Controlling Electricity Consumption by Forecasting its Response to Varying Prices

Olivier Corradi, *Associate Member, IEEE*, Henning Ochsenfeld, Henrik Madsen, and Pierre Pinson, *Member, IEEE*

***Abstract*—In a real-time electricity pricing context where consumers are sensitive to varying prices, having the ability to anticipate their response to a price change is valuable. This paper proposes models for the dynamics of such price-response, and shows how these dynamics can be used to control electricity consumption using a one-way price signal. Estimation of the price-response is based on data measurable at grid level, removing the need to install sensors and communication devices between each individual consumer and the price-generating entity. An application for price-responsive heating systems is studied based on real data, before conducting a control by price experiment using a mixture of real and synthetic data. With the control objective of following a constant consumption reference, peak heating consumption is reduced by nearly 5%, and 11% of the mean daily heating consumption is shifted.**

***Index Terms*—Adaptive estimation, control by price, demand forecasting, predictive control, price-response, real-time pricing.**

I. Introduction

The concept of controlling power systems using electricity prices was first presented by Schweppe *et al.* [1] in a study where price-sensitive generation and load responded by adjusting their production or consumption in order to maximize their revenue or minimize their costs. Glatvitsch and Alvarado [2] and later Alvarado [3] illustrated how broadcasting different prices at each grid node is a means to controlling congestion in the power system using price-sensitive generators. In addition to managing congestion, the balance of supply and demand using real-time prices was investigated by Jokic *et al.* [4]. Furthermore, the use of price-responsive demand to support a high penetration of fluctuating generation (such as wind or solar power) through real-time pricing is studied in [5]–[7]. However, little has been done about the applicability to price-elastic load, including its uncertainty. These studies are often based on a known deterministic model of the price-responsivity without being based on real data.

In power systems characterized by a high penetration of fluctuating generation, demand-side management programs will play a crucial role in providing the flexibility needed to balance the power system and control its congestion [8]–[13]. This flexibility is controllable by price as consumers become significantly more price-elastic when exposed to varying prices [14], provided that the price is efficiently displayed to the user. With the emergence of energy management systems that automatically control the flexible load of end-users [15], predictable price-responsive patterns emerge, as seen in the Olympic Peninsula project [16]. Due to the migration of load across time, being able to forecast the consequences of price changes on future price-elasticities is crucial [17]. This requires the use of cross-time elasticities to model the auto-correlation of price-elasticities [18].

In a novel way, this study uses a data-driven approach to estimate and forecast the dynamics of the price-elasticity, before using the forecasting model in a model predictive controller with the objective of minimizing power imbalances, as presented in Section II. The stochastic nature of future elasticities and its nonstationarity requires the use of proper statistical models, as discussed in Section III. Section IV studies an application for heating systems, and finally, Section V presents an implementation of a price-generator in a simulation framework. The paper ends with conclusions and suggestions for future work in Section VI.

II. Using the Price-response to Control the Aggregated Electricity Consumption

Different entities in the power system are interested in utilizing the flexibility of users for various purposes such as revenue maximization, balancing issues, or grid stabilization. Different strategies can be used to activate this flexibility. Direct control aims at centrally controlling each flexible device, requiring a bi-directional communication interface and knowledge about the end-user's environment. Indirect control on the other hand decentralizes the consumption control task by sending out a generic control signal interpreted by each consumer's energy management system. Each consumer can then react to a control signal, e.g., by installing devices specially optimized for his/her environment. Indirect control however requires a reliable estimation of the consumption's response to the broadcasted control signal in order to reliably activate a given amount of flexibility.

When the generic control signal takes the form of a price signal, the available consumption flexibility is estimated by investigating the response of consumption to price. This price-consumption relationship will be used to generate prices such that a given objective is achieved (Fig. 1). In order to facilitate the integration of large shares of renewables, an objective can be to minimize needs for reserves by adjusting the consumption.

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The authors are with the Department of Informatics and Mathematical Modeling, Technical University of Denmark, Lyngby, Denmark (e-mail: olc@imm.dtu.dk; hpoc@imm.dtu.dk; hm@imm.dtu.dk; pp@imm.dtu.dk).

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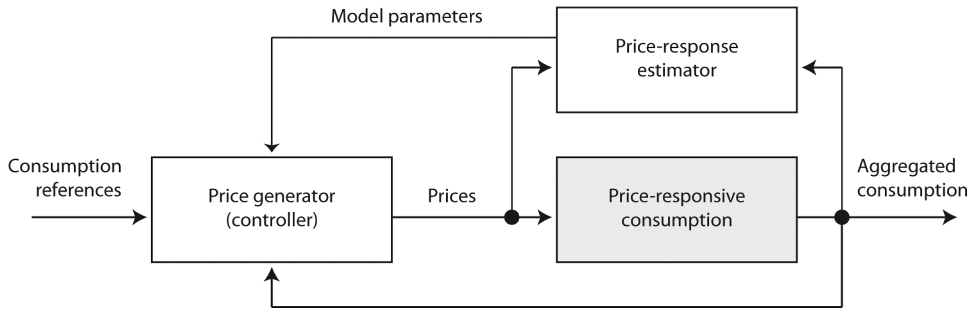

Fig. 1. Controller emitting a price signal is able to influence an aggregate of price-responsive consumption systems. Identification of the relationship between price and consumption (the "price-response") enables the controller to generate prices needed to reach a given consumption target.

Based on knowledge about the price-responsivity of users, the price-generation procedure is then a means to control the consumption.

