Optimized charging of electric drive vehicles in a market environment

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Resumé (abstract in Danish)

El-drevne biler (EDVer) kan komme til at udgøre en betragtelig andel af bilparken inden for de kommende årtier og ses som en stor del af løsningen på problemerne relateret til brugen af flydende brændsler til transport. EDVer interagerer med elsystemet og tilbyder fleksibelt forbrug, da det kan bestemmes, hvornår EDVerne skal lades.

Bestemmelsen af en optimal ladeplan for el-drevne biler kræver kendskab til elpriser og kørselsmønstre. I dette projekt blev en simpel model, der minimerer omkostninger for en flådeoperatør, der har kontrol over opladningen af alle biler i Vestdanmark, udviklet. Modellen adresserer kun handel med el på day-ahead spot markedet.

Fastlæggelsen af den optimale ladeplan kompliceres af, at elpriserne bliver påvirket af flådeoperatørens beslutning om at lade. Sammenhængen mellem el-forbrug og priser blev analyseret ved brug af lineær regression på historiske prisdata for at finde dP/dQ (hvor meget prisen ændrer sig med forbruget). Efter korrektion for faktorer, der driver elprisen på langt sigt, blev en koefficient på 0,1174 DKK/MWh^2 bestemt. En lineær model kunne dog ikke beskrive den komplekse sammenhæng mellem priser og forbrug, særligt ikke ved forbrugsekstremer. En ikke-lineær model, der definerede dP/dQ som en funktion af kvadratet på prisens afvigelse fra en middelpris, blev anvendt. Modellens parametre blev bestemt ved visuel tilpasning af differentialligningens løsning (en tangensfunktion) til data.

Kørselsmønstre blev bestemt ud fra spørgeskemaundersøgelsesdata ved brug af en nyudviklet metode, der anvendte en tilpasset udgave af k-means clustering algoritmen. EDVerne blev modelleret som et sæt af standardbiler. Hver af disse biler repræsenterede et antal faktiske biler med ens kørselsmønstre. Dette tillod en realistisk behandling af batteriladeniveauer. Både rene elbiler og plug-in hybrid biler (PHEVer) blev modeleret.

En række scenarier blev analyseret. I baseline-scenariet med 300.000 EDVer fandt 75 % af ladningen sted om natten. Ladning i dagtimer blev hovedsageligt brugt til PHEVer for at begrænse brugen af flydende brændsler. Det maksimale elforbrug i systemet forblev stort set uændret. Ladning af EDVerne tilføjede ca. 200 MW grundlast om natten, hævede elpriserne med ca. 20 DKK/MWh og sænkede elprisens varians.

EDVerne skal deles om den billige elektricitet om natten, hvilket medfører øgede marginale ladeomkostninger ved indførslen af flere EDVer. Muligheden for at flytte forbrug mellem dage er begrænset pga. flådeoperatørens kontraktlige forpligtelser over for EDV-ejerne. Brugen af V2G (afladning af bilerne til elnettet) var meget begrænset. Prisforskellene er oftest for små og omskostningerne ved slid på batteriet samt de associerede tab for store til, at det er profitabelt. Mens EDVer kan anvendes til at hæve grundlasten er deres rolle som energilagring begrænset, så længe at batteriomkostningerne forbliver høje.

Abstract

Electric drive vehicles (EDVs) could become widespread within the coming decades as a solution to the many problems related to liquid fuels consumption. These vehicles interact with the electricity system and offers, unlike most other electricity consumption, flexibility in demand, since it to a large extent can be decided when the vehicles should be charged.

Deciding on an optimal charging plan for electric drive vehicles requires knowledge of electricity prices and driving patterns. A simple model minimizing costs for a fleet operator charging all vehicles in Western Denmark was developed. The model only addressed the day-ahead spot market.

The problem of finding an optimal charging pattern is complicated by the fact that prices are affected by change in demand caused by the charging. Using linear regression on historical data the relationship between price and demand was investigated in an attempt to find dP/dQ (i.e. how much price changes with a change in demand). After correcting for long term effects (fuel prices etc.) a coefficient of 0.1174 DKK/MWh^2 was found, but a linear model failed to model the complex relationship between price and demand, especially at extreme loads. A non-linear model was proposed which defined dP/dQ as a function of the squared deviation of the price from some mean price. The parameters were estimated by fitting the solution of the differential equation (a tan-function) visually to the data.

Drive patterns were obtained from statistical survey data in a novel approach using a slightly modified version of the k-means clustering algorithm. EDVs were modeled as a set of standard vehicles. Each standard vehicle represented a number of real world vehicles with identical driving patterns. This allowed for a realistic treatment of battery charge levels. Both battery electric vehicles and plug-in hybrid electric vehicles (PHEVs) were modeled.

A set of scenarios were investigated. In the baseline scenario with 300.000 EDVs 75 % of charging takes place at night. Day time charging is primarily used for PHEVs to limit liquid fuels consumption. Peak electricity demand remained almost constant. The charging adds about 200 MW of base load consumption at night and gives rise to an overall average price increase of 20 DKK/MWh, while reducing the price variance.

EDVs compete with each other for cheap electricity, leading to increased marginal charging costs of adding more EDVs. The possibility to shift consumption between days is limited due to the fleet operators contractual obligations to the users. It was found that very little discharging from the vehicle to the grid (V2G) was provided in general. Price differences are most of the time too small for this and the battery wear cost and incurred losses limit the profitability considerably. While EDVs may be used to increase base load consumption their role as energy storage is limited as long as batteries costs remain high.

Preface

This Master Thesis was carried out at Risø National Laboratory of Sustainable energy (Risø•DTU) in the Department of Systems Analysis under the Technical University of Denmark. The project work was carried out in the period from January 1, 2009 until June 27, 2009 and corresponds to 30 ECTS points.

The thesis deals with optimized charging of electric vehicles in Western Denmark accounting for single vehicle battery levels and the influence of electricity demand on spot-prices.

I would like to thank my supervisors at Risø•DTU Trine Kristoffersen and Peter Meibom for their helpfulness and constructive feedback throughout the process, and Pierre Pinson (DTU•IMM) for inspiring discussions on electricity prices.

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Contents

1 Introduction

Much attention has recently been brought towards electric drive vehicles (EDVs) as a solution to the tremendous challenge of reducing transport related CO_2 emissions and geopolitical issues of security of supply from instable regions of the world. Driven by the requirements of mobile applications such as cell phones and laptops battery technology has now reached a point where electric drive vehicles with reasonable performance can be built at a reasonable cost. Several electric vehicles have already been introduced to the market and companies are emerging to bring the new technologies forward. In January 2009 the company Better Place announced a partnership with the largest utility company in Denmark DONG Energy securing 770 Million DKK for investments in charging infrastructure promising widespread deployment of electric vehicles in Denmark from 2011 [1].

The term EDV comprise all vehicles that have an electric drivetrain. This includes battery electric vehicles (BEV), plug-in hybrid electric vehicles (PHEV) and fuel cell vehicles (FCV). BEVs and PHEVs have batteries on board that are charged with grid power

The promise of electric drive vehicles is not only to release us from our dependence of dwindling oil reserves, but also to help integrate large amounts of fluctuating electricity production from renewable sources such as wind power. As pointed out in the two articles by W. Kempton and J. Tomić [2, 3] electric drive vehicles and their batteries serve three purposes. Firstly, they may act as fast-response capacity reserve in case of an unanticipated fall-out of a power plant or transmission line. Secondly, they may provide regulating power in case of deviations from production or consumption plans. Providing these so-called ancillary services helps making the EDVs more economically attractive. The revenues for the vehicle owner lies largely in the capacity payment (being on stand-by) and actual energy payments for charging/discharging make up only a small fraction. With larger penetrations of EDVs the third purpose becomes important. If charged intelligently EDVs can help matching consumption patterns to the production patterns of wind turbines and give a better utilization of base-load capacity. L. Göransson [4] investigated the potential of PHEVs as moderator in the electricity system of Western Denmark (DK-1). She found several positive effects of assuming 12 % (2.5 TWh out of 20.6 TWh) electricity demand being made up by PHEVs with flexible charging patterns and the ability to discharge sending power back to the grid. This latter function is referred to as vehicle-to-grid (V2G). The result was a better utilization of wind power and far fewer start-ups and shut-downs of coal-fired power stations contributing to significantly lower CO_2 emissions in the power sector. It should be noted that this is in comparison to the reference case of no PHEVs in which almost none of the demand is flexible. Introducing EDVs *on top* of current demand *will* increase CO_2 emissions. In the worst case scenario modelled by L. Göransson the vehicles are not charged intelligently, but merely plugged in when returning home. This adds several hundred MW to peak demand and results in an increase in power sector CO_2 emissions larger than what is offset by lower liquid fuel consumption in the vehicles. H. Lund and W. Kempton [5] assessed the effect of various charging schemes of PHEVs introduced to the Danish energy system. They found that additional 10 TWh of annual wind power production (25 % of domestic production) could be introduced if all 1.9 Million cars in Denmark were replaced by PHEVs charged intelligently and with the capability of V2G. Various charging schemes have been proposed, but it remains to be seen how the EDVs will interact with the electric system.

In this thesis I have approached the problem from a fleet operator's point of view. The objective of the fleet operator is to charge as cheaply as possible. A general model is constructed and applied to a hypothetical case of introducing EDVs in DK-1 in the period of May 2, 2006 to October 25, 2007. The time period was chosen based on data availability. The fleet operator is a player on the Nord Pool day-ahead market from which exogenous prices are used. Prices are changed by the fleet operator's decisions as EDV demand increases elec-

tricity prices. The vehicle fleet consists of a number of standard vehicles which have driving patterns resembling those of real world vehicles. This allows for a realistic treatment of battery charge levels. The vehicles are assumed to always be plugged in when not driving and it is assumed that users allow the fleet operator to charge as cheaply as possible with only few contractual constraints.

Section 2 outlines how electricity is traded on the nordic power exchange NordPool and section 3 reviews how the traded electricity is currently being taxed while providing a suggestion for another tax-system. Section 4, 5 and 6 deal with the model inputs: How charging will affect the power price, EDV characteristica, and drive patterns. A review of linear and quadratic programming, which is used to solve the model, is given in section 7 before the model is presented in section 8. The different scenarios modeled and results obtained are discussed in section 9, 10 and 11.

The main contributions of this thesis to the EDV research field are the investigation of how electricity prices depend on demand and the concept of modeling EDVs as a set of standard vehicles. In particular the drive patterns of the standard vehicles have been obtained through a novel approach from statistical data.

Part I
Economic aspects of electricity

2 The Nord Pool markets

Since the liberalization of the power sector during the nineties several electricity exchanges in Europe have emerged. Danish electricity producers and consumers trade either bilaterally or on the Nord Pool exchange [6]. Nord Pool is one of the largest exchanges in Europe covering Norway, Sweden, Finland, Denmark and a part of Northern Germany. It is split into two main sections: **Nord Pool Spot** is the spot power market comprising Elspot (day ahead) and Elbas (intraday/hour-ahead). **Nord Pool ASA** is the market for financial derivatives and emissions trading. This thesis focuses on the day-ahead spot market only.

As of September 2008, 325 participants traded on the Nord Pool Spot markets daily. Traded volumes through Nord Pool Spot in 2007 amounted to 290.6 TWh in Elspot and 1.6 TWh in Elbas. This equals more than 69% of the total consumption of electricity in the Nordic countries.

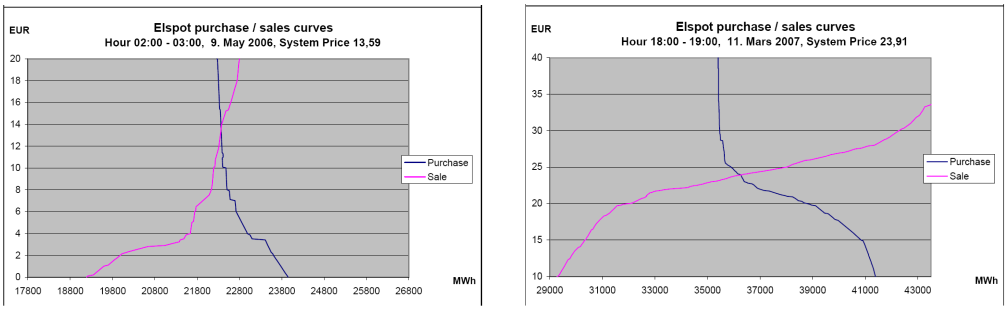
2.1 The Elspot day-ahead market

Market players are to submit their bids for all 24 hours in the following day no later than 12 pm (noon). Sale and purchase bids that are either price dependent or price independent can be placed. Examples of price dependent bids could be "sell 50 MWh if P > 300 DKK/MWh in the hour 5-6 am" or "buy 120 MWh if P < 230 DKK/MWh in the hour 4-5 pm". Usually sale bids will be price dependent (and related to marginal production costs), whereas purchase bids will usually be price independent. This is so because most consumers does not respond to the price signal as they either do not experience it (small consumers) or because they do not want to defer from consumption even in the case of a price spike. Based on the bids Nord Pool Spot constructs a supply and demand curve for each hour. The price where the two curves intersects will be the price for that hour. Two examples of these so-called bid curves can be seen in figures 1a and 1b. Note that on the first graph price independent sales bids of some 19 GW of power have been submitted. These bids could be from wind power production, mandatory hydro power production, coupling to other markets[1]. On the second graph about 6 GW of price dependent purchase bids have been submitted but rejected. These could have been placed by heavy industry (e.g. aluminium smelting) seeking to limit their variable costs of production.

2.1.1 Price areas

The electricity market is constrained by the physical transmission capacities linking the different areas. Due to transmission constraints it is not always possible for producers and consumers to exchange as much electricity as they would like to. The system price is the price that would exist if transmission capacity was infinite. Following the system price calculation transmission constraints are taken into account the following way: If any of the constraints are violated two new bid curves are created. One for each of the two areas between which the congested transmission line exist. The bid curves only contain sale bids

[1] If prices in DK-1 are expected to be higher traders can utilize the transmission capacity between DK-1 and Germany and put price-independent purchase bids in Germany and equally sized price-independent sale bids in Denmark. As of mid 2009 this so-called market-coupling is taken over by European Market Coupling company (EMCC) [7].



(a) 9 May, 2006 at 2-3 am

(b) 11 March, 2007 at 6-7 pm

Figure 1: Supply and demand curve for the entire Nord Pool system. The settlement prices of 13.59 EUR/MWh (Ĩ00 DKK/MWh) and 23.91 EUR/MWh (Ĩ80 DKK/MWh) are relatively low [8]

from producers and purchase bids from consumers within the area. A price-independent sales bid of power corresponding to the transmission capacity is placed in the area to which power flows. Likewise an equal sized purchase bid is placed in the area from which power flows. A new price is then calculated for each affected area. This method ensures that power always flow from an area with low price to an area with high prices. The situation is shown in figure 2. In the case of strong winds at night it could well happen that there is insufficient capacity to export power to Norway and Sweden. This would result in DK-1 having a lower price than the rest of the Nord Pool area.

In another case of low winds at peak demand the desired draw on cheap hydropower from Norway and Sweden exceeds transmission capacity resulting in a high price in DK-1. Due to similar lifestyles and weather peak demand in DK-1 almost always coincides with peak demand of Northern Germany. Therefore transit of power through Denmark to Germany will enhance this effect since much of the power imported from Norway and Sweden to Denmark is simultaneously exported to Germany.

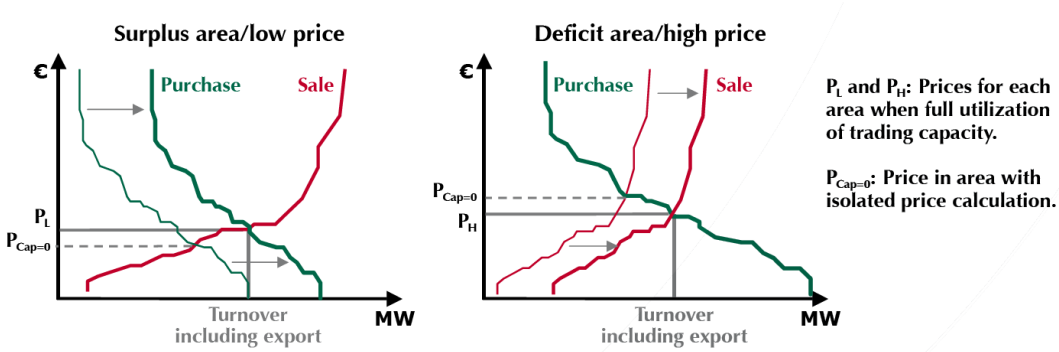


Figure 2: Calculating area prices in two areas between which a congested transmission line exists. A price-independent sales bid is placed in the high-price area and likewise a price-independent purchase bid i placed in the low-price area [9].

2.1.2 Factors driving the price in DK-1

Denmark is situated between two very different physical power systems with two very diffent pricing structures. Whereas the price is almost constant in hydro power based Norway,

large diurnal variations exist in the thermal-based power system in Germany. Both areas of Denmark fall somewhere in between, as can be seen from figure 3. Large hydro power penetration results in very stable electricity prices. If an operator of a hydro power plant expects high prices at a later time of day he should save his dammed water for production at that time. The opportunity cost of producing hydro power is high since for some plants water can stay dammed for time periods longer than those of demand variations. For thermal systems we usually distinguish between base-load and peak-load plants. Base load plants could be coal-fired power plants with low marginal production costs, whereas e.g. gas turbines with high marginal costs serve as peak-load plants covering electricity demand when it is at its highest. Because of the lack of a technology with inherent storage capabilities the sales bid function is very steep in the load interval where the equilibria are found resulting in large diurnal price variations.

The variations from wind power production cannot be seen in the average prices, but contribute to a generally lower level.

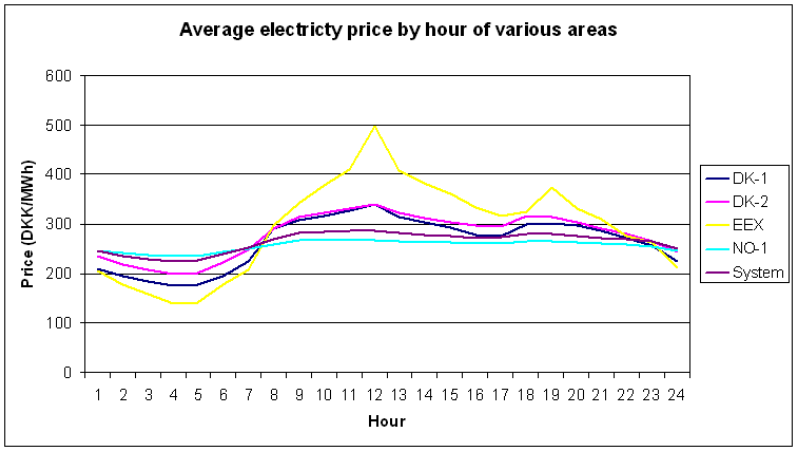


Figure 3: Average electricty spot prices by hour for the two danish price areas, the Nord Pool System price, the price of Norway-1 (Southern Norway) and the German electricity exchange EEX. Data is from 2/5/2006 - 25/10/2007 [10].

