

**ARTICLE TYPE**

# Optimal trading strategies for wind power producers in futures and short-term electricity markets

Athanasios Papakonstantinou\*<sup>1</sup> | Georgia Champeri | Stefanos Delikaraoglou<sup>2</sup> | Pierre Pinson<sup>1</sup>

<sup>1</sup>Department of Electrical Engineering, Technical University of Denmark (DTU), 2800 Kgs. Lyngby, Denmark

<sup>2</sup>EEH-Power Systems Laboratory, Swiss Federal Institute of Technology (ETH), Zurich 8092, Switzerland

**Correspondence**

\*Email: athpapa@dtu.dk

**Summary**

Wind power producers participating in today's electricity markets face significant variability in revenue streams, with potential high losses mostly due to wind's limited predictability and the intermittent nature of the generated electricity. In order to further expand wind power generation despite such challenges it is important to maximise its market value, and move decisively towards economically sustainable and financially viable asset management. In this paper, we introduce a decision-making framework based on stochastic optimisation, that allows wind power producers to hedge their position in the market by trading electricity related derivatives in futures markets in conjunction with their participation in short-term electricity markets. The proposed framework relies on a series of two-stage stochastic optimisation models that identify a combined trading strategy for wind power producers participating in both financial and short-term electricity markets. The proposed models can incorporate different preferences of risk aversion, enabling a trade-off between the expected value and the variability of profits. Empirical analysis based on data from the Nordic region illustrates high efficiency of the stochastic model and reveals increased revenues for both risk neutral and risk averse wind producers opting for combined strategies.

**KEYWORDS:**

Electricity markets, futures markets, options, uncertainty, multi-stage stochastic optimisation.

## 1 | INTRODUCTION

During the past decade wind energy has expanded significantly its share in power systems, to the degree where it can meet consumption for several consecutive days. However, electricity markets were designed with a focus on fuel based conventional generation with minimum exposure in renewable energy sources (RES). This, in turn makes markets and those that participate in them, vulnerable to the limited predictability and increased

variability associated with wind energy<sup>1</sup>. Such exposure can often bring price volatility and increase the cost of regulation due to imbalances between market schedule and realised production; both significant challenges for wind energy's economic sustainability and further expansion.

It is therefore imperative to consider how such challenges can be addressed so that wind energy's full potential is achieved. In the context of the liberalised electricity sector, participating in the futures markets is a viable option that can mitigate exposure to financial risks linked with uncertainty both in generation and consumption. Like with conventional commodities, energy can also be traded in futures markets at a fixed price determined before the settlement of short-term electricity trading floors, i.e. day-ahead, intraday or balancing markets. Specifically, Deng and Oren<sup>2</sup> discuss how several types of financial instruments (e.g. forward contracts, futures and swaps) can be used by generators, load serving entities (utilities) and system operators to mitigate their exposure in market price risks. In the same direction, further integration of options in contracts for demand-side management is proposed in<sup>3</sup>.

Towards a more formal approach that relates to fundamental methods from finance literature, Oren<sup>3</sup> applies the Black-Scholes model by replacing stock prices with the forward market clearing under the assumption of zero dividend (i.e. a zero-risk interest return on the stock). The Black-Scholes model<sup>1</sup> is an analytic tool that can determine theoretical prices of specific derivatives over time given the stock price, the associated dividend and a stock volatility measure, but despite its dominance in finance literature, there is not a consensus on how it can be applied on electricity markets. Lane et al.<sup>5</sup> identify a limitation of the Black-Scholes model as it assumes a log-normal distribution of electricity prices. Instead, they propose an agent-based approach using genetic algorithms to simulate the market participants' valuations as they determine whether to buy or sell an option. Similarly, Bhanot<sup>6</sup> simulates option prices based on estimates of electricity market prices to demonstrate the impact of consumption uncertainty on electricity markets, using models of Brownian motion to estimate market prices.

Although the aforementioned approaches clearly show that derivatives can be used to contain exposure in price volatility and hedge against uncertainty in electricity prices, they do not identify the steps that a market participant can follow to achieve the discussed positive results. Liu and Wu<sup>7</sup> move towards this direction by introducing a risk management decision framework for conventional generators participating in multiple trading floors (i.e. day-ahead and real-time) of an electricity market. Pineda and Conejo<sup>8,9</sup> further contribute in this line of work by looking at how conventional generators determine an optimal portfolio of forward contracts and options while considering specific operational constraints such as the availability of production units. They propose a stochastic framework which takes into account three sources of uncertainty: derivatives and day-ahead market clearing prices, and the probability of failure for the production units. Despite the significance of such contributions, they overlook stochastic generation by restricting their analysis to conventional generators. Designing a trading framework to assist wind power producers in their decision making poses a challenging problem as it introduces new sources of uncertainty and additional market layers i.e. real-time balancing markets.

Against this background, the contribution of this paper lies in formulating a trading framework based on stochastic optimisation that determines the optimal trading strategy for wind power producers with varying risk sensitivity, actively trading options in addition to participating in a day-ahead electricity market. This paper extends the state of the art by accounting for more dimensions of uncertainty and considering a realistic

---

<sup>1</sup>An analysis of the Black-Scholes model exceeds the scope of this paper; as such, further information can be found in finance related literature cf. <sup>4</sup>.

representation of an electricity market that consists of multiple trading floors i.e. futures markets, day-ahead scheduling and real-time balancing. Modelling this complex decision-making process would naturally require a multi-stage optimisation framework that could be practically intractable. To address this challenge we decompose a typically multi-stage stochastic optimisation problem to two two-stage stochastic problems. The proposed formulation provides a computationally tractable model, while it establishes an implicit link between trading electricity through derivatives in futures markets and day-ahead scheduling and physical delivery of electricity. An empirical analysis of the proposed framework for a realistic test case utilising historical data of options, electricity market prices and wind power generation for the Nordic region, demonstrates the efficiency of the stochastic model and illustrates the advantage of combined trading in futures and short-term electricity markets, as opposed to only participating in the day-ahead market. The numerical evaluation concludes with an analysis of a risk averse producer's exposure to different levels of risk which reinforces trading options as a risk diminishing strategy that limits exposure to regulation costs by favouring futures to day-ahead markets.

The rest of the paper is organised as follows. In Section 2 we introduce the basic concepts from futures and short-term electricity markets and outline the decision-making process of a wind power producer actively participating in these markets. In Section 3 we identify the sources of uncertainty involved in our problem and outline the process to formulate statistical scenarios compatible with the stochastic optimisation problems also introduced in this section. These stochastic optimisation problems are numerically evaluated in Section 4, showing how the proposed framework can be used for wind power producers operating in the Nordic region. Finally, we conclude in Section 5.