The price-responsivity of users must be adaptively estimated as its population and characteristics change over time, reflecting, e.g., the demographic development or the adoption rate of energy management systems. This means the price-response model must be updated every time a new measurement is available. When identifying the price-consumption relationship, special care has to be taken when price and consumption are interdependent, as they affect each other in a closed loop system. In this case, the forward relationship, being the response of consumption to price, must be decoupled from the feedback, being the response of price to consumption, using a multivariate time series model [19] as investigated in Section III.

As some price-responsive devices might have very fast responses, consumption must be measured close to real-time in order to take full benefits from price responsive users. The resulting prices must then accordingly be generated and sent out at the same time scale, putting heavy constraints on the IT and communication infrastructure. To minimize the infrastructure costs, this concept works aims at controlling consumption by price using *external* variables measured on an *aggregated* level. External variables are measurements available from outside the household, e.g., like the aggregated electricity consumption, excluding variables requiring sensors inside the house, e.g., like the inside air temperature. Measurements on an aggregated level are measurements of a whole population in contrast to an individual device or consumer. For example, the sum of every household's consumption in a certain district is an aggregated variable, measurable at grid level. Here, only a one-way communication system is needed between the price-emitting entity and the price-responsive devices, given that the aggregated consumption is measurable at grid level.

III. Mathematical Formulation of the Control by Price Concept

In this paper, the grid dynamics have been ignored, meaning that infinite grid capacity is assumed at every point. Also, instantaneous communication of the electricity price and of the aggregated consumption metering has been assumed. This implies that additional dynamics and/or delays may appear when dealing with a real power and communication system.

A. Identification of the Price-Consumption Relationship

The price influence can be seen as a change in consumption added to the nonresponsive consumption. We will here make the assumption that consumption can be separated into nonresponsive and responsive parts, such that

$$c_t = f(\tilde{c}_{t-1}, \tilde{\mathbf{z}}_t) + g(\tilde{c}_{t-1}, \tilde{p}_t, \tilde{\mathbf{z}}_t) \tag{1}$$

with

$$\begin{aligned} \tilde{c}_{t-1} &= c_{t-1}, \ldots, c_{t-n_c} \\ \tilde{p}_t &= p_t, \ldots, p_{t-n_p} \\ \tilde{\mathbf{z}}_t &= \mathbf{z}_t, \ldots, \mathbf{z}_{t-n_z} \end{aligned}$$

where n_c, n_p, and n_z denote the finite range of past values of consumption c, price p, and externals $\mathbf{z}$ influencing the current consumption. The price-consumption relationship is described by the function g, whereas the traditional load is described by f. In the following, statistical consumption models which can be separated into responsive and unresponsive loads are proposed. Even though these models focus on predicting both types of load, only the responsive part is to be controlled. These models will then be used in Section III-B to control the consumption by price.

1) Finite Impulse Response Model: As a first approach, the price-consumption relationship and its influence by a given set of external variables are assumed to be linear. A *finite impulse response* (FIR) model is proposed [19], linearly combining past and present values of price and externals to form the output consumption, independently of past consumption values. In that sense, dependence on price and on external variables (such as outside temperature) are decoupled, and the price-responsivity can be isolated. The input variable $\mathbf{x}_t$ is constructed as a vector containing a bias term followed by the present input values and a finite range of n_p and n_z past input values. The FIR model describing the output consumption y_t is then expressed as a general linear model

$$y_t = \mathbf{x}_t^\top \boldsymbol{\theta} + \epsilon_t \tag{2}$$

where ϵ_t is a zero-mean Gaussian random variable.

For a data set of input-output measurements pairs $(\mathbf{x}_t, y_t)$, all observations are vertically concatenated into an input matrix $\mathbf{X}$

and an output vector $\mathbf{y}$ in order to estimate the associated model parameters in $\boldsymbol{\theta}$ using the least squares estimator [19]

$$\hat{\boldsymbol{\theta}} = \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \mathbf{X}^\top \mathbf{y}. \tag{3}$$

The estimated vector $\hat{\boldsymbol{\theta}}$ consists of the coefficients describing the contribution of each input variable at each time delay (or lag). The coefficients a_i corresponding to the price input variable define the impulse response function from price to consumption, characterizing the price-response g

$$g(p_t, \dots, p_{t-n_p}) = \sum_{i=0}^{n_p} a_i p_{t-i}. \tag{4}$$

At time t, the optimal k-step consumption prediction minimizing squared errors is the conditional expectation [19]. For the FIR model (2), the forecasting model is

$$\hat{y}_{t+k|t} = \mathbb{E}\left[y_{t+k} | \hat{\mathbf{x}}_{t+k|t}\right] = \hat{\mathbf{x}}_{t+k|t}^\top \boldsymbol{\theta} \tag{5}$$

requiring a *forecast* $\hat{\mathbf{x}}_{t+k|t}$ made at time t of the external variables up to time $t+k$.

Recursive and adaptive estimation, needed in order to account for evolution of the population of price-responsive systems, is done by re-estimating the model parameters $\boldsymbol{\theta}_t$ every time a new observation $(\mathbf{x}_t, y_t)$ is obtained [19], such that

$$\begin{aligned} \mathbf{R}_t &= \alpha \mathbf{R}_{t-1} + \mathbf{x}_t \mathbf{x}_t^\top & (6)\\ \hat{\boldsymbol{\theta}}_t &= \hat{\boldsymbol{\theta}}_{t-1} + \mathbf{R}_t^{-1} \mathbf{x}_t \left(y_t - \mathbf{x}_t^\top \hat{\boldsymbol{\theta}}_{t-1}\right) & (7) \end{aligned}$$

where α is the forgetting factor. In order to avoid inversion problems, inputs $\mathbf{x}$ should be normalized and the matrix $\mathbf{R}$ should be initialized as a diagonal matrix with sufficiently small values.

