

# Loss Allocation in Joint Transmission and Distribution Peer-to-Peer Markets

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**Abstract**—Large deployment of distribute energy resources and the increasing awareness of end-users towards their energy procurement are challenging current practices of electricity markets. A change of paradigm, from a top-down hierarchical approach to a more decentralized framework, has been recently researched, with market structures relying on multi-bilateral trades among market participants. In order to guarantee feasibility in power system operation, it is crucial to rethink the interaction with system operators and the way operational costs are shared in such decentralized markets. We propose here to include system operators, both at transmission and distribution level, as active actors of the market, accounting for power grid constraints and line losses. Moreover, to avoid market outcomes that discriminate agents for their geographical location, we analyze loss allocation policies and their impact on market outcomes and prices.

**Index Terms**—Peer-to-peer trading, Loss allocation, Optimal power flow, Network charges.

## I. INTRODUCTION

The fast development of Distributed Energy Resources (DERs) and the increasing flexibility of energy management systems for residential homes and commercial buildings have paved the way to new opportunities for energy end-users. Thanks to the recent development of Information and Communication Technology (ICT) and in line with the ongoing evolution of the economy towards sharing systems [1], a shift from centralized to decentralized electricity markets is expected to happen. Scenarios of how these electricity markets will take place have been proposed in [2], ranging from peer-to-peer (P2P), community-based and microgrid structures. Multi-lateral electricity trades were already proposed in [3], but only recently a new branch of literature has started to address how these markets would be designed and implemented in practice [4]–[6]. However, the independent evolution of electricity markets and power systems, operated by Transmission (TSO) or Distribution (DSO) System Operators, raises concerns on the ability of these new market proposals to coexist with reliable power system management.

Pool-based electricity markets have the advantage to gather all the information needed to find market equilibria within one platform. Consequently, TSOs can interface directly with the market operator to guarantee feasibility in power system operation. This is usually done by accounting for power flows during the market clearing, like in the case of the American locational marginal pricing system, or by redispatch actions, i.e., ex-post corrective actions, as it happens in Europe.

When conceiving future electricity markets that operate in a decentralized manner, the challenges are to redefine the current interactions with TSOs and to include DSOs in the market mechanism, since most of end-users are connected to the distribution level. Several solution methods to integrate System Operators (SOs) in decentralized frameworks have been proposed, mainly including two approaches: an iterative process in which SOs accept or reject multi-lateral energy trades depending on whether network constraints are violated [7]–[9] and the usage of pre-defined network charges, e.g., based on the electrical distance between peers [10].

All the proposed solutions have two main limitations. First, they do not account for a coordination between TSOs and DSOs, a crucial challenge in modern power systems, as analyzed in [11], [12]. Second, power losses occurring in the network are often overlooked, since their procurement costs are currently included in grid tariffs of end-users. More recently, the authors of [13] proposed a peer-to-peer electricity market accounting for power losses, where network charges are dynamically adapted to each trade based on its grid usage. However, no attention is given to how loss allocation policies impact fairness of market equilibria.

In this work, we argue that loss allocation policies directly impact market outcomes and prices. Therefore, carefully designing these policies could enhance fairness of market equilibria, avoiding, for instance, discrimination of agents based on their geographical location. To address this issue, we first extend the unified market formulation of decentralized electricity markets proposed in [14], where the market architecture is automatically defined by the communication links among its participants, to account for the interactions with multiple TSOs and DSOs. We model additional market products not only to enhance coordination among different SOs, but also to dynamically compute network charges of each energy trade. We then focus on three different loss allocation policies based on geographical location, on grid usage and on socialization. In line with the existing literature on transmission cost allocation [15], we extend the models of system operators with loss allocation policies and investigate their impact on fairness of market outcomes.

The remainder of the paper is structured as follows. In Section II, the optimization models of each market actors are presented and then combined into a single market clearing formulation. Section III addresses the definition of different loss allocation policies and their impact on energy trades and prices in the form of network charges. A dedicated test case and simulation results are then described in Section IV. Finally, Section V gathers conclusions and perspectives regarding further works.

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## II. MARKET CLEARING FORMULATION

We first lay down the optimization problems of each market participant: namely, prosumers, system operators (both at transmission and distribution level) and the market operator. We then consider the market clearing as an equilibrium problem, where all agents negotiate their energy procurement, while maximizing their profits. Eventually, we will show that this negotiation mechanism is a convex potential game, hence there exists a Nash equilibrium that can be found by solving the equivalent optimization problem.

Since the focus of this study is to assess the properties of the proposed market design, we find the market equilibrium by directly solving the equivalent optimization problem. Distributed optimization techniques could also be readily applied to clear the market, as largely reviewed in [16]–[18]. In this way, one would simulate an actual implementation of decentralized electricity markets, where each agent individually optimizes her energy procurement and exchange information with the others to reach consensus.

### A. Preliminaries and Notation

The scope of this study is to integrate SOs in the negotiation process of decentralized electricity markets and to investigate properties of the consequent market outcomes. This justifies the need for the following assumptions. Market actors are considered rational and truthful, thus no strategic behaviour is taken into account. Any non-convexity of their optimization problems is neglected or relaxed to convex approximations, in particular the optimal power flow equations of SOs. For clarity of notation, we present the mathematical formulation of a market with one TSO and one DSO, but the extension to multiple TSOs and DSOs is straightforward: in fact, the numerical results presented in Section IV consider multiple DSOs. We then restrict the market clearing to a single instance: no ramping and time coupling constraints are included in the model. Finally, we assume perfect communication among market actors, to guarantee that in case the market is cleared in a decentralized fashion, the exact same market equilibrium is attained.

In each formulation, we use lower and upper case symbols respectively for variables and parameters, in bold if matrices or vectors. Dual variables, which in an equilibrium problem are decision variables of the market operator and parameters for the other agents, are expressed with Greek letters. We use  $i$  as index for market participants  $\mathcal{I}$ , which can communicate and trade with a set of trading partners  $j \in \Omega_i$ . Each agent is connected to a bus of the power grid: for simplicity of notation we distinguish between TSO nodes, indexed  $n \in \mathcal{T}$ , and DSO buses,  $r, s \in \mathcal{D}$ . We identify the set of agents connected at bus  $n$  (or  $r$ ) by subscripting the set of agents  $\mathcal{I}_n$ . Additionally, for each bus at TSO level  $n$  we split the lines connected to it in two sets, AC  $k \in \mathcal{L}_n^{\text{AC}}$  and HVDC  $h \in \mathcal{L}_n^{\text{DC}}$ , while  $\mathcal{L}^{\text{TSO}}$  is the set of all lines at TSO level. Lines at DSO level are grouped in the set of the pairs of receiving and sending nodes  $d = (r, s) \in \mathcal{L}^{\text{DSO}}$ . All the lines are then grouped in the set  $\mathcal{L}$ . Finally, we define a set of nodes  $c \in \mathcal{C}$ , where DSOs and TSOs are connected.

