# Information Uncertainty in Electricity Markets: Introducing Probabilistic Offers

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***Abstract*—We propose a shift from the current paradigm of electricity markets treating stochastic producers similarly to conventional ones in terms of their offers. We argue that the producers' offers should be probabilistic to reflect the limited predictability of renewable energy generation, while we should design market mechanisms to accommodate such offers. We argue that the transition from deterministic offers is a natural next step in electricity markets, by analytically proving our proposal's equivalence with a two-price conventional market.**

***Index Terms*—Electricity markets, renewable energy, imperfect information**

## I. Introduction

As generation from renewable sources of energy (RES) reaches grid parity, the support mechanisms that were put in place to promote their development become obsolete, while RES producers are asked to participate in electricity markets under the same rules as conventional generators. This has allowed them to exploit market inefficiencies and to design trading strategies in order to hedge their positions between day-ahead and real-time trading floors[1], [2]. A way to address this issue has been the development of electricity markets where the day-ahead dispatch and real-time reserve are jointly optimized [3]. Still, as the expected real-time outcomes are calculated based on scenarios of stochastic production these approaches are vulnerable to forecasts of poor quality. Given that the senarios would have to be derived from producers' deterministic offers, we find markets with stochastic clearing but deterministic offers contradicting.

We here propose and facilitate the use of probabilistic offers in markets, by showing analytically how a producer's revenue in a two-price real-time market can be expressed as an affine transformation of strictly proper scoring rules (i.e. Brier and Continuous Ranked Probability Scores). Such functions were designed to elicit *accurate and precise* probabilistic estimates from forecasters [4] and using them as the foundation of an electricity market achieves the same for stochastic producers. This opens the door to appropriate valuation and use of the uncertainty information revealed through market participation.

## II. From a Deterministic to a Probabilistic Electricity Market

We model wind power production as a possible realization $y$ of a random variable $Y$, scaled in $[0, 1]$. Let $Y$ follow a distribution with density $g(y)$ and $y_0$ denote the actual wind power production. Wind power producers generate forecasts with distribution densities denoted as $f(y)$ to represent the probability distribution, $g(y)$. It is important to clarify that the producers' forecasted densities may not be equal to the distribution of the actual production i.e. $f \neq g$.

We consider a day-ahead market, where the clearing price is denoted by $\lambda^{\text{DA}}$ followed by a two-price real-time market. Deviations are priced differently depending on the sign of the imbalance, there is an up-regulation $\lambda^{\text{UP}}$ or a down-regulation $\lambda^{\text{DN}}$ price depending upon overall system balance. In a market with deterministic offers, the potential imbalance for stochastic producers is between their reported day-ahead offer $\widehat{y}$ (i.e. a point forecast) and the realized production $y_0$ estimated by $f(y)$. A stochastic producer's revenue is given by

$$P(\widehat{y}) = \lambda^{\text{DA}} y_0 + \begin{cases} (\lambda^{\text{UP}} - \lambda^{\text{DA}})(y_0 - \widehat{y}) & \text{if} \;\; y_0 \leq \widehat{y} \\ (\lambda^{\text{DN}} - \lambda^{\text{DA}})(y_0 - \widehat{y}) & \text{if} \;\; y_0 > \widehat{y} \end{cases} \quad (1)$$

In this context, a risk neutral stochastic producer determines his offer in order to maximize expected revenue. Under the common practice in the literature, e.g. [1], [2], that the producer is a price-taker and therefore $\lambda^{\text{DA}}$ is independent of $\widehat{y}$, the optimal offer is: $y^* = F^{-1}(\alpha)$, where $F$ is the cumulative distribution function of wind power production $y$ and $\alpha \leq 1$ and equal to $\Lambda^{\text{DN}}/(\Lambda^{\text{DN}} + \Lambda^{\text{UP}})$, with $\Lambda^{\text{UP}} = \lambda^{\text{UP}} - \lambda^{\text{DA}}$ and $\Lambda^{\text{DN}} = \lambda^{\text{DA}} - \lambda^{\text{DN}}$ denoting the unit regulation cost for positive and negative imbalances. Given the above definitions of $\alpha$, $\Lambda^{\text{UP}}$ and $\Lambda^{\text{DN}}$: $1 - \alpha = \Lambda^{\text{UP}}/(\Lambda^{\text{DN}+\Lambda^{\text{UP}}})$.

Eq. (1) takes the following form after being multiplied by 2 and divided by $\Lambda^{\text{DN}} + \Lambda^{\text{UP}}$,

$$\begin{aligned} P'(q) &= \frac{2\lambda^{\text{DA}} y_0}{\Lambda^{\text{DN}} + \Lambda^{\text{UP}}} + 2(\alpha - \mathbb{I}\{y_0 < q\})(q - y_0) \\ &= \frac{2\lambda^{\text{DA}} y_0}{\Lambda^{\text{DN}} + \Lambda^{\text{UP}}} - \text{QS}_\alpha(q, y_0) \end{aligned} \quad (2)$$

where $q$ is a quantile forecast at level $\alpha$ and $\text{QS}_\alpha(q, y_0)$ is the Quantile Score (QS), a strictly proper scoring rule measuring *in-sample goodness of fit and out-of-sample forecast performance* [5]. QS is equivalent to the Brier Score (BS), a strictly proper scoring rule used to verify predictions of the occurance of a specific event,

$$\text{BS}(p, y_0) = (p - \mathbb{I}\{y_0 \leq y\})^2 \quad (3)$$

where $p = F(y)$ is the probability of occurrence of a binary event $\{Y \leq y\}$ as reported by the forecaster. The predicted event is characterized by a threshold value $y$, which divides a real line in two intervals s.t. $I_1 = (-\infty, y]$ and $I_2 = (y, \infty)$. The event is said to occur if $y_0 \leq y$, i.e. the actual production $y_0$ belongs in the interval $I_1$.

