An Integrated Multiperiod OPF Model with Demand Response and Renewable Generation Uncertainty

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***Abstract*—Renewable energy sources such as wind and solar have received much attention in recent years, and large amount of renewable generation is being integrated to the electricity networks. A fundamental challenge in a power system operation is to handle the intermittent nature of the renewable generation. In this paper we present a stochastic programming approach to solve a multiperiod optimal power flow problem under renewable generation uncertainty. The proposed approach consists of two stages. In the first stage, operating points for the conventional power plants are determined. The second stage realizes generation from the renewable resources and optimally accommodates it by relying on the demand-side flexibilities and limited available flexibilities from the conventional generating units. The proposed model is illustrated on a 4-bus and a 39-bus system. Numerical results show that with small flexibility on the demand-side substantial benefits in terms of re-dispatch costs can be achieved. The proposed approach is tested on all standard IEEE test cases upto 300 buses for a wide variety of scenarios.**

***Index Terms*—Demand response; optimal power flow; power system modelling; linear stochastic programming; smart grids; uncertainty; wind energy.**

NOMENCLATURE

Sets

$\mathscr{B}$	Buses, indexed by b.
$\mathscr{L}$	Lines (edges), indexed by l.
$\mathscr{G}$	Generators, indexed by g.
$\mathscr{W}$	Renewable generators, indexed by w.
$\mathscr{D}$	Loads, indexed by d.
$\mathscr{D}_0$	Flexible loads, $\mathscr{D}_0 \subseteq \mathscr{D}$.
$\mathscr{B}_l$	Buses connected by line l.
$\mathscr{L}_b$	Lines connected to bus b.
$\mathscr{G}_b$	Generators located at bus b.
$\mathscr{D}_b$	Loads located at bus b.
$\mathscr{S}$	Scenarios, indexed by s.
$\mathscr{T}$	Discrete set of time intervals, indexed by t.
$\mathscr{T}_d^{\mathrm{F}}$	Flexibility windows for demand d, $\mathscr{T}_d^{\mathrm{F}} \subset \mathscr{T} \times \mathscr{T}$.

Parameters

b_l	Susceptance of line l.
τ_l	Off-nominal tap ratio of line l.
$P_g^{\mathrm{G}-}, P_g^{\mathrm{G}+}$	Min., max. real power outputs of conventional generator g.
$P_{d,t}^{\mathrm{D}}$	Real power demand of load d.
$f_{g,t}(p_{g,t}^{\mathrm{G}})$	Cost function for generator g.
$P_{w,t}^{\mathrm{W}}$	Initial forecast for real power generation availability from generator w in time period t.
$\Delta P_{w,s,t}^{\mathrm{W}}$	Change in generation availability under scenario s from generator w in time period t.
$\lambda_{w,s}$	Probability of scenario s.
$C_{w,t}^{\mathrm{W}}$	Cost of renewable generation spillage.
$F_{d,t}^{-}, F_{d,t}^{+}$	Min., max. load flexibility of demand at bus d.
$\Delta P_{g,t}^{-}, \Delta P_{g,t}^{+}$	Min., max. change in operating point of generator g during time period $[t, t+1]$.
$R_{g,t}^{-}, R_{g,t}^{+}$	Min., max. regulation of generator g.
$C_{g,t}^{R-}, C_{g,t}^{R+}$	Downward, upward regulation cost for generator g.
$C_{d,t}^{D-}, C_{d,t}^{D+}$	Cost of decreasing, increasing demand in the time period t.
$P_l^{\max}$	Max power flow capacity of line l.

Variables

$p_{g,t}^{\mathrm{G}}$	Real power output of generator g.
$\Delta p_{g,s,t}^{\mathrm{G}}$	Second stage recourse variable for real power output of generator g.
$\Delta p_{g,s,t}^{\mathrm{G+}}$, $\Delta p_{g,s,t}^{\mathrm{G-}}$	Upward, downward regulation variables for real power output of generator g.
$p_{w,s,t}^{\mathrm{W}}$	Real power output of renewable generator w.

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$\theta_{b,s,t}$	Voltage phase angle at bus b.
$p^{\mathrm{L}}_{l,s,t}$	Real power injection at bus b into line l (which connects buses b and b').
$p^{\mathrm{D}}_{d,s,t}$	Real power delivered at bus d.
$\alpha_{d,s,t}$	Proportion of load delivered at bus d.
$\alpha^{+}_{d,s,t}$, $\alpha^{-}_{d,s,t}$	Variables for increase, decrease in demand supply at bus d.

Acronyms

(M)OPF	(Multiperiod) optimal power flow.
(S)UC	(Stochastic) unit commitment.
RES	Renewable energy sources.
TSO	Distribution system operator.
DSO	Distribution system operator.
LMP	Locational marginal price.
ILC	Interpretable load control.
DR	Demand response.

I. Introduction

ELECTRICITY networks around the world are evolving at a rapid pace. This change is happening because of the increased emphasis on clean and renewable energy sources. Large-scale renewable energy sources (RES) are encouraged by different incentive schemes to mitigate the climate related issues. Many countries are investing substantial resources in planning and expanding current infrastructure to cope with the RES integration. Wind power generation is the most widely used source of renewable energy and it has been integrated in many power systems around the world [1], [2], while solar power is catching up at a rapid pace.

Non-dispatchable nature of wind power introduces additional costs stemming from the management of intermittency [3], [4]. Extra reserves are required, at an additional cost, in order to hedge against the uncertainty from the partly predictable wind power generation. Despite the advancements in forecasting methodologies and tools, on average hour-ahead forecast errors for a single wind farm may be as high as 10%-15% of its expected output [5]. This effect of forecast errors is expected to become more pronounced as the share of renewable energy increase in the power networks.