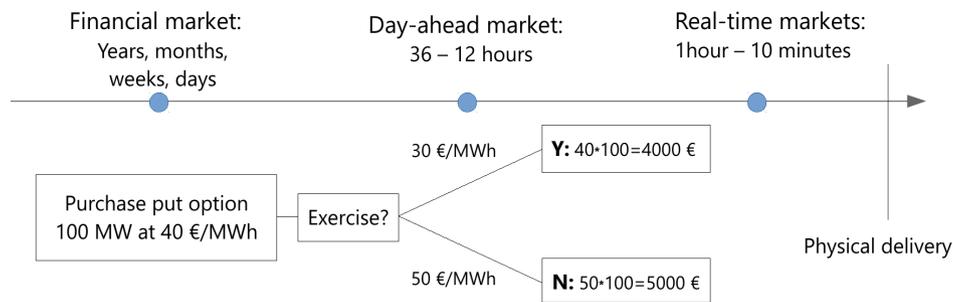
## 2 | THE MARKET FRAMEWORK

This section introduces the decision-making process for a wind power producer participating in both futures and short-term electricity markets. Prior to that, we relay the basic principles behind the financial products and describe a typical market setup that consists of a day-ahead and a real-time trading floor.

### 2.1 | Derivatives based on electricity

Derivatives are financial contracts that draw their value from the value of an underlying asset such as a commodity or even another financial instrument. Having electricity as the underlying asset can lead to derivatives that provide hedging opportunities and can reduce the risk of exposure to balancing costs often associated with renewable energy. There are several types of derivatives that offer various advantages and can mitigate different types of risk depending on the terms of the contract.

In this context, a forward contract is an agreement to buy or sell a specific amount of electricity at a certain price at an arranged date in the future known as *expiration* or *maturity* date. Any transaction involves two parties, with the party selling the electricity assuming a *short* position and the party buying it assuming a *long* position. A producer may accept a contract at a price that is higher than the one expected at the electricity market several days, weeks, months or even years ahead and profit by exercising that contract instead of participating in the actual market. Still, under the terms of the agreement, it is mandatory for the short party to sell the agreed quantity, even if it is not possible to produce it. This can



**FIGURE 1** Timeline for a producer participating in multiple trading floors.

expose producers to additional risks, which however that can be addressed by introducing more complex financial instruments such as options. As opposed to forward contracts, options are *non-mandatory* agreements that allow their holders to either buy (i.e. *call* options) or sell (i.e. *put* options) a specific amount of energy at the expiration date for a fixed price known as *exercise* or *strike* price. Both call and put options have long and short sides depending on whether they are bought or sold.

## 2.2 | The electricity markets

Electricity markets are more complex than the commodity markets that finance literature refers as *spot* markets. The real-time component and the very nature of electricity differentiate commodity markets from electricity markets. They typically consist of trading floors sequentially cleared at different time horizons. The largest volumes of electricity are traded today in day-ahead markets, cleared 12 to 36 hours before the physical delivery of electricity. Producers and consumers submit offers in the form of price-quantity bids. Once the market is cleared, the operator announces the day-ahead price and an initial generation schedule. Wind power producers comply to the same rules as the conventional ones, only in their case, actual production may deviate from day-ahead schedule given that their offers are based on forecasts.

Naturally, as shares of renewable energy increase, so do deviations between day-ahead scheduling and real-time dispatch. In practice though, forecast errors in estimating generation from renewable sources are not the sole source of deviations. For example, there can be outages in generation units and transmission lines, or network congestion issues that were not considered a day before they occur. Irrespective of the deviations' source, power balance has to be maintained at real-time operation with the market that compensates the flexible generators and penalises the sources of deviation referred as a real-time market. To cope effectively with the uncertainty in a market driven power system, producers have to consider complex trading strategies that span through the various trading floors in order to mitigate their exposure to increased balancing costs and financial losses due to the deployment of typically expensive reserves.

The simplified overview that follows, is based on the components used for this specific work; a detailed analysis of electricity markets and the role of various actors from market operators to transmission and distribution operators can be found in <sup>10</sup> and <sup>11</sup>.

## 2.3 | Decision making for wind power producers

Against this background, we outline the decision-making process of a wind power producer actively participating in both futures and short-term electricity markets. We focus on a simple market setup which consists of a day-ahead market followed by a real-time market to settle energy deviations with respect to the day-ahead schedule.

In this basic framework, a wind power producer aims to maximise its revenue by determining its optimal involvement in both futures and short-term electricity markets. At the beginning of the planning period, the producer decides on the purchase of a set of financial contracts, with the decision on whether to exercise them or not left closer to delivery, at the day-ahead market. Noting that the day-ahead decisions can be executed at a rolling horizon on a daily basis with hourly resolution before the day-ahead market clears, the two-step decision process is given by

- At a planning period ranging from years, months, weeks or days prior to the day of delivery of each contract, the producer decides on the purchase of derivatives which may include a variety of financial instruments at different maturity dates covering an extended period.
- For each hour prior to the day-ahead market clearing, the producer decides whether or not to exercise the derivative if that is applicable (i.e. option) based on estimates of its actual production and market prices. In case of futures at their expiration date, they are exercised based on the terms of the contract.

This process is described in Figure 1 which provides the timeline for participating in all three trading floors. In this context, let a producer sign a put option on selling 100 MW at 40 €/MWh. At the time of maturity the producer chooses whether to exercise it or not depending on its estimate of the day-ahead market clearing. If the market price is expected at 30 €/MWh, then the producer will exercise it to receive a payment of 4000 € while participating only in the electricity market would have led to a payment of 3000 €. For a day-ahead market price estimated at 50 €/MWh the producer expects to yield higher profit if it does not exercise the option and therefore chooses to sell its production in the market instead.

Increased profits are a clear benefit but they come at a significant risk as it is highly unlikely that the option or market prices will be known prior to the decision-making process. Irrespective of the derivative a wind power producer selects, and despite the wide range of available financial instruments<sup>2</sup> there will always be financial risks when trading electricity, as long as there is uncertainty in generating and consuming it. Price volatility follows and even if literature shows that futures markets can reduce exposures to such risks, it acknowledges that additional ones are brought as new sources of uncertainty are introduced.

To address the challenges faced by wind power producers participating in both futures and short-term electricity markets requires a decision-making process based on stochastic optimisation that accounts for multiple sources of uncertainty. Still, the proposed framework takes into consideration uncertainty that not only relates to wind energy's stochastic nature, but also to limited information on market outcomes.

---

<sup>2</sup>Further definitions and examples of several types of derivatives incl. forward contracts and options are found in<sup>4</sup>. Moreover, Deng and Oren<sup>2</sup> provide a detailed overview of applications in electricity markets.

### 3 | OFFERING STRATEGY FOR A WIND POWER PRODUCER

In this section we describe how the two-step decision process expands to a multi-stage optimisation problem with each stage corresponding to different points in time when uncertainty is unveiled and different trading floors. We present the intuition behind the reduction of a typically multi-stage problem to a series of two-stage stochastic optimisation problems and then introduce the mathematical formulation of the trading framework for both risk neutral and risk averse producers.