2) Nonlinear Finite Impulse Response Model: For some external variables, nonlinearities might be inaccurately described by a linear model. A solution is to describe such a variable u by a linear approximation $g(u) \approx \mathbf{b}(u)^\top \boldsymbol{\theta}_b$. As an example, the $\mathbf{b}(u)$ function could consist of a polynomial, Fourier or exponential series. Assuming that the same transformation of u is used at each time delay (in order to lower the number of coefficients needed), the FIR model (2) becomes

$$\begin{aligned} y_t &= \mathbf{x}_t^T \boldsymbol{\theta}_x + \left(g(u_t) \quad \dots \quad g(u_{t-n_u}) \right) \begin{pmatrix} \theta_{u,0} \\ \vdots \\ \theta_{u,n_u} \end{pmatrix} \\ &= \mathbf{x}_t^T \boldsymbol{\theta}_x + \left(\mathbf{b}(u_t)^T \boldsymbol{\theta}_b \quad \dots \quad \mathbf{b}(u_{t-n_u})^T \boldsymbol{\theta}_b \right) \begin{pmatrix} \theta_{u,0} \\ \vdots \\ \theta_{u,n_u} \end{pmatrix} \\ &= \mathbf{x}_t^T \boldsymbol{\theta}_x + \mathbf{B}_t^T \begin{pmatrix} \boldsymbol{\theta}_b & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \boldsymbol{\theta}_b \end{pmatrix} \boldsymbol{\theta}_u \end{aligned} \tag{8}$$

where $\mathbf{B}_t^\top = \left(\mathbf{b}(u_t)^T \ \dots \ \mathbf{b}(u_{t-n_u})^T\right)$ and n_u denotes the finite range of past values of u. The least squares minimization problem is established by vertically concatenating observations of y, $\mathbf{x}_t$, and $\mathbf{B}_t$. Given a specific function $g(u)$, the model parameters are found using a nonlinear solver such as the Levenberg-Marquardt algorithm [20], as the product of the linear model coefficients $\boldsymbol{\theta}_b$ with the lag coefficients $\boldsymbol{\theta}_u$ makes (8) nonlinear in the parameters.

Forecasting is achieved by using the conditional expectation [19], where an additional forecast of u_t up to time $t+k$ is required in order to construct $\mathbf{B}_{t+k|t}$. Recursive and adaptive estimation is achieved for the nonlinear case with the recursive equations

$$\begin{aligned} \mathbf{R}_t &= \alpha \mathbf{R}_{t-1} + \boldsymbol{\psi}_t \boldsymbol{\psi}_t^\top & (9)\\ \boldsymbol{\theta}_t &= \boldsymbol{\theta}_{t-1} + \mathbf{R}_t^{-1} \boldsymbol{\psi}_t \boldsymbol{\epsilon}_t & (10) \end{aligned}$$

where α is the forgetting factor, $\boldsymbol{\psi}_t$ is the gradient with respect to $\boldsymbol{\theta}_t$ of the model output (8), and $\boldsymbol{\epsilon}_t$ is a vector containing residuals at time t; see [21].

3) Auto-Regressive Model With eXogeneous Inputs: An FIR model does not include contributions from past consumption values. In the case where the present output depends on its previous values, autoregressive terms need to be added. The *auto-regressive* model with *eXogeneous* inputs (ARX) is then formed by adding a linear contribution of past output values such that

$$y_t = \left(\mathbf{y}_{t-1}^\top \quad \mathbf{x}_t^\top \right) \begin{pmatrix} \boldsymbol{\theta}_y \\ \boldsymbol{\theta}_x \end{pmatrix} + \epsilon_t \tag{11}$$

where $\mathbf{y}_{t-1}^\top = (y_{t-1}, \dots, y_{t-n_c})$ is the finite range of n_c past outputs and $\boldsymbol{\theta}_y$ its associated coefficients. Using $(\mathbf{y}_{t-1}^\top, \mathbf{x}_t^\top)$ as input vector and $(\boldsymbol{\theta}_y^\top, \boldsymbol{\theta}_x^\top)^T$ as parameter vector, vertically concatenated observations permits the use of the same least squares estimator (3) and adaptive estimator (7). Forecasting is done by taking the conditional expectation [19], i.e.,

$$\begin{aligned} \hat{y}_{t+k|t} &= \mathbb{E}\left[y_{t+k} | \mathbf{y}_t, \hat{\mathbf{x}}_{t+k|t}\right] \\ &= \left(\mathbf{y}_{t+k-1|t}^\top \quad \hat{\mathbf{x}}_{t+k|t}^\top \right) \begin{pmatrix} \boldsymbol{\theta}_y \\ \boldsymbol{\theta}_x \end{pmatrix} \end{aligned} \tag{12}$$

noting that future values (up to time $t+k-1$) of $\mathbf{y}$ are needed, obtained by iteratively generating forecasts for $t+1, t+2, \dots, t+k-1$.

The models above are based on the important assumption that the input prices are independent of the output consumption, technically meaning that there is no feedback from consumption to price. In situations where the price is issued by a market, price and consumption affect each other in a closed-loop system. Decoupling these variables is necessary in order to obtain the price-response. This is achieved by expanding the ARX structure to bivariate outputs (y_1, y_2) (the outputs being consumption and price) [19]

$$\begin{aligned} \left(y_1 \quad y_2 \right)_t = \left(-\mathbf{y}_1^\top \quad -\mathbf{y}_2^\top \right)_{t-1} &\begin{pmatrix} \boldsymbol{\theta}_{y_1 y_1} & \boldsymbol{\theta}_{y_1 y_2} \\ \boldsymbol{\theta}_{y_2 y_1} & \boldsymbol{\theta}_{y_2 y_2} \end{pmatrix} \\ &+ \mathbf{x}_t^\top \left(\boldsymbol{\theta}_{x y_1} \quad \boldsymbol{\theta}_{x y_2} \right) + \boldsymbol{\epsilon}_t. \end{aligned} \tag{13}$$

This can be rewritten into the general linear model (2) where the least squares and adaptive estimators (3) and (7) can be used. This model can easily be extended to include nonlinearly transformed inputs $\mathbf{x}_t$ following the same methodology used to form the nonlinear FIR model (8).