In contrast, demand at a transmission level has a large base component that can be predicted accurately. In power systems optimization problems, electricity demands typically are modelled as inelastic. However in reality a substantial amount of electricity demand is elastic [6]. Electric demands such as plug-in electric vehicles (PEV) charging, district heating and heating ventilation and air conditioning (HVAC) systems are elastic demands which constitute considerable percentage of the total demand *e.g.*, more than one third of the US residential demand is flexible [7]. Majority of these demands are *deferrable* meaning that part of a demand can be shifted in time while respecting deadlines and rate constraints [6].

Demand response (DR) is a way to utilize electricity demand as a resource to increase efficiency and reliability of an electricity network [8]. In the helms of power systems research DR is an active area of research and there is a vast amount of prior work in this domain (*e.g.* see [9] and references therein). Demand response is generally characterized as *price-based* DR and *incentive-based* DR [10]. Demand response programs are generally managed by the distribution companies (DSO) or entities other than transmission system operator (TSO). Electricity is traded on a transmission level and therefore linking demands through DR programs with the decisions made at the transmission level is of vital importance. Most of the current literature, if not all, focus on price based DR while modelling transmission level optimization problems [9], [11]–[14]. Practicality and benefits of such optimization models incorporating price based DR are not very clear, especially in view of reliability and volatility of power systems [15].

In this paper we revisit the multi-period optimal power flow problem and propose a two-stage stochastic model that incorporates demand as a flexible asset and minimize total cost of generation while considering uncertainties in generation from the renewable sources. Next two subsections give an overview of the relevant literature and our contribution in this area, respectively.

A. Literature review

In existing literature, traditional formulations of the OPF problem has been extended to account for variable and partly-predictable nature of the wind power generation [3], [16]–[18]. These papers capture the intermittent nature of wind power generation using different probabilistic techniques and determine a robust operating point of the conventional generating units to meet a constant demand. With stronger focus on the demand side, authors in [19] consider demand-side participation as well as uncertainty in the demand bids. Authors in [20] extended the optimal power flow problem to a two-stage stochastic optimization problem, where the decision problem is to find a steady-state operating point for large generation units in the first stage, while scheduling fast-response generation in the second stage of the problem. Uncertainty in renewable generation is captured by using a set of scenarios. Demand is assumed to be deterministic and the problem is not time coupled. This means that the optimal operating point is independent of the temporal characteristics of the system.

A stochastic unit commitment model is presented in [21] in which authors investigate the impact of large-scale wind power integration into a power system. Uncertainty in wind generation is addressed by means of scenarios and demand is considered to be fixed. Recently, a very flexible approach is presented in [22]. Authors of [22] present a stochastic optimization framework for day-ahead operation of a electricity network. Their model includes unit-commitment, (N-1)-security constraints and a model of electricity storage. Flexible demands are modelled as dispatchable loads, which means that a demand can be shed at a price. We note

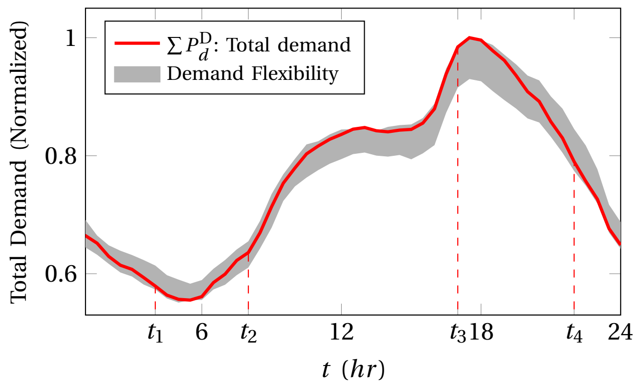


Fig. 1. Flexibility in an aggregated demand. Total demand should be conserved in the time windows $[t_1, t_2]$ and $[t_3, t_4]$.

that this is equivalent to load shedding and not to demand response.

Much of the relevant work in this area focus on either modelling of uncertainty in renewable generation or modelling of demand response. To best of our knowledge, there are very few papers which deals with both of these aspects in a general framework. Authors in [22] also allude to the sparse literature on this problem.

B. Our contributions

In this paper, we present an optimization model that considers the flexibility offered in demand bids from DSOs and optimally utilizes this flexibility by minimizing the total cost of generation. We propose that DSOs within a transmission network provide inelastic demand bids along with a flexibility interval; which means that the demand bids are elastic to a certain level. Such flexibility can be achieved by a DSO's own DR programs. Fig. 1 shows a possible scenario of such a flexibility on an aggregated demand from a single DSO. If such information is available to a TSO then the decision problem is to optimally utilize the generation from renewable sources while given flexibility in demand.

The main contributions of this paper are twofold:

- a revisited multi-period OPF formulation with integrated demand response and renewable generation uncertainty;
- a rigorous mathematical model of demand response.

The optimization based formulation concerns demand response and uncertainty in generation. In contrast to recent approaches, we give a complete mathematical model of demand response and also formulate constraints for considering demand shift and conservation of demand for a given time period. In spite of the generality, the proposed model is computationally efficient and the approach holds promise. The proposed approach can also be used as a tool to project future LMPs given demand side flexibilities. The projected prices are useful information for distribution companies, and they can use this information to plan their demand response strategies [10]. Finally we provide wind scenarios and network data of all the numerical results presented in this paper in an online archive at [23].