As can be seen in figure 4, the price level of DK-1 varies quite considerably from year to year. This is mainly driven by the hydropower availability, which is dependent on the amount of rainfall in Norway and Sweden and fuel prices. Peak prices are 1.5-2.2 times higher than night time prices indicating that price-flexible demand should usually take place at night.

Wind power does influence the price and wind patterns could therefore change the time of day at which electricity is cheapest. Figure 5 shows an example of how prices are affected by wind power production. Price data has been showed along with demand and wind power production for three days in DK-1. These three days are not typical, since prices are usually not this extreme, but have been chosen since the effect of wind power and demand are more evident here. During Friday and Saturday winds are low and high consumption levels trigger a price spike in the morning (1642.83 DKK/MWh at 10-11 am). There is a small price spike at cooking time friday evening before price levels return to normal in the evening. Through Saturday prices are at a typical level, with slightly lower prices during the night than day-time. High wind power production (>1000 MW) during the night of Sunday cause the electricity price to drop to zero. Higher day-time consumption levels brings electricity prices back to a somewhat normal level even with extremely high wind power production (>1500 MW) before the price goes to zero again at midnight.

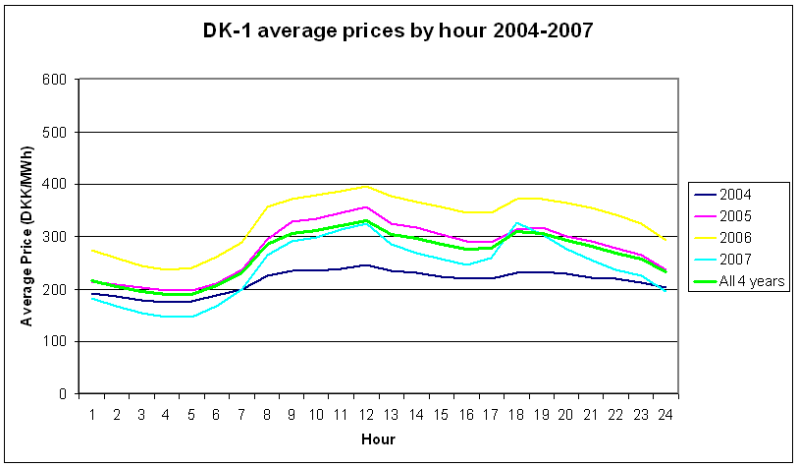


Figure 4: Average electricty spot prices by hour for DK-1 each year and total during the period of 2004-2007. The price level varies, but the pattern of peak prices being 1.5-2.2 times higher than night time prices on average repeats itself [10].

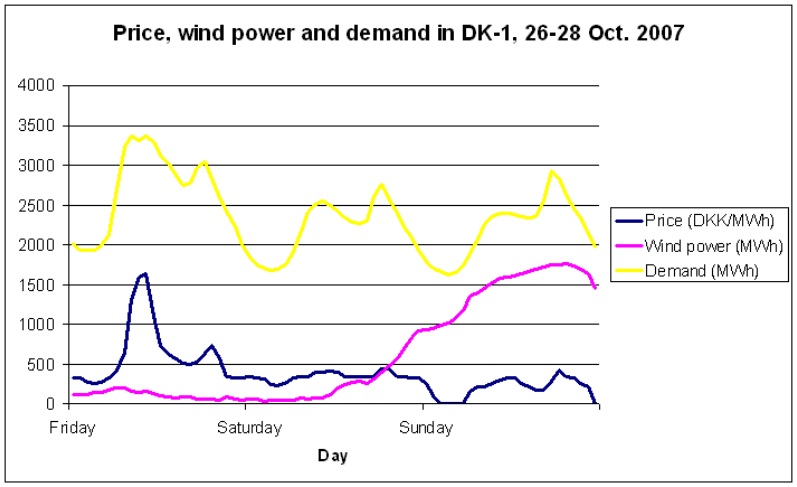


Figure 5: Spot price, Wind Power and electricity Demand of Western Denmark during three days of October 2007 [10]

3 Taxes and charges

When assessing optimal consumer behaviour one must include the taxes and charges that the electricity prices are subject to. What may seem feasible if only accounting for spot-prices on the day ahead market may not be so when including taxes and charges on retail electricity. This could for example be the case with charging and subsequent use of V2G. If a high tax on energy consumption exist the losses incurred when charging and discharging would be more expensive to pay for. Other tax systems could be thought of that would make the use of V2G and price-flexible demand more attractive. The incentives to introduce and use price-flexible demand will be highly dependent on how the tax system is constructed.

Price-flexible demand is desirable from an economic and technical point of view, since it could help balancing supply and demand. This would give a better utilization of the existing capacity. Furthermore it would greatly reduce the need for extra transmission and power plant capacity that would otherwise need to accompany large-scale wind power integration and use of electric vehicles. However, price-flexible demand is only desirable to the extent that social welfare gains are positive when shifting consumption to hours with lower prices. Taxes should reflect actual costs on society from using electricity. From a politicial point of view, electricity taxes are associated with large revenues to the state which means that any tax reform will most likely have to be revenue neutral in order to be politically feasible.

This section adresses the current system and outlines two other systems discussing their benefits and drawbacks.

3.1 The current system

Today for private households the market price of electricity only make up a fraction of the final cost of electricity. Denmark has one of the worlds highest tax rates on electricity for private households. All taxes and charges except value added tax (VAT) are currently fixed and relate to consumption only. They are listed in table 1.

Table 1: Taxes and charges on electicity [11] and surcharge for the market player purchasing electricity on NordPool [12]

Price element	DKK / MWh
Electricity tax	541
CO_2-tax	88
Electricity distribution tax	40
Electricity savings contributions	6
Total taxes	675
Public service obligations	43
Energinet.dk transmission (TSO)	55
Regional transmission	6.1
Local network tariffs	86.8
Total transmission and distribution	190.9
Spotprice surcharge	14.5
Total surcharge on marginal electricity excl. VAT	880.4

The costs listed in table 1 are only the marginal costs. Consumers must also pay subscription fees to their utility company. These have been omitted since they would have to be paid

regardless of how much electricity is consumed. VAT is currently 25 % in Denmark. The conversion of the spot price to that experienced by the user is:

$$P_{user} = (P_{spot} + 880.4\text{DKK/MWh}) \cdot 1.25 \tag{1}$$

Except for the small VAT of 25 % the current tax system does not give any extra incentives to consume electricity at a different time of day and will penalize the resulting losses from electricity storage severely if EDV users would choose to use V2G.

3.2 Turning the fixed tax into a VAT

In order to provide stronger incentives for price-flexible demand the energy-tax could be substituted partly or fully by a VAT. Changing all or part of the taxes from being based on consumption to being relative as a percentage on the price of electricity would amplify the price signal from a market price, which in its outset internalizes many of the effects of energy production and transmission. Having a stronger response to this signal could be beneficial to society. However, too strong incentives could result in in-optimal behaviour and/or investments in and use of technical equipment which serves a function that could be obtained at a lower cost elsewhere. If market prices are amplified beyond what can be justified by externalities a VAT would function as a state subsidy to electricity storage in home-owned batteries. Buying electricity at a low price P_{low} and selling it back in an hour with high price P_{high} would not only yield the profit of $P_{high} - P_{low}$ (excluding losses) but also $t(P_{high} - P_{low})$, where t is the tax rate. t is relatively large for the residental sector which currently pays the highest tax. If all taxes are turned into VAT the residental sector and hence the home-charging stations would get a far higher price when providing electricity stored in their batteries since it counts as negative demand. Providing V2G could be seen as equivalent to the household and its neighbours not using electricity from the grid in the given hour.

If all electricity-taxes were to be VAT what should the rate then be? As discussed previously a politically feasible tax reform would probably be revenue neutral. Market prices varies fairly much from year to year (see figure 4) and a drawback of this is that electricity tax would be a less stable source of income to the state. In practise the tax rate could be adjusted yearly or quarterly to secure revenues, but in this thesis a single tax rate was used. The value of this was found based on the four years 2004-2007. For each year the market value of electricity bought by households was calculated using historical spot prices of DK-1 and DK-2 [10] and household consumption patterns [13]. This yields the market value, which can be seen in table 2.

$$\text{Market Value} = \sum_t P(t) \cdot C(t) \tag{2}$$

where P(t) is the spot price in hour t and C(t) is the house-hold consumption in hour t.

Table 2: Total market value of electricity consumed by private households. Based on consumption patterns in 2007 and market prices for the four years [10, 13]

Year	Market value / MDKK
2004	2112
2005	2761
2006	3298
2007	2535
Average	2676

Dividing the average yearly expenditures by the total annual consumption in the household sector (9746 GWh [14]) gives a weighted average price of 274.6 DKK/MWh. Comparing this figure to the total taxes yields a tax rate of:

$$T = \frac{675 \text{ DKK/MWh}}{274.6 \text{ DKK/MWh}} = 246\% \tag{3}$$

On top of this tax the transmission and distribution charges as well as the spotprice surcharge would have to be added. Furthermore the regular VAT of 25 % should be added to the final price. The conversion from spot-market price to the price experinced by the user is

$$P_{user} = (3.46 P_{spot} + 190.9\text{DKK/MWh} + 14.5\text{DKK/MWh}) \cdot 1.25 \tag{4}$$

A number of examples of spotprices and prices experienced by the consumer are given in table 3.

Table 3: Examples of prices experienced by the consumers under the current system with fixed taxes and the VAT system desribed. All prices are in DKK/MWh and have been calculated using equation (1) and (4).

Spot price	Current system	All VAT
0	1101	257
100	1226	689
200	1351	1122
300	1476	1554
400	1601	1987
500	1726	2419
1000	2351	4582
2000	3601	8907

4 Electricity prices

4.1 Price forecasting

Many factors influence the spotprice. When market players place their bids before 12 pm (noon) they act on the best available knowledge to them at that time. Therefore wind power and demand forecasts, fuel prices, emission allowance prices, expected future electrictiy prices as well as knowledge about transmission capacities, availability of other power plants and market power all influence the bids. Forecasting electricity prices is a complex issue and the simple picture of a constant supply curve with the different technologies intersected by a horizontally shifting demand curve is insufficient to describe the dynamics involved.

Price forecasting is a valuable tool since it give market players the possibility to make strategic bidding decisions. Good estimates of or information on what bids other players in the market are going to place will allow a market player to increase profits [15].

T. Jónsson [16] developed several different models to explain the short term price variations in DK-1 accounting for wind power production and time of day and obtained fairly good results. He focused on an integrated view of both wind-induced dynamics and own dynamics of elspot prices. Most of the models were autoregressive and therefore eliminated the long term drivers of electricity prices by constantly adjusting the price level to that of the same hour the day and/or week before. The autoregressive approach also to a large extent factors in demand as a price-determining variable since the demand patterns of two consecutive weeks usually are very similar. With the expected expansion of wind power in the coming decades good wind prognoses and short term price forecasting is going to be of even more interest.

Long term price forecasting takes into account investments in new generation and transmission capacity and marginal costs of production (dependent on fuel prices and environmental regulation) in so-called system models. An example of such a system model is Balmorel [17], which determines the optimal development of the Baltic Sea region's heat and electricity system and yields long term electricity prices. However, system models, as they assume power is dispatched at marginal production costs, fail to explain market power, which may influence price substantially [15].

Good short term price forecasting is essential when optimizing EDV charging patterns, and long term forecasting is essential when evaluating the feasibility of EDVs compared to liquid fueled vehicles.

4.2 Optimal bidding

In this thesis I will seek to derive a model optimizing charging from a fleet operators point of view. This fleet operator is assumed to be in control of all EDVs in DK-1. The consequences of this approach is discussed below.

The fleet operator must know how much he wants to charge and when before placing a bid on the electricity exchange. If the bids submitted were price-dependent (e.g. "purchase if $P < 200$ DKK/MWh") then the fleet operator would not be certain to receive the needed power. Price dependent bids are only applicable if the operator would be willing to defer from charging. As electric vehicle users demand their vehicles to be charged before use, unsafe price-dependent bids, which may not be activated, are not an option. Plug-in hybrid electric vehicles does not need to be charged and can run on liquid fuels instead if it is economically favourable over charging. Therefore price dependent bids could be placed for these. On the other hand, it is very rarely the case that electricity prices are so high that using the engine will be economical. The problem of the EDV-fleet operator therefore is

to estimate when electricity prices are lowest and purchase power accordingly, while taking into account charging demand and when vehicles are plugged in. The problem is further complicated by the fact that price is dependent of demand. Say the fleet operator estimates that prices are lowest at 2-3 am. Knowing that he will charge 100.000 vehicles at full power ($\tilde{1}0$ kW) in that hour, he then places a bid of 1 GWh for that hour. This 1 GWh (on top of approximately 2 GWh base consumption) would surely increase the price, and probably also quite considerably. The result would be that electricity in this hour would end up being more expensive than in the neighbouring hours giving a bad result compared to a more distributed charging. Likewise for V2G if an hour with high prices are expected it may not make economic sense to discharge all vehicles simultaneously since supplying all this extra power would cause electricity prices to drop.

It is important to note the consequences of using the single fleet operator approach. The fleet operator acts like a monopolist seeking to maximize profits (by minimizing costs) for the entire EDV fleet. This implies that the fleet operator may make charging plans in which some vehicles are not charged in the hour with the lowest price. As an example consider a very simple model where prices increase by 1 for every vehicle put to charge in an hour. Two hours exist with one having a price of zero without EDV load, the other 4. The total cost function is then:

$$Cost = n_1(0 + n_1) + n_2(4 + n_2) \tag{5}$$

where n_1 and n_2 are the number of vehicles set to charge in hour 1 and 2 respectively.

If the fleet operator needs to allocate four vehicles to charge in these two hours he would allocate three vehicles to charge in the first hour and one in the second. The total cost of this is $3 \cdot (0 + 3) + 1 \cdot (4 + 1) = 14$, which implies one vehicle paying more in order to let the other three charge cheaper. In free competition none of the vehicles would take on the burden of limiting prices for the others and therefore all four vehicles would charge in the same hour at a cost of 4 per vehicle and a higher total cost of 16.

In the future such a single fleet operator may not exist. Given the economic incentives for single EDV owners to defer from the plan given by the fleet operator a competition case could arise and strategic bidding could be profitable. The treatment of a system of several market players in free competition would require application of game theory and falls beyond the scope of this thesis. Game theory has been applied to power markets, but mainly to the producer side [15].

4.3 Price dependency on demand

Investigating how changes in demand affect the price is for the above mentioned reason of great importance when optimizing charging patterns. Electricity as a commodity is unique by the fact that it cannot easily be stored and therefore supply and demand must match each other at all times. Furthermore, although electricity can be transported instantaneously over very large distances, markets are somewhat geographically limited by transmission constraints. This combined with the nature of a supply curve changing with wind and hydropower production and an almost inelastic demand results in very volatile prices. Much literature can be found on how prices and demand correlates. Most of the work utilize advanced models. M. Barlow includes the prediction of price spikes in a mean-reverting diffusion model [18]. Diffusion here refers to the model being jumpless and *mean-reverting* that equation terms dependent on the deviation from some mean price is included in the model. The model, which was fairly good at estimating price spikes in the electricity markets of California and Alberta, from which data was tested, uses a fixed supply curve and a demand function with stochastic features. It should be noted that wind power production and small combined heat and power plants, which particularly affects the assumption of constant sup-

ply curve, were small in the period analysed (1996-2000) in these power market areas. This is not the case of Western Denmark.

X. Lu et al. attempts to account for asymmetry in price response to increasing/decreasing demand using a modified autoregressive model [19]. In contrast to the model of M. Barlow jump terms were used as well as conditional variance. The latter allowed for asymmetric price response to demand changes. As was the case with the price-forecasting model of T. Jónsson [16] long term effects are cancelled out by using autoregressive models and/or only comparing data from same time of day.

How a change in demand affects the electricity price depends on the shape of both the supply and demand function. A change in the price independent quantities bid on the market would shift the demand curve horizontally as shown in figure 6.

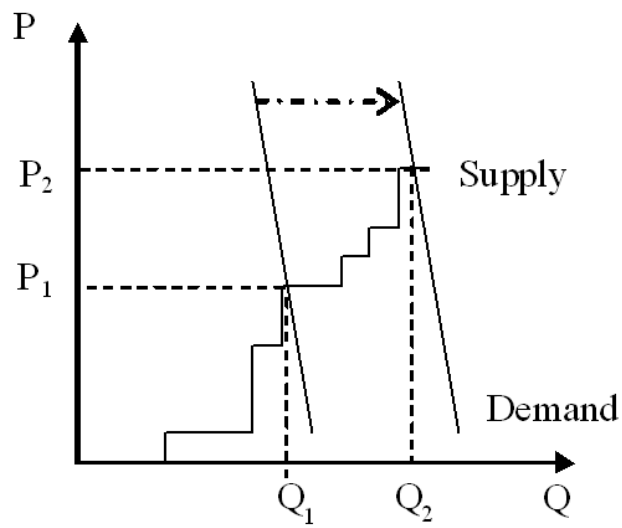


Figure 6: An increase in the price independent bids will result in the demand curve shifting outwards and a higher price. The change depends on the shape of both the supply and demand function.

More formally the change in price with a change in demand is described by equation 6.

$$\frac{dP}{dQ} = \left(\frac{dQ}{dP}\right)^{-1} = \left(\frac{dQ_S}{dP} - \frac{dQ_D}{dP}\right)^{-1} \tag{6}$$

where Q is the traded quantity (demand), $Q_S(P)$ is the supply function consisting of all the sales bids and $Q_D(P)$ is the demand function made up by the purchase bids. Both of these are assumed to be smooth functions, which is an approximation to reality where the curves are made up by discrete steps. When assessing dP/dQ the differential coefficients are to be evalulated at the equilibrium from which the change in demand takes place.

Assuming smooth supply and demand functions the first equality holds due to the inverse function theorem[2] The correctness of the second equality can be seen from figure 7. Placing a price independent bid of Q_{indp} is conceptually equivalent to removing Q_{indp} from the market.

[2]For functions of a single variable, the inverse function theorem states that, if f is a continuously differentiable function and f has a nonzero derivative at a, then f is invertible in a neighborhood of a, the inverse is continuously differentiable, and $\left(f^{-1}\right)'(b) = \frac{1}{f'(a)}$ where $b = f(a)$.

This has two effects:

1. A decrease in the number of accepted price-dependent purchase bids
2. An increase in accepted price-dependent sales bids

The two effects are taken into account by the two differentials dQ_S/dP and dQ_D/dP.