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B. Design of a Price Generator (Controller)

With the objective of adjusting the consumption in real-time using the electricity price, future *expected* deviations from a given reference are sought to be minimized. Because the cost of deviating from the consumption reference varies with time, future expected deviations are penalized with different weights. Furthermore, in order to keep the generated prices constrained, a penalty is associated with deviations from a given price level. This prevents extreme prices when unreachable consumption references are used.

Every time t a control action is to be taken, prices are to be generated up to a horizon K with the objective of following future consumption targets $c^*_{t,k},\ k = 1,\ldots,K$. Even though only the first price is broadcast, generating prices for the whole horizon enables the system to continue broadcasting prices in the event of failure of the price-generator, for example due to unavailable measurements.

Given the set of information $\mathcal{F}_t$ available at time t, minimizing future expected costs over the horizon can be formulated as the following stochastic optimization problem:

$$\begin{aligned}\min_{p_{t,1},\ldots,p_{t,K}} \mathbb{E}\Bigg[\sum_{k=1}^{K} w_{t,k} \left\| c_{t,k}\left(\mathbf{p}_{t,k}, \mathcal{F}_t\right) - c^*_{t,k} \right\|^2 & \\ + \lambda_{t,k} \left\| p_{t,k} - p^*_{t,k} \right\|^2 \Bigg| \mathcal{F}_t \Bigg] & \\ \text{with} \quad \mathbf{p}_{t,k} = p_{t,1},\ldots,p_{t,k} & \end{aligned} \tag{14}$$

where $w_{t,k}$ and $\lambda_{t,k}$ are weights reflecting penalties associated with consumption and price deviations from their respective reference at each look-ahead time k. The coefficient $\lambda_{t,k}$ is introduced in order to stabilize the controller, by penalizing deviations from the reference price. The random variable $c_{t,k}\left(\mathbf{p}_{t,k}, \mathcal{F}_t\right)$ describes the consumption at look-ahead time k as a function of generated prices up to look-ahead time k, given all the information $\mathcal{F}_t$ available at scheduling time t. This information set contains measurements of explanatory variables needed to describe the future consumption $c_{t,k}$, including previously generated prices.

Using the expectation property $\mathbb{E}[\cdot^2] = \mathbb{E}[\cdot]^2 + \mathrm{Var}[\cdot]$ (from the definition of variance) and as the expectation is a linear operator, the stochastic optimization problem (15) can be rewritten as

$$\begin{aligned}\min_{p_{t,1},\ldots,p_{t,K}} \sum_{k=1}^{K} \Bigg(& w_{t,k} \Big(\mathbb{E}\left[c_{t,k}\left(\mathbf{p}_{t,k}, \mathcal{F}_t\right) | \mathcal{F}_t\right] - c^*_{t,k} \Big)^2 \\ & + w_{t,k} \Big(\mathrm{Var}\left[c_{t,k}\left(\mathbf{p}_{t,k}, \mathcal{F}_t\right) | \mathcal{F}_t\right] \Big) \\ & + \lambda_{t,k} \left(p_{t,k} - p^*_{t,k}\right)^2 \Bigg). \end{aligned} \tag{15}$$

The conditional expectation $\mathbb{E}\left[c_{t,k}(\cdot)|\mathcal{F}_t\right]$ is the optimal predictor under such a quadratic loss function, while the conditional variance $\mathrm{Var}\left[c_{t,k}(\cdot)|\mathcal{F}_t\right]$ gives its associated uncertainty [19]. Minimizing (15) therefore implies finding the optimal predictor (forecasting model) for the consumption together with its associated uncertainty. Assuming that the uncertainty is the same for every price value, the variance term can be ignored, and (15) can be solved by setting the price derivative to zero, yielding a solution equivalent to a generalized predictive controller [22]; see [21] for details.

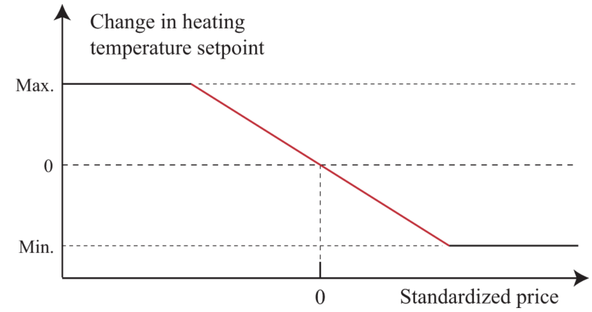

Fig. 2. Change in price yields a change of the heating setpoint within a certain comfort zone. Transformation of the price into a *standardized price*, centred around zero, is needed in order to assess how high or low a price is.

The resulting controller is a closed-loop controller, which can have a potentially long (and maybe unknown) time delay as grid measurements might not instantaneously be available for the controller. This can be handled by considering controllers specially designed for this; see [23].

Note that the consumption forecasting model is not required to be separable into responsive and unresponsive load, but rather must have an explicit dependency on the price variable.

IV. Application to Price-Responsive Heating Systems

Inspired by concepts and ideas developed during the Olympic Peninsula project [16], we here present a possible approach to how heating systems can be used to bring flexibility into the power system.