The remaining sections of this paper are outlined as follows. Section II gives the formulation of the problem. Numerical results are given in section III. We give conclusions in section IV.

II. Problem Formulation

We propose a two-stage stochastic programming formulation of a multiperiod OPF problem. In this paper the two-stage stochastic formulation is presented in a deterministic equivalent form [24]. In the first stage, decisions are made about the dispatch from conventional generators. The second stage realizes the generation from renewable sources. Any resulting supply-demand mismatch is alleviated by the DR from flexible demands and slight adjustments of the operating points of conventional generators. We assume that DSOs can bid demand along with the flexibility for each time interval in a given time horizon. Transmission system operator can either meet the demand or can use the flexibility (by paying a price of using flexibility of a demand) to accommodate the uncertainty from renewable generation.

Consider a power network with the set of buses $\mathscr{B}$. Let $\mathscr{W}$ denote the set of renewable generators in the network. Since the real power generation from the renewable generators is uncertain, let $\mathscr{S}$ be the set of real power generation scenarios of these generators. We assume zero marginal price of the generation from renewable generators. Let $\mathscr{G}$ be the set of conventional power plants. Let $\mathscr{T} := \{1, 2, \cdots, T\}$ be the set of give time horizon. Following we give constraints and objective function of our two stage stochastic multiperiod OPF problem.

A. Power flow

Let $p^{\mathrm{G}}_{g,t}$ be the real power generation from the conventional generator g in the time interval t. The power balance equations are given as, $\forall b \in \mathscr{B}, s \in \mathscr{S}, t \in \mathscr{T}$:

$$\sum_{g \in \mathscr{G}_b} \left(p^{\mathrm{G}}_{g,t} + \Delta p^{\mathrm{G}}_{g,s,t} \right) + \sum_{w \in \mathscr{W}_b} p^{\mathrm{W}}_{w,s,t} = \sum_{d \in \mathscr{D}_b} p^{\mathrm{D}}_{d,s,t} + \sum_{l \in \mathscr{L}_b} p^{\mathrm{L}}_{l,s,t} \quad (1)$$

where $p^{\mathrm{W}}_{w,s,t}$ denotes the real power output taken from the renewable generator w, $p^{\mathrm{D}}_{d,s,t}$ denotes the real power delivered to the demand d and $p^{\mathrm{L}}_{l,s,t}$ is the flow of real power in the line l in the time period t in the case when scenario s is realized, respectively. The power flow equations are given as, $\forall l \in \mathscr{L}, s \in \mathscr{S}, t \in \mathscr{T}$:

$$p^{\mathrm{L}}_{l,s,t} = -\frac{b_l}{\tau_l} \left(\theta_{b,s,t} - \theta_{b',s,t} \right) \quad (2)$$

where b and b' are two ends of the line l. Voltage angles at the two ends of the line $l = (b, b')$ are denoted by $\theta_{b,s,t}$ and $\theta_{b',s,t}$, respectively. We consider the DC model of line flow [25]. This model ignores the line losses and the reactive power in a network. We have made this assumption in order to keep the formulation linear. Second stage recourse variables $\Delta p^{\mathrm{G}}_{g,s,t}$ in (1) are modelled in terms of the upward and the downward regulation variables as follows:

$$\Delta p^{\mathrm{G}}_{g,s,t} = \Delta p^{\mathrm{G+}}_{g,s,t} - \Delta p^{\mathrm{G-}}_{g,s,t} \tag{3a}$$

$$0 \le \Delta p^{\mathrm{G+}}_{g,s,t} \le R^{+}_{g,t} \tag{3b}$$

$$0 \le \Delta p^{\mathrm{G-}}_{g,s,t} \le R^{-}_{g,t} \tag{3c}$$

where $R^{+}_{g,t}$, $R^{-}_{g,t}$ are the permissible upward and downward regulation of generator g in the time period t, respectively.

B. Demand Model

Let $\mathscr{D}$ denote the set of real power demands and we assume that a distribution network is attached to each bus $d \in \mathscr{D}$. The demand at distribution network is aggregated and is denoted by $P^{\mathrm{D}}_{d,t}$. We assume that each distribution company at the demand bus d know about the flexibility of their demand during the time interval t. This flexibility can either come from distribution company's direct control over some demands or from its DR programs.

Let $\alpha_{d,s,t}$ be the proportion of load delivered at the bus d at the time period t if the scenario s is realized. Let $[F^{-}_{d,t}, F^{+}_{d,t}]$ be the flexibility interval of the demand at bus d and at the time period t. The flexibility interval is defined around $\alpha_{d,s,t} = 1$ and therefore $0 \le F^{-}_{d,t} \le 1$ and $F^{+}_{d,t} \ge 1$. If demand at bus d is not flexible then $F^{-}_{d,t} = F^{+}_{d,t} = 1$ are used. If demand at bus d is flexible then it is placed in the set $\mathscr{D}_0 \subseteq \mathscr{D}$.

The demand model is given by following set of constraints:

$$p^{\mathrm{D}}_{d,s,t} = \alpha_{d,s,t} P^{\mathrm{D}}_{d,t} \tag{4a}$$

$$0 \le F^{-}_{d,t} \le \alpha_{d,s,t} \le F^{+}_{d,t} \tag{4b}$$

$$\alpha_{d,s,t} = 1, \forall d \in \mathscr{D} \setminus \mathscr{D}_0 \tag{4c}$$

where $(1 - F^{-}_{d,t})$ is the proportion of demand d which is flexible in the time interval t, and $(F^{+}_{d,t} - 1)$ is the proportion of load that can be increased in the time interval t.