### B. Prosumers

The objective of each prosumer  $i$  is to minimize their energy procurement costs and can be modelled as an optimization problem,

$$\min_{\Gamma_p} f_i(p_i, q_i) - \sum_{j \in \Omega_i} (\tau_{ij}^T + \tau_{ij}^Z) t_{ij} - (\tau_{ij}^Z + \tau_{ij}^L) w_{ij} - \lambda_r |_{i \in \mathcal{I}_r} q_i \quad (1a)$$

$$\text{s.t.} \quad \sum_{j \in \Omega_i} (t_{ij} + w_{ij}) = p_i \quad : \pi_i \quad (1b)$$

$$\underline{P}_i \leq p_i \leq \bar{P}_i \quad : \underline{\gamma}_i^P, \bar{\gamma}_i^P \quad (1c)$$

$$\underline{Q}_i \leq q_i \leq \bar{Q}_i \quad : \underline{\gamma}_i^Q, \bar{\gamma}_i^Q \quad (1d)$$

$$w_{ij} \geq 0, t_{ij} \in \mathbb{R} \quad (1e)$$

where  $\Gamma_p = \{p_i, q_i, \mathbf{t}_i, \mathbf{w}_i\}$  is the set of variables, with  $p_i$  the net active power of agent  $i$  (positive if generated and negative if consumed) and  $q_i$  the reactive power. The vector  $\mathbf{t}_i$  is the collection of  $t_{ij}$ , i.e., the energy traded between agent  $i$  and each of the  $j$  trading partners in  $\Omega_i$  at price  $\tau_{ij}^T$ . As in [14], by changing the communication topology among market participants, one could simulate different market structures, from pool-based to community-based and peer-to-peer markets.

Additionally, each trade impacts the power grid both in term of congestions, reflected by the price charge  $\tau_{ij}^Z$ , and by creating power losses. These losses  $w_{ij}$ , caused by the trade between agents  $i$  and  $j$ , need to be procured at price  $\tau_{ij}^Z + \tau_i^L$ , i.e. grid plus loss price, with the respective system operator. The sum of the trades balances out with the net generation and the losses as in (1b) at a perceived price  $\pi_i$ .  $\bar{P}_i$  and  $\underline{P}_i$  are respectively the maximum and minimum active power set-points while  $\underline{\gamma}_i$  and  $\bar{\gamma}_i$  are the dual variables associated with these limits. Respectively, (1d) bounds reactive power, whose grid costs are enforced by means of the reactive nodal price  $\lambda_r$ . The objective function (1a) is to minimize procurement costs, as a sum of generation costs (and consumption utility), energy and loss trades and grid costs.

### C. Transmission System Operator

We formulate the optimization problem of the Transmission System Operator (TSO) as

$$\min_{\Gamma_T} \sum_{c \in \mathcal{C}} \tau_c^E e_c^T + \sum_{n \in \mathcal{T}} \sum_{i \in \mathcal{I}_n} \tau_{ij}^Z z_{ij} + \sum_{l \in \mathcal{L}^{\text{TSO}}} \sum_{i | \mathcal{I}_n \in \mathcal{T}} \tau_{ij}^L A_{(i,j),l} w_l \quad (2a)$$

$$\text{s.t.} \quad f_k = \sum_{n \in \mathcal{T}} N_{kn} \left( \sum_{i \in \mathcal{I}_n} z_{ij} - \sum_{h \in \mathcal{L}_n^{\text{DC}}} I_{nh}^{\text{DC}} f_h - \sum_{c \in \mathcal{C}_n} e_c^T - \sum_{l \in \mathcal{L}} D_{nl} w_l \right) \quad \forall k \in \mathcal{L}^{\text{AC}} : \varphi_k \quad (2b)$$

$$-\bar{F}_l \leq f_l \leq \bar{F}_l \quad \forall l \in \mathcal{L}^{\text{TSO}} : \underline{\mu}_l, \bar{\mu}_l \quad (2c)$$

$$w_l = M_l^T |f_l| + Q_l^T \quad \forall l \in \mathcal{L}^{\text{TSO}} : \phi_l \quad (2d)$$

$$\mathbf{z}, \mathbf{e}^T \in \mathbb{R}, \mathbf{w} \geq 0 \quad (2e)$$

where  $\Gamma_T = \{\mathbf{f}, \mathbf{w}, \mathbf{z}, \mathbf{e}^T\}$  is the set of decision variables of the TSO, with  $\mathbf{f}$  the vector of line flows both in AC and DC,  $\mathbf{w}$  the vector of line losses and  $\mathbf{e}^T$  the vector of power exchanges from TSO to DSOs. The vector  $\mathbf{z}$  gathers all the injections

corresponding to each trade, including the associated losses. The objective function (2a) represents the cost of the energy exchanged with all DSOs at price  $\tau_c^E$ , the sum of the congestion rent over AC and HVDC lines and the cost of losses. The subset  $\mathcal{C}_n$  is used to identify all the DSOs connected to node  $n$ . On the one hand, the congestion rent is expressed as the sum over all nodes of all injections  $z_{ij}$  times the respective grid price  $\tau_{ij}^Z$ . On the other hand, the cost of losses is calculated as the sum over the losses of each line  $l$  at price  $\tau_{ij}^L$ .

Power flows over AC lines are derived using the *Power Transfer Distribution Factors (PTDF) matrix*, represented by the elements  $N_{kn}$  of  $\mathbf{N}$  in (2b), and the nodal injection of each trade, with  $\varphi$  being the respective dual variable. Nodal injections are calculated as the sum of the injections corresponding to each energy trade, HVDC line flows, energy exchange with DSOs and line losses. Line losses are transformed into nodal losses by allocating half of the line losses to the respective receiving and sending node,  $D_{nl} = 0.5$  if node  $n$  is connected line  $l$  and zero otherwise. The PTDF matrix identifies how power flows are distributed across the network, showing the marginal variation in the power flows due to a marginal variation in the power injections. This matrix  $\mathbf{N}$  is an  $|\mathcal{L}^{\text{TSO}}| \times |\mathcal{T}|$  matrix, with  $|\cdot|$  the cardinality of the sets, and can be calculated as

$$\mathbf{N} = \mathbf{B}_{line} \tilde{\mathbf{B}}_{bus}^{-1} \quad (3)$$

where  $\tilde{\mathbf{B}}_{bus}^{-1}$  is the inverse of the bus susceptance matrix after removing the row and the column corresponding to the slack bus, and  $\tilde{\mathbf{B}}_{line}$  the line susceptance matrix. We define AC flows by means of the PTDF matrix and not through voltage angle differences, in order to avoid multiple dual solutions. In fact, in the B-theta formulation, power flows are calculated as the difference of voltage angles: the corresponding KKTs represent an undetermined system of equations with infinite dual variable solutions. The flows over HVDC lines are included in the nodal injections. Indeed, thanks to the flexibility of HVDC lines, those flows can be arbitrarily defined by the TSO and are modeled as positive or negative injections. Constraint (2c) defines the line limits, with  $\bar{F}_l$  being the capacities of the lines, and  $\underline{\mu}_l$  and  $\bar{\mu}_l$  the respective dual variables, in other words the shadow prices of congestion.