A. Flexibility Potential of Heating Systems

Flexible devices are seen as devices with a high inertia, meaning that they can be turned off for a short period with no or very little consequences for the user. Heating systems, such as hot tap water heating and space heating have inertia due to the time constants involved in their dynamics. These systems are actuated by being turned on or off such that their thermostat's temperature follows a certain reference (setpoint), meaning that the device is adequately turned on or off when it reaches extremes of a given temperature band centered around the reference temperature. Controlling the temperature setpoint is therefore a mean to turn on or off such devices during a certain period of time. This period is related to the system's time constant, and therefore characterizes the flexibility potential.

B. Electricity Price Incentive to Activate Flexibility

In a real-time pricing context, users would be willing to adjust their consumption to periods of low prices in order to save money. According to the price of electricity, the temperature setpoint could then be adjusted as long as it stays within a predefined comfort zone (Fig. 2), as inspired by the Olympic Peninsula project [16]. During periods of high price, space heating is for example more likely to be turned off as the temperature reference is lowered. Furthermore, pre-heating takes place during periods of low prices in anticipation of future high prices. As it

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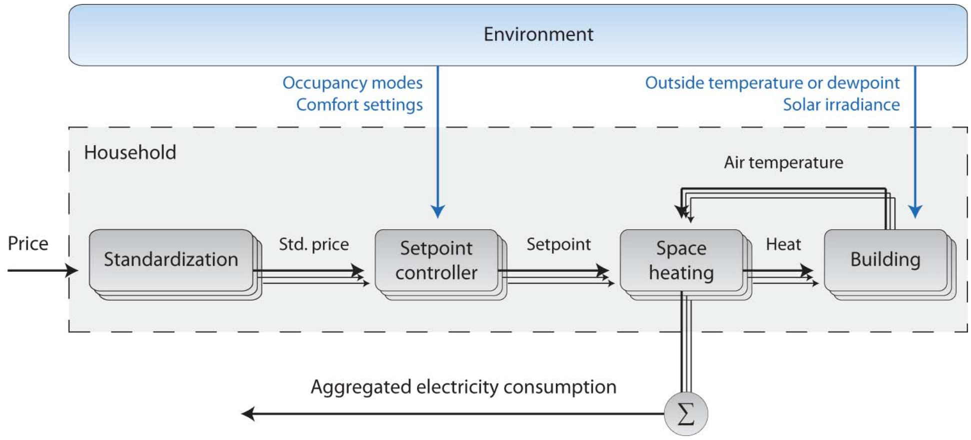

Fig. 3. Price responsive heating systems adjust their consumption based on a filtered price signal, their comfort settings, and their heating needs. The measured aggregated consumption is the sum over all heating systems.

takes a certain time for the system to reach its reference, comfort is barely reduced given that the setpoint is only changed for a short period of time.

Price-responsive devices must assess how high or low the electricity price is, filtering out slow variations (such as monthly or yearly variations). The received price p_t is therefore transformed into a dimensionless *standardized* price ρ_t by comparing it to a reference reflecting the mean price level. Inspired by [10], [16] the standardized price ρ_t is defined as the *increase* in price relative to a reference $\bar{p}_t$

$$\rho_t = \frac{p_t}{\bar{p}_{t-1}} - 1 \tag{16}$$

where the reference $\bar{p}_t$ is computed as an exponentially weighted moving average of past prices

$$\bar{p}_t = \bar{p}_{t-1} + \frac{\Delta t}{\Delta t + \tau} \left(p_t - \bar{p}_{t-1}\right). \tag{17}$$

τ is a smoothing constant accounting for how long a price is remembered and Δt denotes the sampling time. The standardized price has a simple interpretation: when a doubling of the price occurs, the standardized price is +100%. This however assumes that the reference price $\bar{p}_t$ is kept different from zero.

C. Occupancy Modes

Consumers' price responsivity and comfort needs vary during a day, mostly depending on whether or not they are at home. This state is represented by an *occupancy mode*, triggered by the user or according to a schedule [16]. Three occupancy modes are used: night mode, work mode and home mode (Fig. 4). For each device, each occupancy mode defines a setpoint, a price sensitivity and its related comfort bounds (Fig. 2), hereby reflecting varying price responsivities and comfort needs.

The overall setup of a price-responsive heating system is described in Fig. 3.

D. Price-Responsiveness of the Participants of the Olympic Peninsula Project

The Olympic Peninsula project [16] implemented, among others, price-response heating systems as previously described in this section (with a slightly different price standardization method; see [16]).

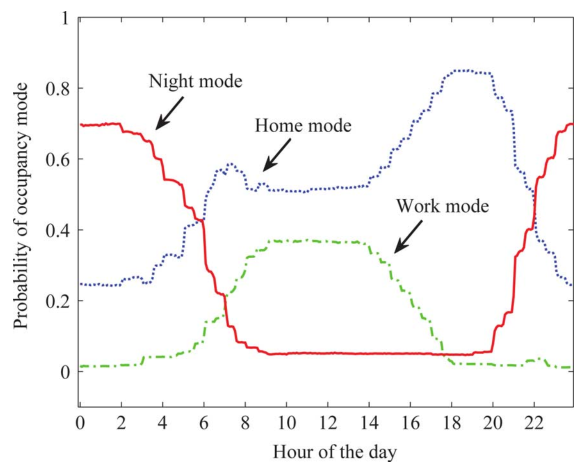

Fig. 4. Time dependent sample distributions approximating the probability of being in one of three occupancy modes from the Olympic Peninsula project [16].

Every 5 min, price-responsive appliances formulated bids expressing their current electricity needs. The aggregated electricity bids (demand) and production capacities of the generators (supply) together with feeder constraints (supply limits) yielded a clearing price that was sent out to customers every 5 min. The calculation of the clearing price was done by intersecting supply and demand curves, leaving the market as the price-generating entity.

