forall i, s \tag{24}$$

The risk-adjusted probabilities ν_i^s are aligned across agents and are equal to the price of the financial contracts.

For complementarity ν_i^s and τ_i^s are non-negative, it follows

$$\lambda_w^s = \begin{cases} \frac{1}{N_S(1-\chi_k)} & \text{if } L_k^s > \eta_k \\ 0 & \text{if } L_k^s \leq \eta_k \end{cases} \tag{25}$$

with $L_k^s = \omega\Delta_k^s + \vartheta(x_k - \tilde{x}_k^s) - w_k^s$ and k the agent with the lowest risk aversion ($\chi_k = \min_i \chi_i$). □

Lemma 2 shows that, when allowed to trade financial contracts, market participants align their risk adjusted probability on the uncertainty scenarios to the one of the least risk averse agent. In other words, the more risk averse agents hedge their risk to comply with the least risk averse one. Note that whenever one market participant is risk neutral, market participants hedge completely their risk. We show that financial contracts in the form of Arrow-Debreu securities restore risk-fairness by aligning risk adjusted probabilities across market participants.

Proposition 3. *In a risk-adjusted electricity market, risk-fairness is restored asymptotically, as market participants with heterogeneous risk attitudes attain a market equilibrium that converges to the case of homogeneous risk preferences.*

Proof. Let us assume a continuous description of each stochastic process and an infinite number of market participants. Under these assumptions, Arrow-Debreu securities can be traded to fully hedge agent risks and, as for Lemma 2, the risk-adjusted probabilities of each agent are aligned. Each market participant dispatches her uncertain asset according to the risk aversion of the least risk averse agents, resulting in equal dispatched quantiles. From (21), it follows that equal quantiles $\boldsymbol{\delta}$ correspond to equal risk attitudes $\boldsymbol{\chi}$ across agents. Therefore, market participants procure their energy as if under homogeneous risk preferences. This behaviour is asymptotic since, in practice, stochastic processes are described by a discrete number of scenarios and the number of market participants is limited. $\square$

From Proposition 3, it follows that financial contracts yield a non-discriminating dispatch of resources and attain the best total social welfare by restoring risk-fairness. However, financial contracts in the form of Arrow-Debreu securities, are an arguably unattainable ideal risk hedging product, since they rely on several unrealistic assumptions. Each stochastic process is assumed to be described by the same number of scenarios N_S across agents with its realization included in the employed scenarios (assumption of continuity in Proposition 3). Additionally, the correlation across stochastic processes can weaken the effect of financial contracts, as market participants might want to all buy or sell contracts at specific scenarios, reducing the liquidity (assumption of infinite agents in Proposition 3). Finally, including financial contract trades adds N_S complicating constraints to the problem, making largely complex, if not intractable, the market clearing, via decomposition techniques. Thus, we consider this product as an ideal benchmark for future design of other products readily applicable to real-life systems.

4.2. Fairness assessment in risky environments

In order to quantify the impact of risk hedging mechanisms on market dynamics, we propose a fairness indicator as a possible metric of a fair cost allocation. A straightforward computation of a fairness indicator would be a direct verification of Definition 3. We could therefore compute several market equilibria under different sets of risk attitudes and calculate the risk cost allocation as in Definition 2. Another variation could include a calculation of the perceived price of risk, i.e. the risk cost allocation divided by the change in dispatched quantity, and its spread and correlation to the set of risk attitudes. However, to compute both these indicators, one would need to clear the market several times, which is an impractical and often not available procedure in real-life applications.

Therefore, we propose a fairness indicator that only requires as input the outcome of a single market equilibria. In this way, it could be used as a metric to check performances of ongoing decentralized markets, or as a feedback signal to manage fairness of payment allocations. To do this we leverage on the definition of risk-fairness. From Definition 4, risk-fairness is achieved when all market participants act upon a set of homogeneous risk preferences. This is equivalent to an homogeneous set of quantiles $\boldsymbol{\delta}$, as proven in Proposition 2. Therefore, the more the quantiles differ among agents, the less fair the risk cost allocation will be. We adapt the Jain index, most notably used to quantify fairness in communication networks [12], to evaluate the uncertain assets in the context of risk-augmented electricity markets. The proposed fairness indicator is defined as follows

$$J(\boldsymbol{\delta}) = \frac{(\mathbf{1}^\intercal \boldsymbol{\delta})^2}{N\, \boldsymbol{\delta}^\intercal \boldsymbol{\delta}} \tag{26}$$

This indicator has a maximum in 1 (indicating perfect risk-fairness), when each agent decides to dispatch its uncertain assets at the same quantile. This happens in the ideal case where every agent has the same risk attitude (not necessarily risk neutral) and no bounds on the asset capacities are hit. In a more realistic case, e.g. with heterogeneous risk attitudes and binding bounds on agent variables, this metric ensures higher fairness whenever agents can dispatch their uncertain assets with a similar quantile, reducing the price increase within the community. As discussed in Proposition 3, including financial contracts incentivizes the market participants, even those with heterogenous risk attitudes, to dispatch their assets at a similar quantile, granting fairness properties similar to a market clearing with homogeneous preferences. In this case, the proposed indicator assumes values closer to 1 the more the financial contracts are used to hedge the risk of market participants.