In order to define the losses allocated to each trade, we first model line losses  $w_l$  as a linear function of the line flows  $f_l$ , constraint (2d). Loss functions are included in the form of two inequality constraints as in [19]; in this way it is possible to properly calculate losses without considering the direction of the flow (that would require the introduction of binary variables). The coefficients of the linear approximation are computed using the least squares method as in [19], where the authors propose an extension to piece-wise linear approximation that is added to this model for the test case analysis. Line losses are then allocated to market participants, connected at TSO  $i|\mathcal{I}_n \in \mathcal{T}$ , by means of allocation policies, modelled as coefficients of the matrix  $\mathbf{A}$ . We will deeper investigate different policies and their impact on market equilibria in the following sections.

#### D. Distribution System Operator

A distribution system operator participates to the negotiation mechanism with objectives similar to a TSO. Given the lower voltage level, we extend the power flow equations to include reactive power, resulting in

$$\min_{\Gamma_D} \sum_{r \in \mathcal{D}} \sum_{s \in \mathcal{D}_r} \lambda_r f_{(r,s)}^Q - \tau_c^E e_c^D + \sum_{r \in \mathcal{D}} \sum_{i \in \mathcal{I}_r} \tau_{ij}^Z z_{ij} \quad (4a)$$

$$+ \sum_{d \in \mathcal{L}^{\text{DSO}}} \sum_{i|\mathcal{I}_r \in \mathcal{D}} \tau_{ij}^L A_{(i,j),d} w_d$$

$$\text{s.t. } f_d^P = B_{rs}(\vartheta_r - \vartheta_s) - G_{rs}(v_r - v_s) \quad \forall d = (r,s) \in \mathcal{L}^{\text{DSO}} : \varphi_d^P \quad (4b)$$

$$f_d^Q = B_{rs}^*(v_r - v_s) + G_{rs}(\vartheta_r - \vartheta_s) - b_{rs}^0 \quad \forall d = (r,s) \in \mathcal{L}^{\text{DSO}} : \varphi_d^Q \quad (4c)$$

$$(f_d^P)^2 + (f_d^Q)^2 \leq (\bar{S}_d)^2 \quad \forall d \in \mathcal{L}^{\text{DSO}} : \eta_d^{\text{AC}} \quad (4d)$$

$$e_c^D = - \sum_{r \in \mathcal{D}} \sum_{i \in \mathcal{I}_r} z_{ij} + \sum_{d \in \mathcal{L}^{\text{DSO}}} w_d \quad : \eta_c^E \quad (4e)$$

$$\vartheta_{\text{ref}} = 0 \quad (4f)$$

$$\underline{\Theta}_r \leq \vartheta_r \leq \bar{\Theta}_r \quad \forall r \in \mathcal{D} : \underline{\eta}_r^\theta, \bar{\eta}_r^\theta \quad (4g)$$

$$\underline{V}_r \leq v_r \leq \bar{V}_r \quad \forall r \in \mathcal{D} : \underline{\eta}_r^V, \bar{\eta}_r^V \quad (4h)$$

$$w_d = M_d^D |f_d^P| + Q_d^D \quad \forall d \in \mathcal{L}^{\text{DSO}} : \phi_d \quad (4i)$$

$$\mathbf{f}^P, \mathbf{f}^Q, \mathbf{z}, \mathbf{e}^T \in \mathbb{R}, \mathbf{w} \geq 0 \quad (4j)$$

with  $\Gamma_D = \{\mathbf{f}^P, \mathbf{f}^Q, \mathbf{z}, \mathbf{w}, e^D, \vartheta, \mathbf{v}\}$  is the set of decision variables of the DSO, with  $\mathbf{f}^P$  and  $\mathbf{f}^Q$  respectively the line active and reactive flows,  $\mathbf{z}$  the injections corresponding to agent trades and losses,  $\mathbf{w}$  the line losses,  $e^D$  the energy flow from DSO to TSO and  $\vartheta, \mathbf{v}$  respectively the voltage angles and magnitudes. As already presented for the TSO model, the objective function (4a) represents the sum of the congestion rent (both for active and reactive flows), the costs of trades with the TSO and the cost of losses. The losses are traded between market participants, at distribution level  $i|\mathcal{I}_r \in \mathcal{D}$ , and the DSO, who enforces loss allocation policies, summarized with the matrix  $\mathbf{A}$ , further discussed in the following section.

The active and reactive power flows are derived through constraints (4b) and (4c) using a linear approximation of an AC power flow [20], where  $b_{rs}^0$  is the shunt susceptance,  $G_{rs}$  the conductance and  $B_{rs}$  the susceptance of line between  $d$  nodes  $r$  and  $s$  ( $B_{rs}^* = B_{rs} + 2b_{rs}^0$ ), with  $\varphi_d^P$  and  $\varphi_d^Q$  the associated dual variables. Line capacities are enforced by the conic constraint (4d).

Similarly to the formulation of the TSO problem, power losses in the distribution grid are calculated for each line through constraint (4i), by means of a linear approximation. The energy flow between TSO and DSO is calculated as the sum of all the trades originated at DSO level and the losses on all DSO lines, as in constraint (4e). The case of one connection node is presented above only for simplicity of indexing and notation, however several connection points can be readily considered. Finally, constraints (4h) and (4g) ensure voltage standard compliance at each node  $r$ , with  $\underline{V}_r, \bar{V}_r, \underline{\Theta}_r$  and  $\bar{\Theta}_r$  being the maximum deviation of magnitudes and angles, and  $\underline{\eta}_r^V, \bar{\eta}_r^V, \underline{\eta}_r^\theta, \bar{\eta}_r^\theta$  the corresponding dual variables.