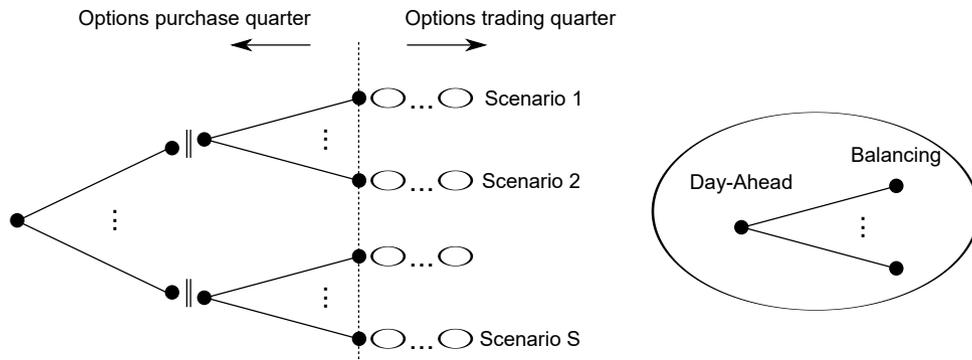
We consider a wind power producer trading in all three markets introduced in Section 2. The producer's objective is to maximise profits from purchasing options in the futures markets while actively trading in the day-ahead market based on forecasts of its production and consequently susceptible to losses in real-time market due to potential imbalances.

#### 3.1 | An optimisation framework for trading wind energy under uncertainty

The structure of this problem resembles the decision-making process of energy investment and capacity expansion problems, common in the technical literature (for an in-depth introduction to this topic cf. <sup>12</sup>). Specifically, the options purchase phase in the wind power trading problem is analogous to the investment decision of a capacity expansion problem, while participation in the short-term electricity market is modelled similarly in both problems. In addition, both problems involve high level of uncertainty, e.g. in future electricity prices that requires proper risk management, while in both cases the decisions to be made are dynamic in a sense that are made throughout a long planning horizon as more uncertain parameters become known, e.g. the options strike price or the fuel cost of new generation assets.

Given that the problem at hand involves three trading floors spanning an horizon up to four months, a reasonable approach would be to formulate a multi-stage stochastic optimisation problem akin to investment decision problems <sup>13,14</sup>. However, there are two notable differences between these problems in terms of uncertainty modelling and timing of decisions that prevent the application of a stochastic multi-stage optimisation formulation in practice. Regarding the former aspect, the variability and uncertainty of wind energy in investment problems are typically approximated throughout the planning horizon by duration curves derived from historical data, which are divided into time segments of different production levels. While this approach captures adequately the characteristics of wind energy in a very long-term horizon, e.g. 20 years as the typical lifetime of wind farms, it poorly describes uncertainty in the shorter time horizons that are more common in trading applications. In such cases, profitability is largely driven by real-time forecast errors and the associated imbalance penalties and not long-term investment returns.

Considering the timing of decisions, typical investment problems adopt a static approach, whereby the optimal expansion decisions are obtained for a single representative year, assuming that the rest of the power system characteristics remain unchanged in the future. Unlike power system configuration that may remain relatively unchanged, option prices are much more volatile and thus it is highly essential for the proposed trading strategy to capture this dynamic behaviour. On the other hand, an attempt to model all uncertainty sources involved in our options purchase and energy trading problem, i.e. market prices and real-time production, in an accurate probabilistic framework preserving also the timing of decision-making would yield a computationally intractable multi-stage stochastic optimisation problem.



**FIGURE 2** Schematic representation of the full scale multi-stage stochastic optimisation problem for a wind power producer trading in futures, day-ahead and balancing markets.

To overcome these complexities we propose an alternative approach, exploiting some particular properties that stem from the practical implications of the proposed framework. That is, we consider that options which are purchased during a quarter to be exercised at a following one, can be exercised at any trading day during that quarter, under the common assumption that the options' purchase fee is negligible compared to the strike price. Therefore, the amount of options bought during a quarter is not a hard constraint for our problem, as long as we manage to compose an options portfolio with favourable strike prices that can be exercised in the next quarter.

The aforementioned practical attributes of the employed financial instruments allow us to decouple the multi-stage optimisation problem and reduce it in a series of two two-stage problems linking trading futures with the day-ahead market and the day-ahead with the real-time market respectively; a process schematically shown in Figure 2. In line with existing literature cf. <sup>15</sup> and the decision process outlined in Section 2.3 both problems are modelled as stochastic optimisation programs. Each stage represents a step from the two-step decision process with some decisions made during the planning period under uncertainty (here-and-now decisions) with the rest taken after the stochastic processes are realised (wait-and-see decisions).

Specifically, the first problem focuses on the options' purchase, with the here-and-now decision determining the amount of options bought the first day of the planning period and the wait-and-see decision variables being the options to be bought at the consequent days based on the statistical estimation of option strike prices. The decision outcome is then used as an input for the second optimisation problem which determines whether to exercise the options and the amount of electricity to be offered in the day-ahead market anticipating the balancing actions at real-time operation.

### 3.2 | Modelling uncertainty

The efficiency of the proposed trading framework and consequently its usefulness to a wind power producer heavily depends on the accuracy of models used to describe the underlying stochastic processes. The two main uncertainty sources considered in this work, i.e. option strike prices and wind power production, are modelled based on state-of-the-art methodologies in the form of scenarios to be compatible with the stochastic programming setup we develop in this paper.

As shown in Figure 2 we generate two distinct scenario sets that represent different realisations of the random variables that represent sources of uncertainty in our model. In this context, for the option purchase problem an option's strike price is the stochastic process that has to be estimated by the decision maker, while the day-ahead market problem is more complex with stochastic processes describing day-ahead and real-time market clearing prices and wind power production.

In more detail, a set of trajectories generated by an auto-regressive integrated moving average (ARIMA) model is used to describe options' price dynamics and uncertainty over time. The ARIMA model characterises the uncertainty regarding the strike price of the derivative of interest (i.e. a quarterly put option on price area DK1) based on historical data. This approach is widely adopted in energy price forecasting applications and formally defined in the context of scenarios for stochastic optimisation in<sup>8</sup> and<sup>16</sup> among several sources.

Furthermore, we model wind power uncertainty in the form of scenarios using the techniques proposed in<sup>17</sup> and<sup>18</sup>. Wind power scenarios are issued based on non-parametric probabilistic forecasts while the interdependent structure of forecast errors among a set of prediction horizons is captured using Copula theory<sup>19,17</sup>. These statistical scenarios are derived from probabilistic forecasts of wind power production. The temporal interdependence structure of forecast errors is captured by a covariance matrix estimated using a transformation of prediction errors to Gaussian multivariate random variables. Long-term variations of this covariance matrix are accounted using an exponential forgetting scheme.

### 3.3 | Mathematical formulation

Despite modelling the decision process as two distinct optimisation problems, there are some assumptions that apply to the whole model, in addition to the assumptions for each stage. Specifically, we assume that producers behave as price-takers in markets exhibiting perfect competition. That is, options' prices, day-ahead balancing market clearing are not affected by the producers' decisions.