5. Numerical Results

In this section we simulate a risk-augmented and risk-adjusted electricity market with thousands participants in-line with the applications we expect such markets to have as these technologies become more accessible. However, the sheer number of possible interactions among thousands participants and the number of possible equilibria make the task of analyzing the dynamics in such markets very complex. To address this challenge we start our analysis with an illustrative example which, while it preserves the market structure, has only 3 agents as market participants. Through this example, we show in detail market mechanisms and cost allocations of agents under different sets of risk attitudes and the impact of financial contracts on market equilibria. We then employ a larger case study to analyze market dynamics on a heterogeneous population and to verify our analytical results on a large scale application.

5.1. Illustrative example

We first analyze a small scale decentralized electricity market to demonstrate how financial contracts allow agents to hedge their risk, while restoring risk-fairness and the maximum social welfare. The market consists of 3 agents each equipped with fixed load and uncertain PV generation, described by 1 000 Gaussian scenarios. Retail prices are fixed both in first and second stage.

We identify and investigate market equilibria under different sets of risk attitudes for the market participants, as summarized in Table 1 and Table 2. Following the convention used in our analytical derivation, Table 1 shows agent costs (revenues in case of negative figures). We refer to system total expected costs as the sum across agents of their first- and expected second-stage costs.

We first compare the risk neutral (RN) case, $\chi_i = 0 \ \forall i$, with an homogeneous risk averse (RA) case, $\chi_i = 0.3 \ \forall i$. As proven in Proposition 2, the system total expected costs and the dispatched quantiles are the same in both cases. What changes is the costs allocation among agents. Recalling (20) with $\partial\delta_i/\partial\chi_i = 0$, agents that assume the role of net generators (in this case agents 1 and 3), get better off and all risk cost is pushed to the agent being the net consumer (agent 2). The same allocation pattern occurs, even more amplified, in the case of heterogeneous risk aversion

Table 1: Summary of agent and system total expected costs [$] for different sets of risk attitudes.

Agent	**RN**	**RA**	**RHe**	**RA3**	**RHe FC**	**RA3 FC**
1	-7.363	-8.288	-13.759	-11.440	-27.778	-7.240
2	69.549	82.913	113.072	98.260	98.325	74.666
3	-71.680	-84.119	-106.631	-92.768	-80.033	-76.918
Tot	-9.495	-9.495	-7.318	-5.948	-9.485	-9.491

Table 2: Summary of agent dispatched quantiles and Jain index for different sets of risk attitudes.

Agent	**RN**	**RA**	**RHe**	**RA3**	**RHe FC**	**RA3 FC**
1	0.247	0.247	0.373	0.353	0.261	0.255
2	0.247	0.247	0.311	0.353	0.239	0.241
3	0.247	0.247	0.125	0.101	0.246	0.245
Jain	1.000	1.000	0.868	0.837	0.999	0.999

(RHe), where the risk attitude of each market participant is set respectively to 0.4, 0.5 and 0.8. Additionally, this simulation shows a loss of optimality in terms of system total expected costs as risk-adjusted probabilities are misaligned. Consequently, risk-fairness, with a Jain index on the dispatched quantiles of 0.868. We also investigate a variation of the risk attitudes in RA, where only agent 3 deviates from her risk preference to $\chi_3 = 0.8$ (RA3). Since the net generation $q_3 = 1.959$ [MW] exceeds the threshold $T_3 = 0.230$ [MW], computed as in (20), agent 3 gets better off at the expense of others even if she is the only one increasing her risk aversion. As discussed in Section 3, this cost allocation, besides not being fair, provides an incentive to agent 3 to misrepresent her risk attitude and becoming even more risk averse, as stated in Corollary 2.

We then proceed to simulate financial contracts (FCs) and analyze their impact on the market outcome. In particular, we evaluate these products in a market which consists of agents with heterogeneous risk attitudes (RHe FC) and for a case study where only one agent (agent 3) deviates from a homogeneous set of risk preferences (RA3 FC). Focusing on the case with heterogeneous set of risk attitudes (RHe FC), one can notice from Table 1 that the cost allocation across agents is

Table 3: Summary of agent and system cost variation [$] when including financial contracts under heterogeneous risk attitudes (RHe FC - RHe).