### E. Market Operator

The market constraints ensure trade reciprocity of energy and losses, as well the allocation policies for losses and power balances at TSO and DSO level. The respective optimization problem becomes

$$\begin{aligned} \min_{\Gamma_M} \quad & \sum_{i \in \mathcal{I}} \sum_{j \in \Omega_i} \left[ - (t_{ij} + t_{ji}) \tau_{ij}^T - (t_{ij} + w_{ij} - z_{ij}) \tau_{ij}^Z \right. \\ & \left. - \left( w_{ij} - \sum_{l \in \mathcal{L}} A_{(i,j),l} w_l \tau_{ij}^L \right) \tau_{ij}^L \right] + \sum_{c \in \mathcal{C}} (e_c^T - e_c^D) \tau_c^E \\ & + \sum_{r \in \mathcal{D}} \sum_{s \in \mathcal{D}_r} (f_{rs}^Q - \sum_{i \in \mathcal{I}_r} q_i) \lambda_r \end{aligned} \quad (5a)$$

$$\text{s.t. } \boldsymbol{\tau}^T, \boldsymbol{\tau}^Z, \boldsymbol{\tau}^E, \boldsymbol{\tau}^L, \boldsymbol{\lambda} \in \mathbb{R} \quad (5b)$$

where  $\Gamma_M = \{\boldsymbol{\tau}^T, \boldsymbol{\tau}^Z, \boldsymbol{\tau}^E, \boldsymbol{\tau}^L, \boldsymbol{\lambda}\}$  is the set of decision variables of the market operator, indicating respectively the prices of the energy trades among market participants, of grid usage for each injection caused by the energy trades, of energy exchange between TSO and DSOs, of losses and the nodal prices for reactive power. The objective function of the market operator (5a) is to enforce shared constraints by setting all the mentioned prices.

### F. Equivalent Optimization Problem

The optimization problems of all market players are convex problems, as long as the cost or utility function  $f_i$  of each prosumer  $i$  is convex. Note that a usual formulation of a DC power flow via the PTDF matrix involves employing the sum of agent injections  $p_i$  in constraint (2b). This, however, results in a generalized Nash equilibrium, since prosumer actions limit the feasible space of the TSO. By introducing the additional variables  $z$ , no variable of a market player appears in the constraints of other agents, refining the generalized Nash equilibrium to a Nash equilibrium. It is straightforward to show that the Jacobian of the game map is symmetric, hence the game map is integrable (Theorem 1.3.1 in [21]). Therefore, a Nash equilibrium exists and we can write the following equivalent optimization problem,

$$\min_{\Gamma_p, \Gamma_T, \Gamma_D} \sum_{i \in \mathcal{I}} f_i(p_i, q_i) \quad (6a)$$

$$\text{s.t. } (1b) - (1e) \quad (6b)$$

$$(2b) - (2e) \quad (6c)$$

$$(4b) - (4j) \quad (6d)$$

$$t_{ij} = t_{ji} \quad \forall i \in \mathcal{I}, \forall j \in \Omega_i : \tau_{ij}^T \quad (6e)$$

$$t_{ij} + w_{ji} = z_{ij} \quad \forall i \in \mathcal{I}, \forall j \in \Omega_i : \tau_{ij}^Z \quad (6f)$$

$$w_{ij} = \sum_{l \in \mathcal{L}} A_{(i,j),l} w_l \quad \forall i \in \mathcal{I}, \forall j \in \Omega_i : \tau_{ij}^L \quad (6g)$$

$$e_c^T = e_c^D \quad \forall c \in \mathcal{C} : \tau_c^E \quad (6h)$$

$$\sum_{s \in \mathcal{D}_r} f_{rs}^Q - \sum_{i \in \mathcal{I}_r} q_i = 0 \quad \forall r \in \mathcal{D} : \lambda_r^Q \quad (6i)$$

Note that (6) can be solved by means of decomposition techniques, where market actors trade energy and losses by seeking consensus on constraints (6e) - (6i).

## III. LOSS ALLOCATION POLICIES

When including losses in decentralized electricity markets, fairness of agent payments may be undermined. To verify this, we analyze the price formation mechanism and the consequent effect of different loss allocation policies.

### A. Price Formation

Trade prices, for both energy and losses, can be extracted as dual variables of (6e) - (6i) or can be analytically derived by computing the first-order conditions of (6). We hereby presents only the most meaningful among all these equations to understand the impact of grid constraints and losses on trade prices. These are

$$p_i : \frac{\partial f_i}{\partial p_i} - \pi_i + \bar{\gamma}_i^p - \underline{\gamma}_i^p = 0 \quad (7a)$$

$$t_{ij} : \pi_i - \tau_{ij}^T - \tau_{ij}^Z = 0 \quad (7b)$$

$$w_{ij} : \pi_i - \tau_{ij}^Z - \tau_{ij}^L = 0 \quad (7c)$$

$$z_{ij} : \tau_{ij}^Z - \sum_{k \in \mathcal{L}} \text{PTDF}_{kn|n \in \mathcal{I}_n \cup \mathcal{C}} \varphi_k = 0 \quad (7d)$$

$$w_l : \sum_{i \in \mathcal{I}} A_{(i,j),l} \tau_i^L + \phi_l + \sum_{k \in \mathcal{L}} \text{PTDF}_{kn} D_{nl} \varphi_k = 0 \quad (7e)$$

From (7a), we can identify  $\pi_i$  as the price perceived by agent  $i$ , driving her dispatch decision, limited by asset capacities. From (7b) and (7c), it follows that the perceived price equals each trade price  $\tau_{ij}^T$ , or equivalently the loss price  $\tau_{ij}^L$ , adjusted by  $\tau_{ij}^Z$ , including the impact of all grid constraints on the market equilibrium as in (7d) (it is intuitive that a price difference happens only if some of these constraints are binding). This implies that energy and losses are traded at the same grid-aware price. Therefore, the allocation policies, summarized by  $A_{(i,j),l}$ , do not only impact the definition of the loss price of each trade, as in (7e), but also the energy price of each trade. Hence, loss allocation policies should be carefully designed in order to grant fairness of market participation. Since the concept of fairness can differ among system operators, we propose two policies, that can be independently employed by different SOs or even combined to obtain a milder strategy.

### B. Loss Allocation Policies

We summarize the allocation policy coefficients in a matrix  $\mathbf{A}$  with  $2T$  rows, with  $T$  the number of trades (we define a coefficient for the trade  $(i, j)$  and one for the trade  $(j, i)$ ) and with  $L$  columns, with  $L$  the number of lines. We propose two different allocation policies: one where each System Operator (SO) socializes the losses in its lines ( $\mathbf{A}^{\text{SOC}}$ ) and one reflecting grid usage of individual trades ( $\mathbf{A}^{\text{IND}}$ ). We define the socialization policy coefficients as

$$A_{(i,j),l}^{\text{SOC}} = \begin{cases} \frac{1}{2T^{\text{SO}}} & \text{if } i \in \mathcal{I}^{\text{SO}}, l \in \mathcal{L}^{\text{SO}} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

with  $T^{\text{SO}}$  the number of trades of the SO, that agent  $i$  belongs to. Each agent is equally allocated the losses occurring on the lines of the respective SO. This policy erases geographical

discrimination of market participants, but at the same time trades are not affected by their individual impact on the grid.