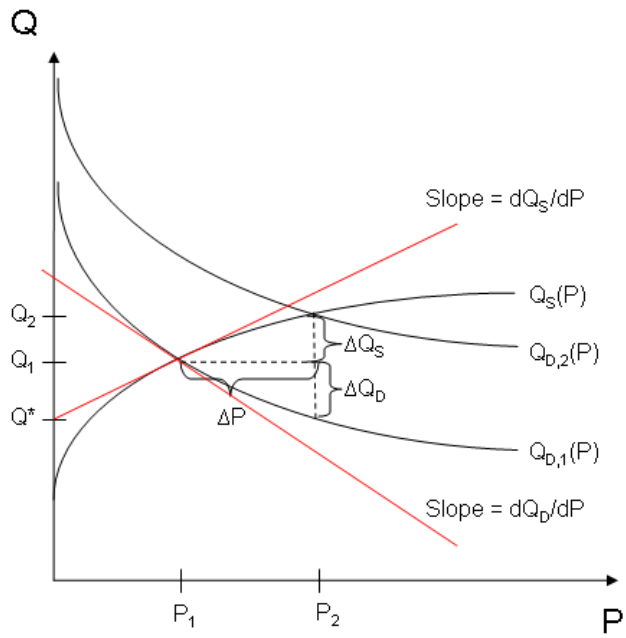


Figure 7: An increase in the price independent bids will result in the demand curve shifting upwards in a Q-P diagram (from $Q_{D,1}(P)$ to $Q_{D,2}(P)$). The effect will be an increase in price from P_1 to P_2, which will result in some price-dependent purchase bids being rejected ($Q_1 - Q*$) and more price dependent sales bids being accepted ($Q_2 - Q_1$)

Assuming linearity in the supply and demand function (or more generally linearity in $Q_S(P) - Q_D(P)$) makes dP/dQ constant. As will be discussed in section 8 this is desirable when modelling the optimal charging pattern. A zero dP/dQ (price independent on demand – infite elasticity in demand and/or supply) yields a linear programming problem and a constant dP/dQ yields a quadratic programming problem, both which can be solved fast whereas other forms of dP/dQ would yield other types of non-linear problems, which would be computationally more tedious to solve. Due to the complexity of electricity markets $Q_S(P) - Q_D(P)$ is most likely non-linear and dP/dQ is hence not constant. This is the case in the example in figure 7 where the actual price-effect of a demand increase would be underestimated since $-\Delta Q_D/\Delta P < -dQ_D/dP$ and $\Delta Q_S/\Delta P < dQ_S/dP$.

Different approaches of modeling price dependency on demand will be discussed in the following sections.

4.4 Regression analysis on historic data

Regression analysis has been performed on historic data from May 2, 2006 to October 25, 2007 on hourly day-ahead prices and the suspected main factors driving electricity prices in the long and short term in Western Denmark (DK-1):

- Coal prices
- EUA CO_2-quota price
- Season corrected water reservoir levels
- Gas prices
- Demand
- Forecasted wind power production

The regression model is a linear combination of the above:

$$P = P_0 + \alpha Coal + \beta EUA + \gamma Water + \delta Gas + \phi D + \sigma \overline{WP} \tag{7}$$

where P_0 is the intercept and α, β, γ, δ, ϕ and σ are the regression coefficents for the variables.

Confidential data on forecasted wind power production of DK-1, not including offshore production from Horns Rev was used in the analysis. However, under the assumption that the production pattern of Horns Rev follows that of the onshore wind turbines in DK-1 the resulting factor of wind power can be scaled to account for this. Horns Rev has 160 MW installed capacity compared to 2513 MW installed onshore capacity in DK-1 during 2006-2007. The price of one-month futures of Coal prices traded on the German energy exchange EEX was used [20]. Spot prices on gas with next-day delivery were obtained from APX [21]. Deviations of water reservoir levels from the three year median for each week were used to describe the divergence from normal reservoir levels [22]. Prices of EUA CO_2 emission allowances under the EU emission trading scheme were downloaded from the Nord Pool website [23]. Preferably demand forecasts should have been used since these forecasts are being used when placing bids on the day-ahead market. It was not possible to obtain these, but the error resulting from using actual consumption data instead is limited since the forecast issued by Energinet.dk is within 3 % of the actual consumption [24]. Both price and demand data was downloaded from Energinet.dk [10].

Since a linear regression model will fail to explain price spikes these has been removed by only selecting data points with $P < 600$ DKK/MWh. This accounts for 99.2 % of the observations. Using linear regression in sas[3] on all data yields a $R^2 = 0.736$ and a F-value of 6001. All explaining variables are statistically significant with the price of gas being least significant and demand being most with a t-value of 122. The results are displayed in table 4.

As expected, price correlates negatively with predicted wind power and water reservoir levels. Price correlates positively with demand, fuel prices and emission allowance price. The intercept is notably far from zero, which would be the price at zero demand, wind production, fuel prices, and emission prices and normal reservoir levels, showing that fitting a generalized linear model of this type on price data is a very crude approximation.

Also worth noting is the fact that $\overline{WP}$ and D does *not* have the same parameter estimate (with opposite signs). Since shifting the supply curve to the left corresponds to shifting the demand curve to the right (and vice versa) it would be expected that they had the same

[3] SAS 9.1 by SAS Institute Inc.

Table 4: Regression results on data covering the period May 2, 2006 to Oct 25, 2007

Variable	Parameter	Unit	Parameter estimate	Standard error	t-value
Intercept	P_0	DKK/MWh	-333.1	6.42	-52
$\overline{WP}$	σ	$\frac{\text{DKK/MWh}}{\text{MWh}}$	-0.0668	0.0012	-55
D	ϕ	$\frac{\text{DKK/MWh}}{\text{MWh}}$	0.1174	0.0096	122
EUA	β	$\frac{\text{DKK/MWh}}{\text{DKK/ton CO}_2}$	1.2044	0.0267	45
Coal	α	$\frac{\text{DKK/MWh}}{\text{DKK/ton coal}}$	0.6475	0.0156	42
Gas	δ	$\frac{\text{DKK/MWh}}{\text{DKK/MWh}_{gas}}$	0.1282	0.0262	4.9
Water	γ	$\frac{\text{DKK/MWh}}{\%deviation}$	-1.812	0.0835	-22

absolute value. Reasons for this not being the case could be that hydro power in Norway and Sweden to a large extent backs up the wind power, thereby smoothing the effect on prices. When winds are strong electricity is exported to Norway and Sweden. The hydro power operators then save the water for production at later times when winds are low, prices are higher and power can be exported to Denmark. Since demand patterns in the entire Nordpool area and Germany roughly follows demand of DK-1 hydropower exports has a limited potential to damp demand induced price variations.

Reducing the units reveals that EUA has a greater effect than what would be expected. 1.20 ton CO_2/MWh exceeds the about 0.5 ton CO_2/MWh that was average emissions of DK-1 in 2007 [25] and even the 0.8 ton CO_2/MWh that was emissions from Enstedværket, one of the largest coal fired power plants in DK-1 [26]. Also the parameter estimate for coal prices seems slightly too high. Enstedsværket, has a thermal efficiency of about 40 % and uses approximately 0.4 tonnes of coal per produced MWh [26]. Reducing the gas units one sees that δ should be the reciprocal value of the efficiency of gas turbines. This is far from the case with $\delta = 0.1282$ hinting to gas usually not being price setting in DK-1 or that the use of gas in combined heat and power plants complicates the relationship.

In the remainder of this section focus will only be on the short term effects. Therefore a corrected price is introduced in order to remove the effect of factors influencing the price in the long term term. In order to graphically show the correlation between price, forecasted windpower and demand three new variables are introduced P_{corr}, DWP and χ.

$$P_{corr} = P - \alpha Coal - \beta EUA - \gamma Water - \delta Gas - P_0 \tag{8}$$

$$DWP = D - \chi\overline{WP} \tag{9}$$

$$\chi = -\frac{\sigma}{\phi} = 0.569 \tag{10}$$

where P_{corr} is the electricity price corrected for long term effects and DWP is the demand reduced by the fraction χ of predicted wind power. The factor χ is defined in equation (10) as the negative of ratio of the regression coefficients for forecasted wind power and demand. Comparing equation (7) with (8), (9) and (10) one can see that the regression result is

$$P_{corr} = \phi DWP \tag{11}$$

The two variables are plotted against each other in figure 8. First of all, it can be seen that the linear model fails to explain all effects. The points make up a "double trumpet" and the regression model would hence fail a test for heteroscadicity. However, it can be seen that they are fairly well correlated in the region 1500 MWh $< DWP <$ 2800 MWh. Besides the general trend of rising prices with higher demand and lower predicted wind power, two effects are evident from the graph. For $DWP <$ 1500 MWh there are a number of points

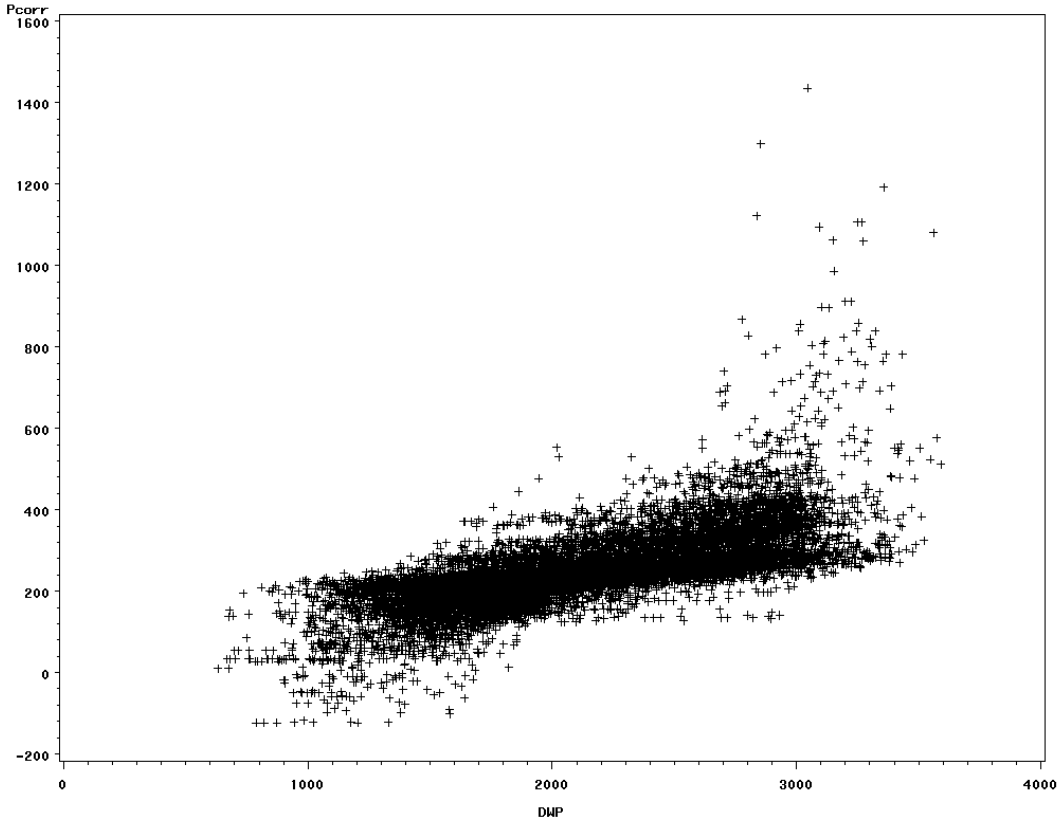


Figure 8: Price of electricity corrected for long term effects vs. demand reduced by part of forecasted wind power production. Points with negative prices occur in the plot. Although the real price is never negative the corrected price can be. Note that the price spikes have been shown, although they were not included in the regression analysis.

that 'drop of out the bottom'. For $DWP > 2800$ MWh there are a number of points that 'jump out of the top'. These two effects can be explained by the transmission contraints. For very low demand and high winds the export constraint results in a greater effect on the price area of DK-1 since no more power can be exported and the wind power has to be sold on the market in DK-1 with very low demand. Similarly the import constraint causes prices to increase significantly when winds are low and demand very high. It should be noted that Germany contributes to reaching the transmission constraint since Germany usually also indirectly draws on the power from Norway and Sweden during peak demand through imports from DK-1, as described in section 2.1.2. During the period on which data was analysed the system price only rised above 700 DKK/MWh once [10] and only surpassed 600 DKK/MWh in 203 hours (out of 13109), which means that extra import capacity would eliminate price spikes and give a more linear result.

Figure 9 shows the points on which regression analysis was performed along with the regression result $P_{corr} = 0.1174\text{DKK/MWh}^2 \cdot DWP$.

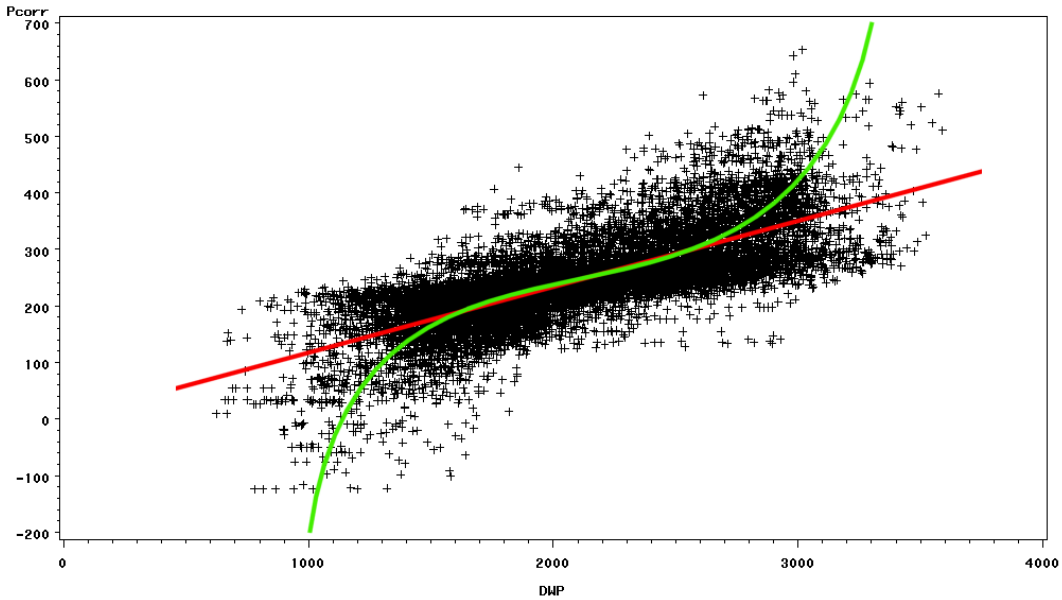


Figure 9: Price of electricity corrected for long term effects vs. demand reduced by part of forecasted wind power production. In the plot only points where P < 600 DKK/MWh are shown. This is the data used for the regression analysis. To graphs have been added:
1. The linear regression result (red line): $P_{corr} = 0.1174\text{DKK/MWh}^2 \cdot DWP$.
2. The tan-function described in section 4.5 (green line).

4.5 Other approaches to dealing with price dependency on demand

The restriction of dP/dQ being constant is quite severe, but also nescessary if the optimization problem is to be formulated as a quadratic one. As is evident from figure 8 there is a strong non-linear behavior between price and demand when the system is either at high or low load. A major shortcoming of the above model is that it fails to model price spikes appropriately. As an example consider a price spike at noon of 1000 DKK/MWh. Even with 1 GWh of V2G covering about one third of the entire demand and lowering load levels to those typical at midnight the price would only fall 117 DKK/MWh to 883 DKK/MWh.

Two other approaches to modelling the price dependency will be proposed here

1. Selecting a mean-point (Q^*, P^*) between which and the starting (Q, P) point a slope is calculated.

2. Fitting a non-linear function to the data and using the derivative of this function as dP/dQ assuming that it remains constant over the range in which the EDV load can affect the total load.

As the linear model described in the previous section the above two methods have advantages and drawbacks. None of them provide a satisfactory solution to modelling the price dependency on demand. In this thesis only the second of the two proposed methods will be investigated in detail.

4.5.1 The mean-point method

To briefly outline the first method, it involves selecting some mean-point (Q^*, P^*), which could be in the center of the cloud of points (e.g. (2200 MWh, 250 DKK/MWh)). The slope

$\Delta P/\Delta Q$ is then calculated between this point and the starting point (Q, P) of the hour in question. The starting point being the load and price without EDVs. There are two major drawbacks of this method. The first being that there is a substantial spread in prices at a given load (see figure 9) which could result in very large and/or negative values of $\Delta P/\Delta Q$. As an example consider a starting point of (2150 MWh, 350 DKK/MWh). With the above example of a mean point one would get $\Delta P/\Delta Q =$ -2 DKK/MWh^2. This would be disastrous in a model, since it would predict providing a very large amount of V2G in this hour would raise the price considerably and be very economically attractive. The second drawback is less severe and deals with the modelling of extreme prices. The mean-point method offers a better treatment of price spikes than the linear model, but the calculated slope will still be too small to correctly model the drop in prices as a result of providing V2G.

4.5.2 The non-linear function approach

It is important to note that dP/dQ does *not* need to be the same for all values of Q, it only needs to be constant over the range of interest. That is from Q^* to $Q^* + \Delta Q$, where Q^* is the load without EDVs and ΔQ is the change in load caused by the EDVs. It is possible for each Q^* to calculate dP/dQ and then approximate this as the constant price dependency on demand factor for a hour having this load without EDVs. The major drawback of this is (as can be seen from figure 8) that there is no guarantee that a price spike will occur at high load or a very low price will occur at low loads. As said in the previous sections many factors influence the spot price besides those that were made an attempt to remove. Examples of these are limitations in transmission capacity, unforseen shut-downs of power plants, exports to Germany etc. Furthermore, the marginal costs of production and thereby the sales bid function $P_S(Q)$ increases dramatically as the system is pushed to its limits. This is examplified by the points "jumping out" of the top on the right side of figure 8. What the limit actually is changes based on the above described influencing factors. In conclusion, DWP is a poor measure of what the price and dP/dQ should be (especially in terms of predicting price spikes). Instead one could use the price to predict dP/dQ instead.

It was attempted to fit a non-linear function for $P(Q)$ and expressing its derivative (dP/dQ) as a function of itself. Tan-functions satisfy this condition if assuming that dP/dQ depends on the square of the deviation of the price from some mid-price. The resulting differential equation is:

$$\frac{dP}{dQ} = a(P_{corr} - P_0)^2 + b \tag{12}$$

this differential equation has the solution

$$P_{corr}(Q) = P_0 + \sqrt{\frac{b}{a}} \tan\left((Q - Q_{mid})\sqrt{ab}\right) \tag{13}$$

Getting a reasonable result out of fitting such a function by ordinary sum of squares regression analysis is made impossible by the fact that the function goes to infinity when the argument for the tan function approaches $\pi/2$ or negative inifity at $-\pi/2$. This results in very large squares for the points at high or low Q. In order to avoid these large squares the regression analysis would attempt to fit a very broad tan-function with a very large value of $\sqrt{ab}$. Broad understood in the sense that it would go to infinity only at a *very* high value of Q and negative infinity at a *very* low value of Q. This would result in a flat function (in the Q-range of interest) resembling the linear function found in the last section.

A model that would resemble reality needs to have a function increasing rapidly at high loads and go to zero or negative values at low loads[4]. It is impossible to find the expression of

[4]Currently the minimum price in NordPool is zero, but the market will be opened for negative prices within 2009.

such a function by a statistical method since the range of interest have *not* been uniformly sampled. That is the historical data available have been obtained by testing the system at moderate conditions only. Very few of the points arise from extreme conditions where the system is pushed towards its limts. These points were referred to as outliers, but are not errors. Instead they are observations of what happens when the system is subjected to more extreme conditions.