1) Cost of demand response: We introduce two positive continuous variables $\alpha^{+}_{d,s,t}$, $\alpha^{-}_{d,s,t}$ which gives the p.u. increase and decrease in the amount of real power delivered to the demand bus d respectively. These variables are modelled linearly as:

$$\alpha_{d,s,t} \quad = 1 + \Delta\alpha_{d,s,t} \tag{5a}$$

$$\Delta\alpha_{d,s,t} = \alpha^{+}_{d,s,t} - \alpha^{-}_{d,s,t} \tag{5b}$$

$$0 \le \alpha^{+}_{d,s,t} \le F^{+}_{d,t} - 1 \tag{5c}$$

$$0 \le \alpha^{-}_{d,s,t} \le 1 - F^{-}_{d,t} \tag{5d}$$

Let $C^{\mathrm{D+}}_{d,t}$ and $C^{\mathrm{D-}}_{d,t}$ be the cost of upward and downward regulation of the demand d during the time interval t, respectively. Cost of the demand response for a single time period is given by $(C^{\mathrm{D+}}_{d,t}\alpha^{+}_{d,s,t} + C^{\mathrm{D-}}_{d,t}\alpha^{-}_{d,s,t})P^{\mathrm{D}}_{d,t}$. Since the cost of upward/downward regulation is strictly positive and one of the objectives in our optimization problem is to minimize the cost of demand response therefore both the upward and the downward regulation variables for a demand cannot be nonzero during any time period at an optimal solution. For example, if an optimal decision is to increase a demand at bus d by 10% during the time period t then the optimal decision variables would take up the values $\alpha^{+}_{d,s,t} = 0.1$ and $\alpha^{-}_{d,s,t} = 0.0$ with the cost of $0.1C^{\mathrm{D+}}_{d,t}$. A feasible solution for this situation could be $\alpha^{+}_{d,s,t} = 0.2$ and $\alpha^{-}_{d,s,t} = 0.1$ but then the cost of this solution is $0.2C^{\mathrm{D+}}_{d,t} + 0.1C^{\mathrm{D-}}_{d,t}$ which is obviously higher then the optimal solution cost of $0.1C^{\mathrm{D+}}_{d,t}$.

2) Conservation of demand: If a demand at a bus d is flexible in the time window $[t_s, t_f]$ and it is required that the total consumption over a time period is kept constant then this situation can be modelled using following linear equations: $\forall d \in D_0$, $[t_s, t_f] \in \mathscr{T}^{\mathrm{F}}_{d} = \{[t_s, t_f] : t_s, t_f \in \mathscr{T}, t_s < t_f\}$:

$$\sum_{t=t_s}^{t_f} p^{\mathrm{D}}_{d,s,t} = \sum_{t=t_s}^{t_f} P^{\mathrm{D}}_{d,t} \tag{6}$$

where $\mathscr{T}^{\mathrm{F}}_{d}$ is the set of flexibility windows for demand at bus d. For example in Fig. 1 we have $\mathscr{T}^{\mathrm{F}}_{d} = \{[t_1, t_2], [t_3, t_4]\}$.

Optimization model would decide the amount of demand to be consumed in each time interval. Note that we assume that there is enough power to support a task which requires more than one time interval to finish. This assumption is justifiable because of the lower bound on the value of $\alpha_{d,s,t}$. Otherwise it is possible to impose a constraint coupled in time. We have assumed that flexibility can be utilized in any way across the time interval. In practice the flexibilities depend on the type of demands *e.g.,* some demands might need up and down times, and charging/discharging rates. All these technical details can be modelled using linear constraints. However technical details and discussion on this subject is out of the scope of this paper.

C. Operating constraints

Generation from the conventional generators is bounded by the following inequality constraints:

$$P^{\mathrm{G-}}_{g} \le p^{\mathrm{G}}_{g,t} + \Delta p^{\mathrm{G}}_{g,s,t} \le P^{\mathrm{G+}}_{g} \tag{7}$$

where $P^{\mathrm{G-}}_{g}, P^{\mathrm{G+}}_{g}$ are the lower and upper bounds on the generation output of generator g, respectively.

In the short time scales it is not possible for a conventional generator g to considerably deviate from its current operating point [20]. Therefore we limit the amount of change in generation depending on the ramp rate of individual generators. The ramp rate constraints are given as:

$$\Delta P^{-}_{g,t} \le p^{\mathrm{G}}_{g,t+1} - p^{\mathrm{G}}_{g,t} \tag{8a}$$

$$p^{\mathrm{G}}_{g,t+1} - p^{\mathrm{G}}_{g,t} \le \Delta P^{+}_{g,t} \tag{8b}$$

The line flow limits are given by following set of constraints: $\forall l \in \mathscr{L}, t \in \mathscr{T}, s \in \mathscr{S}$

$$-P^{\max}_{l} \le p^{\mathrm{L}}_{l,s,t} \le P^{\max}_{l} \tag{9}$$

where $P^{\max}_{l,t}$ is the real power capacity limit of the line l.

D. Scenarios of renewable energy generation

Forecasting of renewable energy generation is a very active area of research, especially for wind and solar energy applications. While forecasts were traditionally provided in the form of single-valued trajectory informing of expected generation for every lead time and location of interest, individually, emphasis is now placed on probabilistic forecasts in various forms [26]. For decision problems where the space-time dependence structure of the uncertainty is important, forecasts should optimally take the form of space-time trajectories.