In the option purchase problem, we omit more speculative financial instruments by only considering long positions (i.e. purchases) on put options. Given that our analysis centres on wind power producers trading their actual production in electricity markets, we focus on the financial instruments that will give them additional choices by allowing them to sell their estimated production either on futures or day-ahead market.

Finally, regarding real-time markets, we assume that wind power producers are not capable of offering ancillary services. That is, they do not actively bid in regulation or reserve markets and do not apply control strategies on their real-time production.

#### The option purchase problem

In the beginning of each day  $d_1$ , a wind power producer has to decide on the number of options that should be purchased for the next quarter. Although the producer has access to an option's strike price  $\lambda_{d_1}^S$  for the present day, it still faces uncertainty regarding the evolution of option prices  $\lambda_d^S$  in the remaining days of the quarter  $d = \{2, \dots, D\}$ . To account for this source of uncertainty, future options' strike prices are considered as random variables and modelled using a set of scenarios  $\Omega$  such that  $\lambda_{d\omega}^S$  is the strike price at day  $d$  and scenario  $\omega$  with corresponding probability of occurrence  $\pi_\omega$ . Then, the optimal number of options  $x_{d_1}$  to be bought in day  $d_1$  as well as the amount of options  $x_{d\omega}$  that can be purchased over

the following days depending on price uncertainty outcome  $\omega$ , are given as the solution to the following two-stage optimisation problem

$$\max_{x_{d_1}, x_{d\omega}} \lambda_{d_1}^S x_{d_1} E_o + \sum_{d=2}^D \sum_{\omega=1}^{\Omega} \pi_{\omega} \lambda_{d\omega}^S x_{d\omega} E_o \quad (1a)$$

subject to

$$0 \leq x_{d_1} \leq X, \quad (1b)$$

$$0 \leq x_{d\omega} \leq X, \quad \forall d, \forall \omega, \quad (1c)$$

$$x_{d_1} + \sum_{d=2}^D x_{d\omega} \leq X^{\text{tot}}, \quad \forall \omega. \quad (1d)$$

The objective function (1a) to be maximised is the expected revenue from exercising options, where  $E_o$  denotes the size (in terms of energy volume) of option  $o \in O$ . Constraints (1b) and (1c) set an upper limit  $X$  on the amount of options purchased each day. Constraint (1d) states that the total number of options purchased for the whole planning horizon and for every scenario  $\omega$  must be at most equal to predefined parameter  $X^{\text{tot}}$ .

There are two important points that have to be clarified regarding optimisation model (1). First, considering that option fee  $\lambda^O$  paid to buy an option is negligible compared to the strike price, model (1) aims at optimising the options' purchase schedule, i.e. the number of options bought each day, rather than the total amount of options  $X^{\text{tot}}$ . In practice, the wind power producer can specify parameter  $X^{\text{tot}}$  according to the size and the capacity factor of its portfolio as well as the options' size  $E_o$ . In addition, model (1) is executed in a daily rolling horizon until the end of the decision period or until the maximum number of purchased options  $X^{\text{tot}}$  is reached. This allows to refine strike price information over time and incorporate it in decision-making using an updated set of price scenarios. Given the lengthy decision horizon of the overall purchase process, i.e. up to four months, it is plausible that long-term strike price dynamics are not accurately captured in the scenario set. This flaw, which stems from the nature of problem, can be addressed by the proper selection of the daily purchase limit  $X$  to exploit favourable short-term strike prices but leaving a buffer on the remaining number of options to be purchased at better prices which are not captured by the current scenario set. According to this procedure, the value of parameter  $X^{\text{tot}}$  has to be updated on a daily basis subtracting the number of options bought the previous day.

### Short-term trading strategy of a risk neutral wind power producer

Given the portfolio of options bought during the previous quarter based on model (1), the wind power producer should decide how to trade its energy production for the next day. In particular, the producer has to determine the amount of energy to be sold either in the day-ahead market or through options, taking into account the penalties from the balancing market due to wind power production uncertainty. Aiming at maximising its expected profit from participating in these trading floors, a wind producer solves the following two-stage stochastic program

$$\max_{p_t^o, p_t^D, \Delta p_{t\omega}^+, \Delta p_{t\omega}^-} \sum_{t=1}^T \left[ \lambda_t^S p_t^O + \hat{\lambda}_t^D p_t^D + \sum_{\omega=1}^{\Omega} \pi_{\omega} \left( \lambda_{t\omega}^D r_{t\omega}^+ \Delta p_{t\omega}^+ - \lambda_{t\omega}^D r_{t\omega}^- \Delta p_{t\omega}^- \right) \right] \quad (2a)$$

subject to

$$p_t^D + p_t^O \leq W, \forall t, \quad (2b)$$

$$p_t^O = \sum_{o=1}^O u_{to} n_{to} E_o, \forall t, \quad (2c)$$

$$n_{to} \leq N_o, \forall t, \forall o, \quad (2d)$$

$$\Delta p_{t\omega}^+ + \Delta p_{t\omega}^- = P_{t\omega} - (p_t^D + p_t^O), \forall t, \forall \omega, \quad (2e)$$

$$\Delta p_{t\omega}^+ \leq P_{t\omega}, \forall t, \forall \omega, \quad (2f)$$

$$\Delta p_{t\omega}^- \leq W, \forall t, \forall \omega, \quad (2g)$$

$$p_t^O, p_t^D \geq 0, n_{to}, \forall t, \forall o, \quad (2h)$$

$$\Delta p_{t\omega}^+, \Delta p_{t\omega}^-, \forall t, \forall \omega, \quad (2i)$$

$$u_{to} \in \{0, 1\}, \forall o. \quad (2j)$$

The objective function (2a) to be maximised is the total expected profit of the wind power producer. The first stage decision  $p_t^O$  is the amount of energy to be sold exercising available options with strike price  $\lambda^S$ . Similarly, the decision variable  $p_t^D$  is the energy volume traded in the day-ahead market at the expected spot price  $\hat{\lambda}_t^D$ . Note that regarding first-stage decisions, spot price uncertainty can be summarised in a single-valued forecast, i.e. expected value, under the assumption that wind power production and day-ahead market price are not correlated<sup>20</sup>. The second-stage variables  $\Delta p_{t\omega}^+$  and  $\Delta p_{t\omega}^-$  represent the excess and deficit of energy production with respect to the scheduled quantities, respectively. Parameters  $r_{t\omega}^+$  and  $r_{t\omega}^-$  are the ratio of the positive and negative imbalance prices respectively with the day-ahead market price  $\lambda_{t\omega}^D$  in period  $t$  and scenario  $\omega$ .