The model used should reflect the physical reality and therefore instead of using a statistical method the parameters were varied until reaching an acceptable visual fit. The resulting parameters were

$$P_0 = 270\frac{\text{DKK}}{\text{MWh}}, \qquad Q_{mid} = 2150\text{MWh}, \qquad \sqrt{ab} = 0.0011\text{MWh}^{-1}, \qquad \sqrt{\frac{b}{a}} = 105\frac{\text{DKK}}{\text{MWh}} \tag{14}$$

Solving for a and b one gets

$$a = 1.05 \cdot 10^{-5}\text{DKK}^{-1}, \qquad b = 0.116\frac{\text{DKK}}{\text{MWh}^2} \tag{15}$$

It can be seen that b, which is the minimum dP/dQ (at $P_{corr} = P_0$), has almost the same value as ϕ (0.1174 DKK/MWh^2). dP/dQ is plotted against P_{corr} in figure 10.

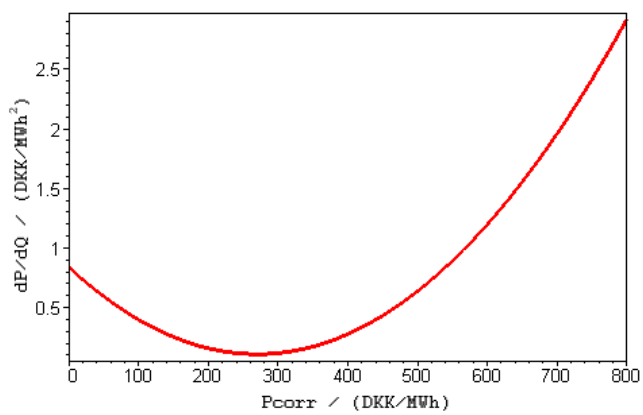


Figure 10: dP/dQ as a function of P_{corr}

As can be seen from equation (12) dP/dQ doubles (to $2b$) at $P_{corr} - P_0 = \sqrt{b/a} = 105$ DKK/MWh.

At $P_{corr} - P_0 = 1000$ DKK/MWh ($P_{corr} = 1270$ DKK/MWh – a price spike) $dP/dQ = 10.6$ DKK/MWh^2. The model would predict that providing 100 MWh of V2G in this hour with a price spike, would reduce the price by 1060 DKK/MWh. Additional 20 MWh of V2G would make the predicted price negative. Although $dP/dQ = 10.6$ DKK/MWh^2 could well be a correct price dependency on demand at the specific load level the assumption of it remaining constant over a range of Q seems highly unlikely.

As opposed to the method described in the previous section, using constant dP/dQ for all (Q, P), this method will result in too little V2G at high prices and too little charging at low prices. A number of examples are shown in figure 11. Each arrow in the figure shows in which direction one moves on the (Q, P) graph. The lengths of the arrows are arbitrary. It should be noted that one does not have to move towards more moderate loads. The same dP/dQ applies when moving towards more extreme load conditions. If moving towards extreme Q the method underestimates dP/dQ.

Using the price corrected for long term effects to determine a good measure for dP/dQ gives rise to two concerns. Firstly, are the long term effects removed properly? Looking at the

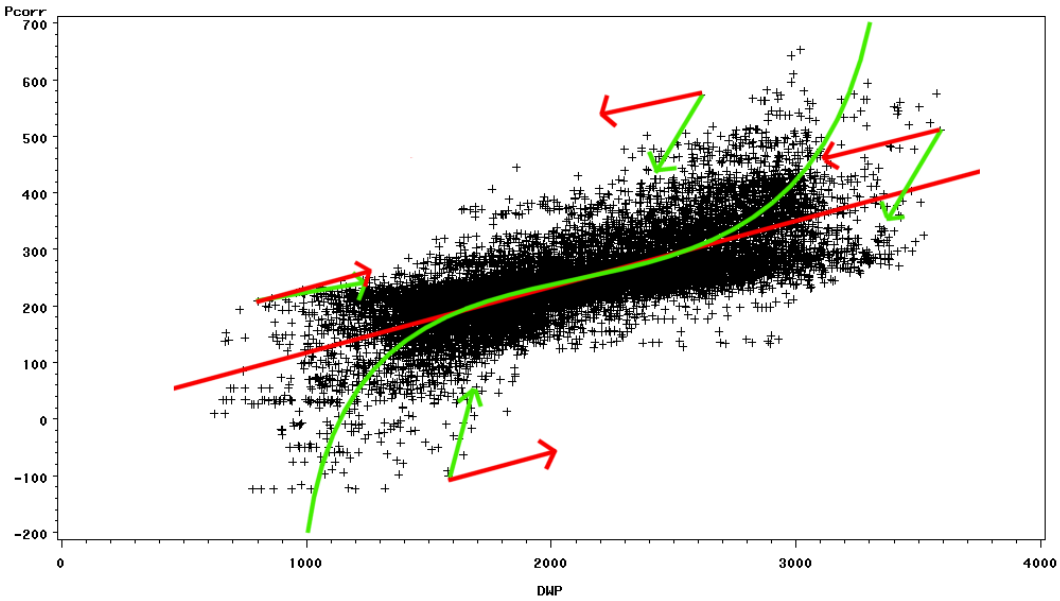


Figure 11: Figure 9 with examples of how a change in demand would affect the price using the two methods described. The color of the arrows correspond to those of the fitted functions.

cloud of points in figure 8 the answer is probably no. This is not surprising since modeling the price as linearly dependent on each of the factors driving the electricity price in the long run is a quite crude assumption. Secondly, is dP/dQ independent on the long term effects? Changing fuel prices, emission rights and water reservoir levels affect the supply curve. In a period of high prices the supply curve will be steeper and hence dP/dQ would be higher. A way to deal with this could be to include the long term effects by using a moving average of the price. Fitting several non-linear functions would give values of a and b dependent on P_0. This would allow the optimization model to move on a more correct (Q, P) curve.

4.5.3 Using iterations to model price-demand relations better

As dP/dQ is rising when P goes towards its extremes the constant dP/dQ method will always predict a too low dP/dQ whereas the non-linear function approach will overestimate dP/dQ when returning to a normal price level and underestimate it if deviating further from the normal level. A solution to this shortcoming of the model while still keeping the problem as a quadratic programming one, could be to iterate changing the values of dP/dQ based on the result of the optimization (see section 8 for a model description). As an example consider figure 12. The starting point is a hour with a low price. The non-linear model predicts an optimal level of charging $(Q_1 - Q_0)$ increasing the price to some level P. Based on the value of Q_1, and the non-linear function a new value of $\Delta P/\Delta Q < dP/dQ$ for that hour is estimated to account for the curve flattening out as prices increase upwards towards extreme prices. This is done iteratively until the values of Q, P, and $\Delta P/\Delta Q$ converge to each their value. In figure 12 P is shown as being constant. This is an approximation, since adding to the load in one hour would decrease the load in another making it less expensive to charge in that hour whereby competition between the hours would reduce the price in all hours. This is not a problem since the method should secure convergence regardless of the price changing or not.

A potentially faster method of reaching convergence could be to calculate the new $\Delta P/\Delta Q$ based on the new P. The only input needed would be ΔP given as the price change in an hour induced by the EDVs, which is easily obtained from the output data of the first iteration. Equation (16) could then be used to calculate $\Delta P/\Delta Q$

$$\frac{\Delta P}{\Delta Q} = \frac{\Delta P}{Q(P_{corr} + \Delta P) - Q(P_{corr})} \tag{16}$$

where $Q(P)$ is the inverse of $P_{corr}(Q)$ in equation 13

$$Q(P) = Q_{mid} + \frac{1}{\sqrt{ab}} \arctan\left((P - P_0)\sqrt{\frac{a}{b}}\right) \tag{17}$$

The calculated value of ΔQ is the EDV load that would be required to change the price to the value it had in the first iteration if the non-linear curve had been followed.

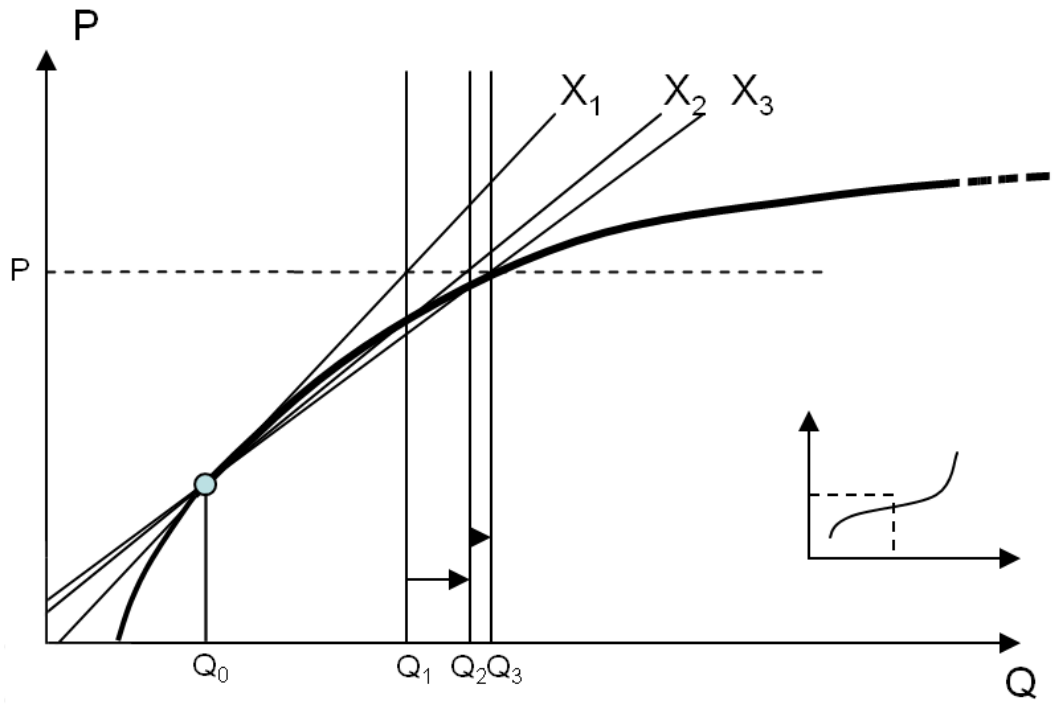


Figure 12: The iterative process of finding an optimal charge level by changing dP/dQ. The graph is the lower left corner of figure 9 as indicated by the small graph. The three first iteration steps are shown. X denotes $\Delta P/\Delta Q$. Since the non-linear approach only uses the first iteration the result obtained is Q_1.

The approach was tested on the baseline scenario (defined in section 10) testing 300,000 EDVs. Figure 13 shows the result of the iterative approach for four example days. In these four days prices were generally low and no V2G was provided. All large values of dP/dQ are caused by low prices. dP/dQ converges very fast. Using equation (16) dP/dQ is slightly underestimated due to the general price level being overestimated in the first simulation.

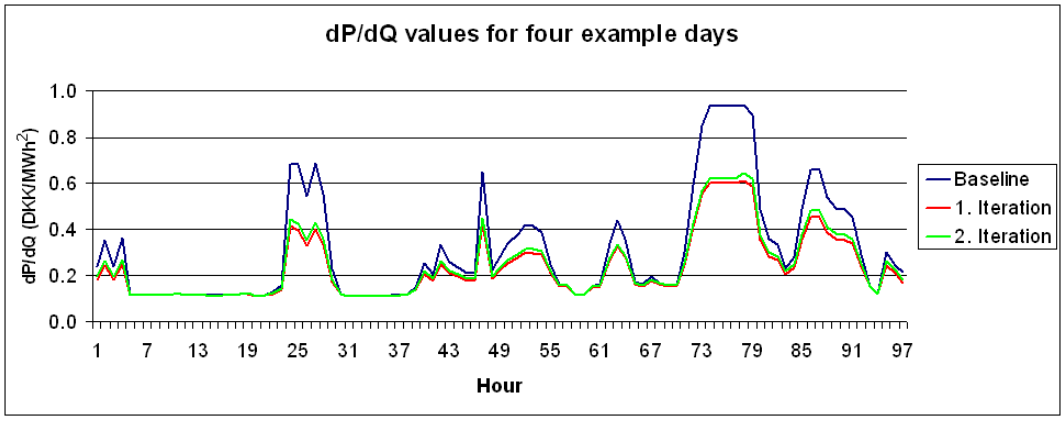


Figure 13: The resulting values of dP/dQ when using the iterative approach shown for four example days (May 4-7, 2006). Baseline refers to the values obtained using equation (12).

Part II
Vehicles and users

5 Electric drive vehicles

The term Electric drive vehicles (EDV) comprise all vehicles that have an electric drive train. This includes battery electric vehicles (BEV), plug-in hybrid electric vehicles (PHEV) and fuel cell vehicles (FCV). BEVs and PHEVs have batteries on board that are charged with grid power. If the right power electronics are present in the vehicle there is the possibility to discharge the batteries providing power from vehicle to grid (V2G) [27]. FCVs are fueled by hydrogen and therefore does not need to be charged. Since the fuel cells generate electricity for the vehicle FCVs also have the option to do V2G. However, FCVs will not be adressed in this thesis. Due to their low energy efficiency[5] and limited state of development it seems highly unlikely that FCVs will make up a larger fraction of the vehicle fleet in next few decades. J. Romm [28] states that it is unlikely that hydrogen vehicles will achieve significant (>5%) market penetration by 2030. And furthermore, if deployed, FCVs may actually cause increased CO_2 emissions in the near- and medium-term.

5.1 Battery Electric Vehicles

BEVs rely solely on battery power for propulsion. Although BEVs have been on the market for several decades and some of the first vehicles produced were BEVs, the internal combustion engine (ICE) has vastly outperformed the batteries during the last century. The emergence of new battery technologies such as Li-ion and Ni-MH has allowed reasonable energy and power densities to produce vehicles of similar performance to their ICE counterparts. Significant cost-reductions, particularly for Li-ion batteries, driven by the demand for mobile electronic applications, has brought prices to a level where BEV could soon be cost-competitive with ICE vehicles.

One of the main advantages of BEVs is that they are "fueled" by electricity which is significantly cheaper, often cleaner and comes with a higher security of supply than liquid fuels. Other benefits include zero local emissions and noise reductions as well as the added value of not having to go to the gas station. The major drawback of BEVs is that they have limited range and long charge times render them poorly suited for long distance trips. Fast-charge infrastructure and/or battery-swap stations could alleviate this problem. These solutions fall beyond the scope of this thesis and will not be adressed. Since by far the largest fraction of driving need is less than what would deplete the batteries in one day home and/or workplace charging seems likely to be the dominant method in the future.

5.2 Plug-in hybrid electric vehicles

Whereas BEVs are all-electric, PHEVs also have an ICE using liquid fuels, which functions as a range-extender allowing longer trips. Different drive modes for PHEV have been proposed. Charge depleting, in which the batteries are used as much as possible until depleted is one. The other is charge-sustaining or blended mode in which the liquid fueled engine assists the electric motor or even charges the vehicle while driving [29]. Which mode of operation is optimal and how far the vehicle can drive in all-electric mode (if at all) is defined by the

[5] Large energy losses are associated with the production of hydrogen by electrolysis, compression of hydrogen for fuelling and reconversion of hydrogen to electricity in the fuel cell.

size of the battery. PHEVs differ from hybrid electric vehicles (HEV) in that they can be charged with grid electricity from the grid. HEVs are already on the streets with the Toyota Prius probably being the most known. Several large car manufacturers have announced the introduction of PHEVs in the coming years.

5.3 Parameters used for modeling

Since the battery is a very expensive component it seems reasonable to expect that both BEVs and PHEVs will be marketed with several different battery sizes. Users can then purchase a battery size fitting their needs. In order to keep things simple only one type of PHEV and one type of BEV were modeled in this thesis. PHEVs are modeled under an assumption of 65 km all-electric range. Most demonstration PHEVs built in the last decade features batteries of about this size [30]. BEVs are modeled assuming an 150 km range. EDV electric drive efficiencies used when modeling electric vehicles differ substantially from 3.2 km/kWh in [31] to 10 km/kWh in [5]. Many articles including [32] use about 6 km/kWh. A vehicle efficiency of 6 km/kWh was used when modelling both PHEVs and BEVs. This give battery sizes of 10.8 kWh for PHEV and 25 kWh for BEV.

The assumed year 2015 peak efficiency of ICEs (39 %) [29] was used in the model. This is a high estimate since the combustion engine will not always be used at the peak efficiency power level. One litre of diesel is estimated to cost 10 DKK and have the energy content of 10 kWh (36 MJ). This yields an energy cost of 1000 DKK/MWh primary energy. The assumed vehicle efficiencies correponds to a fuel economy of 23.4 km/litre for PHEVs when using the ICE. Charging efficiencies were assumed to be 0.90 and V2G efficiency to be 0.93 [2]. These are both high figures giving an grid-to-grid energy storage efficiency of 0.837.

5.4 Grid connection systems

In order to charge electric vehicles and allowing for V2G grid connection systems must exist. Widespread home- and workplace-charging seems reasonable since EDV users could then charge their vehicles where they park. Using the current grid infrastructure would avoid excessive investment costs of introducing electric vehicles. Therefore a three-phase 16 A charging system at 230 V voltage (same voltage and fuses that are currently used in Danish homes) would be good estimate. This gives a charging and V2G capacity of:

$$C_{max} = V2G_{max} = 0.23\text{kV} \cdot 3 \cdot 16\text{A} = 11.1\text{kW} \tag{18}$$

This figure is in the range of 10-15 kW used in [2], which was found for American systems.

The AC-150 Gen-2 EV Power System [33], which is already on sale allows for 20 kW charging and V2G. This means that the power line and not the vehicle sets the limit on power flow from the grid to the battery and vice versa.

5.5 Cost of V2G

Much uncertainty exists on how widespread use of V2G will prove to be. A very large part of the cost of an EDV is the battery. If wear on the battery leads to it having to be replaced in the vehicles lifetime the cost of this wear should be taken into account. Based on a NiMH battery Kempton and Tomić found the cost of V2G to be 75 $/MWh V2G energy [2]. Using a currency exchange rate of 5.25 DKK/USD one gets

$$V2G_{cost} = 394\text{DKK/MWh} \tag{19}$$

Having to pay this cost will limit the attractiveness of using V2G for energy storage purposes considerably since the diurnal variations in power prices rarely exceeds this. Figure 4 shows the average picture but hints to this point.

6 Driving patterns

Accurate prediction of driving demand and availability to charge will be paramount to the success of optimized charging. If one knows the exact driving and plug-in pattern of an EDV and the future electricity prices one is able to plan the charging perfectly. Fortunately, driving patterns are fairly predictable. Many users are commuters with a fixed weekly schedule of working hours and leisure time activities. Some EDVs could be used for business purposes where driving schedules are very predictable since they usually follow a schedule and a specific route. Examples of such vehicles are postal-service or elder-care vehicles. In this thesis such vehicles will not be addressed specifically, although the work could fairly easily be expanded to include a large shift in propulsion technology to electric drive for these vehicle types.

Instead, an attempt will be made to model the entire car fleet under the assumption that there will be no bias in deciding to purchase an EDV or an ICE vehicle based on driving patterns. It is important to note that EDVs are expensive to buy (because of the battery) but are cheap to drive compared to ICE vehicles (because electricty is cheaper than liquid fuels). Therefore it could be expected that users with a medium-high constant daily driving demand would be more likely to purchase EDVs than users with a low or very varying driving demand. Other factors may, however, also influence the purchase decision. An interest in new technology, environmental concerns and/or special local regulation or incentives could be correlated to place of residence and driving patterns. The assumption of no user bias is one of many needed to model EDV driving demand before specific statistical data hopefully becomes available once EDVs have been introduced.