In this paper, scenarios of wind power generation are used as input to the stochastic programming approach to solve the multiperiod OPF problem. The exact setup, data and methods of [26] are employed. A sample of 100 space-time scenarios are generated which will be used for the simulation of results. The scenarios are made available online at [23].

In our simulations we assume zero cost of wind power production [27]. Moreover wind power from source w in time period t can be spilled continuously to zero at the price of $C^{\mathrm{W}}_{w,t}$. Let $P^{\mathrm{W}}_{w,t}$ be the initial forecast of power generation availability and let $\Delta P^{\mathrm{W}}_{w,s,t}$ be the changes in generation availability corresponding to s scenarios for generator w in time period t respectively. Wind power output in each time period t for generator w is modelled as follows:

$$0 \le p^{\mathrm{W}}_{w,s,t} \le P^{\mathrm{W}}_{w,t} + \Delta P^{\mathrm{W}}_{w,s,t} \tag{10}$$

E. Objective function

Let $\lambda_{w,s}$ be the probability of scenario s for the renewable generator w. Objective is to minimize the cost of generation from conventional generators, and optimally utilize the generation from renewable resources while initiating demand response from the distribution system operators. Overall the objective function is to minimize the following over the given time horizon:

$$\begin{aligned} z = \sum_{g\in\mathcal{G}} f(p^{\mathrm{G}}_{g,t}) + \sum_{s\in\mathcal{S}} \lambda_{w,s} \Bigg(& \underbrace{\sum_{w\in\mathcal{W}} C^{\mathrm{W}}_{w,t}\left(P^{\mathrm{W}}_{w,t} - p^{\mathrm{W}}_{w,s,t}\right)}_{\text{Cost of wind spillage}} \\ & + \underbrace{\sum_{d\in\mathcal{D}} \left(C^{\text{D+}}_{d,t}\alpha^{+}_{d,s,t} + C^{\text{D-}}_{d,t}\alpha^{-}_{d,s,t}\right) P^{\mathrm{D}}_{d,t}}_{\text{Cost of demand response}} \\ & + \underbrace{\sum_{g\in\mathcal{G}} \left(C^{\text{R+}}_{g,t}\Delta p^{\text{G+}}_{g,s,t} + C^{\text{R-}}_{g,t}\Delta p^{\text{G-}}_{g,s,t}\right)}_{\text{Cost of generation regulation}} \Bigg) \end{aligned} \tag{11}$$

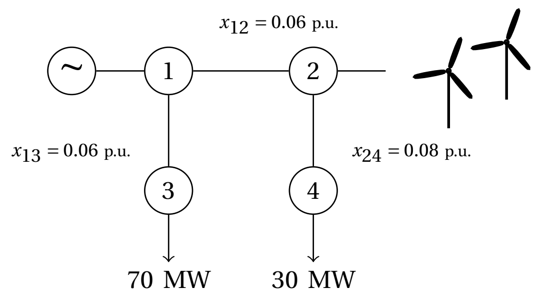


Fig. 2. 4 bus network, with a conventional generator at bus 1 and a wind farm at bus 2.

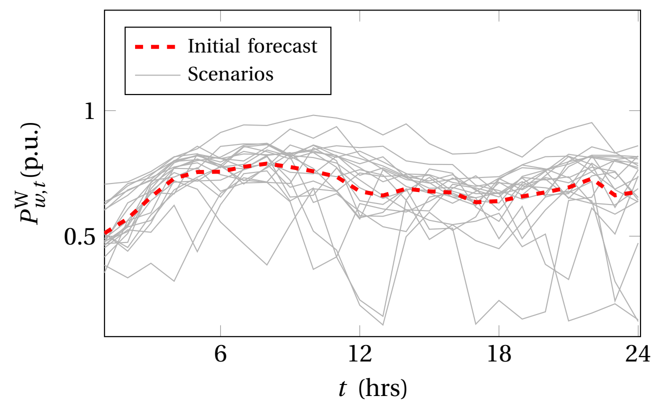


Fig. 3. Initial forecast and 20 scenarios for wind power generation at bus 2.

F. Overall formulation

Overall formulation of the multiperiod optimal power flow problem is given as follows:

$$\min \sum_{t\in\mathcal{T}} z\left(p^{\mathrm{G}}_{g,t}, p^{\mathrm{W}}_{w,s,t}, \alpha_{d,s,t}, \Delta p^{\mathrm{G}}_{g,s,t}\right) \tag{12a}$$

subject to

$$(1-10) \tag{12b}$$

The overall problem is then, depending on the objective function $f(p^{\mathrm{G}}_{g,t})$ is linear or quadratic program (LP or QP). We use CPLEX 12.06 [28] called from an AMPL [29] model to solve the problem.

In principle, other physical and operational constraints such as spinning reserve requirements can be included in the current formulation, and the solution approach that we describe here remains valid.

III. Numerical Example

A. An illustrative example: 4 Bus Case

Consider a small 4 bus network as shown in Fig. 2. The network consists of a generator at bus 1 and a wind farm at bus 2. Total demand of the network is 100 MW. Complete data of this network is available online at [23].

We assume that the time horizon consists of twenty four time periods *i.e.* $\mathcal{T} = \{1, 2, \cdots, 24\}$, as shown in Fig. 1. We assume 20 different scenarios for wind power generation at bus 2 as shown in Fig. 3.