Constraint (2b) ensures that the level of contracted energy through options and the day-ahead market is lower or equal to the installed capacity  $W$  of the wind energy portfolio. The energy sold through options is defined according to constraint (2c), where  $n_{to}$  and  $E_o$  denote the number and size of options units and the binary variable  $u_{to}$  is equal to 1 if option  $o$  is exercised in period  $t$  or 0 otherwise. Constraint (2d) limits the number of options exercised at each time period to the number  $N_o$  of the corresponding options purchased. The positive  $\Delta p_{t\omega}^+$  or negative  $\Delta p_{t\omega}^-$  deviations from the contracted amount of energy are calculated according to equation (2e), based on the realised wind power production  $P_{t\omega}$  for each scenario  $\omega$  and period  $t$ . The split of energy imbalance to positive and negative deviations requires that the imbalance penalties are  $r_{t\omega}^+ \geq 1$  and  $r_{t\omega}^- \leq 1$ . This property, which follows from the Nordic balancing market structure, ensures that at the optimal solution either  $\Delta p_{t\omega}^+$  or  $\Delta p_{t\omega}^-$  will be equal to zero. Constraints (2f) and (2g) set the upper bounds of the positive and negative imbalance equal to the wind power realisation and the installed capacity, respectively. Finally, constraints (2h)-(2j) are variable declarations.

The stochastic model (2) is executed on a daily basis in a rolling planning fashion, with look-ahead optimisation horizon of one day, using updated price and wind power production scenarios. In the beginning of each quarter, the maximum number of options  $N_o$  that can be exercised are equal to the total units purchased during the previous quarter according to model (1). For every consecutive day this parameter is reduced by the number of units exercised the day before.

### Short-term trading strategy of a risk averse wind power producer

Trading model (2) presented above aims at maximising the expected profit of a risk neutral wind power producer. This formulation does not account though for the variance of revenue as well as about its dispersion from the expected value. However, a wind power producer might not be interested only in the expected value of its profit but also in the probability of experiencing heavy losses due to highly imbalance penalties.

A common risk measure to account for the risk aversion in trading strategies is the conditional value at risk (CVaR). The CVaR at confidence level  $\alpha \in [0, 1)$  is defined as the mean value of the  $1 - \alpha$  cases with lowest profit. The main advantage of CVaR compared to other risk measures proposed in the technical literature, is its mathematical coherence<sup>21</sup> and its ability to take into account fat tails, i.e. probability distributions with large skewness or kurtosis. CVaR is incorporated in the trading strategy model as follows

$$\max_{p_t^O, p_t^D, \Delta p_{t\omega}^+, \Delta p_{t\omega}^-} \Pi_\omega + \beta \left( \zeta - \frac{1}{1 - \alpha} \sum_{\omega=1}^{\Omega} \pi_\omega \eta_\omega \right) \quad (3a)$$

subject to

$$\text{constrains (2b) - (2j)}, \quad (3b)$$

$$\Pi_\omega = \sum_{t=1}^T \left[ \lambda_t^S p_t^O + \hat{\lambda}_t^D p_t^D + \sum_{\omega=1}^{\Omega} \pi_\omega \left( \lambda_{t\omega}^D r_{t\omega}^+ \Delta p_{t\omega}^+ - \lambda_{t\omega}^D r_{t\omega}^- \Delta p_{t\omega}^- \right) \right], \quad (3c)$$

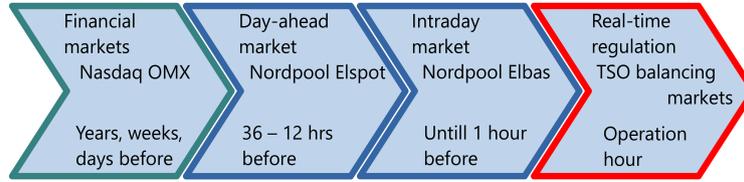
$$- \Pi_\omega + \zeta - \eta_\omega \leq 0, \quad \forall \omega, \quad (3d)$$

$$\eta_\omega \geq 0, \quad \forall \omega. \quad (3e)$$

The new objective function (3a) is a trade-off between the expected profit and the CVaR of the profit distribution. The decision maker can adjust this trade-off by selecting the value of the weighting parameter  $\beta$  according to its risk aversion level. In fact, for  $\beta = 0$  the objective function (3a) becomes identical to one of the risk neutral formulation in (2a). On the contrary, higher values of  $\beta$  indicate more risk averse behaviour, since the CVaR term becomes more important. Choosing different values of  $\beta$  allows to draw the efficient frontier, i.e. a set of points (expected profit, risk) that none of them is strictly dominant to the others. Constraint (3c) defines the profit for each scenario  $\omega$  and constraints (3d) and (3e) along with the auxiliary variables  $\zeta$  and  $\eta_\omega$  are used to model the CVaR.

### Short-term trading strategy as a deterministic problem

For the formulation of the day-ahead market participation as a deterministic problem we summarise wind power's uncertainty into a single-valued forecast, such as the stochastic process's conditional expectation  $\hat{W}_t$ . While such trivialisation of the forecasting process cannot be considered as an original contribution, it does act as a benchmark for the proposed trading framework given that the deterministic model does not account for potential imbalance penalties due to forecast errors.



**FIGURE 3** The structure of the Nordic electricity markets.

In this context, participation in the day-ahead market is formulated as the following optimisation problem

$$\max_{p_t^O, p_t^D} \sum_{t=1}^T [\lambda^S p_t^O + \hat{\lambda}_t^D p_t^D] \quad (4a)$$

subject to

$$p_t^D + p_t^O = \hat{W}_t, \quad \forall t, \quad (4b)$$

$$p_t^O = \sum_{o=1}^O u_{to} n_{to} E_o, \quad \forall t, \quad (4c)$$

$$n_{to} \leq N_o, \quad \forall t, \quad \forall o, \quad (4d)$$

$$p_t^O, p_t^D \geq 0, n_{to}, \quad \forall t, \quad \forall o, \quad (4e)$$

$$u_{to} \in \{0, 1\}, \quad \forall o. \quad (4f)$$

The objective function (4a) to be maximised is the profit of the wind power producer from trading its production in the day-ahead market and through options. Constraint (4b) states that the amount of energy sold in the day-ahead market plus the total energy sold through options each time period  $t$  is equal to the expected wind power production  $\hat{W}_t$ . Constraints (4c)-(4f) remain the same as in the stochastic trading strategy (2).

## 4 | A CASE STUDY FOR THE NORDIC MARKETS

In this section we evaluate the proposed model based on a case study for the Nordic region. We first outline the Nordic electricity market setup and then discuss how data requirements were met regarding scenario generation given the multiple sources of uncertainty. We then provide a detailed evaluation of the decision-making process for the time period that includes the last three quarters of 2006 and the first three of 2007 and outline the benefits of combined trading in both futures and day-ahead markets. Finally, we provide a risk analysis and demonstrate the impact of risk aversion in the producers' revenue.

### 4.1 | The Nordic electricity markets

The Nordic electricity market consists of multiple trading floors depending on the time-horizon at which electricity is traded in connection with physical power delivery. As shown in Figure 3, there are four types of markets that define the Nordic setting.

At a long-term horizon spanning from years to days prior to delivery, electricity is traded through financial instruments which are bought and sold in futures markets. In such markets participants trade financial products linked to a specific commodity. Electricity is treated as such a commodity bereft of the characteristics that are associated with the power system (i.e. power balance, voltage, frequency control). Electricity derivatives for the Nordic region, are traded in Nasdaq Nordic (formerly known as Nasdaq OMX).