6.1 Survey data

Statistical data on transportation patterns have been recorded in the investigations on travel activities (Transportvaneundersøgelsen (TU) in Danish), which is an interview based survey of the personal transport activities in Denmark [34]. Respondents are asked about their travel activities of the previous day: which mode of transportation, duration, time of day, distance, etc. Personal information on the respondents age, home-town, number of vehicles in the household etc. is gathered as well. The following variables have been extracted from TU of 2006 and 2007:

- id (User id)
- trmid (Means of transportation)
- totkm (Distance travelled in single journey)
- start (Starting time - hour)
- weekday (Day of week or holiday)

From this data it is possible to generate a 24-hour driving pattern for each id with the distance driven specified for each hour. The distance travelled is registered in the hour where the journey has commenced. It has been assumed that a vehicle cannot drive more than 70 km in an hour and therefore distances in excess of 70 km will be split onto the following hours. As an example 160 km registered in hour 8 will be split as 70 km in hour 8 and 9 and 20 km in hour 10. It should be noted that it is assumed that the respondents driving patterns (as drivers, not as passengers) correspond to vehicle driving patterns. This may well be a very crude assumption. If more than one person is using the vehicle too little driving will be registered. As an example if two persons go on a trip and one drives out and the other back home this would appear as two driving patterns although it is the same vehicle.

Also a person could use more than one vehicle per day, but compared to the problem of more persons sharing the same vehicle the error resulting from this is probably small. Furthermore, it is assumed that all vehicles drives at least once a day. All patterns will therefore be used. In the real world some vehicles stay parked for a day or more so the error arising from this assumption will contribute to increase the modeled driving demand. Weekday information is used to distinguish the driving patterns from day to day. No information was extracted on what time of year the data is from and hence weekly driving patterns are assumed to be constant throughout the year.

Figure 14 shows that Monday, Tuesday, Wednesday and Thursday resemble each other with respect to driving patterns. Friday is somewhat similar, but many commuters leave their work place earlier for the weekend, which shows up in the second peak being shifted slightly to the left. Saturdays, Sundays and the holidays are different in that there are no peaks. Driving starts later than on weekdays and driving is distributed fairly evenly in the hours from 10 to 17. Aggregated evening driving demand in weekends resembles that of the weekdays.

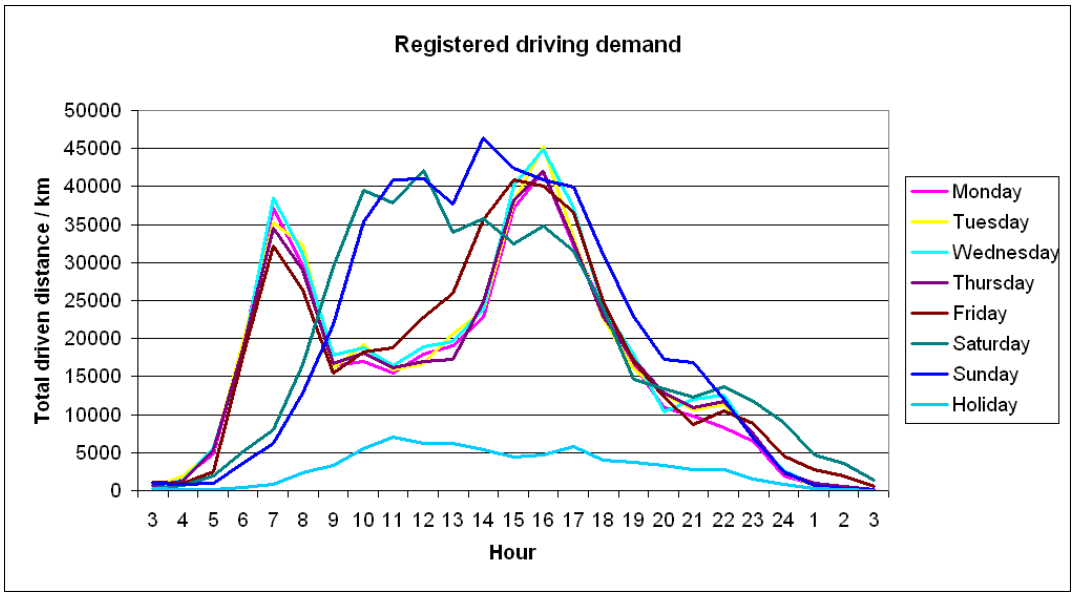


Figure 14: Total surveyed driving demand in Denmark in each hour. Data from 1998 to 2007 [34].

Plotting the number of trips and distance travelled with respect to time of first and second trip (figure 15) shows clear evidence of a commuter pattern. Drivers departing between 6-9 am and returning between 3-6 pm make up about 45 % of the transportation demand on weekdays. Other driving demand is fairly dispersed over the day. The distance driven per day varies substantially from respondent to respondent.

Figure 16 shows how large a fraction of the driving demand is made up by vehicles travelling less than a certain distance. On weekdays half the driving demand takes place in those 85 % of the vehicles that drive less than 92 km per day. 31 % of all driving demand takes place in the 6.5 % of vehicles driving more than 150 km per day. As both graphs are fairly smooth there is no natural grouping of vehicles driving short and long distances.

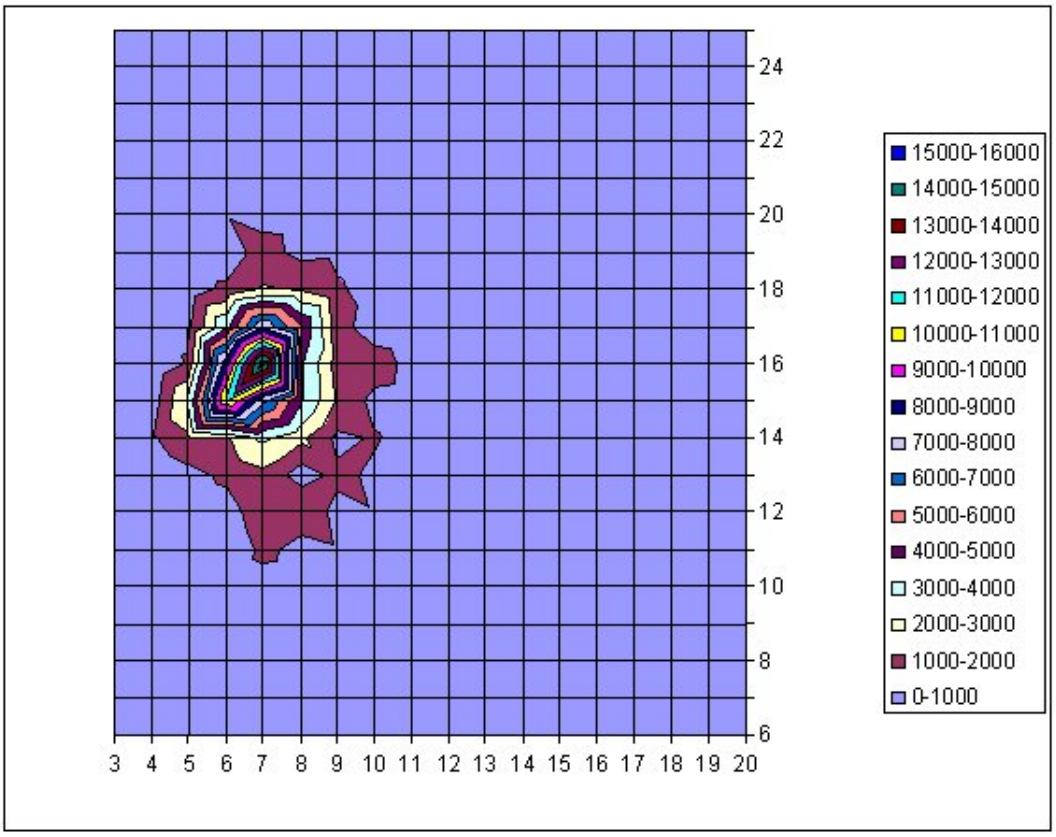


Figure 15: Contourplot showing sum of distances driven on weekdays reported by respondents. First axis shows time of first trip (departure) while the second axis shows time of second trip (often return).

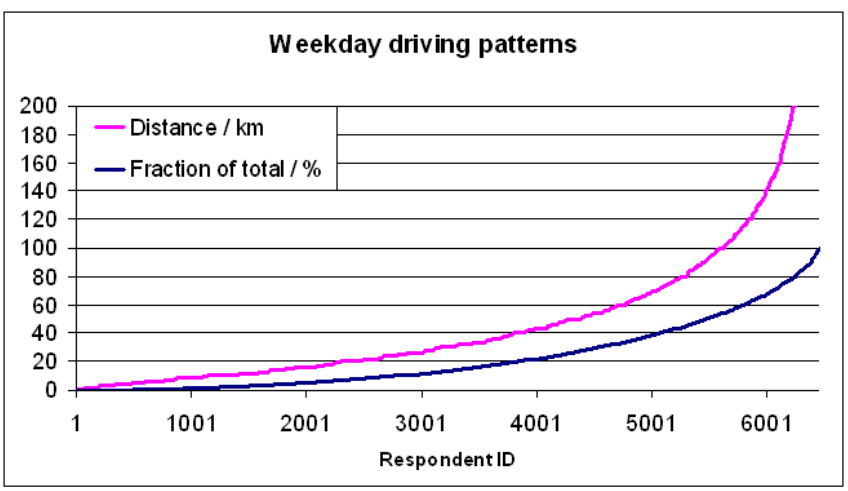


Figure 16: Respondents ordered by total distance driven in a day. The blue line shows the cumulative fraction of total driving demand made up by respondents. The vertical axis and distance graph is cut-off at 200 km.

6.2 Using standard vehicles to model drive demand

Other approaches in EDV-modelling [35] describes the system as a large battery from which a part is taken by the vehicles departing. Upon return, the part with low state-of-charge (SOC) is mixed with the large battery resulting in a large battery with a slightly lowered SOC. Maximum charging power is calculated as if all vehicles could charge simultaneously. This gives a too large potential for charging, since it corresponds to all vehicles being able to charge on the same battery. Furthermore it allows vehicles just having returned from a long trip to depart again or provide V2G although their SOC in the real world would not permit this. As an example consider figure 17 and 18. If the batteries are aggregated vehicle B would recharge much faster than in the real world.

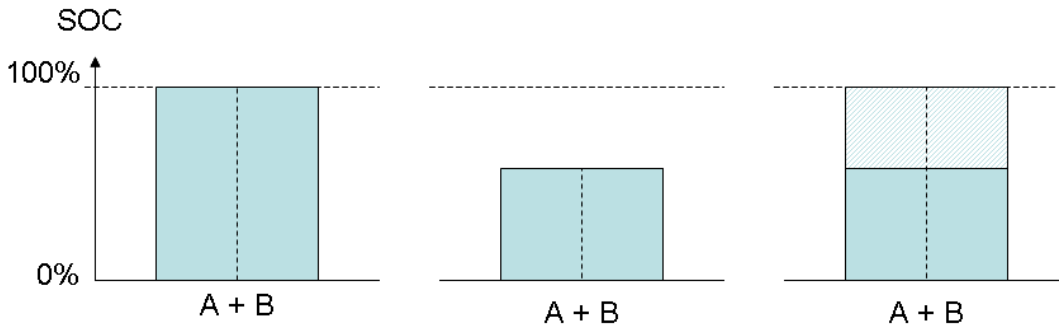


Figure 17: The two batteries are considered as one. Vehicle A uses 10 % and vehicle B uses 80 % of their battery capacities resulting in an overall SOC of 55 %. After some time has passed both vehicles are fully charged.

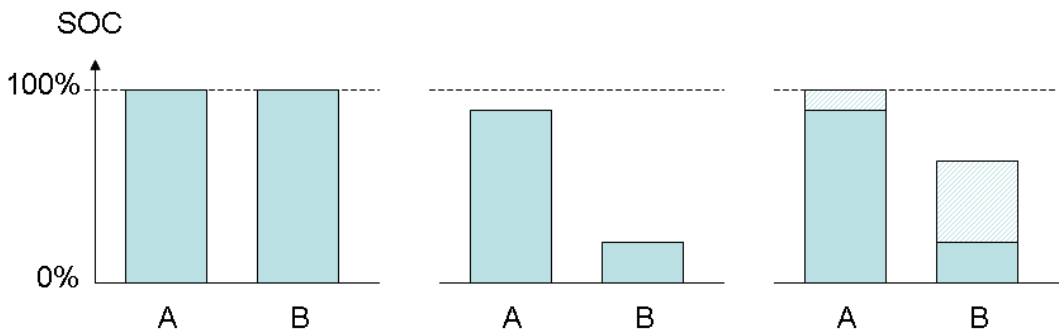


Figure 18: The two batteries are considered seperately. Vehicle A uses 10 % and vehicle B uses 80 % of their battery capacities. After the same time period has passed as in figure 17 vehicle A is fully charged, but vehicle B is only at 65 % SOC.

In this thesis an approach will be made to model the real world with a set of standard vehicles. Each of these standard vehicles represents a number of real world vehicles all having similar driving patterns. The benefit of this approach is a more realistic representation of the potential to charge vehicles and do V2G. The need for using standard vehicles arises from the fact that the simplex algorithm used to optimize the charging patterns has exponential time in the worst case[6]. Finding an optimal solution for several thousand vehicles would take too long. In order to determine the driving pattern of each of these standard vehicles and

[6]Exponential time means that calculation time grows exponentially with the number of variables.

Table 5: Examples of 7 hour driving patterns. Using the euclidean distance measure all patterns are equally similar to pattern A.

Pattern \ hour	1	2	3	4	5	6	7
A	0	20	0	0	0	0	0
B	0	20	20	0	0	0	0
C	0	0	0	0	0	0	0
D	0	20	0	0	0	0	20
E	0	40	0	0	0	0	0
F	10	10	0	0	10	0	10

how many vehicles each of the standard vehicles should represent, clustering of single-vehicle patterns must be carried out.

6.3 Clustering of driving patterns

Clustering is a method to group a set of patterns or points based on similarity. Several different clustering techniques exist with various benefits and drawbacks. A review of clustering methods are given in [36].

Driving survey data is only available on a single-day basis. Therefore a weekly pattern would have to be composed of several different vehicles. It is assumed that for each vehicle the driving patterns of all weekdays are the same and driving patterns for weekend-days and holidays are the same as well. Thereby only two types of patterns (weekday/weekend) have to be merged. The merging will be discussed in section 6.4.

Each driving pattern is a 24-dimensional vector with the distance driven in each hour as its elements. In order to cluster the driving patterns a measure of similarity must be found to compare two patterns. One of the most typical methods of clustering vectors or points is to use the euclidean distance as defined by equation (20).

$$D_{ab} = ||\mathbf{a} - \mathbf{b}|| = \sqrt{\sum_i (a_i - b_i)^2} \tag{20}$$

where D_{ab} is the distance between points $\mathbf{a}$ and $\mathbf{b}$ and the a_is and b_is are the elements of the two vectors.

This measure was used to cluster driving patterns of hybrid electric vehicles (HEV) by S. Ichikawa et al. [37] although in a slightly different context than here. The clustering was used to reduce the number of driving patterns on a commuting route in order to create input for an optimal energy management system in the HEV. The patterns were composed of velocities at different distances and showed far higher similarity than those used in this project.

Euclidean distance provides a good measure of distances between points in a multidimensional space, but unfortunately viewing daily driving patterns as points gives a poor result as shall be seen. Driving patterns vary substantially in both distances and which hours the vehicles are driven. Furthermore the 24 dimensions are not independent as in euclidean space. Two adjacent hours are more similar than two hours that are far appart. As an example consider the patterns in table 5. All six driving patterns B-F are equally similar to A with the euclidean distance of 20. In terms of driving patterns it would make sense to consider B and E to be more similar to A than C, D and F is.

In order to cluster data appropriately two main challenges must be solved. One is defining a good distance measure the other is defining a center for the new cluster. If the euclidean distance measure is used an obvious choice for a center is the spatial center of all points in the cluster.

6.3.1 The k-means algorithm

The efficient algorithm k-means [38] has been devised to cluster n points into k clusters based on euclidean distance measure. It works as described in box 1 below. It is an iterative procedure that usually reaches convergence fairly fast. There is no guarantee that the result obtained is the globally optimal solution, since the result is dependent on the initial points selected. The output of k-means can be dependent on the selection of initial centroids. Therefore a number of clustering attemps should be carried out and the result used be the one with the lowest overall error.

Box 1: The k-means algorithm

1. Randomly select k points and use them as intial centroids (cluster centers).
2. Determine the closest centroid for each of the other $n - k$ points and assign them to a cluster each.
3. Loop:
   - Define new centroids calculated as the spatial center of all points in a cluster.
   - Assign each of all n points to new cluster
4. Quit loop when difference between the error (sum of all distances between points and their respective cluster centers) of the current iteration and the last is less than a threshold value.

The k-means algorithm was implemented using C♯ and the code can be seen in appendix C. The main() function sending arguments to the k-means algorithm and the functions handling input and output of data have been omitted.

The result of clustering 6455 weekday driving patterns using the traditional k-means algorithm is shown in figure 19. Although the total distance driven in each hour is unchanged by the clustering the resulting patterns are useless. For all patterns driving is dispersed on most hours of the day. Two patterns make up by far the largest driving demand and does not resemble any real world driving pattern. Most of the time none of the vehicles would be available for charging if these patterns were used.

As seen, defining the centroid as the spatial mean of all points within the cluster yields centroids that are unlikely to be points in the original data. An alternative centroid definition is the point in the cluster which has the lowest sum of distances to all other points in the cluster. The major drawback of this method is that the distances between all points in a cluster have to be calculated. The major advantage is that one can use other distance measures than the euclidean one, which is the only distance measure for which the spatial mean makes sense. Using this centroid definition and euclidean distance as a measure can be seen as an improvement, since the resulting patterns are actual patterns in which driving only takes place some of the hours. Unfortunately the centroids found by this method

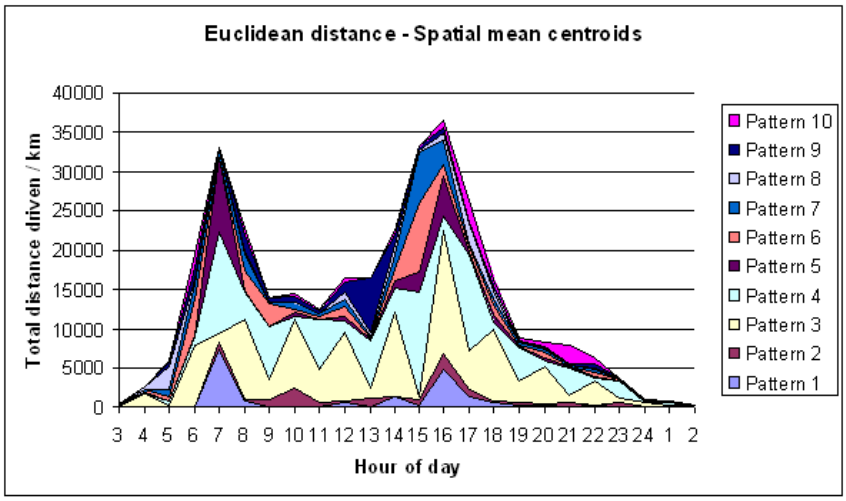


Figure 19: Result of clustering using euclidean distance and defining centroids as the spatial mean. The patterns are stacked. Note that the sum is identical to the weekday driving patterns in figure 14.

generally contains very little overall driving. Patterns in which long distances are driven are unlikely to be picked as centroids since driving must match all other patterns both in time and distance in order not to give a large error. The squaring of the difference in distance driven (see equation (20)) contributes further to this. Also driving patterns in which vehicles depart several hours during the day are more prevalent in the resulting centroids than in the patterns clustered. The results of clustering using euclidean distance and centroids defined as the pattern with lowest sum of intra-cluster distances can be seen in figure 20. The driving demand has not been scaled to fit the overall driving demand. Instead, total driving demand has been calculated by multiplying the driving demand of each standard vehicle by the number of vehicles in the cluster it represents.