	First-stage cost variation		Second-stage cost variation		Total
Agent	**Energy**	**Financial Contracts**	**Energy**	**Financial Contracts**	**Variation**
1	10.905	-46.654	-5.788	27.518	-14.019
2	-19.357	9.763	-7.255	2.102	-14.747
3	8.452	36.891	10.875	-29.620	26.598
Tot	-	-	-2.167	-	-2.167

different compared to the case without financial contracts (RHe), while the system total expected costs are almost the same as the homogeneous case (RA and RN). In Proposition 3, we prove that financial contracts restore risk-fairness only asymptotically: in fact, when only few agents participate in the market, it is unlikely that each market participant can fully hedge her risk. With only three agents there can exist few situations where each market participant would like to buy financial contracts, but no one is willing to sell any. This is also reflected in Table 2, where the dispatched quantiles are not exactly the same, but the spread across agents is largely decreased, resulting in a Jain index of almost 1. Below, we verify that full recovery of the best social welfare can be achieved in case of a larger case study.

Looking deeper into the cost allocation of market participants when financial contracts are included, Table 3 shows the difference of costs (RHe FC - RHe) for first and second stage. One should note how financial contracts, in the first stage, are sold by the least risk averse agent (agent 1) and bought by the others proportionally to their risk attitude ($\chi_2 = 0.5$ and $\chi_3 = 0.8$). These products allow market participants to hedge their risk in second stage. First-stage trades are all budget balanced, while the expected second-stage costs are not, as financial contracts impact differently agents across different scenarios. This difference induces an overall improvement of the system expected costs, recovering almost the same social welfare of a market equilibrium with homogeneous risk attitudes.

The effect of financial contracts is highlighted in Figure 2, which displays the Cumulative Distribution Function (CDF) of second-stage losses of each market participant. Purchasing financial contracts, as proved in Lemma 2, alters the loss functions of agents such that their risk is hedged and their risk-adjusted probabilities are aligned to the one of the least risk averse agent. We visualize this, by plotting the CDF of the second-stage losses for equilibria with and without financial contracts, respectively in black and red, together with the respective Value at Risk (VaR), plotted as vertical lines. The risk attitude upon which each market participant acts, corresponds to the probability mass not considered by market participants for their perceptions of second-stage costs. Recalling the definition of cVaR in Section 2.3, this probability mass can be identified by the intersection of each CDF and the VaR. Figure 2 shows that financial contracts align the perception of risk across agents to the smallest risk attitude, $\chi_1 = 0.4$.

We now follow the case where only agent 3 changes her risk attitude, with all market participants able to trade financial contracts (RA3 FC). Assuming that all agents have risk homogeneous attitudes, as in RA, if agent 3 chooses to act upon a different risk preference, her total expected costs are increased when including financial contracts, as displayed in Table 1. Since these products allow agents to hedge their risk by purchasing contracts in first-stage, the more risk averse the agents become, the more contracts they will have to purchase to hedge their risk. Further work is needed to analytically prove this, however the intuition, verified by our simulations, is that financial contracts remove agent incentives to exercise market power by misrepresenting her risk attitudes.

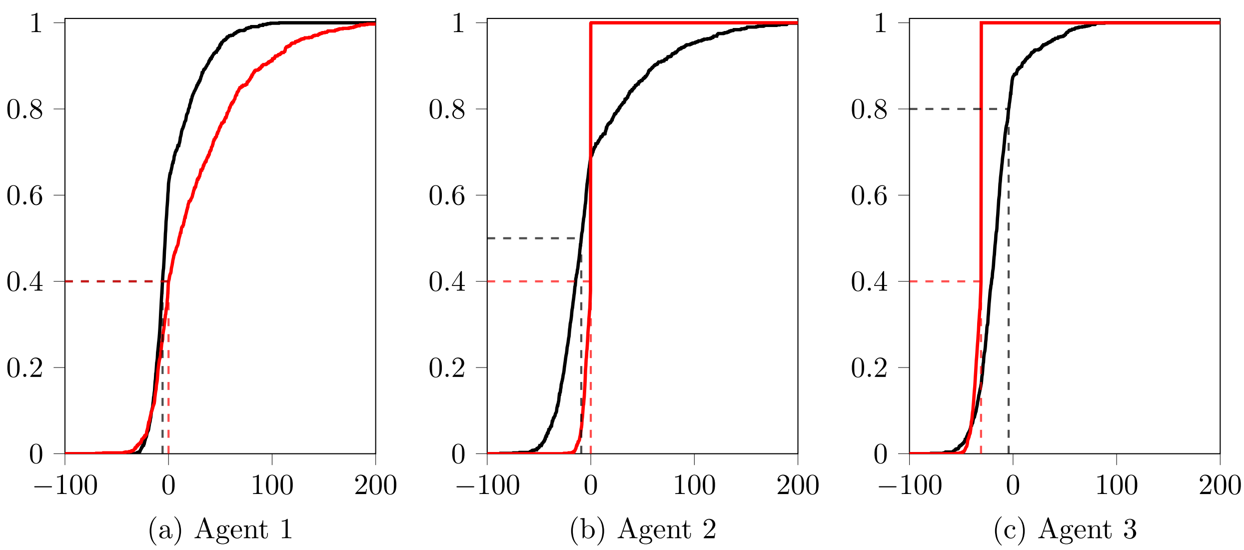

Figure 2: Cumulative distribution function of second-stage losses, with (red) and without (black) financial contracts, and respective Value at Risk (vertical dashed lines).