The second proposed policy attains the opposite effect. By allocating to each trade the losses produced by the flows it generates, this policy discriminates agents based on their geographical location. The coefficients of this policy are defined as

$$A_{(i,j),l}^{\text{IND}} = \frac{|\text{TF}_{ln|i \in \mathcal{I}_n} - \text{TF}_{lm|j \in \mathcal{I}_m}|}{\sum_{i,j \in \mathcal{I}} |\text{TF}_{ln|i \in \mathcal{I}_n} - \text{TF}_{lm|j \in \mathcal{I}_m}|} \quad (9)$$

where  $\text{TF}_{ln|i \in \mathcal{I}_n}$  is a modified version of the PTDF matrix for line  $l$  and node  $n$  where agent  $i$  is connected. The PTDF matrix is modified to include DSO nodes and lines. This is done by defining the PTDF matrix as in (3) for each block of nodes and lines of each SO. Then each node at DSO has the same values for the TSO lines as the node of connection between TSO and DSO. To compute the allocation policy coefficients for trade  $(i,j)$ , we employ the difference of the modified PTDF at the respective nodes in absolute value, to avoid negative values while accounting for grid usage at the same time. Finally, the values are normalized to guarantee that all the coefficients for each line sum to 1.

Ideally, this individual allocation policy should account for the amount of energy traded, to allocate more losses to larger trades and vice-versa. However, this would make appear bilinear terms in the optimization problem of the SOs, which is highly non-practical. Therefore, we propose to account for the size of the market agents, by adjusting the allocation policy coefficients by the agent capacities ( $K_i$ ). The policy in (9) then becomes

$$A_{(i,j),l}^{\text{IND}} = \frac{|\text{TF}_{ln|i \in \mathcal{I}_n} - \text{TF}_{lm|j \in \mathcal{I}_m}|}{\sum_{i,j \in \mathcal{I}} |\text{TF}_{ln|i \in \mathcal{I}_n} - \text{TF}_{lm|j \in \mathcal{I}_m}|} \frac{K_i}{\sum_{i \in \mathcal{I}} K_i} \quad (10)$$

where

$$K_i = \max\{|P_i|, |\bar{P}_i|\} \quad (11)$$

Arguments for fairness and unfairness of both policies exist. On the one hand, a socialization policy equally distributes total loss costs across market participants without discrimination on geographical location. At the same time, agents are not incentivized to trade locally, as they are charged the same no matter the distance with their trading partners. On the other hand, an individual policy makes sure that each agent pays for the losses she causes in the system, incentivizing local trading. However, this policy may discriminate, for instance, one agent at the end of a DSO feeder that gets allocated the losses of all the lines to reach the feeder, compared to a market participant close to the connection with the TSO. Therefore, we propose a linear combination of the two strategies,

$$A_{(i,j),l} = \chi A_{(i,j),l}^{\text{SOC}} + (1 - \chi) A_{(i,j),l}^{\text{IND}} \quad (12)$$

with  $\chi$  a socialization factor, to find a trade-off between the advantages and caveats of both policies. One should notice that different SOs can have different allocation policies. In fact in the following section, we investigate a test case with both TSO and DSOs over different combination of allocation policies to show how they impact market outcomes and payments.

### C. Illustrative Example

We hereby propose a small test case to provide in-depth insight on the impacts of different loss allocation policies. We consider a 5 bus system, including both a meshed (TSO) and a radial (DSO) network, with only one generator and three loads, as pictured in Figure 1. On the one hand, a consumption unit (agent 2) belongs to the TSO grid and is connected to the generator by long lines. On the other hand, agents 3 and 4 are connected at DSO level, respectively close to and far from the feeder.

We simulate this illustrative example with 3 different loss allocation policies: socialization ( $\chi = 1$ ), individual ( $\chi = 0$  w/out  $K$ ) and individual scaled by capacities ( $\chi = 0$  w/  $K$ ). We then investigate the impact of different policies in terms of payment increase in absolute ( $\Delta C_i$ ) and percentage ( $\Delta \% C_i$ ) values,

$$\Delta C_i = C_i - C_i^0 \quad (13)$$

$$\Delta \% C_i = \frac{C_i - C_i^0}{|C_i^0|} \quad (14)$$

with  $C_i = \sum_{j \in \omega_i} t_{ij} (\tau_{ij}^T + \tau_{ij}^Z)$  the payments of agent  $i$ . We use the superscript 0 to identify the reference case, where losses are not considered while clearing the market.

Figure 2 shows that the socialization policy achieves an equal split of losses across market participants. One should note that the revenues of the generator (agent 1) increase, since, in this small case, it is the only generator, hence the marginal one providing the extra power to compensate for the losses that are not considered in the reference case. However, when looking at the results for the same policy in percentage terms, it is clear that the smaller agents (3 and 4) are strongly discriminated. On the other hand, the individual policy allocates larger payments to agents 2 and 4, corresponding to the units further away from the generator. In percentage terms,

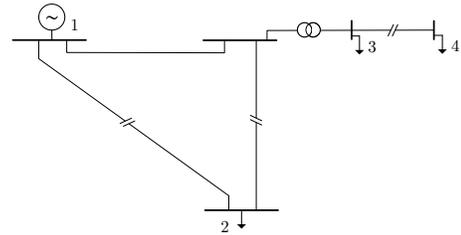


Fig. 1. Illustrative 5-bus test case, with a TSO and a DSO.

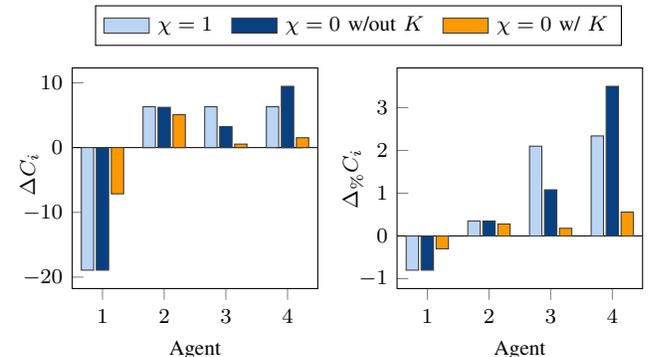


Fig. 2. Increase of payments in absolute (left) and percentage (right) values.

however, agent 3 is still discriminated over agent 2 because of her size. When including the capacity proportionality to the individual allocation policy, the payments become more uniform across agents in percentage terms. As for the actual payments, they also decrease in magnitude as the marginal generator, having a larger capacity than the others, gets more losses allocated reducing the need for the loads to procure power.