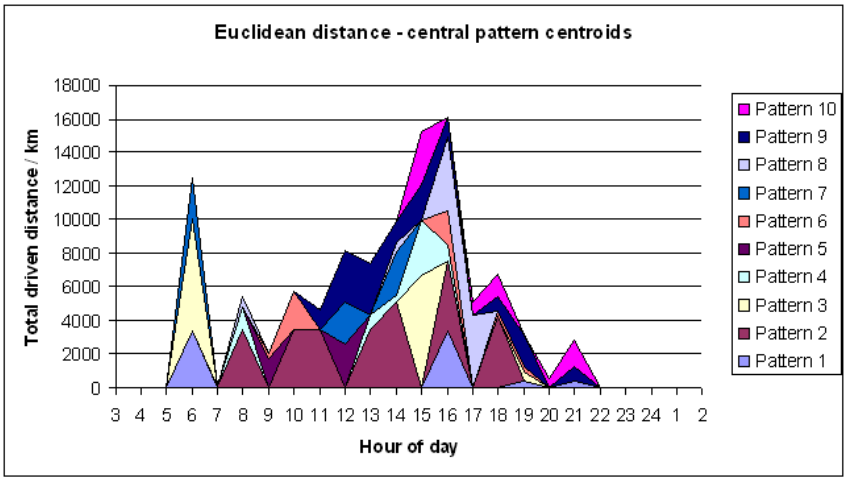


Figure 20: Result of clustering using euclidean distance and defining centroids as the pattern with lowest sum of distances to all other patterns in cluster. The patterns are stacked. Note that the sum is *not* identical to the weekday driving patterns in figure 14.

6.3.2 Defining another distance measure

Using the example shown in table 5 it can be seen that there is a need to take into account that some hours are more similar than others. One requirement for a distance measure in order to get consistent results is that the distance from A to B must be the same as the distance from B to A. A proposal for how to calculate a distance between two patterns, while taking into account similarity of adjacent hours, is to compare the hours two by two. This could be done by searching through the patterns until a non-zero value is reached. Then the sum of the value found and that of the next hour is compared to the same two hours in the other pattern. Summing the absolute values of differences between the two yields the distance measure.

Pseudocode for the distance function can be seen in box 2, where N_hours is the number of hours in each pattern (24) and pat1[] and pat2[] are Nhours-dimensional arrays holding the values of distance driven in each hour for the two patterns. The += operator adds the right hand side to the left hand side. Abs(x) returns the absolute value of x.

As can be seen the last hour is never investigated if the second-to-last hour does not contain any driving. Since little driving takes place in the middle of the night the error arising from this is small. Furthermore, as another drawback of this method, it should be noted that the result may not be independent of which direction the driving pattern is searched.

An example of a cluster and the standard vehicle representing it is given in figure 21 and the aggregated picture is shown in figure 22. These figures are discussed in more detail in the next section.

When selecting centroids this method has a tendency to pick driving patterns in which driving takes place in fewer hours than average in the sample. The effect of this is that the standard vehicles are parked more often than in the real world fleet. However, the effect of this might be a better result, since it will even out the crude assumption that vehicles are driving the entire hour when driving.

Box 2: Pseudocode for alternative distance function

```
dist = 0;
j = 0;
while (j < N_hours - 1)
{
  if (pat1[j] != 0 or pat2[j] != 0)
  {
    dist += Abs(pat1[j] + pat1[j+1] - pat2[j+1] - pat2[j]);
    j = j + 1;
  }
  j = j + 1;
}
return dist;
```

Refining the described alternative distance measure or investigating other distance measures from other branches of science (e.g. the Levenshtein distance using string matching [39]) could potentially improve the clustering result. This falls beyond the scope of this thesis.

6.3.3 Splitting the patterns on vehicle technology

The driving patterns are very dissimilar. Although drivers commuting creates some order in the patterns (as seen from figure 14 and 15) clustering the patterns is a difficult exercise, which will reduce the quality of the input data. There was no natural grouping of long- and short-distance drivers (as seen from figure 16) and therefore 150 km per day was assumed arbitrarily to be the distance at and over which vehicles are assumed to be PHEV. All vehicles driving less than 150 km were modeled as BEVs. This puts about one-third of driving demand onto PHEVs and two-thirds onto BEVs. The split ensures that no BEV drives so far that batteries have the risk of being depleted. If they did, it would result in no feasible solutions to the optimization problem. PHEVs can use their ICE as backup when the battery is empty and therefore the same problem does not exist for these.

PHEV vehicles were modeled by 10 standard vehicles whose driving patterns were obtained by clustering the 415 available driving patterns. Following the clustering the driving patterns were scaled in order to ensure representation of the total driving demand of a group in each standard vehicle. This was done by finding the average vehicle driving distance in a cluster. The distances driven by the standard vehicle were then scaled so the sum would be the average of the group it represented. When modeled each standard vehicle will be weighted by the number of vehicles in the cluster it represents.

The result of the PHEV weekday clustering was 10 driving patterns of standard vehicles representing between 11 and 68 vehicles from TU and between 6551 and 17099 km of total daily driving. This result will be fed into the model described in section 8. The remaining 6039 weekday patterns with a daily total driving distance less than 150 km were clustered into 20 patterns. The number of vehicles represented by each standard vehicle ranges from 28 to 1102 and driving demand representation ranges from 2244 to 21486 km. As an example of a standard vehicle consider figure 21. The vehicles that have been grouped in the cluster all drive between 7 and 9 am and/or 2 and 5 pm. This cluster, containing 472 vehicles driving 35 km in a day on average, could contain many commuters. They are represented by a vehicle that drives at 8 am and 3 pm.

Clustering 4039 vehicles into 20 will of course distort the overall picture as can be seen from figure 22. This figure shows the sum of all patterns compared to all the standard vehicles. Although it may not seem so at first, it should be noted that the two graphs follow each other fairly closely and could be made almost equivalent by shifting demand to adjacent hours. Driving demand is therefore fairly correctly distributed over the course of the day.

6.4 Merging weekend and weekday driving patterns

In order to model weekend driving demand it was assumed that the same ratio of BEV to PHEV vehicles would be driving on weekdays and weekends. That is 415 to 6039. This is a result of the general assumption that all vehicles drive every day. 2262 weekend driving patterns were available. The 2116 vehicles with the lowest total daily driving demand were assumed to be BEVs whereas the remaining 146 vehicles were modeled as PHEVs. The underlying assumption is that the group who drive most during the weekdays also has the highest driving demand in the weekend. This seems reasonable if this group live in areas that necessitates driving far in order to reach points of interest. By chance, this splitting also happened to result in two groups of vehicles driving less and more than 150 km per day.

When using the k-means algorithm it is only possible to decide the number of clusters, but not how many patterns should go in each. Therefore clustering will give a discrepancy between the number of driving patterns in each group for weekdays and weekends respectively. Table 6 compares the number of PHEV patterns when sorted from lowest to highest. It can be seen that the ratio of patterns from weekends and weekdays is fairly close to the overall ratio

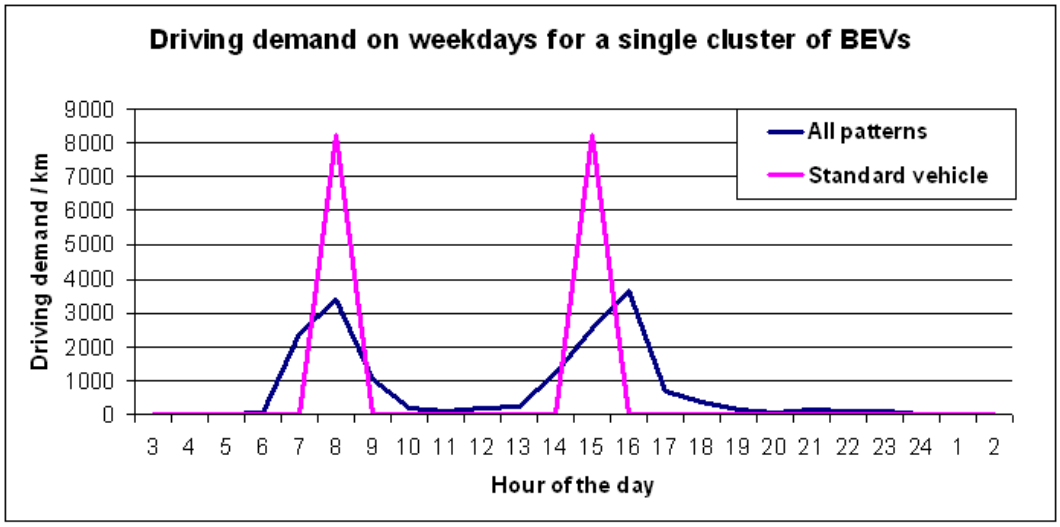


Figure 21: Summed hourly driving demand for BEVs (<150 km/day) that are grouped in a cluster. The driving demand of the standard vehicles they are represented as in the model is shown as well. Standard vehicles have been found by clustering using the method described in section 6.3.2.

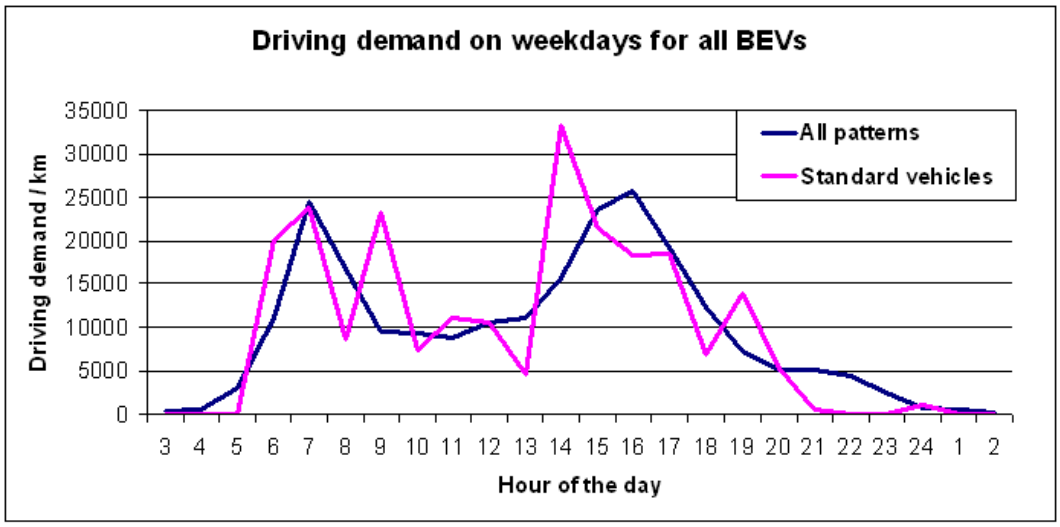


Figure 22: Sum of all patterns and all standard vehicles. See figure 21 for further explanation.

of 2.84. When using the resulting driving patterns in the model the various patterns are weighted by the number of weekday patterns in each group.

Two approaches to merging weekday and weekend driving patterns can be made. The two methods are shown in figure 23. On the left are the patterns to be merged. The letter denotes the standard vehicle and the number is the number of vehicles in each cluster. There is a discrepancy in the number of vehicles in cluster A and X as well as B and Y. The first method, *scale and merge*, always results in the same number of clusters as the input. The weekend driving patterns are scaled such that the distance driven by the weekend standard vehicle is multiplied by the ratio of weekend to weekday driving patterns in the two clusters

that are merged. The * on the AX* and BY* patterns denotes that X and Y have been scaled. Vehicle X* drives 5/4 longer than X in each hour whereas Y* only drives 5/6 of what Y does. When modeled the two vehicles will be weighted by 4 and 6 respectively and the total driving demand will be modeled correctly. In *match and merge* the number of merged patterns could be as many as double the number of weekday patterns. Each weekday pattern is merged with the equivalent number of weekend patterns, when no more weekend patterns are left in one cluster weekend patterns are taken from the next cluster. This results in the AX, BX, BY and CZ clusters.

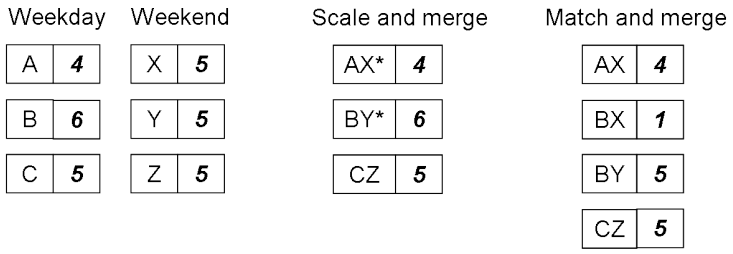


Figure 23: Two methods of merging weekday and weekend clusters/standard vehicles. The letter denotes the cluster/vehicle whereas the number shows how many vehicles it represents. Further explanation in text.

The *scale and merge* method was chosen in this project in order to limit the number of resulting standard vehicles. As was written in the beginning of this section it is assumed that each pattern corresponds to a vehicle. Since there are no patterns with zero driving demand all vehicles are expected to be driving every day. This assumption could particularly overestimate weekend driving demand. With TU having uniform sampling of days the number of patterns hints to this error. The ratio of patterns of 2.84 exceeds the ratio of weekdays to weekend-days and holidays, which is less than 2.5.

Table 6: Number of patterns in each cluster represented by a standard PHEV vehicle (>150 km/day) along with the ratio of the number of weekday and weekend patterns.

Standard vehicle	Weekday	Weekend	Ratio
A	11	3	3.67
B	24	8	3.00
C	27	10	2.70
D	31	12	2.58
E	42	13	3.23
F	48	17	2.82
G	50	18	2.78
H	51	18	2.83
I	63	23	2.74
J	68	24	2.83
All vehicles	415	146	2.84

6.5 Charging preferences and plug-in patterns

In this project it is assumed that vehicles are plugged in when not driving. Also users are assumed willing to let go of the control of charging. The only two constraints modeled will be a minimum SOC and a contractual obligation of the fleet operator to have the vehicles charged to a minimum level in the morning. This obligation is discussed in section 8.3. In the real world EDV users will probably demand some control of how their vehicles are charged. Forced charging where the user can choose to override the fleet operators instructions and charge the vehicle immediately could be a solution. Also it may well be the case that many EDV users do not plug in when parking their vehicles. Either because there is no plug, due to forgetfulness or because of the inconvenience. If the user knows that there is enough energy to complete the next trip he may choose not to plug-in upon arrival. User behaviour with regard to EDVs is very difficult to predict, since there is no previous experience with introducing EDVs in the mass market. How to expand the model to include user behaviour is addressed in section 12.1.

Part III
Optimization of charging

7 Mathematical programming

Mathematical programming derives from operation research and deals with the solution of optimization problems that can be expressed by an objective function to be maximized or minimized subject to a set of constraints that can be formulated as mathematical equations or inequalities. Two very important cases of mathematical programming are linear programming (LP) and quadratic programming (QP). The latter is used to solve the model presented in section 8. LP is closely related to QP and solving QP problems involves converting them to a modified LP form. Both LP and QP have linear constraints and the terms linear and quadratic are referring to the form of the objective function.

The set of points that lie within the constraint boundaries is called the *feasible region*. Every point in this set corresponds to a feasible solution to the optimization problem. Finding the optimal solution requires searching through this set. The task is usually made much simpler if the feasible region is a *convex set*. A convex set is defined as a set in which every point on the line between any two points in the set is also in the set.

Sets bounded by linear functions are convex. Therefore both linear programming and quadratic programming problems have convex set feasible regions. Some necessary conditions for optimality apply to the objective function. These will be described in the following sections.

Most of this section is based on the textbook Introduction to Operations Research by Hillier and Lieberman [40]. Vectors are indentified by bold font lower case letters (e.g. $\mathbf{x} = (x_1, x_2, ..., x_n)^T$). Matrices are denoted by bold font capital letters. A superscript T denotes the transpose.

7.1 Linear programming

Any optimization problem that can be formulated as a linear objective function to be optimized subject to a set of linear constraints is a linear programming problem and has (at least) one optimal solution if the constraints are bounding a feasible region not being the empty set. Any linear programming problem can be expressed in its so-called standard form as:

$$\begin{aligned} &\text{Maximize} \\ Z = \mathbf{c}^T\mathbf{x} &= \sum_i c_i x_i \\ &\text{Subject to the constraints} \\ \mathbf{A}\mathbf{x} &\leq \mathbf{b} \\ \mathbf{x} &\geq \mathbf{0} \end{aligned} \tag{21}$$

Where Z is the objective function which in a economic problem could be the sum of profits of each activity c_i multiplied by the activity level x_i, this could e.g. be quantity produced. The variables x_i are bounded by two types of constraints resulting in these not being able to take any value: Functional and non-negativity constraints. $\mathbf{A}$ is the matrix containing the coefficients of the x_is in the functional constraint equations bounded by the b_is. As an example if x_i is the output of product i then $\mathbf{A}$ could describe how many resources are used to produce each product and $\mathbf{b}$ the available resources.

Geometrically finding the optimal solution can be thought of as moving the hypersurface described by the objective function (by changing Z) out towards the edges of the feasible region (bounded by the constraints). When the hypersurface only intersect the feasible region at one point the optimal solution has been found. An example of this is shown in figure 24 in a case of two variables and three functional constraints. The feasible region is bounded by the functional constraints and the nonnegativity constraints. The line $Z = f(x_1, x_2)$ is moved outwards by increasing Z until it is intersecting only one point. This point is the optimal solution since the value of Z cannot be increased further while staying within the feasible region. Z has been maximized subject to the constraints. In the case where the objective function is parralel to one of the constraint equations an infinite number of optimal solutions can be found to the problem. Since the objective function and the functional constraint are identical but a constant Z will have the same value in all points on the constraint boundary. If this happens any of the optimal solutions can be used.