The link with the power system is established closer to delivery for those trading in the day-ahead market. For the Nordic region such market is operated by Nord Pool. In Nord Pool's day-ahead market (Elspot) electricity suppliers and consumers with physical access to the power grid place their offers which are aggregated to supply and demand curves. The market clearing price for each of the following day's 24 hours is the intersection of the two curves.

The significant time interval of 12 to 36 hours between Elspot participation and physical operation for the renewable heavy Nordic countries can potentially lead to significant deviations between day-ahead scheduling and physical delivery of electricity. In order to alleviate such deviations, Nord Pool also operates an intraday market (Elbas), a continuous matching market operating at first-come, first-serve basis, which nevertheless exhibits limited liquidity<sup>22</sup>.

Once Elspot and Elbas markets are processed, there can be still power imbalances between electricity generation and consumption as we move towards the delivery time. Responsible for mitigating such deviations and restoring system balance is the Transmission System Operator (TSO) through the real-time market.

## 4.2 | Data requirement and scenario generation

The proposed trading framework links trading futures with the day-ahead and real-time markets. As expected, the scenarios for both the option purchase and the day-ahead offering problem require data from all three markets.

### Trading futures

We consider six quarterly option contracts from Q2 2006 to Q3 2007. The volume of each option is set equal to 2190 MWh for the the quarterly interval which in turn consists of around 2190 hours given the number of days in each month including winter time or summer time and leap years, and a base contract size of 1MW<sup>23</sup>.

The scenarios for the expected daily option strike prices are generated using the ARIMA model described in Section 3.2. Based on historical data from 02/01/2004 to 31/03/2006 we generate 20 scenarios for the daily strike prices for each decision period. For each of the quarterly intervals, the expected prices are generated based on the data available prior to that day with each new day's scenarios taking into account the realisation of the price the day before.

### Day-ahead market

We consider a set of expected prices to model the uncertainty a wind power producer faces regarding the day-ahead market clearing price, given

that it has to make a decision on its position prior to market clearing. Therefore, we generate 20 scenarios for the day-ahead market participation stochastic problem based on historical Elspot prices for the period 01/04/2006 until 30/09/2007 using a simple approach inspired by the literature related to wind energy trading under uncertainty in which price stochasticity is estimated based on historical information without significant loss in forecast accuracy<sup>24, 1</sup>. For the purposes of the proposed setting, each one of the 20 day-ahead market price scenarios can be traced back to a respective week using a sliding window of 20 weeks. For example, scenario  $\omega_1$  considers Elspot prices one week prior to the decision, scenario  $\omega_2$  goes two weeks back and so on.

### Real-time market

Regarding real-time operation the model requires forecasts of prices for upward and downward regulation and realisations of wind power production. In principle, the ARIMA model could have been used to generate scenarios for the regulation prices based on historical data which could then be used to calculate the ratio between regulation prices and the day-ahead market price. However, data was not available for the period before the planning and settlement stages as required for training the ARIMA model. Instead, we exploit the seasonality in regulation prices and generated data for 2005 based on histograms of the ratios for 2006, 2007 and 2008. We generate the scenarios by sampling from a Beta distribution that fitted the synthesised data for 2005.

Finally, for the wind power production for the period between 01/01/2006 and 24/10/2007 we use data from the Western Denmark area (DK1) collected by the Danish TSO (Energinet.dk) and processed in<sup>25</sup>. Wind power measurements were originally available for approximately 400 geographically distributed grid connection points aggregated into 5 zones with the data set consisting of point forecasts, quantile forecasts and the actual power measurements for each zone and for the whole DK1 area, normalised by the available capacity of each zone at each time. In this paper we consider aggregated zone 5 as a single wind power producer and use the quantile forecasts to generate scenarios based on the methodology outlined in<sup>17</sup> and<sup>18</sup>.

It should be noted that while the contribution of this paper lies in the trading framework itself, we acknowledge that there are several approaches of different complexity that can be used to generate scenarios. For example,<sup>1</sup> generate price scenarios based on historical data to estimate the mean, variance and correlation between prices of different trading floors, a simpler method to the introduced ARIMA process. Still, irrespective of the methods used to generate scenarios, their complexity and the accuracy they may bring in generating realistic outcomes for the decision makers, sometimes it may be impossible to employ them due to their reliance on historical data which may be unavailable for a specific time period. In such cases, decision makers have to make assumptions and synthesise data so that the generated scenarios maintain statistical significance. In this context,<sup>8</sup> generate prices for forward contracts based on their scenarios for market prices under the assumption that close to power delivery the prices of forward contracts converge to the market prices while taking note that more advanced forecasting methods can generate more accurate scenarios.

Having derived the scenarios for the two stochastic optimisation problems we now proceed to evaluate the model.

**TABLE 1** Total profit for wind power producers.

	Day-ahead market	Options and day-ahead market
Deterministic model [m €]	21.09	27.83
Stochastic model [m €]	23.39	29.26
Perfect info. [m €]	24.26	29.90

**TABLE 2** An overview of the combined trading strategy.

	Deterministic model	Stochastic model	Perfect info.
Total scheduled energy [GWh]	650.59	727.86	676.88
Energy sold in the day-ahead market [GWh]	277.42	255.68	246.75
Energy sold through options [GWh]	373.17	472.18	430.13
Number of options exercised	171	215	183
Positive imbalance [MWh]	118.04	164.79	-
Negative imbalance [MWh]	91.76	215.77	-
Down-regulation costs [m €]	3.56	5.50	-
Up-regulation costs [m €]	3.53	6.42	-
Total revenue [m €]	27.87	29.26	29.90
Performance ratio ( $\gamma$ ) [%]	93.21	97.86	100

**TABLE 3** Total revenues for various quarterly and daily purchase limits.

Daily / Quarterly limits	60 options	100 options	200 options
2 options	29.93	32.65	-
5 options	29.24	30.40	32.00
10 options	29.21	29.63	30.59
20 options	29.11	29.61	29.69