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Substituting the QS with the BS and solving Eq. (2) to $P()$ yields the following expression for a stochastic producer's revenue,

$$P(F, y_0) = \lambda^{\mathrm{DA}} y_0 - \frac{\Lambda^{\mathrm{DN}} + \Lambda^{\mathrm{UP}}}{2} (F(y) - \mathbb{I}\{y_0 \leq y\})^2 \quad (4)$$

To this end, formulating a stochastic producer's revenue as a linear transformation of a strictly proper scoring rule shows that it is possible to design a market which accepts probabilistic offers without a complete overhaul of existing structures. In fact, for those producers opting to use deterministic offers or those offering perfect estimates (i.e. $\widehat{y} = y_0$), the Brier Score payment reverts to the current two-price payment.

## III. Evaluation of Probabilistic Offers through the Continuous Ranked Probability Score

The design of an electricity market based on a linear transformation of the CRPS allows for offers of a *simpler structure*. For example, stochastic producers can now report a CDF or more realistically a set of quantiles. Following [6] and [5] we construct the CRPS by calculating the average BS over all possible $y$ values, while taking note that the CRPS can be written as a kernel score [5]:

$$\mathrm{CRPS}(F, y_0) = -\int_{-\infty}^{\infty} (F(y) - \mathbb{I}\{y_0 \leq y\})^2 dy \quad (5)$$

$$= \frac{1}{2} E_F |Y - Y'| - E_F |Y - y_0| \quad (6)$$

where $Y$ and $Y'$ are independent copies of the random variable with distribution $F$. In this formulation the CRPS consists of two parts: the *measure of uncertainty*, $U(F)$, and the *divergence* between predictive and actual distributions $D(F, G)$.

In the context of electricity markets this decomposition of the CRPS suggests that $U(F)$ can be associated with the day-ahead stage of the mechanism, while $D(F, G)$ associates with real-time balancing. We calculate the average revenue $\overline{P}$ by integrating Eq. (4) withe respect to the value of $y$,

$$\overline{P}(F, y_0) = \int_{-\infty}^{\infty} P(F(y), y_0) dy$$

$$= \lambda^{\mathrm{DA}} y_0 + \frac{\Lambda^{\mathrm{DN}} + \Lambda^{\mathrm{UP}}}{2} \mathrm{CRPS}(F, y_0) \quad (7)$$

## IV. A Probabilistic Two-Price Electricity Market

Now, based on the analytically proven relation between the BS, the CRPS and the two-price system, we propose an electricity market which asks stochastic producers to submit their predictive distributions instead of deterministic offers. Alternatively, stochastic producers can report a CDF, or a set of quantiles, noting that in case of parametric distributions, producers can report the parameters e.g., mean and variance, of a known distribution. Both day-ahead and real-time markets are designed based on this type of offers to achieve the following objectives:

1) The day-ahead market holds them accountable for generating forecasts of low predictive value i.e. low precision or high dispersion.
2) The real-time market now addionally rewards or penalizes the stochastic producers based on the goodness of fit of the distributions and the quality of the estimate i.e. divergence from the realized production.

Regarding supply the market model is defined as follows:

**Day-ahead market**:

1) Each producer $i \in N$ submits his quantity-price offer $(\widehat{y}_i, c_i)$, with stochastic producers submitting their distribution functions $F_i(y)$ instead;
2) The market price $\lambda^{\mathrm{DA}}$ is determined based on the mean of the reported distribution following conventional market clearing rules;
3) Each producer receives the following payment,

$$P_i^{\mathrm{DA}} = \lambda^{\mathrm{DA}} \overline{y}_i - \lambda^{\mathrm{DA}} U(F_i)$$

**Real-time regulation market**:

1) Each stochastic producers generates outputs $y_{0_i}$ and up-regulation $\lambda^{\mathrm{UP}}$ or down-regulation $\lambda^{\mathrm{DN}}$ prices are determined as in the two-price system;
2) Each producer receives the following payment,

$$P_i^{\mathrm{RT}} = \frac{1}{2} (\lambda^{\mathrm{UP}} - \lambda^{\mathrm{DN}}) \mathrm{BS}(F_i, y_{0_i}) + \lambda^{\mathrm{DA}} (y_{0_i} - \overline{y}_i) + \lambda^{\mathrm{DA}} U(F_i)$$

**Total Payment**: Based on the realization of each producer's production $y_{0_i}$, the total revenue is given by

$$P(F, y_{0_i}) = \frac{1}{2} (\lambda^{\mathrm{UP}} - \lambda^{\mathrm{DN}}) \mathrm{BS}(F_i, y_{0_i}) + \lambda^{\mathrm{DA}} y_{0_i}$$

Now, since $\overline{\mathrm{BS}} = \mathrm{CRPS}$, the expected revenue is given by Equation (7). Due to its dependence on the CPRS, the proposed market satisfies the required objectives of precision and accuracy, and a stochastic producer is held *accountable* for the uncertainty it introduces in the system. Finally, it should be noted that as for stochastic producers reporting their point forecasts the proposed mechanism reverts to current practices, so does for conventional producers reporting in the day-ahead market their productions.

## V. Conclusion

This letter proves that submitting probabilistic offers in a day-ahead market can lead to the development of market mechanisms which can evaluate, and consequently reward or penalize, stochastic producers regarding the precision and accuracy of the estimates of their production. We show a straight forward process which can transform a deterministic market to a fully stochastic one through the utilization of strictly proper scoring rules.

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