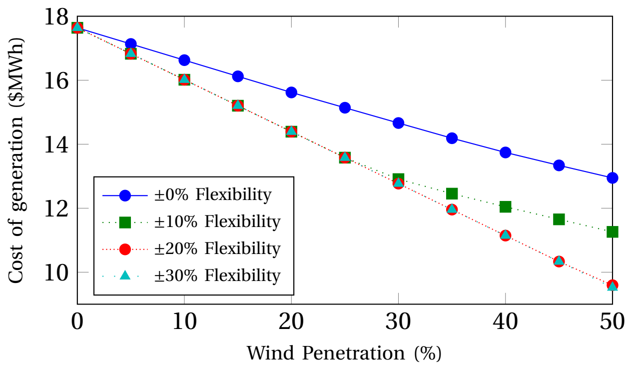
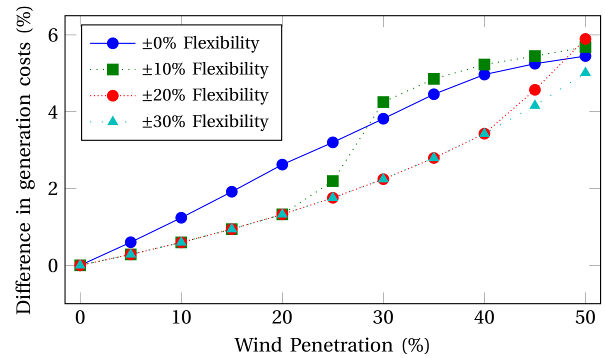
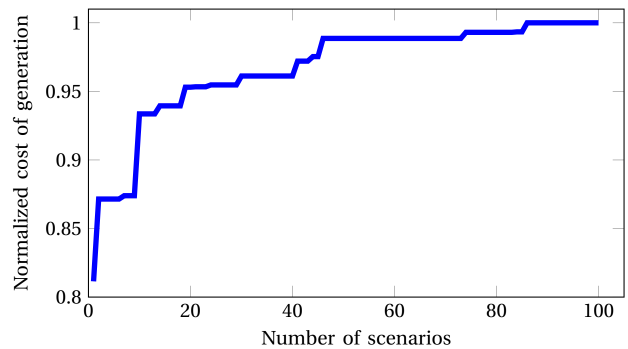


Fig. 4. Generation cost vs wind power penetration for 4 bus network.



(a) Value of Stochastic Solution (20 scenarios vs 100 Scenarios).



(b) Robustness of stochastic solution.

Fig. 5. Robustness of the solutions of 4 bus network with respect to uncertainty in the wind power generation.

Marginal price of the conventional generator at bus 1 is quadratic monotonically increasing function of the real power generation. We assume the cost of wind spillage to be unity and the ramp rate of the generator at bus 1 to be ±10%. It is important to note that if there is no flexibility in the demand then the ramp rate of the generator at bus 1 should be equal or greater than the max rate of change in demand during any given time interval to ensure feasibility of the optimization problem.

Regulation cost of the generator at bus 1 is assumed to be $C^{\text{R+}}_{1,t} = 1.4 > 0.8 = C^{\text{R-}}_{1,t}$. We further impose the constraint that the total demand should be conserved over the given time horizon and the cost of demand response is considered to be $C^{\text{D+}}_{d,t} = C^{\text{D-}}_{d,t} = 0.5$. Fig. 4 shows the cost of generation as a function of wind power penetration in the system. We can observe a general trend that the cost of generation is monotonically decreasing as the wind power penetration in the system is increased. This follows from the fact that we have assumed zero cost of generation from the wind. Even if the cost of wind power generation is non-zero it is much less than the cost of conventional power generation and hence this assumption is justifiable. Further we note that when there is no flexibility from the demand, uncertainty from wind power generation can only be managed by adjusting the generator outputs in the second stage of the problem. In this case as wind power penetration is increased more wind is spilled because the generators cannot be regulated cheaply and rapidly to accommodate the variations from the wind power as they are restricted by the ramping constraints. As we make the demand flexible it can be utilized as a resource for accommodating uncertainties from the wind power generation. Cost of generation decreases when the demand is made more flexible. There is no difference in the cost of generation between ±20% and ±30% demand flexibility. This is because of the fact that tapping on demand as a resource is not economical any more. For this example we can say that for the given ramp rate of ±10% and the wind generation uncertainties, the optimal demand flexibility needed to fully utilize the wind power is ±20%.

Uncertainty in the wind power generation affects the total operating cost of network as uncertainty in wind power generation is balanced by the generation from the conventional power plants. This implies that increasing the number of scenarios (which is equivalent to increasing the uncertainty) also leads to greater operating costs. Fig. 5 shows the robustness of solution depending on the number of scenarios. Fig. 5(a) shows the difference in cost of generation when scenarios are increased from 20 to 100. The difference in cost of generation between 20 and 100 scenarios increases as the wind penetration in the system increase. This is because there is more uncertainty in generation from wind for 100 scenarios as compared to 20 scenarios. However the difference between cost of generation, for given demand flexibilities and penetration levels, is always less than 6% (corresponding to 200% increase in number of scenarios), which shows that solution corresponding to 20 scenarios is quite robust to the changes in wind generation scenarios. Fig. 5(b) shows the monotonic increasing trend of generation cost as the number of scenarios are increased. There is a trade-off between the number of scenarios and capturing uncertainty of the renewable generation. We note that the curve in Fig. 5(b) smooths off as the number of scenarios are increased, which shows that after a certain point including more scenarios will not have significant effect on the optimal solution.