As prices were issued by a market, consumption and price both influence one another. Isolating the response of consumption to price is therefore achieved by utilizing the bivariate ARX model (13), using consumption and price as output variables. Price, dewpoint,[1] sun irradiance, and heating temperature setpoint are used as input variables. The model complexity is chosen such that an increase in the number of lags does not significantly improve performances, measured as the coefficient of determination. Hence, $n_c = n_p = 40$ autoregressive lags (10 hours) are used with $n_z = 40$ lags for the external variables, yielding a coefficient of determination of 75%. The

[1]The dewpoint is a measure of temperature associated with humidity.

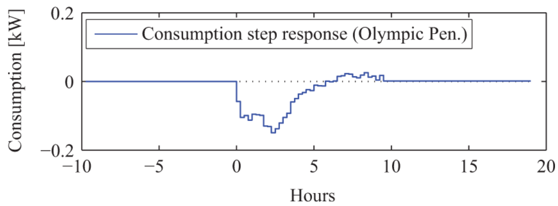

Fig. 5. Response of consumption to a price step after removing the consumption to price feedback. A model complexity of $n_c = n_p = n_z = 40$ lags (10 hours) yields a coefficient of determination of 75%. The selected model shows a slight rebound after 6 h, which in our opinion is negligible. However, situations where a significant rebound is present cannot be excluded, as seen later in Fig. 10. In this sense, the step response has a significant duration of 5–6 h.

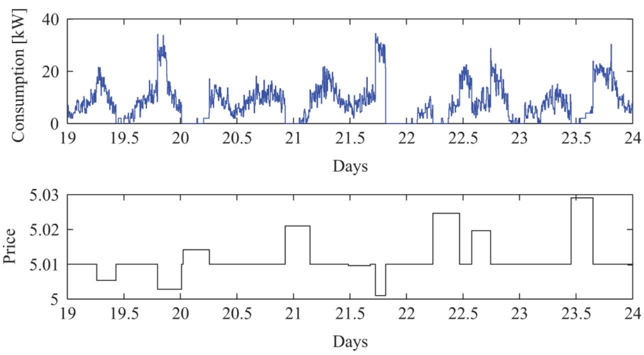

Fig. 6. Consumption's response to price of the heating system is investigated by stimulating the system with price steps of random durations and magnitudes.

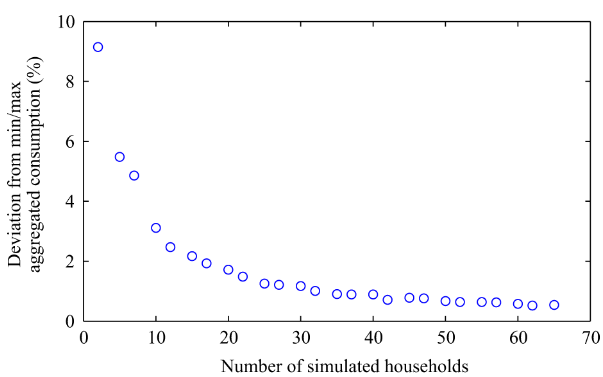

Fig. 7. Standard deviation of the population of household consumptions is averaged over the whole time series, and compared to the span (max-min) of the aggregated consumption, in order to give an indicator of the variability in the data. Twenty houses are considered to be enough to obtain consumption patterns having a mean representative for the population.

found consumption response to a price step has a significant duration of 5–6 h (see Fig. 5).

V. Control-by-Price Experiment

A. Experimental Setup

In the light of testing a consumption control system, a simulation framework is developed implementing the concept described in Section IV. Focussing on the flexible consumption of households, a thermal model for a household is established based on [10], [24], and [25] in the form of a system of differential equations having as states indoor air and building thermal mass temperatures. These are influenced by the outside temperature, sun radiation and space heating system of the households.

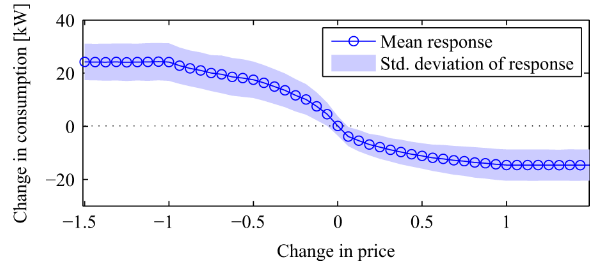

Fig. 8. Mean consumption change after a change in price, averaged over different times of the day, displayed with one standard deviation.

Thermal parameters are functions of the living area, therefore representative populations of the Danish society can be simulated based on its distribution of living areas,[2] The space heating system is assumed to instantly and uniformly heat up the indoor air. Heat pumps are implemented having their output and consumption depending on outside temperature, as seen in [24]. Price-responsivity is implemented by controlling the temperature setpoint as described in Section IV, and human behavior is introduced by randomizing the occupancy modes extracted from the Olympic Peninsula project. The simulated results are then validated against real consumption data; see [21].

Based on weather data from the Danish Meteorological Institute (DMI) for January 2009, the heating consumption of 20 Danish households equipped with heat pumps is simulated with prices broadcasted every 5 min. Price sensitivities and comfort bounds are kept constant over time and across the population of households. Price steps of random durations and magnitudes are generated in order to obtain representative responses in consumption (Fig. 6). Twenty houses are considered to be enough in order to obtain consumption patterns having a mean representative for the population (Fig. 7).

B. Estimation of the Price-Response

The real response of the system can be investigated by exciting the simulated system with different prices. Keeping the price constant, two days are simulated in order to ensure convergence from initial conditions. Using the final state of the system, various changes of prices are simulated for one sample ahead. As a reference case, no price change is performed. The resulting consumption is measured, assuming that all internal variables have not had time to change significantly during one sample, and compared to the reference case (Fig. 8). Because the price during the two first days was kept constant to 1, the change in price is actually the standardized price.