5.2. Case study

We test our market formulation on a large population, in order to illustrate how the market mechanisms and properties scale up. In particular, we focus on group fairness, i.e. if there are patterns in the market outcomes that show a discrimination of certain subsets of agents. We employ a synthetic dataset of 10 000 agents, each with uncertain PV generation described by 50 scenarios and fixed load. We uniformly sample the mean and standard deviation of PV generation, as well as electricity consumption and risk attitudes. We benchmark the results to the same dataset but with all risk attitudes set to zero, i.e. the risk neutral case, to calculate risk payments as difference of costs between the heterogeneous risk averse and the risk neutral case.

Figure 3 displays a summary of the results. The black dots represent the risk payments of agents as function of their net generation, while the density of the data points is displayed by isobars (black lines). It is clear that risk-fairness is not respected as agents with similar resources (in this case all agents) are allocated largely different risk payments. The risk cost distributions for net producers and net consumers separately (Figure 3 right) clearly show that the market is

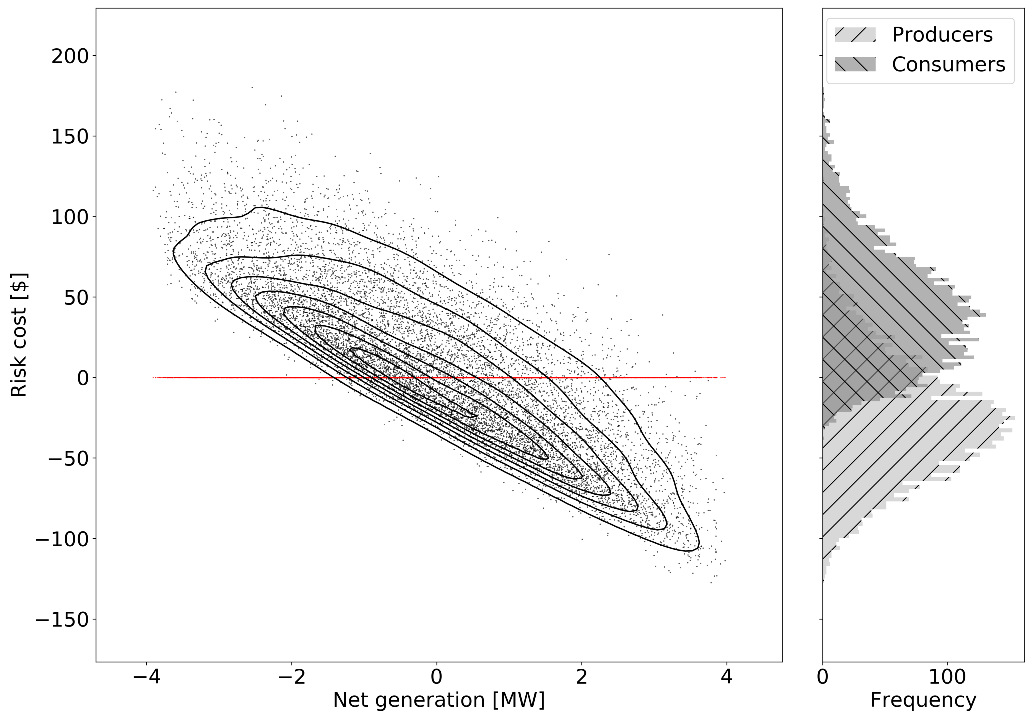

Figure 3: Risk cost as function of net generation (left black), with the respective distribution for net producers and consumers (right), and equivalent risk cost when including financial contracts (left red).

strongly biased to penalize more net consumers. The probability of net consumers to be penalized is of around 88% compared to the 22% for net producers.

When including financial contracts, the difference of risk costs across agents are smoothed out, as depicted in Figure 3 by the red dots. Additionally, the risk cost magnitudes are largely reduced, from costs between $[-127.37, 180.20]$ \$ to costs in the range of $[-0.93, 0.01]$ \$. Risk-fairness is perfectly restored, from a Jain index of 0.823 for the risk-augmented market to a Jain index of 1.0 for the risk-adjusted market. At the same time, market outcomes are not biased between net consumers and net producers, as both have a probability of 57% to be penalized. The results also verify that the same social welfare attained in the risk neutral case is recovered.