#### IV. NUMERICAL RESULTS

This section evaluates the impact of grid constraints and the performance of the different loss allocation policies on a test case, including a TSO network with several connected DSOs. First, the test case is described. Then, the numerical results are investigated to compare market equilibria with and without grid constraints, followed by an analysis on the impact of different loss allocation policies on market outcomes and prices.

##### A. Test Case Description

We simulate our market formulation on a modified RTS '96 IEEE test system presented in [22]. Not being the focus of this paper, the overall synchronicity of the test case is guaranteed by replacing the HVDC line between nodes 123 and 323 by an AC line of the same capacity. To match the formulation of this paper, wind farms are considered as prosumers such that  $\underline{P}_i$  and  $\overline{P}_i$  are equal to the power production. Focusing on the analysis of the proposed loss allocation policies, the results presented in this paper are obtained for a single time step, September 2<sup>nd</sup> of the original dataset. Finally, 12 energy communities are considered at DSO level, whose grid parameters are based on the IEEE 33-bus distribution network<sup>1</sup> and are connected to transmission nodes 111, 112, 120, 211, 214, 303, 311, 312, 411, 412 and 414. DSO agents are considered as prosumers with lower bound of their power asset  $\underline{P}$  corresponding to the power demand of the IEEE 33-bus distribution network (with a change of the sign convention), while their upper bounds  $\overline{P}$  are computed by randomly sampling the difference  $\overline{P} - \underline{P}$  from a uniform distribution  $\mathcal{U}(0,1)$ . Similarly, cost curves are sampled uniformly between 10 and 50 €/MWh. The final test case as well as the code used to run the simulations are available on GitLab<sup>2</sup>.

##### B. Impact of Grid Constraints

We first investigate the impact of power flow constraints on the feasibility of market outcomes, by simulating the market clearing (6) with and without including grid constraints. Power losses and their allocation policies are not yet considered, to only analyze the effect of the network on the market equilibrium. The results of our simulations verify that all power grid variables respect their limits, once included in the model. For instance, Figure 3 shows the loading of transmission

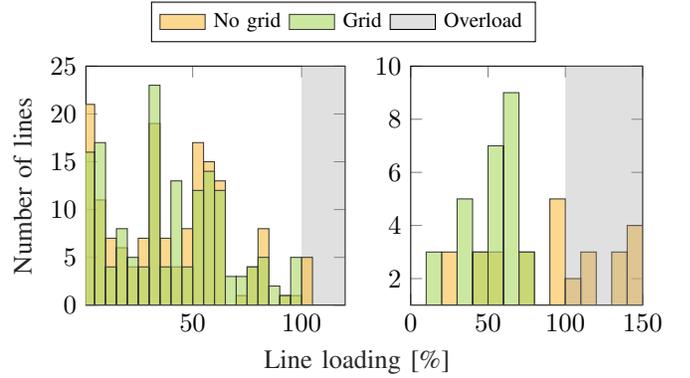


Fig. 3. Line loading at TSO (left) and DSO (right) level.

and distribution lines for a market clearing with and without grid constraints. The results clearly show violation of line capacities for both system operators, if grid constraints are not included in the negotiation process, in particular at DSO level where reactive flows contribute to the overload of lines. Optimizing energy trades while accounting for grid feasibility become fundamental in view of an even larger penetration of DERs, causing considerable reversed (from DSO to TSO) and reactive power flows. By including grid constraints in the market clearing, system operators will reduce the need for grid reinforcement and reactive compensation measures.

From (7), we extracted how grid constraints modify trade prices to account for power flow feasibility, e.g., line and voltage limits. Similarly to a market based on locational marginal pricing, one line congestion impacts the price of each bus. Note that agents connected at the same bus trade at the same price if no economic preference is included. Figure 4 displays the percentage increase, compared to the market clearing without grid constraints, of the price of agents at each TSO bus. For clarity of representation, DSOs (represented with a squared marker) are grouped at their connection node

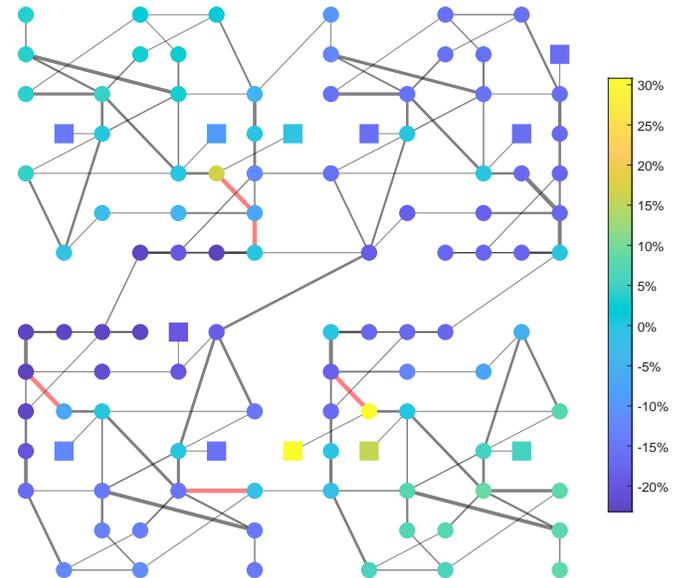


Fig. 4. Price percentage increase on the TSO network when including grid constraints.

<sup>1</sup>Line capacities of the distribution systems are based on <https://portfolio.du.edu/downloadItem/358246>

<sup>2</sup>[https://gitlab.com/fmoret/p2p\\_loss](https://gitlab.com/fmoret/p2p_loss)

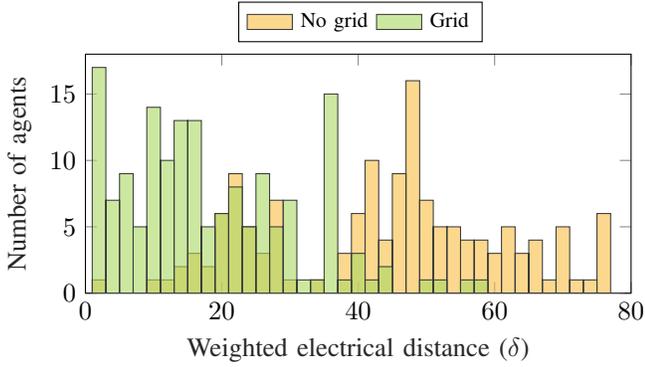


Fig. 5. Weighted average electric distance of trades per TSO prosumer.