A nonlinear immediate price response is observed, with responsivity saturations for price changes higher than 1 and lower than -1. The response also depends on time of the day, with maximum responsivity during periods of high heating demands (the morning and evening peak) [21].

An FIR model (2) is sufficient to describe the price response of the aggregate of households, found to depend on outdoor temperature, sun irradiance and heating temperature setpoint. A coefficient of determination of 59% is achieved using $n_p = n_z =$

[2]Available at Statistics Denmark http://www.dst.dk/HomeUK.aspx.

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

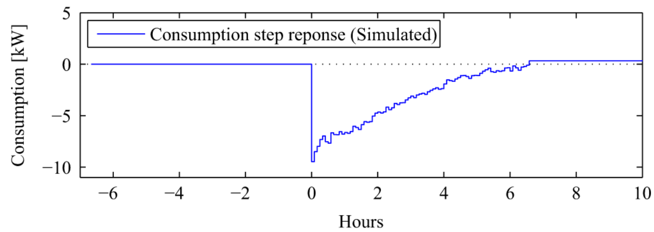

Fig. 9. Extracted response of consumption to a step of price with an FIR model using price, outside temperature, sun irradiance, and heating temperature setpoint as inputs. A model complexity of $n_p = n_z = 80$ lags (6.7 h) yields a coefficient of determination of 59% with a noticeable price effect of approximately 5–6 h.

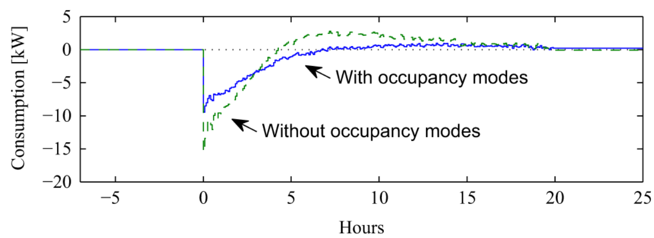

Fig. 10. Using a model complexity of $n_p = n_z = 240$ lags (20 h), it is observed that occupancy modes reduce the effect of consumption shifting.

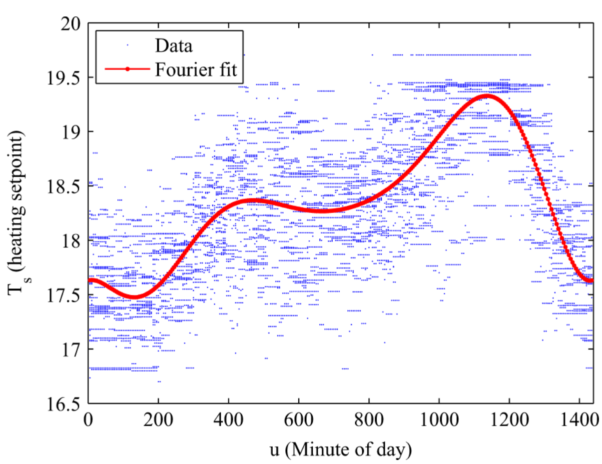

Fig. 11. Linear model with Fourier basis expansion using the four first harmonics corresponding to periods of 24, 12, 6, and 3 h sufficiently represents the heating setpoint by minute of the day (Olympic Peninsula data).

80 lags (6.7 h). The noticeable duration of the response is also found to be 5–6 h (Fig. 9).

It should however be noted that this response contains both the price standardization procedure of Section IV and the physical response of the system. Furthermore, occupancy modes enabled a consumption reduction during periods of high prices, without entailing a noticeable increase at a later time.

The models used for the price-responsivity are based on the heating setpoint variable, which is an internal variable, thus requiring a two-way communication system with households. As the heating temperature setpoint exhibits a functional relation to the minute of the day variable (Fig. 11), it can be replaced by the latter, being an external variable. The minute of day variable is therefore nonlinearly transformed, and is approximated by a linear model using a Fourier series with 4 harmonics corresponding to periods of 24, 12, 6, and 3 h (Fig. 11).

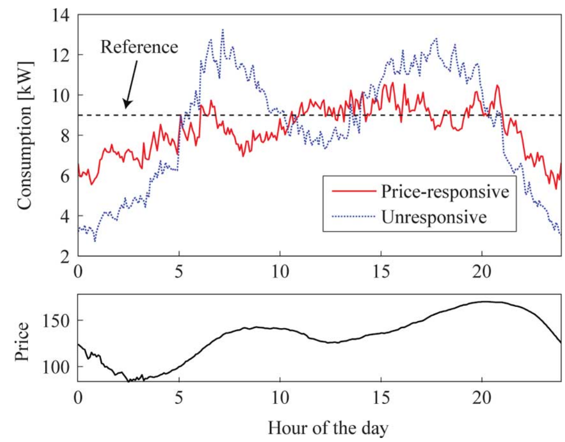

Fig. 12. Proof-of-concept is illustrated by a constant consumption reference, yielding a reduction in peak of nearly 5% and a mean daily shifted quantity of 11%. The price-responsive and unresponsive groups are comparable as controlling by price only decreased the overall consumption over two months simulated by 1% for the unresponsive group.

The FIR model (8) that uses a transformed input can then be used to extract a price-response exclusively based on external variables. Using 40 lags (3.5 h) for every input, the coefficient of determination is reduced only from 59% to 56%, while the number of parameters is halved (from 312 to 168) without any significant change in the response structure.