B. 39 Bus Case

Consider the 39 bus New England test network obtained from [30]. This test network consists of 39 buses, 10 generators, and 46 transmission lines. We modify the network as follows. We consider 8 conventional generators, and

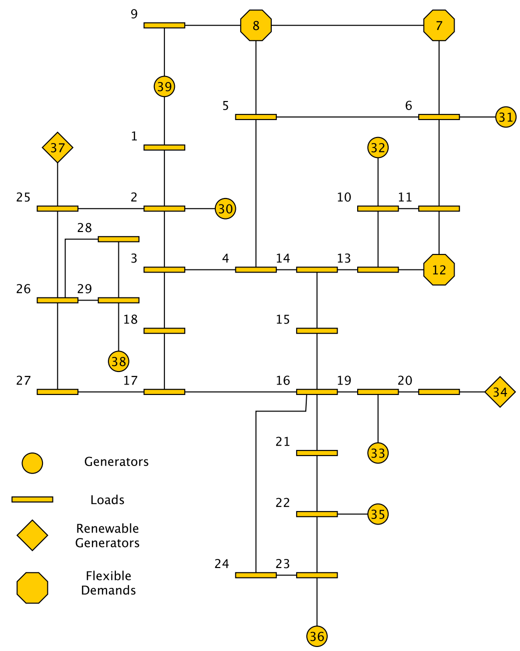


Fig. 6. Modified 39 bus system with 8 conventional generators, 3 flexible demands, 18 inflexible demands and 2 renewable generators.

two renewable generation sources at buses 34 and 37, respectively. Demands at buses 7, 8 and 12 are considered to be flexible *i.e.*, $\mathscr{D}_0 = \{7, 8, 12\}$. The topology of the network is shown in Fig. 6. Default data from [30] assume same cost of generation for all the generators. We take more realistic generation cost data from [31] to use in our simulations. Modified data of this network is available at [23].

Let the time horizon be $\mathscr{T} = \{1, 2, \cdots, 12\}$ (first 12 time periods from Fig. 1). We consider 100 independent scenarios for the renewable generators at the buses 34 and 37. The total demand in the network is 6254.23 MW. Approximately 12% of this demand is at the flexible demand buses 7, 8 and 12. Total generation capacity of the network is 7367 MW, and approximately 15% of the total capacity is from the renewable generators at the buses 34 and 37. We assume that the ramp rate of all the conventional generators are ±5%. Cost of the generator regulation is $C^{\text{R+}}_{g,t} = 1.8, C^{\text{R-}}_{g,t} = 0.5, \ \forall t \in \mathscr{T}, g \in \mathscr{G}$.

We further impose a constraint that the demand at the flexible bus 8 is conserved between the time intervals $[4, 8]$. Cost of using demand flexibility is $C^{\text{D+}}_{d,t} = 1.1, C^{\text{D-}}_{d,t} = 0.7$ for all the demand buses except for bus 8 where the cost when demand is conserved is $C^{\text{D+}}_{8,t} = C^{\text{D-}}_{8,t} = 0.5, 4 \le t \le 8$.

Fig. 7 shows the result of our model on 39 bus case as the flexibility of the demand is increased. Line limits were not active at the optimal solution, therefore the locational marginal price at all the buses were equal. The solid (blue) line shows the results when demand at buses 7, 8 and 12 is not flexible. In this case the marginal prices follow the behaviour of the demand curve *i.e.*, prices are high when the demand is high and the prices decrease with the decrease in demand. If demand is ±10% flexible then the marginal prices are low but this flexibility (coupled with ±5% ramp rate) is not enough to have constant system price. We observed that with ±10% demand flexibility, the cost of generation is decreased by 3.9%. Further as the flexibility of demand is increased, the system price tends toward a constant function. It is interesting to note that the difference in system prices is very small for the demand flexibilities of ±40% and ±100%. This is because that constant system price is the optimal solution which can be achieved by having ±40% flexibility on the demand side. Note that 40% flexibility is for flexible demands which constitutes 12% of the total demand. In other words 40% demand flexibility in the flexible demands corresponds to 4.8% flexibility of the total demand.

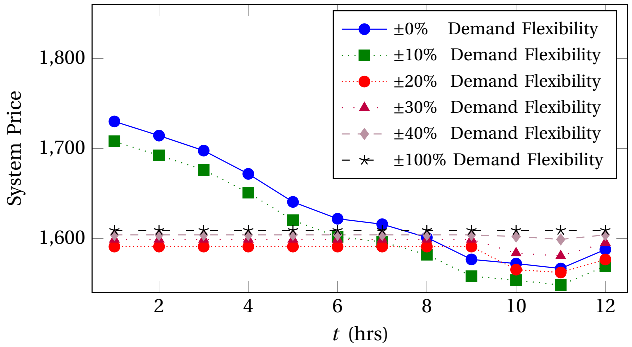
Another important point to note in Fig. 7 is that demand curve is not constant which shows that optimal solution to accommodate wind uncertainty is not peak shaving or valley filling but it is to have a demand curve which yields constant system prices. Our optimization approach optimally shifts demand to the time periods where power generation is cheaper and reduce the demand in the time periods where power generation is expensive. This shift is done considering the prices of demand and generation regulation. We would like to emphasize that this optimization approach is more optimal and generic then the valley filling and peak shaving approaches which aims to minimize the transmission losses.

Another interesting point to observe is that since we consider the linear model of the system, the results are generally independent of the flexibility *i.e.*, the flexibility can come from any node of the network as long as line limits are respected. In practice the transmission system are lossy, so the results would depend on the line losses however the effect of the line losses is expected to be very small. Active line limits will constraint the shift of demand from one time period to another because the demand might not be increased in certain time period due to active line limits. This situation is similar to a constrained dispatch of a generator where a generator cannot be utilized to its maximum capacity due to the active line limits.