6. Conclusions and Future Work

The role of uncertainties in decentralized electricity markets is fundamental, as most of the distributed energy resources of prosumers are uncertain, yet the impact their risk preferences may have on the market outcome is not deeply researched. In this paper, we investigated the impact of uncertainties and the corresponding risk attitudes in community-based markets. We modelled risk costs by means of the cVaR and analyzed cost allocations in risky environments. In particular, we focused on how the costs of including risk in the market clearing are allocated to the agents and proposed a definition of fairness of risk cost allocation. To do this, we derived the analytical relation between payments and risk attitudes of the community members in the form of comparative statics and adapted the well-known Jain index as a fairness metric for cost allocations in decentralized electricity markets. We verified that including risk in community-based market mechanism compromises fairness of cost allocation among participants and incentivizes large producers to strategically misrepresent their risk preferences. Therefore, we investigated risk hedging mechanisms, in the form of financial contracts, not only as a way to shield agents from second-stage losses but also as a product to restore fairness within the community. Furthermore, we showed that including financial contracts removes incentives from market participants to misrepresent their risk aversion, but further work is needed to analytically prove this property.

By completing the market with Arrow-Debreu securities, we set an ideal benchmark for full risk hedging, a largely non practical set-up when solving the problem via decomposition techniques as it adds a number of complicating constraints equal to the number of the scenarios employed to describe the stochastic processes. Moreover, Arrow-Debreu securities, although ideal, may not be attainable, as they imply a certain level of coordination across market participants. Hence, future research should aim at designing hedging mechanisms that approximate the effect of the proposed benchmark, while preserving the decomposition properties of the problem and reducing the need of coordination among uncertainty descriptions.

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7. Appendix

7.1. *Proof of Lemma 1*

We look for an analytical expression of total costs $C_j \; \forall j \in \mathcal{I}$ as function of the risk parameter of one agent $i \in \mathcal{I}$. We begin by deriving the KKT conditions of (4) as follows

$$-\pi_i + \bar{\mu}_i - \underline{\mu}_i + \sum_{s \in S_i} [\omega \, \text{sign}(x_i - x_i^s) + \vartheta] \, \nu_i^s = 0 \qquad \forall i \quad [x_i] \tag{27a}$$

$$-\pi_i + \lambda_c = 0 \qquad \forall i \quad [q_i] \tag{27b}$$

$$\gamma_b - \lambda_c - \mu_b = 0 \qquad [q_b] \tag{27c}$$

$$-\gamma_s + \lambda_c - \mu_s = 0 \qquad [q_s] \tag{27d}$$

$$\epsilon_i - \sum_{s \in S_i} \nu_i^s = 0 \qquad \forall i \quad [\eta_i] \tag{27e}$$

$$\frac{\epsilon_i}{N_{S_i}(1-\chi_i)} - \nu_i^s - \tau_i^s = 0 \qquad \forall i, s \quad [u_i^s] \tag{27f}$$

$$-x_i - q_i = 0 \qquad \forall i \tag{27g}$$

$$\sum_{i \in \mathcal{I}} q_i = 0 \tag{27h}$$

$$0 \le \overline{X}_i - x_i \perp \bar{\mu}_i \ge 0 \qquad \forall i \tag{27i}$$

$$0 \le x_i - \underline{X}_i \perp \underline{\mu}_i \ge 0 \qquad \forall i \tag{27j}$$

$$0 \ge \eta_i + u_i^s - \omega|(x_i - \tilde{x}_i^s)| - \vartheta(x_i - \tilde{x}_i^s) \perp \nu_i^s \ge 0 \qquad \forall i, s \tag{27k}$$

$$0 \le u_i^s \perp \tau_i^s \ge 0 \qquad \forall i, s \tag{27l}$$

As displayed in Figure 4, the dispatch set-point x_i, and its corresponding cumulative distribution function $\delta_i = \Phi(x_i)$, define the scenarios where the loss function will be positive ($\tilde{x}_i^s \ge x_i$, i.e. agent i will sell the excess generation at price λ_s) and negative ($\tilde{x}_i^s \le x_i$, i.e. agent i will buy the lack generation at price λ_b), hence the quantile of the negative loss function is $1-\delta_i$ (and vice-versa δ_i for the positive loss function), as represented by the black lines in Figure 5. By manipulating the KKT conditions, in particular accounting for the complementarity conditions (27k) and (27l) as displayed in Figure 5, we derive

$$\sum_{s \in S_i} [\omega \, \text{sign}(x_i - x_i^s) + \vartheta] \, \nu_i^s = \begin{cases} 1 & \eta_i \ge 0 \\ \frac{2\delta_i}{1-\chi_i} - 1 & \eta_i \le 0 \end{cases} \tag{28}$$

From Figure 5, one can notice that the condition $\eta_i \ge 0$ happens when $\delta_i \ge 1 - \chi_i$ (and vice-versa). The continuity of (28) can be easily verified for $\delta_i = 1 - \chi_i$. Under the assumption that the