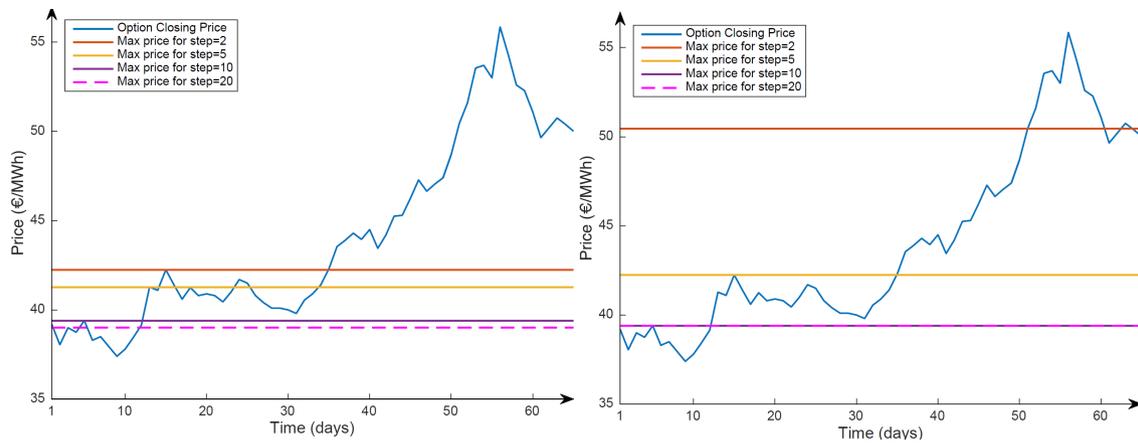
### 4.3 | Empirical evaluation

The numerical simulation of the trading framework consists of two parts. For the first part, we assume a risk neutral decision maker and assess the impact of its participation in both futures and day-ahead markets against participating only in the day-ahead market. We evaluate the proposed framework described by optimisation problems (1) and (2) using two cases to denote a lower and an upper bound for the performance of the model. The lower bound is defined by the deterministic optimisation model (4), while for the upper bound we assume that it is possible for the decision maker to have access to perfect information regarding the involved uncertainties, thus eliminate regulation costs. For the second part, we evaluate the option purchase and day-ahead participation problem assuming a risk averse decision maker as described by optimisation models (1) and (3) to demonstrate the impact of risk control on the decision-making process.

To evaluate the model's efficiency against uncertainty, we introduce a simple two-step process. First, we obtain the decisions made at time step  $t$  by solving the main stochastic optimisation models (1) and (2) for all time steps leading up to  $t - 1$ . For the second step, we use the solution already acquired for time step  $t$  against the observed outcome from the given data. By iteratively implementing this process for time steps  $t > 1$  we obtain a set of revenues that includes the estimated outcome of the model and the actual ones based on the observed outcomes. Based on a process described in <sup>24</sup> we use these revenues to assess the trading strategies through the 'performance ratio' given by

$$\gamma = \frac{R_O + R_{DA} + R_B}{R_{DA+O}^*} \quad (5)$$

where  $R_O$ ,  $R_{DA}$ ,  $R_B$ , are the revenues from trading options and participating in day-ahead and real-time markets respectively.  $R_{DA+O}^*$  is the normalisation factor that denotes the revenue a producer would have obtained had its forecast been equal to the actual realisation. Due to this normalisation,  $\gamma$  takes values in  $(-\infty, 1]$ . Naturally, the ideal outcome would be  $\gamma = 1$  corresponding to perfect forecasts whereby all market deviations are equal to 0. In the Danish context, for a quarterly wind production of 125 GWh at 2006 and 2007 and a typical wind capacity factor around 20% and 25%



**FIGURE 4** Option closing prices for quarterly total limits of 60 and 100 options and various purchase steps.

respectively the nominal capacity of the wind power portfolio is set at 258 MW. Finally, we assume that a producer can purchase up to 60 options each quarter, capped at 5 options daily, i.e.  $X^{\text{tot}} = 60$ ,  $X = 5$ . The model's sensitivity to both assumptions is further examined.

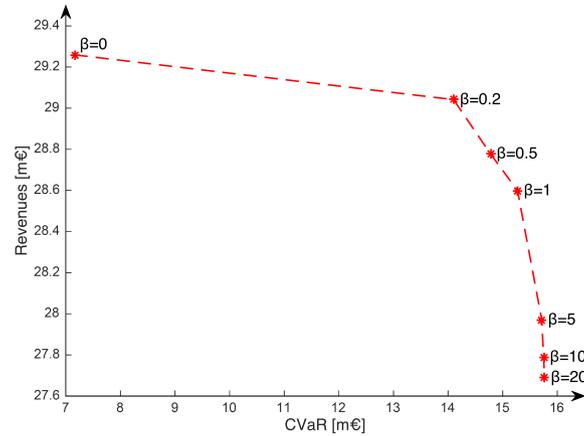
Initially, the main benefits of purchasing electricity options in addition to participating in the day-ahead market are outlined in Table 1 which shows a clear advantage of combined trading for all models, irrespective of how (or whether) they model uncertainty. In particular, combined trading yields 32.5% and 25.1% additional revenue for the deterministic and stochastic models respectively and an increase of 23.2% for decision makers with access to perfect information.

Combined participation in both futures and day-ahead markets and the impact of uncertainty are detailed in Table 2. The amount of energy traded in each market under perfect foresight is less compared to the energy traded in the stochastic model and more compared to the deterministic model, mainly due to the number of exercised options and consequently due to the energy traded through options. Still, a producer maximises its revenue under perfect foresight by avoiding the regulation costs that are bound to occur under uncertainty. Overall, the stochastic model achieves 2% less revenues compared to perfect information, with the deterministic model resulting in 7% less revenues. Consequently this translates to performance ratio equal to 97.86% and 93.21% for the stochastic and deterministic models, respectively. For the numerical evaluation of the option purchase problem (cf. model (1)) quarterly purchase limit  $X^{\text{tot}}$  and daily limit  $X$  have been arbitrarily set equal to 60 and 5 respectively. Determining these values reflects on the intuition regarding physical and economic aspects of the option purchase problem and has a clear impact on the outcome of the model. Figure 4 shows sensitivity on the daily purchase limit, with the strike price increasing as the daily limit decreases. Furthermore, prices increase as the block of options purchased at the beginning of each quarter increases. However, this clear trend regarding the strike prices is not reflected on the total revenues. Solving again the options purchase and day-ahead market problems from models (1) and (2) for various quarterly and daily limits, shows that for all values of quarterly blocks, revenues decrease as the daily limit increases, while revenues increase as quarterly blocks increase (Table 3).

For the second part of the analysis we consider different risk preferences in the decision-making process with a producer now considering the impact of extreme scenarios on its expected profit due to very high imbalance costs. Replacing the day-ahead purchase model (2) with (3) leads to risk aware trading strategies for risk averse decision makers. The trading framework, now consisting of models (1) and (3), is evaluated with

**TABLE 4** Amount of energy traded in options and day-ahead market based on the parameter  $\beta$ .

Weighting factor $\beta$	Total traded energy GWh	Energy traded as options GWh	Energy traded in day-ahead market GWh
0	727.86	472.18 (64.9%)	255.68 (35.1%)
0.2	696.09	463.60 (66.6%)	232.49 (33.4%)
0.5	666.54	455.90 (68.4%)	210.63 (31.6%)
1	638.54	450.80 (70.6%)	187.74 (29.4%)
5	573.40	425.75 (74.2%)	147.65 (25.8%)
10	757.80	420.15 (75.3%)	137.64 (24.7%)
20	549.02	416.84 (75.9%)	132.18 (24.1%)

**FIGURE 5** The impact of risk aversion parameter  $\beta$  on total revenues.

$\alpha = 0.95$  against the weight parameter  $\beta$  that measures risk aversion. As shown in Figure 5 and Table 4, increasing risk aversion decreases the expected total revenues and increases the CVaR. Specifically, the expected revenue of a risk averse producer ( $\beta = 20$ ) is 1.57 m. € or 5.7% less than that of a risk neutral one ( $\beta = 0$ ), although the risk of profit variability is reduced as indicated by the significantly larger (120%) value of the CVaR term (Figure 5). Further analysis of how energy is distributed between options and the day-ahead market (Table 4) reveals that as risk aversion increases, more energy is traded as options and less is traded in the day-ahead market. This is justified by risk averse producers preference in trading options given that it is an action that incurs no losses at that stage. On contrary, participating in the day-ahead market may lead to losses if the clearing price is lower than the option strike price. In addition, by reducing its participation in the day-ahead, a producer reduces its exposure to regulation costs due to imbalances between day-ahead scheduling and real-time operation.