C. Control by Price Using a Constant Consumption Objective

Two months of consumption data (November and December 2008) are simulated as training data for the forecasting models. A comparative nonresponsive group is simulated in parallel. The consumption target c^* is kept constant, equal to the mean of the training data, but could be chosen equal to the generation of renewables, in order to maximize its penetration. The price level p^* is also kept constant, chosen as the mean of the training prices. Based on the step response extracted, a horizon K of 3.5 h is used, taken as the amount of time during which a change of price significantly influences the consumption. Price and consumption penalties $\lambda_{t,k}$ and $w_{t,k}$ are kept constant during the whole horizon. The penalties $\lambda_{t,k}$ on the deviations from reference prices were chosen sufficiently large to prevent prediction errors from causing instabilities in the controller.

As a result, a reduction in peak consumption of nearly 5% is observed together with a mean daily consumption shift of 11% (Fig. 12). The price-responsive and unresponsive groups are comparable because controlling by price only decreased the total consumption by 1%.

VI. Conclusions and Further Work

In this paper, several models are proposed to describe the dynamics of the price-elasticity of consumers, used to control their consumption using prices. The estimation of the price-response is carried out using data from the Olympic Peninsula project [16], demonstrating the applicability of the method. A price generator (controller) is designed based on a forecasting model of the price-response, continuously updated to track its time-varying changes. With the objective of maintaining constant consumption, a significant amount is shifted and peaks are flattened (Fig. 12). Even though the control signal is not followed perfectly, this presents a significant reduction of back-up power capacity that is necessary for integrating large amounts of fluctuating energy, and reduces the grid reinforcement investments required.

The presented control-by-price approach has been conceptualized with flexibility, implementability, and cost-efficiency in mind. It can be used in a restricted environment between, e.g., an electricity trader and his flexible customers, or in an larger environment between, e.g., the TSO and flexible customers (provided that the proper market structures exist). It can be extended to include price-responsive generation, or to generate nodal prices reflecting grid congestion. As no *a priori* knowledge about consumers is required, if the aggregated consumption is measurable at grid level, then a one-way communication system suffices (namely the broadcasting of prices), thereby reducing deployment costs. As long as adaptive estimation is used to track the time-varying changes, the system is self-learning, meaning, e.g., that the population of consumers and devices can grow and vary in diversity.

From a TSO or DSO perspective, the fact that price signals should be used to optimize power system operations in real time should act as a strong incentive for them to account for new information on the system's state (grid failures, generation forecasts, etc.) as quickly as possible. Therefore, it is expected that prices would adjust accordingly so as to reflect all available information, public and/or private, in the sense of an efficient market.

Future developments are envisaged in three complementary directions. First of all on the prediction side, the models proposed for the price-response of the consumers may be improved by generalizing the assumed linear relationship between price and consumption, for instance in order to account for saturations in the measured price-response (Fig. 8). A measure of the amount of available flexibility in the system would then be given by the estimated saturation bounds, representing the maximum increase and decrease in consumption that can be achieved at all times. These models should also be generalized to apply to other load types with more complex consumption patterns, e.g., electric vehicle charging systems.

Secondly concerning the control aspects, the risk associated with the uncertainty of the users' response to prices should be taken into account when generating price signals. This means that uncertainties in the predicted explanatory variables, and subsequently the uncertainty in the price-elasticity forecast itself, need to be accounted for. Resulting controllers would then not only seek to minimize expected costs, but also integrate a risk-related measure, e.g., the value at risk (VAR).

Finally, the development of better price-forecasting models and price-generating controllers calls for new experiments with a wide-range of demand-side management set-ups exposed to a real-time price signal, as simulations are limiting because the human factor needs to be accounted for. Such new experiments could reveal ways to improve consumers' flexibility. Furthermore, user acceptance of such a technology might require specific pricing contracts that reduce the financial risk taken by consumers exposed to varying prices. This, however must take into account consumers' loss of interest in participating when the price-generating entity takes on the risk.

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Olivier Corradi (A’12) was born in Neuilly, France, in 1987. He received the B.Sc. degree in mathematics and technology from the Technical University of Denmark, Lyngby, Denmark, in 2009, the M.Sc. degree in mathematical modelling and computation from the Department of Informatics and Mathematical Modelling of the Technical University of Denmark in 2011, and graduated from Ecole Centrale Paris, Paris, France, as part of a dual degree in 2011.

He is currently working as a research intern at the IBM Zürich Research Lab. His research interests include among others pattern recognition, forecasting, optimization, and renewable energies.

Henning Ochsenfeld was born in Winterberg, Germany, in 1984. He received the B.Eng. degree in electrical engineering from the University of Applied Sciences, Oldenburg, Germany, in 2009 and the M.Sc. degree in mathematical modelling and computation from the Department of Informatics and Mathematical Modelling of the Technical University of Denmark, Lyngby, Denmark, in 2011.

He is currently working within the unit of Energy Automation of the Siemens Smart Grid division. His research interests include digital signal processing, machine learning, and pattern recognition.

Henrik Madsen received the Ph.D. degree in statistics at the Technical University of Denmark, Lyngby, Denmark, in 1986.

He was appointed Professor in mathematical statistics in 1999. He is an elected member of the International Statistical Institute (ISI), and he has participated in the development of several ISO and CEN standards. His research interests includes analysis and modeling of stochastic dynamics systems, signal processing, time series analysis, identification, estimation, grey-box modeling, prediction, optimization, and control, with applications mostly related to energy systems, informatics, environmental systems, bioinformatics, biostatistics, process modeling, and finance.

Pierre Pinson (M’11) received the M.Sc. degree in applied mathematics from the National Institute for Applied Sciences (INSA Toulouse, France) and the Ph.D. degree in energy from Ecole des Mines de Paris, Paris, France.

He is an Associate Professor in stochastic energy systems at the Informatics and Mathematical Modeling Department of the Technical University of Denmark, Lyngby, Denmark. His research interests include among others forecasting, uncertainty estimation, optimization under uncertainty, decision sciences, and renewable energies.