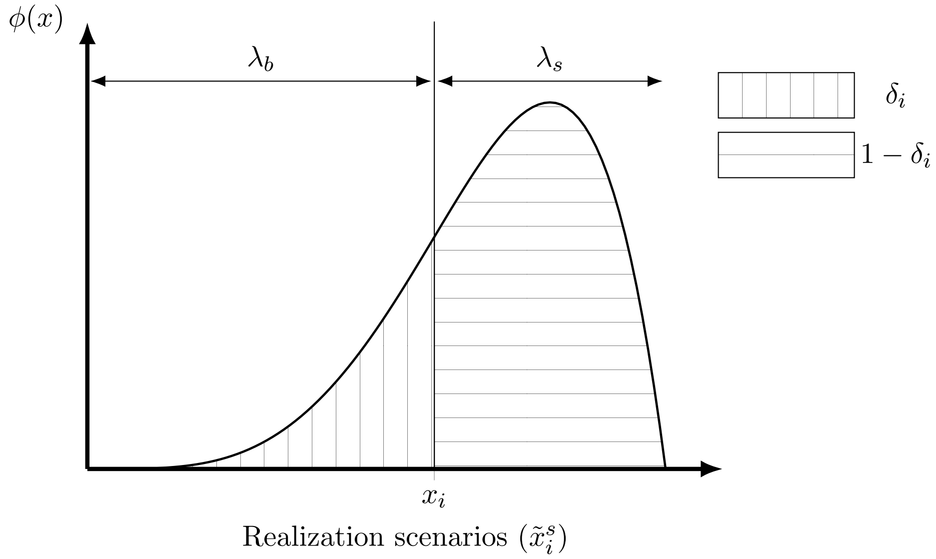

Figure 4: Probability distribution function of uncertain generation scenarios.

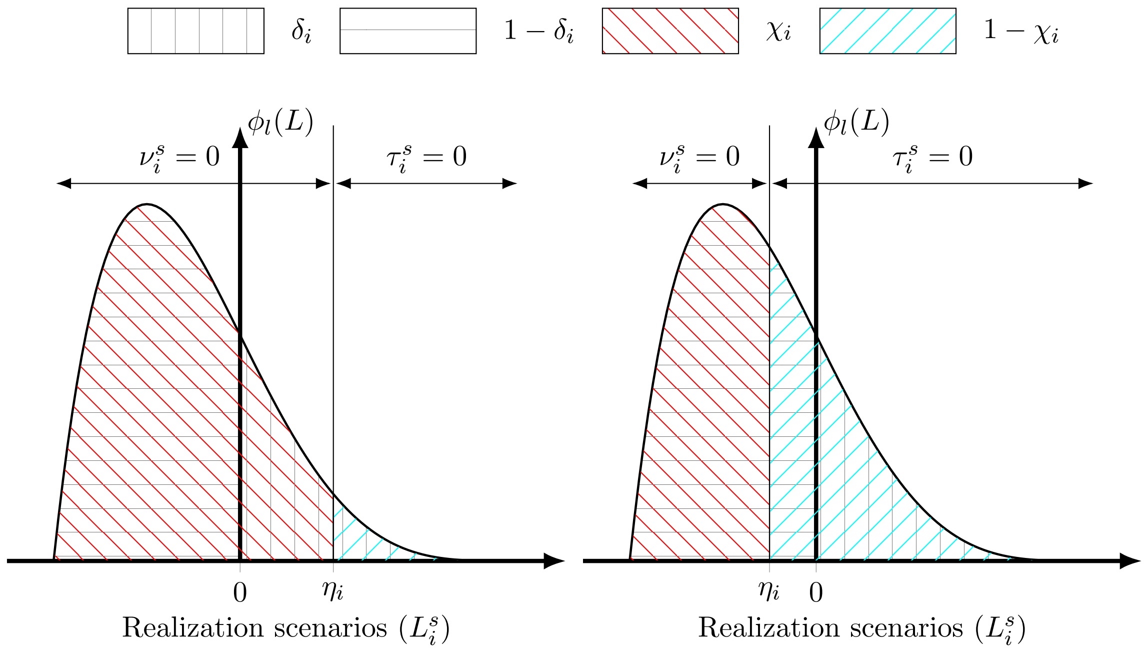

Figure 5: Probability distribution function of loss scenarios, $\eta_i > 0$ (left) and $\eta_i < 0$ (right).

assets affected by uncertainties do not hit their maximum and minimum capacity ($\bar{\mu}_i, \underline{\mu}_i = 0 \; \forall i \in \mathcal{I}$), we substitute (28) in (27a) to derive an expression of the price perceived by each agent π_i as follows

$$\pi_i = \begin{cases} \lambda_b & \text{if } \delta_i \geq 1 - \chi_i \\ \lambda_s + \frac{2\omega\delta_i}{1-\chi_i} & \text{if } \delta_i < 1 - \chi_i \end{cases} \tag{29}$$