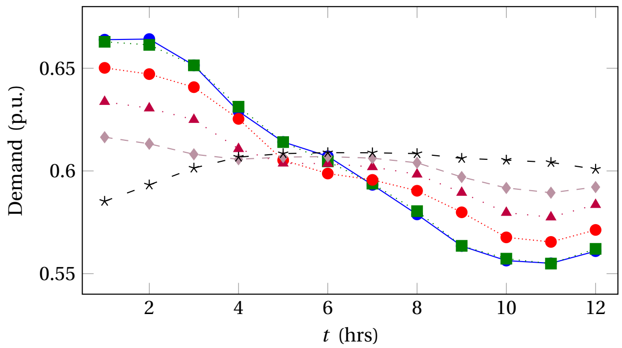
C. Larger test cases

We consider the standard IEEE test networks consisting of 14, 30, 57, 118 and 300 buses from the test archive at [32]. We also consider 9, 24 and 39 bus test cases from [30]. For all test cases, we assumed ramp rate of conventional generators to be ±10%, number of scenarios to be 50 and 12 time intervals. We generated large number of scenarios by considering different demand flexibilities and choices of wind generation buses. To keep consistency across all scenarios we considered that for all cases wind power penetration is always less than or equal to 25%. For all the instances total demand across the time horizon is constrained to be conserved.

Tab. I gives the results of some of the scenarios on 57, 118 and 300 bus networks. Second column in this table gives the set of buses where wind power generation is



(a) System prices.



(b) Expected demand depending on flexibility.

Fig. 7. Numerical results for 39 bus system.

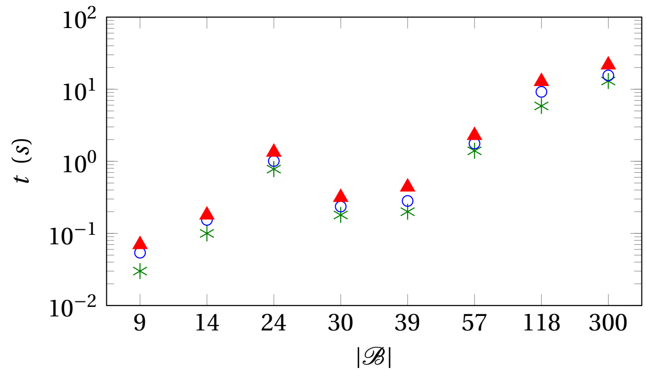


Fig. 8. Min., mean and max. solution times for solving multiperiod OPF problem with different demand flexibilities and wind penetration levels.

assumed. Third column gives the percentage of the wind power penetration in the system. Column four and five gives the set of buses which are flexible and their percentage of load in the system respectively. Second last column gives the assumed flexibility in the set $\mathscr{D}_0$. Last column shows the improvement in the cost of generation when compared to solving the problem with inflexible loads.

Results in Tab. I shows that considerable savings can be made in the generation cost if the demands are flexible. For example consider the 57 bus case with $W = \{3\}$ and $\mathscr{D}_0 = \{12\}$. In this case the load at bus 12 is approximately 30% of the total load of the network. The result shows that if the demand at bus 12 is ±10% flexible that the cost of generation can be improved by 4%, *i.e.*, approximately 3% (10% of 30%) flexibility in demand results in 4% reduction in cost of generation.

Fig. 8 gives the run times on all standard test cases. Problems were solved on a single core 64 bit Linux machine with 8 GiB RAM, using AMPL 11.0 with CPLEX 12.6 to solve LP and QP problems. The results are for large number of scenarios for wind power penetration (less than 25%) and demand flexibilities. Fig. 8 shows that the solution times scale well with increase in the size of the network. Note that solution times for 24 bus case is higher than 39 and 57 network. This is because of the reason that 24 bus network has more generators than 39 and 57 bus networks and hence the size of the problem is bigger.

IV. Conclusions

In this paper we presented a two stage stochastic programming approach to solve a multiperiod OPF problem with flexible demands. Demand response is integrated into the model as well to capture demand as a flexible asset. We observed that considerable savings in power generation costs can be made if a small proportion of the demand is flexible. The flexibility of the demand can come from any node of the network provided it respects the network constraints. Numerical results show that the uncertain wind power generation can be optimally utilized using flexibilities from demand and generation side. Computational times show the promise of the proposed approach.

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TABLE I
RESULTS ON LARGER TEST NETWORKS.

	$\mathscr{W}$	Wind Penetration (%)	$\mathscr{D}_0$	$\frac{\mathscr{D}_0}{\mathscr{D}}$ (%)	$\mathscr{D}_0$ Flexibility (±%)	Improvement (%)
`case57`	{3}	7.1	{8}	12.0	10	1.65
			{12}	30.1	10	4.13
	{12}	20.7	{8}	12.0	10	2.00
			{9}	9.7	20	3.15
`case118`	{10}	5.5	{80, 116}	7.4	20	1.97
			{54}	2.6	10	0.36
	{69, 89}	15.2	{42, 59, 90}	12.6	20	4.25
			{54}	2.6	10	0.45
`case300`	{186, 191}	10.3	{5, 20}	4.1	20	1.13
			{120, 138, 192}	11.0	20	3.03
	{191, 7003, 7049, 7130}	21.9	{10,44}	1.5	10	0.24
			{120, 138, 192}	6.2	20	3.45

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