## 5 | CONCLUSIONS

This paper presents a trading framework to support decision making of wind power producers participating in futures and day-ahead markets. Based on stochastic programming, the model accounts for three sources of uncertainty, i.e. option prices, day-ahead market prices and wind power production, while it spans through three trading floors and their corresponding time horizons. Ideally, these decision-making processes can be formulated as multi-stage problems, albeit specific complexities related to the dynamic interaction between futures and short-term electricity markets called for a different approach.

The main contribution of this work is the formulation of an optimisation framework that incorporates both financial and short-term electricity markets accounting for various sources of uncertainty involved in each stage. Exploiting the properties of the futures market, we tackle the wind

power offering problem in a practical manner solving a series of two-stage stochastic programs that preserve an implicit link between the different types of markets in a computationally tractable formulation. Initially, the option purchase problem determines the number of options to be purchased each quarter. In turn, the day-ahead participation problem determines the number of options to be exercised and the day-ahead market offers each day of the following quarter.

In addition to its methodological merit, the proposed formulation leads to a computational tractable model that allows an extensive empirical evaluation of the decision-making framework. In this context, numerical simulations based on data from the Nordic region in the period during 2006-2007, show that a wind power producer, irrespective of its risk sensitivity, can increase its revenues by implementing a trading strategy that combines participation in both futures and day-ahead markets. Moreover, for risk averse producers, it is shown that they increase their revenues by trading heavily in the futures market and limiting their position in the day-ahead market. The scenario generating methods used for the numerical solution of the stochastic optimisation problems represent with adequate accuracy the involved uncertainty. While we acknowledge that more advanced scenario generation methods could have been considered, there is a clear indication on the positive impact of combined participation in both markets.

Nevertheless, for future lines of work the option purchase and day-ahead participation models can be further improved so that they provide a more accurate and more realistic decision framework. Further future work involves the option purchase problem, which instead of a fixed daily option purchase step, quantitative methods based on historical data could determine whether to purchase all available options based on the daily prices.

## ACKNOWLEDGMENT

Athanasios Papakonstantinou and Pierre Pinson are partly supported by the Danish Council for Strategic Research (DSF) through the project '5s-Future Electricity Markets', No. 12-132636/DSF. The authors are very grateful to Janne Peljo and Anna Sahari for sharing with us access to the data gathered for Janne's MSc Thesis<sup>26</sup>.

## References

1. Botterud A, Zhou Z, Wang J, et al. Wind power trading under uncertainty in LMP markets. *IEEE Transactions on Power Systems*. 2012;27(2):894–903.
2. Deng SJ, Oren SS. Electricity derivatives and risk management. *Energy Economics*. 2006;31(6-7):940–953.
3. Oren SS. Integrating real and financial options in demand-side electricity contracts. *Decision Support Systems*. 2001;30:279–288.
4. Hull JC. *Option, Futures and Other Derivatives*. Pearson; 9 ed.2015.

5. Lane DW, Richter CW, Sheblè GB. Modeling and evaluating electricity options markets with intelligent agents. In: ; 2000.
6. Bhanot K. Value of an option to purchase electric power - the case of uncertain consumption. *Energy Economics*. 2002;24:121–137.
7. Liu M, Wu FF. Risk management in a competitive electricity market. *Energy Economics*. 2007;29(9):690–697.
8. Pineda S, Conejo AJ. Managing the financial risks of electricity producers. *Energy Economics*. 2013;34(6):2216–2227.
9. Pineda S, Conejo AJ. Using electricity options to hedge against financial risks of power producers. *Journal of Modern Power Systems and Clean Energy*. 2013;1(2):101–109.
10. Conejo AJ, Carrión M, Morales JM. *Decision Making Under Uncertainty in Electricity Markets*. Springer; 2010.
11. Kirschen D, Strbac G. *Fundamentals of Power System Economics*. John Wiley & Sons, Ltd; 2004.
12. Siddiqui AS, Kazempour SJ, Conejo AJ, Baringo L. *Investment in Electricity Generation and Transmission: Decision Making Under Uncertainty*. Springer; 2016.
13. Shiina T, Birge JR. Multistage stochastic programming model for electric power capacity expansion problem. *Japan journal of industrial and applied mathematics*. 2003;20(3):379–397.
14. Baringo L, Conejo AJ. Wind power investment within a market environment. *Applied Energy*. 2011;88(9):3239–3247.
15. Birge JR, Louveaux F. *Introduction to Stochastic Programming*. Springer; 1997.
16. Conejo AJ, Carrión M, Morales JM. *Decision Making Under Uncertainty in Electricity Markets* International Series in Operations Research & Management Science, vol. 180: . Springer; 2010.
17. Pinson P, Papaefthymiou G, Klockl B, Nielsen HAa, Madsen H. From probabilistic forecasts to statistical scenarios of short-term wind power production. *Wind Energy*. 2009;12:51–62.
18. Pinson P, Girard R. Evaluating the quality of scenarios of short-term wind power generation. *Applied Energy*. 2012;96:12–20.
19. Hagspiel S, Papaemannouil A, Schmid M, Andersson G. Copula-based modeling of stochastic wind power in Europe and implications for the Swiss power grid. *Applied Energy*. 2012;96:33–44.
20. Morales JM, Conejo AJ, Madsen H, Pinson P, Zugno M. *Integrating renewables in electricity markets: operational problems*. Springer Science & Business Media; 2013.
21. Rockafellar RT, Uryasev S. Optimization of conditional value-at-risk. *Journal of risk*. 2000;2(3):21–42.
22. Weber C. Adequate intraday market design to enable the integration of wind energy into the European power systems. *Energy Policy*. 2010;38(7):3155–3163.

23. *Trading Appendix 2 / Clearing Appendix 2: Contract Specifications*. : Nasdaq Oslo ASA and Nasdaq Clearing AB; 2016.
24. Pinson P, Chevallier C., Kariniotakis N.G.. Trading wind generation from short-term probabilistic forecasts.. *IEEE Transactions on Power Systems*. 2007;22(3):1148-1156.
25. Pinson P. Wind Energy: Forecasting challenges for its operational management. *Statistical Science*. 2013;28(4):564–585.
26. Peljo J. Futures Pricing in the Nordic Electricity Market. Master's thesis Dept. of Finance, School of Business, Aalto University Finland 2013.