Whenever $\delta_i \geq 1 - \chi_i$, the probability for which agent i needs to buy energy in the second-stage decision δ_i is larger than the probability mass of the worst case scenarios considered by the cVaR $(1-\chi_i)$. Therefore the perceived price in this case is simply the buying price in second stage λ_b and no major cost changes happen for small deviation of risk attitudes around this market equilibrium. For this reason, we focus on the less trivial case, where $\delta_i < 1 - \chi_i$ and the perceived price is function of the power set-point of the uncertain asset and the agent risk attitudes. From (27b), we derive that $\lambda_c = \pi_i = \pi_j,\ \forall i, j$. It follows

$$\frac{\delta_i}{1 - \chi_i} = \frac{\delta_j}{1 - \chi_j} \quad \forall i, j \in \mathcal{I} \tag{30}$$

In this way, we can express the cost functions of all agents j with dependency only on the risk preference of agent j. Since the power set-point $x_i = \Phi^{-1}(\delta_i)$, and its quantile, is a non-straightforward function of the risk attitude, we consider the symbolical function $\delta_i = \delta_i(\chi_i)$ to represent this relation.

We first rewrite (6) as

$$\begin{aligned} \mathbb{E}\left[C_i^{\text{II}}\right] &= \int_{-\infty}^{0} l_i^- \,\psi_i(l_i^-) dl_i^- + \int_{0}^{+\infty} l_i^+ \,\psi_i(l_i^+) dl_i^+ \\ &= \int_{\overline{X}_i}^{x_i} \lambda_s(x_i - x)\frac{\phi_i(x)}{\lambda_s}(-\lambda_s dx) + \int_{x_i}^{\underline{X}_i} \lambda_b(x_i - x)\frac{\phi_i(x)}{\lambda_b}(-\lambda_b dx) \\ &= x_i\left[\lambda_s(1-\delta_i) + \lambda_b\delta_i\right] + \lambda_s \int_{\overline{X}_i}^{x_i} x\,\phi_i(x)dx + \lambda_b \int_{x_i}^{\underline{X}_i} x\phi_i(x)dx \end{aligned} \tag{31}$$

Using (29) and (30) as well as the definition of λ_c and x_i, we rewrite (5) and (31) as follows

$$\begin{aligned} C_i = &\left(\lambda_s + \frac{2\omega\delta_i}{1-\chi_i}\right)\left[D_i - \Phi^{-1}(\delta_i)\right] + \Phi^{-1}(\delta_i)\left[\lambda_s(1-\delta_i) + \lambda_b\delta_i\right] \\ &+ \lambda_s \int_{\overline{X}_i}^{\Phi^{-1}(\delta_i)} x\,\phi_i(x)dx + \lambda_b \int_{\Phi^{-1}(\delta_i)}^{\underline{X}_i} x\phi_i(x)dx \end{aligned} \tag{32a}$$

$$\begin{aligned} C_j = &\left(\lambda_s + \frac{2\omega\delta_i}{1-\chi_i}\right)\left[D_j - \Phi^{-1}\left(\frac{\delta_i(1-\chi_j)}{1-\chi_i}\right)\right] + \Phi^{-1}\left(\frac{\delta_i(1-\chi_j)}{1-\chi_i}\right)\left[\lambda_s(1-\delta_j) + \lambda_b\delta_j\right] \\ &+ \lambda_s \int_{\overline{X}_j}^{\Phi^{-1}\left(\frac{\delta_i(1-\chi_j)}{1-\chi_i}\right)} x\,\phi_j(x)dx + \lambda_b \int_{\Phi^{-1}\left(\frac{\delta_i(1-\chi_j)}{1-\chi_i}\right)}^{\underline{X}_j} x\phi_j(x)dx \qquad \forall j \in \mathcal{I} \setminus i \end{aligned} \tag{32b}$$

We then calculate the derivatives of (32) over χ_i as

$$\frac{\partial C_i}{\partial \chi_i} = 2\omega\left(\frac{\partial \delta_i}{\partial \chi_i}(1-\chi_i)+\delta_i\right)\frac{D_i-x_i}{(1-\chi_i)^2} - \frac{\partial \delta_i}{\partial \chi_i}\frac{1}{\phi(x_i)}\left(\lambda_s+\frac{2\omega\delta_i}{1-\chi_i}\right)+\frac{\partial \delta_i}{\partial \chi_i}\frac{\lambda_s+2\omega\delta_i}{\phi(x_i)} \tag{33a}$$

$$\frac{\partial C_j}{\partial \chi_i} = \left(\frac{\partial \delta_i}{\partial \chi_i}(1-\chi_i)+\delta_i\right)\frac{1}{(1-\chi_i)^2}\left[2\omega(D_j-x_j)-\left(\lambda_s+\frac{2\omega\delta_i}{1-\chi_i}\right)\frac{1-\chi_j}{\phi(x_j)}+(\lambda_s+2\omega\delta_j)\frac{1-\chi_j}{\phi(x_j)}\right] \tag{33b}$$