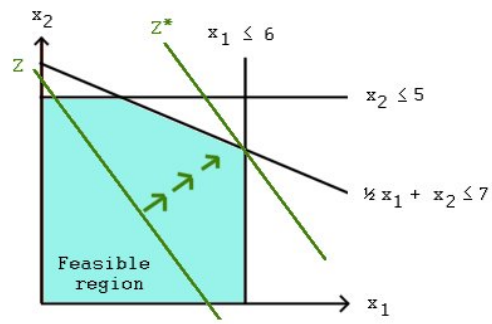


Figure 24: Two dimensional example of linear programming. The line described by the objective function is moved outward by increasing $Z = 4x_1 + 3x_2$ until only intersecting the feasible region in one point $(x_1, x_2) = (6, 4)$: The optimal solution with $Z = Z^*$

7.1.1 The simplex method

The Simplex method was developed by George Dantzig in 1947 and is a very powerful tool to solve linear programming problems. The method exploits the fact that an optimal solution must be a *corner point feasible* (CPF) solution. CPF means that the solution is a corner point of the feasible region. For a problem with n variables it is a point, where, at least, n of the constraint boundaries intersect. This offers the possibility of reducing the problem of searching through an infinite set (the entire feasible region) to an finite set (the CPFs). The example problem in figure 24 has 5 CPFs. The optimal solution is one of them.

The geometric version of the simplex method is as follows:

1. Pick a CPF as starting point.
2. Iterate:
   - → Identify along which constraint boundary to move in order to increase Z the fastest
   - → Move along constraint boundary until next CPF is reached and redo last step.
3. When none of the constraint boundaries emanating from the CPF reached offers an increase in Z the optimal solution has been found.

Implementation of the simplex method requires an algebraic version, which involves the use of so-called slack variables. In case origo is chosen as starting point, the method is equivalent

to a special type of Gauss-Jordan elimination of the matrix shown in equation (22). This is done in the following steps.

1. Find CPF solution as starting point

   → Sometimes origo ($\mathbf{x} = \mathbf{0}$) is a CPF and can be chosen, but otherwise some more sophisticated methods have to be used.

2. Introduce slack variables ($\mathbf{s} \geq \mathbf{0}$) transforming all constraint inequalities into equations.

   → This results in constraints of the form $\mathbf{Ax} + \mathbf{s} = \mathbf{b}$ (all but the first row in equation (22).

3. Iterate:

   → Identify what variable to increase from zero by identifying the variable with the most negative coefficient in the first row of the matrix. This variable is called the entering variable*.

   → Identify which of the rows below to subtract from all other rows in order to get zeros in the column with coefficients for the entering variable.

   → This is done by finding the row in which the ratio between the coefficient and right hand side value is the highest. The one with lowest ratio is picked in order not to get negative variables and thereby infeasible solutions.

   → Doing this will result in another variable becoming zero (leaving variable)

* If no variable can be changed in order to increase Z (i.e. no negative coefficients in the first row), the optimal solution has been found.

$$\begin{bmatrix} 1 & -\mathbf{c}^T & 0 \\ 0 & \mathbf{A} & \mathbf{I} \end{bmatrix} \begin{bmatrix} Z \\ \mathbf{x} \\ \mathbf{s} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{b} \end{bmatrix} \qquad \mathbf{x} \geq \mathbf{0},\, \mathbf{s} \geq \mathbf{0} \tag{22}$$

The example problem of figure 24 is solved in appendix A.

7.2 Non-linear programming

Many optimization problems are such that linearity in both constraints and objective function is too rough an assumption. Introducing non-linearities, however, complicates things. A non-linear objective function could have several local maxima and non-linear constraints could make up a non-convex set, which make it difficult or impossible to make sure that an optimal solution is found. The necessary and sufficient conditions for optimality have been derived independently by Karush [41] and by Kuhn and Tucker [42] and are named the *KKT conditions*. A short derivation of the conditions based on [42] follows here.

7.2.1 The KKT conditions

By introducing a vector of Langrange multipliers $\mathbf{u}$ into the problem of maximizing an objective function $f(\mathbf{x})$ subject to a set of constraints $\mathbf{g}(\mathbf{x}) \geq \mathbf{0}$ we get the Lagrange function of the problem.

$$L(\mathbf{x}, \mathbf{u}) = f(\mathbf{x}) + \mathbf{u}^T \mathbf{g}(\mathbf{x}) \tag{23}$$

The optimal solution $\mathbf{x}^*$ to the primary problem (of maximizing the objective function $f(\mathbf{x})$ subject to the constraints $\mathbf{g}(\mathbf{x}) \geq \mathbf{0}$) requires an nonnegative vector $\mathbf{u}^*$ which is a solution to the saddlepoint problem satisfying

$$L(\mathbf{x}^*, \mathbf{u}) \geq L(\mathbf{x}^*, \mathbf{u}^*) \geq L(\mathbf{x}, \mathbf{u}^*) \tag{24}$$

The term *saddlepoint* arises from the fact that from the optimal point L is decreasing with any change in x and L is increasing with any change in u.

Lemma1 The *necessary* conditions for optimality are

$$\nabla_{\mathbf{x}} L \leq \mathbf{0} \qquad (\nabla_{\mathbf{x}} L)^T \mathbf{x}^* = 0 \qquad \mathbf{x}^* \geq 0 \tag{25}$$
$$\nabla_{\mathbf{u}} L \geq \mathbf{0} \qquad (\nabla_{\mathbf{u}} L)^T \mathbf{u}^* = 0 \qquad \mathbf{u}^* \geq 0 \tag{26}$$

where $\nabla_{\mathbf{x}} = \left[\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, ...\right]^T$ and $\nabla_{\mathbf{u}} = \left[\frac{\partial}{\partial u_1}, \frac{\partial}{\partial u_2}, ...\right]^T$

Proof In order for the solution to be a saddle point the $(\nabla_x L)_i$ and $(\nabla_u L)_i$ must vanish (corresponding to a function extrema) except in the case where x_i or u_i are zero (corresponding to a constraint boundary) where they must be nonpositive and nonnegative respectively.

Lemma2 The *sufficient* conditions for optimality are (25), (26) *and*

$$L(\mathbf{x}, \mathbf{u}^*) \leq L(\mathbf{x}^*, \mathbf{u}^*) + (\nabla_{\mathbf{x}} L)^T (\mathbf{x} - \mathbf{x}^*) \tag{27}$$
$$L(\mathbf{x}^*, \mathbf{u}) \geq L(\mathbf{x}^*, \mathbf{u}^*) + (\nabla_{\mathbf{u}} L)^T (\mathbf{u} - \mathbf{u}^*) \tag{28}$$

for all $\mathbf{x} \geq \mathbf{0}$ and $\mathbf{u} \geq \mathbf{0}$

Proof Equation (25) states that for all i either $(\nabla_x L)_i$ is negative and x_i is zero or $(\nabla_x L)_i$ is zero and x_i is postive. Therefore the last term in equation (27) is always negative or zero. The same argument (with $(\nabla_u L)_i$ being positive instead of negative) holds for equation (28) and hence it can be seen that equation (24) holds if (25-28) are satisfied. Therefore the conditions are sufficient to ensure optimality.

It should be noted that equation (27) is satisfied if the objective function is concave and equation (28) is satisfied if all constraint functions ($\mathbf{g}(\mathbf{x})$) are convex and the feasible region therefore is a convex set.

7.3 Quadratic programming

Quadratic programming problems differ from linear programming problems in the objective function being quadratic. The general form of any quadratic programming problem can be written as

$$\begin{aligned} &\text{Maximize} \\ Z = \mathbf{c}^T\mathbf{x} - \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} = \sum_i c_i x_i - \frac{1}{2}\sum_{i,j} Q_{i,j} x_i x_j \\ &\text{Subject to the constraints} \\ &\mathbf{A}\mathbf{x} \leq \mathbf{b} \\ &\mathbf{x} \geq \mathbf{0} \end{aligned} \tag{29}$$

Due to the quadratic term, it is no longer certain that an optimal solution is a CPF, as is the case with linear programming (see section 7.1.1). Therefore another approach has to be taken. The Lagrangian of the problem is

$$L(\mathbf{x}, \mathbf{u}) = \mathbf{c}^T\mathbf{x} - \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T(\mathbf{b} - \mathbf{A}\mathbf{x}) \tag{30}$$

When applying the KKT conditions to the quadratic programming problem it is seen that the necessary conditions for optimality are

$$\nabla_x L(\mathbf{x}, \mathbf{u}) = \mathbf{c} - \mathbf{Q}\mathbf{x} - \mathbf{A}^T\mathbf{u} \leq \mathbf{0} \tag{31}$$

$$\nabla_u L(\mathbf{x}, \mathbf{u}) = \mathbf{b} - \mathbf{A}\mathbf{x} \geq \mathbf{0} \tag{32}$$

Introducing the nonnegative slack variable vectors $\mathbf{y} = -\nabla_x L(\mathbf{x}, \mathbf{u}) \geq \mathbf{0}$ and $\mathbf{v} = \nabla_u L(\mathbf{x}, \mathbf{u}) \geq \mathbf{0}$ into the equations and rearranging one gets

$$\mathbf{Q}\mathbf{x} + \mathbf{A}^T\mathbf{u} - \mathbf{y} = \mathbf{c} \tag{33}$$

$$\mathbf{A}\mathbf{x} + \mathbf{v} = \mathbf{b} \tag{34}$$

Since the necessary conditions for optimaility (equation (25) and (26)) are $(\nabla_x L)^T\mathbf{x} = (\nabla_u L)^T\mathbf{u} = 0$ it is required that:

$$\mathbf{y}^T\mathbf{x} = \mathbf{v}^T\mathbf{u} = 0 \tag{35}$$

Equation (35) is called the complementarity constraint and is the only non-linear equation that one has to deal with when solving a quadratic programming problem. How this is done is explained in section 7.3.1.

As stated in section 7.2.1 the two other equations making up the sufficient conditions for optimality (equation (27) and (28)) are satisfied if the objective function is concave and the constraint equations are convex. The objective function is concave if $\mathbf{x}^T\mathbf{Q}\mathbf{x} \geq 0$ for all $\mathbf{x}$, that is $\mathbf{Q}$ is positive semidefinite. All the constraints are convex (since they are linear). Therefore, if $\mathbf{Q}$ is positive semidefinite, finding a set of vectors, $\mathbf{y} \geq \mathbf{0}, \mathbf{x} \geq \mathbf{0}, \mathbf{v} \geq \mathbf{0}, \mathbf{u} \geq \mathbf{0}$, which satisfy equations (33), (34) and (35), ensures $\mathbf{x}$ being optimal.

7.3.1 The modified simplex method

Since all constraints except the complementarity constraint are linear it is possible to use the simplex method with a small modification ensuring that equation (35) is not violated. The simplex method requires a feasible solution as starting point and therefore the first task is to identify such a solution.

For every i where $c_i \leq 0$ *and* $b_i \geq 0$ the solution is $x_i = 0, u_i = 0, y_i = -c_i, v_i = b_i$. For every other i it is nescessary to introduce artificial nonnegative variables into each of the equations where $c_i > 0$ *or* $b_i < 0$.

The artificial variables are introduced into the equations as the two column vectors $\mathbf{r}$ and $\mathbf{s}$, adding $\mathbf{r}$ on the left hand side of (33) and adding $\mathbf{s}$ on the right hand side of (34) The feasible solution which will be taken as a starting point has $\mathbf{x} = \mathbf{u} = \mathbf{0}$, $\mathbf{y} = -\mathbf{c} + \mathbf{r}$, where $r_i = c_i$ if $c_i > 0$ and zero otherwise, and $\mathbf{v} = \mathbf{b} - \mathbf{s}$, where $s_i = b_i$ if $b_i < 0$ and zero otherwise. Since these artificial variables are not in the original constraint equations the objective is to make them disappear. When all $r_j = s_j = 0$ in a feasible solution, equation (33) and (34) are satisfied and hence that solution will be optimal. The modified simplex method therefore uses the objective function

$$\text{Maximize} \qquad Z = -\sum_j r_j + s_j \tag{36}$$

Note that the right hand side cannot be positive. The only acceptable solution will be $Z = 0$ that is $\mathbf{r} = \mathbf{0}$ and $\mathbf{s} = \mathbf{0}$. The objective function needs only consider the non-zero r_j and s_j, which reduces the size of the matrix to be eliminated when implementing the method. The modification in the modified simplex method is that the complementary constraint equation is taken into account. Since equation (35) holds for the first found feasible solution to the artificial problem one only needs to make sure that it is not violated by the alogrithm. This is done by making sure that if one variable in any complementary pair (e.g. u_i and v_i) is non-zero, the other cannot be chosen as entering variable. Doing this results in at least one of the two being zero at all times. When setting up the matrix to be eliminated it initially looks like the one in equation 37. First, one must make sure to eliminate the coefficients of $\mathbf{r}$ and $\mathbf{s}$ from the first row, which is done by subtracting one of every row to the first row. Once this is done the standard simplex procedure follows (as described in section 7.1.1) with choosing the variable with the most negative coefficient in the first row and start increasing that from zero until another variable becomes zero. This is done iteratively until the first row is back to the starting point, but with a right hand side of zero. This corresponds to Z being zero and $\mathbf{r} = \mathbf{0}$ and $\mathbf{s} = \mathbf{0}$, which is the case when an optimal solution has been reached.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ \mathbf{0} & \mathbf{Q} & \mathbf{A}^T & -\mathbf{I} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} Z \\ \mathbf{x} \\ \mathbf{u} \\ \mathbf{y} \\ \mathbf{v} \\ \mathbf{r} \\ \mathbf{s} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{c} \\ \mathbf{b} \end{bmatrix} \tag{37}$$

7.4 Working with free variables

Some variables used in a model could be free in the sense that a nonnegativity constraint does not apply. This is e.g. the case of the total load, which can be either positive or negative depending on whether there is a net charge or discharge for the batteries. This is dealt by splitting the variable into two parts, a negative and a postive.

$$x_i = x_i^+ - x_i^- \qquad x_i^+ \geq 0, x_i^- \geq 0 \tag{38}$$

The two parts are modeled as two independent variables and once the optimization is complete the value of x_i is obtained from equation (38).

8 The Optimization model

The problem of optimizing the charging of a number of electric drive vehicles can be formulated as an objective function to be minimized subject to a number of constraints. The objective of the fleet-operator is to minimize total costs, that is charge as cheaply as possible while limiting liquid fuel consumption and use V2G if it is profitable.

To formulate the problem a number of sets, parameters and variables will be defined:

Sets

t	Hour in optimization period
v	Vehicle type
T	set containing all t
V	set containing all v

Parameters

α	Special VAT on electricity
β	Fixed taxes on electricity
γ	General VAT
C_{max}	Maximum charge power
$D(v,t)$	Driven distance
$DCons(v)$	Energy consumption per distance driven
$\eta_C(v)$	Charging efficiency
$\eta_{ICE}(v)$	ICE efficiency
$\eta_{V2G}(v)$	V2G efficiency
$E_{max}(v)$	Maximum state of charge
$E_{min}(v,t)$	Minimum state of charge
$f(v)$	Fraction of vehicles of type v
$ICEcost$	Cost of fuel
λ	Minimum SOC at hour of contractual obligation
μ	SOC at set-point
N_{tot}	Total number of EDVs
$\overline{P}(t)$	Forecasted electricity price without EDV load
$\left(\frac{dP}{dQ}\right)^*(t)$	Price dependency on demand
$V2G_{max}$	Maximum V2G power
$V2Gcost$	Cost of using V2G due to battery wear

Variables

$C(v,t)$	Charge power from grid
$E(v,t)$	Battery state of charge
$ICE(v,t)$	Fuel use in ICE to provide energy
$P(TL(t),\overline{P}(t))$	Electricity price with EDV load
$TL(t)$	Total EDV load
$V2G(v,t)$	V2G power to grid

8.1 The constraints

The optimization problem is constrained by the energy balance, limitations of the power line and limitations of the battery. Unless specifically stated otherwise the constraints apply to all $v \in V$ and $t \in T$. The constraints are as follows:

$$\begin{aligned} E(v,t+1) &= E(v,t) + C(v,t) \cdot \eta_C(v) - V2G(v,t)/\eta_{V2G}(v) \\ &\quad -D(v,t) \cdot DCons(v) + ICE(v,t) \cdot \eta_{ICE}(v) \qquad \forall t \in T \backslash \{t_{last}\} \end{aligned} \tag{39}$$

Equation (39) is the balance equation for the battery. It applies to all $t \in T$ except the last t (since $t_{last} + 1$ is not in T). It states that the amount of energy in the battery at time t+1 is equal to the amount at time t plus the change, which is the sum of four terms. The first term is the product of the energy charged from the grid times the charging efficiency. The second term is subtracted since it is the energy delivered to the grid divided by the efficiency of V2G. The third term accounts for driving and is the distance driven multiplied by the vehicle type specific driving consumption (energy/distance). This term is also subtracted since energy is taken from the battery to drive. The fourth and last term deals with the energy input to the system from the internal combustion engine. This term only has relevance for hybrid vehicles. BEVs were modelled by setting the efficiency of the ICE to zero.

Only <u>one</u> of C(v,t), V2G(v,t) and D(v,t) can be non-zero at time t. (40)

Since the vehicle needs to be plugged in to be charged, it is impossible to both drive and charge at the same time. Also it does not make sense to both charge and do V2G at the same time because energy can only flow one way. This is further discussed in section 8.1.1. The restriction is stated in (40).

$$C(v,t) \leq C_{max} \tag{41}$$

$$V2G(v,t) \leq V2G_{max} \tag{42}$$

Due to power line limitations neither charge nor V2G can exceed a certain value. This is enforced by the constraints in equation (41) and (42).

$$E(v,t) \geq E_{min}(v,t) \tag{43}$$

$$E(v,t) \leq E_{max}(v) \tag{44}$$

Equation (43) and (44) impose the battery limitations, which are physical but in the case of Emin can be higher due to user needs. Emin(v,t) can vary from hour to hour because of requirements set by the user. It is rather likely that EDV users will demand a minimum SOC at all times such that the vehicle is able to be used for unanticipated events.

ICE(v,t) is assumed unconstrained. It would make sense to limit engine use to when the vehicle is in motion, but since fuel costs are independent on what time of day fuel is consumed it makes no difference if the result shows fuel use in the correct hour or not. The only problem which may arise is that the ICE is then allowed to provide energy for V2G power while the vehicle is parked. This would only take place after the battery is depleted, which would require successive hours of extremely high prices, which is a very rare occurrence. ICE(v,t) could be constrained by D(v,t) times some factor/ describing the maximum engine output in the time period in which driving takes place. This factor would be dependent on what type of driving was carried out in the specific hour.

8.1.1 Linearity of the constraints

In order to keep the problem as a standard quadractic programming problem it is required that all the constraints are linear. Equations (39),(41),(42),(43), (44) and (48) are all linear, bounding a feasible region that is a convex set. Equation 40 can be written in the following form:

$$C(v,t) \cdot D(v,t) = 0 \tag{45}$$
$$V2G(v,t) \cdot D(v,t) = 0 \tag{46}$$
$$C(v,t) \cdot V2G(v,t) = 0 \tag{47}$$

$D(v,t)$ is a parameter. Therefore it can be seen that equation (45) and (46) are also linear constraints. The constraint in equation (47) is non-linear and therefore does not fit with the standard linear or quadratic programming form. Fortunately the last constraint can be dropped, since an optimal solution could never have both C(v,t) and V2G(v,t) be non-zero at the same time.