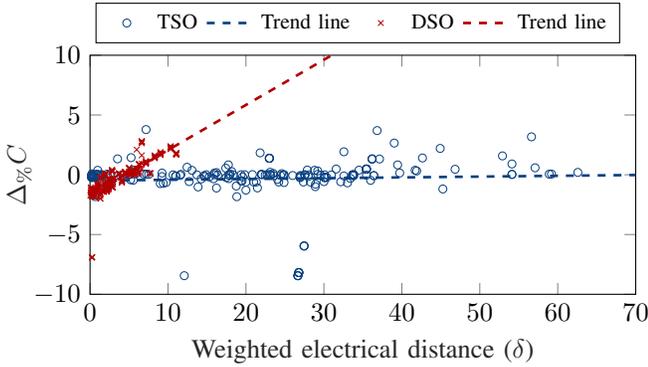


Fig. 6. Price percentage increase as function of the weighted electrical distance. Trend lines (dashed) are calculated without considering outliers.

to the TSO grid. The width of the connecting edges is plotted proportionally to the line flows, with congested lines highlighted in red. The average price per agent decreases from 15.6 €/MWh (the uniform price of the market clearing without grid constraints) to 14.5 €/MWh. However, larger prices, with a peak of 20.4 €/MWh, appear as a consequence of congested lines. Visually, it appears that prices are distributed in high and low price zones: in fact, including grid constraints in the market clearing provides incentives to locally match generation and consumption.

To this extent, Figure 5 shows how the average trading distance of TSO prosumers is largely reduced both in terms of its mean and standard deviation. DSO agents are not considered, since, in this specific market architecture, they only trade with the respective community manager located at the feeder, hence their distance is not impacted. We calculate the average trading distance  $\delta_i$  of each agent  $i$  as the PTDF-based electric distance between two trading agents weighted by the energy traded. It follows

$$\delta_i = \frac{\sum_{j \in \Omega_i} \sum_{l \in \mathcal{L}} |\text{PTDF}_{li} - \text{PTDF}_{lj}| r_l t_{ij}}{\sum_{j \in \Omega_i} t_{ij}} \quad (15)$$

with  $r_l$  the resistance of each line  $l$ . As investigated in [23], the number of trades of each market participant increases the complexity of the market clearing problem, especially if solved by means of distributed optimization techniques. The natural effect of the power grid to incentivize local trades presented here could be employed to increase the sparsity of the communication matrix among agents.

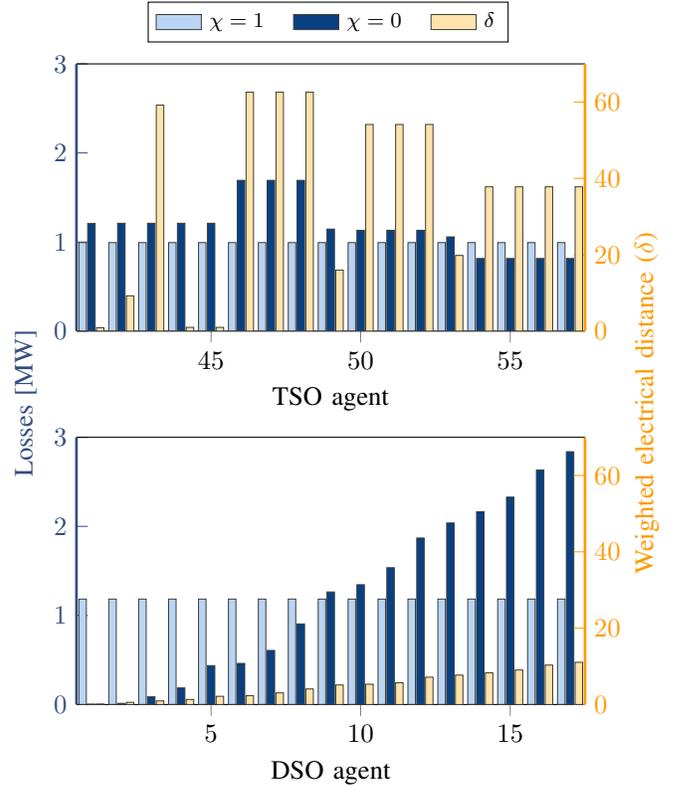


Fig. 7. Losses allocated per agent and weighted electrical distance.

### C. Impact of Loss Allocation Policies

After investigating the impact of grid constraints on market outcomes, we focus our analysis on the different loss allocation policies proposed. We first compare the effects of a socialization (8) and an individual (9) policy. Figure 6 displays the percentage increase of payments as a function of the weighted electrical distance, defined in (15), of both TSO and DSO agents. We use the socialization policy as a reference, where each trade is allocated an equal share of the respective SO losses, and plot the cost increase when an individual policy is employed. It is clear that market participants that engage in distant trades are penalized by larger losses, and consequently payments. This trend is stronger at DSO rather than at TSO level, since agents at the end of a radial grid are naturally further away from the feeder, while it is attenuated in a meshed grid where power flows through several parallel paths.

This can be clearly seen from Figure 7, where losses and electrical distance are displayed for a sample of the simulated agents. While a socialization policy leads to equal losses across all SO agents, an individual policy induces large disparity at DSO level as function of their electrical distance from the feeder. On the other hand, market participants at TSO level are indeed impacted by their trading electrical distance, but the variation is limited thanks to the meshed layout of the power grid. Therefore, we argue that a socialization policy is preferred at DSO level to avoid discriminating agents for their unintentional geographical location, while at TSO level an individual policy can still be used, charging more for trades with higher grid usage, but without creating unfairly large payment differences.

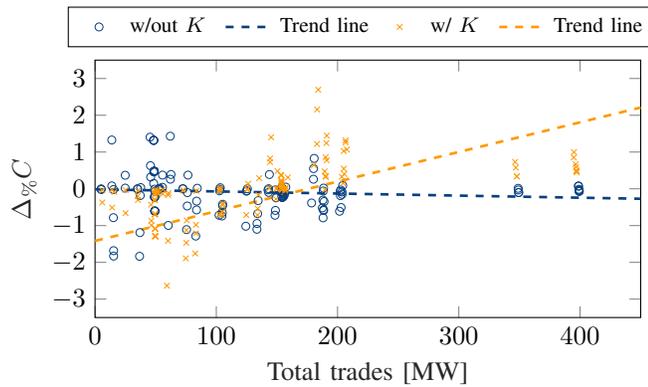


Fig. 8. Price percentage increase as function of the total amount of trades. Trend lines (dashed) are calculated without considering outliers

As pointed out in Section III, an ideal individual policy should use the actual amount of trades to quantify losses, but this would introduce bilinear terms, mining the convexity of the overall problem. We propose to consider the capacity of market participants as a proxy of their trades. Figure 8 shows the impact of considering agent capacities in an individual allocation policy, as in (10), by displaying the percentage payments increase (with the socialization policy as a reference) as function of the overall energy traded by each agent. The clear linear trend obtained when including agent capacities in the allocation policy is evidence that we can approximate the objective of allocating more losses to agents that trade more energy, while keeping the optimization problem convex.