Under the assumption of a non-trivial situation, i.e. $\eta_i \leq 0$, we rewrite (7) as

$$\begin{aligned}\mathbb{R}\left[C_i^{\mathrm{II}}\right] &= \frac{1}{1-\chi_i}\left[\int_{\eta_i}^{0} l_i^- \,\psi_i(l_i^-)dl_i^- + \int_0^{+\infty} l_i^+\,\psi_i(l_i^+)dl_i^+\right] \\ &= \frac{1}{1-\chi_i}\left[\int_{x_i-\frac{\eta_i}{\lambda_s}}^{x_i} \lambda_s(x_i-x)\frac{\phi_i(x)}{\lambda_s}(-\lambda_s dx) + \int_{x_i}^{\underline{\mathrm{X}}_i}\lambda_b(x_i-x)\frac{\phi_i(x)}{\lambda_b}(-\lambda_b dx)\right] \\ &= \frac{1}{1-\chi_i}\left(x_i\left[\lambda_s(1-\delta_i-\chi_i)+\lambda_b\delta_i\right]+\lambda_s\int_{x_i-\frac{\eta_i}{\lambda_s}}^{x_i} x\,\phi_i(x)dx + \lambda_b\int_{x_i}^{\underline{\mathrm{X}}_i} x\phi_i(x)dx\right)\end{aligned} \tag{34}$$

Using (29) and (30) as well as the definition of λ_c and x_i, we rewrite (5) and (34) as follows

$$\begin{aligned}\hat{C}_i = &\left(\lambda_s+\frac{2\omega\delta_i}{1-\chi_i}\right)\left[D_i-\Phi^{-1}(\delta_i)\right]+\frac{1}{1-\chi_i}\Bigg[\Phi^{-1}(\delta_i)\left(\lambda_s(1-\delta_i-\chi_i)+\lambda_b\delta_i\right) \\ &+\lambda_s\int_{\Phi^{-1}(1-\chi_i)}^{\Phi^{-1}(\delta_i)} x\,\phi_i(x)dx+\lambda_b\int_{\Phi^{-1}(\delta_i)}^{\underline{\mathrm{X}}_i} x\phi_i(x)dx\Bigg]\end{aligned} \tag{35a}$$

$$\begin{aligned}\hat{C}_j = &\left(\lambda_s+\frac{2\omega\delta_i}{1-\chi_i}\right)\left[D_j-\Phi^{-1}\left(\frac{\delta_i(1-\chi_j)}{1-\chi_i}\right)\right] \\ &+\frac{1}{1-\chi_i}\Bigg[\Phi^{-1}\left(\frac{\delta_i(1-\chi_j)}{1-\chi_i}\right)\left(\lambda_s\left[1-\frac{\delta_i(1-\chi_j)}{1-\chi_i}-\chi_j\right]+\lambda_b\frac{\delta_i(1-\chi_j)}{1-\chi_i}\right) \\ &+\lambda_s\int_{\Phi^{-1}(1-\chi_j)}^{\Phi^{-1}\left(\frac{\delta_i(1-\chi_j)}{1-\chi_i}\right)} x\,\phi_j(x)dx+\lambda_b\int_{\Phi^{-1}\left(\frac{\delta_i(1-\chi_j)}{1-\chi_i}\right)}^{\underline{\mathrm{X}}_j} x\phi_j(x)dx\Bigg] \qquad \forall j\in\mathcal{I}\setminus i\end{aligned} \tag{35b}$$

We then calculate the derivatives of (35) over χ_i as

$$\begin{aligned}\frac{\partial \hat{C}_i}{\partial \chi_i} = &2\omega\left(\frac{\partial \delta_i}{\partial \chi_i}(1-\chi_i)+\delta_i\right)\frac{D_i-x_i}{(1-\chi_i)^2}-\frac{\partial \delta_i}{\partial \chi_i}\frac{1}{\phi(x_i)}\left(\lambda_s+\frac{2\omega\delta_i}{1-\chi_i}\right) \\ &+\frac{\mathrm{cVaR}_i-\eta_i}{1-\chi_i}+\frac{\partial \delta_i}{\partial \chi_i}\frac{\lambda_s(1-\chi_i)+2\omega\delta_i}{\phi(x_i)(1-\chi_i)}\end{aligned} \tag{36a}$$

$$\frac{\partial \hat{C}_j}{\partial \chi_i} = \left(\frac{\partial \delta_i}{\partial \chi_i}(1-\chi_i)+\delta_i\right)\frac{1}{(1-\chi_i)^2}\left[2\omega(D_j-x_j)-\left(\lambda_s+\frac{2\omega\delta_i}{1-\chi_i}\right)\frac{1-\chi_j}{\phi(x_j)}+\frac{\lambda_s(1-\chi_j)+2\omega\delta_j}{\phi(x_j)}\right] \tag{36b}$$

concluding the proof.