## V. CONCLUSION

In view of decentralized electricity markets, the role of system operators is fundamental to guarantee feasibility in power system operation. In this work, we tackled the coordination of system operators with agents participating in peer-to-peer electricity markets and with other system operators, both at transmission and distribution level. We showed that by including system operators in the negotiation process grid constraints are respected and trade prices are affected by network charges. These additional tariffs are dynamically calculated as an outcome of the negotiation mechanism, reflecting the actual grid usage of each trade and providing incentives to local match of generation and consumption. We then included power losses as an additional market product that system operators trade for compensating transmission and distribution costs. We finally argued that loss allocation policies are needed to guarantee fairness among market participants, that could be discriminated for their geographical location otherwise.

Including system operator in the negotiation mechanism, based on decomposition techniques for large optimization problems, drastically increases the complexity of the algorithm. Even if it is still possible to find the optimal consensus among all these market actors, the time needed to attain it might increase to the point of making such mechanism not suitable for real life implementations. Therefore, this work should be considered as an ideal benchmark for the design of decentralized electricity markets. For instance, it could be used for exploring new opportunities for a decentralized management of the power systems, as well as for better

defining *ex-ante* network charges that approximate the results proposed in this work. Finally, further research should aim for designing negotiation processes with guaranteed trade-off between optimality of market equilibria and algorithmic complexity. Computationally lighter mechanisms may lead to market inefficiency, but this might be the price to pay for real-life implementation of such complex market mechanisms.

## REFERENCES

- [1] D. Selloni, *CoDesign for Public-Interest Services*. Springer, 2017.
- [2] Y. Parag and B. K. Sovacool, "Electricity market design for the prosumer era," *Nature energy*, vol. 1, p. 16032, 2016.
- [3] F. F. Wu and P. Varaiya, "Coordinated multilateral trades for electric power networks: theory and implementation," *International Journal of Electrical Power & Energy Systems*, vol. 21, no. 2, pp. 75–102, 1999.
- [4] C. Giotitsas, A. Pazaitis, and V. Kostakis, "A peer-to-peer approach to energy production," *Technology in Society*, vol. 42, pp. 28–38, 2015.
- [5] F. Moret and P. Pinson, "Energy Collectives: a Community and Fairness based Approach to Future Electricity Markets," *IEEE Transactions on Power Systems*, vol. 34, no. 5, pp. 3994–4004, 2019.
- [6] T. Morstyn, A. Teytelboym, and M. D. McCulloch, "Bilateral Contract Networks for Peer-to-Peer Energy Trading," *IEEE Transactions on Smart Grid*, vol. 10, no. 2, pp. 2026–2035, 2019.
- [7] J. Qin, R. Rajagopal, and P. P. Varaiya, "Flexible Market for Smart Grid: Coordinated Trading of Contingent Contracts," *IEEE Transactions on Control of Network Systems*, vol. 5, no. 4, pp. 1657–1667, 2018.
- [8] T. Morstyn and M. D. McCulloch, "Multiclass energy management for peer-to-peer energy trading driven by prosumer preferences," *IEEE Transactions on Power Systems*, vol. 34, no. 5, pp. 4005–4014, 2019.
- [9] T. Morstyn, A. Teytelboym, and M. D. McCulloch, "Designing Decentralized Markets for Distribution System Flexibility," *IEEE Transactions on Power Systems*, vol. 34, no. 3, pp. 2128–2139, 2019.
- [10] T. Baroche, P. Pinson, R. L. G. Latimier, and H. B. Ahmed, "Exogenous Cost Allocation in Peer-to-Peer Electricity Markets," *IEEE Transactions on Power Systems*, vol. 34, no. 4, pp. 2553–2564, 2019.
- [11] A. Papavasiliou and I. Mezghani, "Coordination Schemes for the Integration of Transmission and Distribution System Operations," in *2018 Power Systems Computation Conference (PSCC)*, 2018, pp. 1–7.
- [12] H. Le Cadre, I. Mezghani, and A. Papavasiliou, "A game-theoretic analysis of transmission-distribution system operator coordination," *European Journal of Operational Research*, vol. 274, no. 1, pp. 317–339, 2019.
- [13] J. Kim and Y. Dvorkin, "A P2P-dominant distribution system architecture," *IEEE Transactions on Power Systems*, pp. 1–1, 2019.
- [14] T. Baroche, F. Moret, and P. Pinson, "Prosumer Markets: A Unified Formulation," in *IEEE PowerTech Conference*, Milano, Italy, 2019, pp. 1–6.
- [15] F. D. Galiana, A. J. Conejo, and H. A. Gil, "Transmission network cost allocation based on equivalent bilateral exchanges," *IEEE Transactions on Power Systems*, vol. 18, no. 4, pp. 1425–1431, 2003.
- [16] D. K. Molzahn, F. Drfler *et al.*, "A survey of distributed optimization and control algorithms for electric power systems," *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 2941–2962, 2017.
- [17] A. Kargarian, J. Mohammadi *et al.*, "Toward Distributed/Decentralized DC Optimal Power Flow Implementation in Future Electric Power Systems," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 2574–2594, 2018.
- [18] G. Hug, S. Kar, and C. Wu, "Consensus + Innovations Approach for Distributed Multiagent Coordination in a Microgrid," *IEEE Transactions on Smart Grid*, vol. 6, no. 4, pp. 1893–1903, 2015.
- [19] A. Tosatto, T. Weckesser, and S. Chatzivasileiadis, "Market Integration of HVDC Lines: Internalizing HVDC Losses in Market Clearing," *IEEE Transactions on Power Systems*, vol. 35, no. 1, pp. 451–461, 2020.
- [20] Z. Yang, K. Xie *et al.*, "A General Formulation of Linear Power Flow Models: Basic Theory and Error Analysis," *IEEE Transactions on Power Systems*, vol. 34, no. 2, pp. 1315–1324, 2019.
- [21] F. Facchinei and J.-S. Pang, *Finite-dimensional variational inequalities and complementarity problems*. Springer Science & Business Media, 2007.
- [22] A. Tosatto, T. Weckesser, and S. Chatzivasileiadis, "A modified version of the IEEE 3-area RTS '96 test case for time series analysis," 2019.
- [23] F. Moret, T. Baroche, E. Sorin, and P. Pinson, "Negotiation Algorithms for Peer-to-Peer Electricity Markets: Computational Properties," in *2018 Power Systems Computation Conference (PSCC)*, 2018, pp. 1–7.