To justify this, suppose $C(v,t) = a + b \geq 0$ and $V2G(v,t) = b \geq 0$. Using the first two terms of equation (39) the change in energy level of the battery will be $a \cdot \eta_C + b \cdot \eta_C - b/\eta_{V2G}$. Charging and V2G efficiencies are both less than unity and so $b \cdot \eta_C - b/\eta_{V2G} < 0$ for $b > 0$. Therefore the net result of having $b > 0$ will be a lower $E(v,t+1)$ compared to the case where $b = 0$. Since the price of electricity is under a non-negativity constraint it will never be desirable to have a lower $E(v,t+1)$. A lower $E(v,t+1)$ will result in more charging be demanded at a later period thereby increasing the objective function. An optimal solution, in which the objective function is minimized, will therefore cause $V2G(v,t) = b = 0$. In the case where $a \leq 0$ ($V2G(v,t) \geq C(v,t)$) a similar argument can be used with the result that $a = -b$ ($C(v,t) = 0$) gives the optimal solution.

8.2 The objective function

The objective function for the fleet operator is to minimize total cost.

The total EDV load in a given hour is the sum of all the individual vehicles' charge minus V2G in that hour. V2G is subtracted since it corresponds to negative load.

$$TL(t) = N_{tot} \sum_v f(v) \cdot (C(v,t) - V2G(v,t)) \tag{48}$$

where each of the standard vehicles are weighted by $N_{tot} f(v)$ – the number of vehicles they represent.

The general objective function, given by equation (49), describes total cost and consists of three terms: Electricity cost, fuel cost and battery wear cost. Electricity cost is the total load times the price of power summed over all hours. Fuel costs are the total fuel use times the price of fuel and battery wear costs are the total use of V2G times the per energy cost of using V2G.

$$\begin{aligned} \min Z &= Z_{el} + Z_{fuel} + Z_{wear} \\ &= \sum_t \left(TL(t) \cdot P(TL(t), \overline{P}(t)) + ICEcost \sum_v ICE(v,t) + V2Gcost \sum_v V2G(v,t) \right) \end{aligned} \tag{49}$$

here price has been written as dependent on total EDV load and the expected power price at zero EDV load. Depending on how $P(TL(t), \overline{P}(t))$ is defined one gets various types of problems to solve. More attention is therefore directed to the first term.

8.2.1 The electricity cost term

In this section attention is brought to the first term in equation (49), Z_{el}. The two last terms are linear and the problem type will be defined by the form of the first term Z_{el}.
Assuming no taxes and that price is independent on the EDV load ($P = \overline{P}$) one gets a linear programming problem with the following electricity cost term

$$Z_{el} = \sum_t TL(t) \cdot \overline{P}(t) \tag{50}$$

Assuming no taxes and the spot price being linearly dependent on total demand with coefficients depending on $\overline{P}(t)$, that is

$$P(TL(t), \overline{P}(t)) = \left(\frac{dP}{dQ}\right)^* (t) \cdot TL(t) + \overline{P}(t) \tag{51}$$

where $\left(\frac{dP}{dQ}\right)^* (t)$ is the derivative of $P(Q)$ evaluated at $TL(t) = 0$, one gets a quadratic expression for the total cost with

$$Z_{el} = \sum_t \left(\left(\frac{dP}{dQ}\right)^* (t) \cdot TL(t)^2 + \overline{P}(t) \cdot TL(t) \right) \tag{52}$$

Including taxes of the form (as described in section 3)

$$P_{user} = \gamma(\alpha P_{spot} + \beta) \tag{53}$$

we arrive at the following electricity cost term, which also results in a quadratic programming problem:

$$Z_{el} = \gamma \sum_t \left(\alpha \left(\frac{dP_{spot}}{dQ}\right)^* (t) \cdot TL(t)^2 + (\alpha \overline{P_{spot}}(t) + \beta) \cdot TL(t) \right) \tag{54}$$

where the subscript *spot* has been added in order to signify that we are dealing with the spot price. In all previous equations P was the same as P_{spot} and hence no subscript was needed. Adding equation (48) as a (linear) constraint completes the problem.

8.2.2 Semi-definiteness of Q-Matrix

As discussed in section 7.3 the requirement for an optimal solution to exist is that the matrix $\mathbf{Q}$ is positive semidefinite. That is:

$$\mathbf{x}^T \mathbf{Q} \mathbf{x} \geq 0, \text{ for all } \mathbf{x} \tag{55}$$

The only quadratic term in the objective function is $TL(t)^2$. There are no terms with the product of two different variables. Therefore the Q-matrix is a diagonal matrix with the values of $\gamma\alpha \left(\frac{dP}{dQ}\right)$ as some of its elements (the rest are zero). Because an increase in demand results in increased prices, the coefficients in the Q-matrix (dP/dQ) are always positive and hence equation (55) will always be satisfied. An optimal solution to the programming problem will therefore always exist and using the modified simplex method (see section 7.3.1) it will always be possible to find this optimal solution.

8.3 Rolling planning

Prediction uncertainty has to be taken into account when determining which time horizon is reasonable to use when planning the charging pattern. At minimum a planning horizon of 12-36 hours ahead, covering the 24 hours of the following day (midnight to midnight), is needed. Knowing the charging pattern of this time period would allow the operator to place his bids on the day-ahead market before gate closure at 12 pm (noon).

A set point for the SOC of the batteries at the end of the optimization period is needed. Solving the optimization problem will always return the vehicles with the lowest possible SOC at the end of the optimization period. Returning the vehicles at a higher SOC would either require more charging and/or imply less use of V2G and hence be more expensive. The only exception here is when the cost of providing V2G exceeds the revenue obtained from selling the energy. It can be difficult to determine a good set-point since the optimal value would depend on what happens in the time following the optimization period. If the operator expects low prices the following day, it could make sense to leave the vehicle batteries at a fairly low SOC at midnight. The importance of defining the optimal set-point can be reduced by expanding the time horizon. The more one expands the planning horizon by using forecasted prices and driving patterns for the following time periods the less important the value of the set point becomes. Increasing the horizon will make the SOC-values at midnight depend more and more on the prices and driving patterns of the following hours and less on the set-point.

Two factors limit the time horizon: Prediction quality falls off fairly rapidly with increasing time horizon and computational time grows. However, poor prediction quality is not a very big problem since what is important is to know the general price level of the following day and prediction errors therefore are of less importance.

Defining a set-point may even be in accordance with the contracts signed with the EDV owners. If the contract ensures the EDV-user that the vehicle will always be fully charged at 7 am a natural set-point exists. Other reasonable set-points could be defined for each vehicle taking its driving pattern into account.

Another approach, which was used in this project, is to define a set-point, but at the same time add a contractual obligation for the fleet operator. This contractual obligation could be to have the vehicle charged to *at least* some level at a certain time, e.g. at least 80 % at 6 am. This is equivalent to adding the two constraints

$$E(v, t_{contr}) \geq \lambda E_{max}(v) \tag{56}$$

$$E(v, t_{set}) = \mu E_{max}(v) \tag{57}$$

where t_{contr} is the hour for which the contractual obligation exists and λ the charge level specified in the contract, e.g. 0.8. t_{set} is the set-point time and μ the charge level at the set point. The optimal solution can take any value between $\lambda Emax(v)$ and $Emax(v)$ depending on what is most economic favourable. The contractual constraint (56) must be added for all the hours in the planning horizon that corresponds to the time of day (e.g. hour 6 and 30). Note that the hours are numbered from midnight onwards, although the plan is made at noon the 12 hours until midnight are not included in the optimization.

Figure 25 and 26 show how the rolling planning works. Once having found an optimal plan the first part of it is carried out. When bids have to be placed for the next day a new optimization is carried out using the SOCs at midnight determined in the last optimization as starting point. This is done by copying the values of $E(v, 25)$ from the last optimization to $E(v, 1)$ in the new for all v.

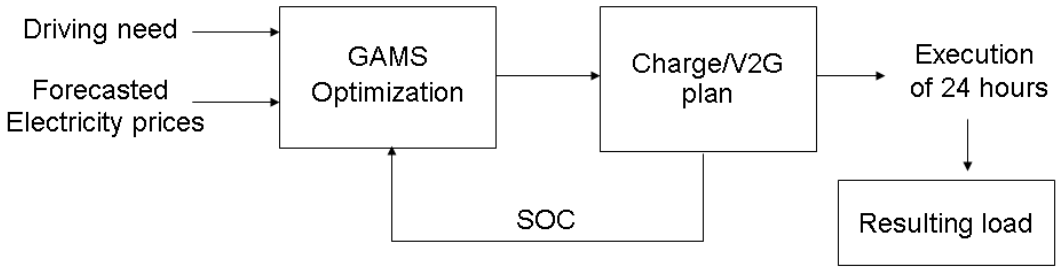


Figure 25: Dataflow in the model

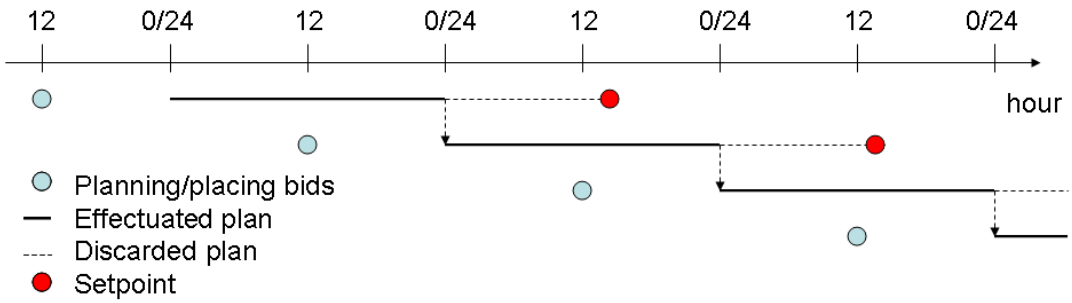


Figure 26: Planning and bid submission for the following day has to be carried out before 12 pm (noon). The planning horizon goes beyond the 24 hours of the following day to avoid the implication of finding an optimal set-point. The plan for the hours following midnight is discarded as the new plan is effectuated.

8.4 Perfect foresight

Section 4 discussed the uncertainty of future prices. When placing bids the fleet operator can only base his bids on forecasted prices. Including the effects of this uncertainty in the modeling could be achieved by using stochastic methods. A simpler approach was chosen in this project. Charging is optimized by deterministic methods under the assumption of perfect foresight. This implies that future prices are known or that one relies fully on the predicted prices to be right. By using historical spotprices it was assumed that the fleet operator knows what the future prices will be without EDV load.

A simple model could be developed to forecast prices based on the work in section 4.4 and 4.5. Equation (7) and (13) could be used to predict a price based on forecasted demand, wind power, fuel prices etc., which would be known at the time where the bid is placed. Forecasted prices from more advanced models like those of T. Jónsson [16] could also be used as input. However, using these forecasted prices instead of the historical price provides little more information. The output of the optimization model presented here is solely a result of the input parameters. Therefore predicting fleet operator behaviour based on predicted prices would yield two results:

1. How the model responds to a given input
2. How well the price forecast model produces the correct input

The first result can be obtained by comparing input data with output data. Whereas the second result could be obtained by comparing historical prices with those predicted. Because the latter would only be an analysis of how well the price forecasting model performs it has little relevance, since that would fall beyond the scope of this thesis.

In order to use the model to predict future behaviour or place actual bids on the power market one would of course need to use predicted prices. The result of the optimization would be strongly dependent on the quality of the forecast and the underlying forecast model.

Also drive patterns are assumed known. This is also a crude approximation to reality, which could be dealt with by using stochastic optimization.

8.5 Implementation in GAMS

The above described model was implemented in GAMS[7] in order to find the optimal charge/discharge plan. The GAMS-code can be found in appendix B. All values of $\left(\frac{dP}{dQ}\right)^*$ and $\overline{P}(t)$ are calculated beforehand and read from an external file. This file also contains the drive patterns obtained from clustering, the fraction of all vehicles represented by each standard vehicle, vehicle type, minimum/maximum SOC and weekday/weekend information for each day in the simulation period. All other parameters were given in the code.

Resulting charging/V2G patterns, spot price, SOC and fuel consumption are output for analysis.

In order to implement rolling planning the following sets and parameters were defined:

Sets

h	Hour in the simulation period
d	Day in the simulation period
dh	Hour in single day driving pattern
H	set containing all h
D	set containing all d
DH	set containing all dh

Parameters

$D_1(v, dh)$	Weekday driving patterns
$D_2(v, dh)$	Weekend driving patterns
$\overline{P}_{all}(h)$	Forecasted electricity price without EDV load
$\left(\frac{dP}{dQ}\right)^*_{all}(h)$	Price dependency on demand
$Dtype(d)$	Daytype (weekday or weekend)

The main simulation loop was on d. An optimization was carried out per day corresponding to placinng bids for 24 hours. Depending on the value of $Dtype(d)$ and $Dtype(d+1)$ driving patterns were either copied from $D_1(v, dh)$ or $D_2(v, dh)$ to $D(v, t)$. In the case of a weekday followed by a weekend this was done by setting

$$D(v,t) = D_1(v,dh) \quad \text{and} \quad D(v,t+24) = D_2(v,dh) \quad \text{for } t = dh, \ \forall dh \in DH \tag{58}$$

In a similar manner prices and values of dP/dQ are copied to the parameter used in a single optimization period. This is done by setting

$$\overline{P}(t) = \overline{P}_{all}(h) \quad \text{for } h = 24(d-1) + t, \ \forall t \in T \tag{59}$$

Values of dP/dQ are copied in the same way.

[7] General Algebraic Modelling System, by GAMS corporation

Part IV
Scenarios and results

9 About the results

The results presented in this section should be viewed as what would have happened had electric drive vehicles already been introduced in the time period of 2 May 2006 - 24 October 2007[8]. This is of course a hypothetical situation, but the results still have relevance for what could be expected in the future. It is important to keep in mind that the Danish electricity system will change in the coming decades towards higher integration of markets, more transmission capacity, more variable production (especially wind power), new power plants and probably also more flexible demand from other sectors than transportation. Examples of flexible demand is heating with heatpumps connected to heat storage tanks or so-called 'intelligent' appliances able to shift consumption to hours with low prices. Fuel prices, and environmental regulation will most likely also change. All these factors affect the spot price in the future. Some will raise it, others will lower it. Some will add to the volatility whereas others will damp the fluctuations. Therefore one should be careful not to interpret the results as what will happen in the future, but instead as a suggestion of what will happen with everything else equal.

9.1 Defining scenarios

Several factors influence the results of the optimization. In order to provide an overview of how and how much changing each of these affects the charging patterns a set of scenarios have been investigated. The relative importance of each of the parameters will be discussed as they are tested.

The scenarios are the result of varying the following parameters:

1. Taxes and charges
2. Set-points and contractual obligations
3. Number of EDVs
4. How demand changes affect power price

Using scenarios will also test the models limitations as will be seen in the following. Given certain parameters the model will break down and produce unrealistic results.

10 The baseline scenario

The baseline scenario investigates the charging pattern of 300,000 EDVs with the split on PHEVs and BEVs as described in section 6.3.3. This is about 25 % of the currently 1.19 million cars in Western Denmark [43]. Price dependency on demand is modeled by using the non-linear function approach as described in section 4.5. Taxes and charges are put on electricity using the current system (described in section 3). Assuming a very long forecast horizon of 60 hours the optimization period will be 48 hours (since bids have to be placed 12 hours before the first hour). A set point of 70 % SOC at midnight the following day and

[8] These 542 days were chosen based on data availability and are the same as those used in section 4.4.

a contractual obligation to have the vehicles charged at least 80 % at 6 am will be used. A minimum charge level of 20 % on the BEVs for all hours is furthermore assumed. This would allow the user to always be able to drive 30 km in case of an unanticipated event.

10.1 Aggregated results

In the baseline scenario a total of 1299 GWh of electricity is purchased at a spot price of 249 DKK/MWh on average is used for charging. Only 0.246 GWh of V2G power is provided at an average price of 857 DKK/MWh. The lowest price at which V2G takes place is 733 DKK/MWh. Looking at this scenario it is clearly evident that EDVs with V2G capability will not provide any means of large scale energy storage on a day-to-day or even hour-to-hour basis[9]. Taxes on associated losses (section 3) and in particular the battery wear cost (section 5.5) makes V2G too expensive for the fleet operator. The highest ramping rate (difference in system load between two consecutive hours) was 679 MWh/h, which is slightly higher than the 663 MWh/h today.

10.2 Distribution of loads

A histogram of the load distribution with and without EDVs are shown in figure 27. The historic reference case with no EDVs is double peaked shifting between a high day-time consumption and a low night-time consumption[10]. Introducing EDVs notably increases base-load shifting the first peak about 200 MW to the right. There is a significant increase in hours with loads between 2200 and 2600 MW. There is a slight increase in hours with high demand, and all-time peak demand is virtually unchanged (3744 MW vs 3739 MW). Since there is almost no increase in peak demand no new investments in transmission infrastructure and production capacity would be needed.

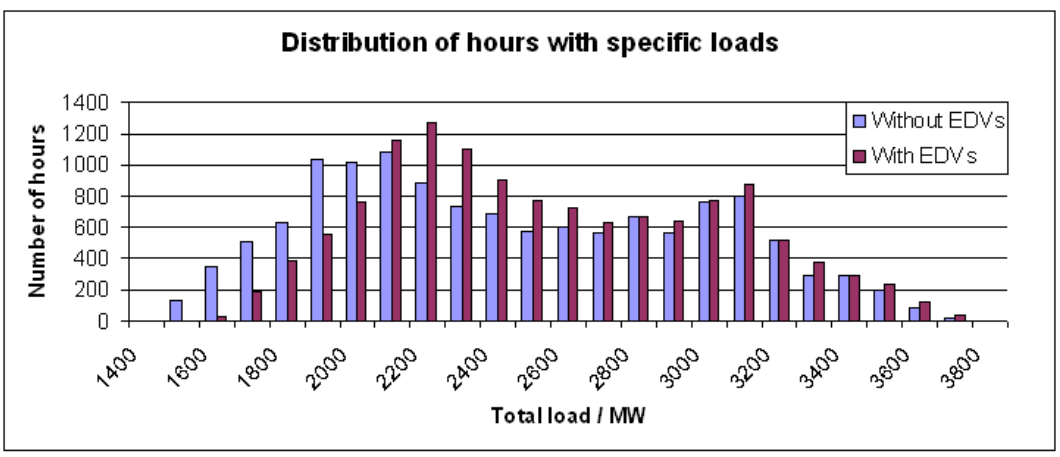


Figure 27: Baseline scenario: Histogram of hours with specific system loads (demand) with and without EDVs.

Looking at the average daily load distribution in figure 28 one finds that charging primarily takes place at night usually adding about 200 MW to base load consumption. Daytime charging (betweem 8 am and 8 pm) makes up 22 % on weekdays and 39 % on weekends.

[9]It should be noted that this thesis only deals with the day-ahead market. Short term balancing of the energy system by providing regulation services might be economically attractive.

[10]Part of this double peak also has to do with seasonal variations.

Charging on weekends is somewhat more predominant with 2634 MWh of charging on average compared to 2305 MWh on weekdays. Weekend load is lower than weekday load due to most people not working, this particularly affect the morning load (and prices), which explains why charging at 8-10 am is much more common on weekends.

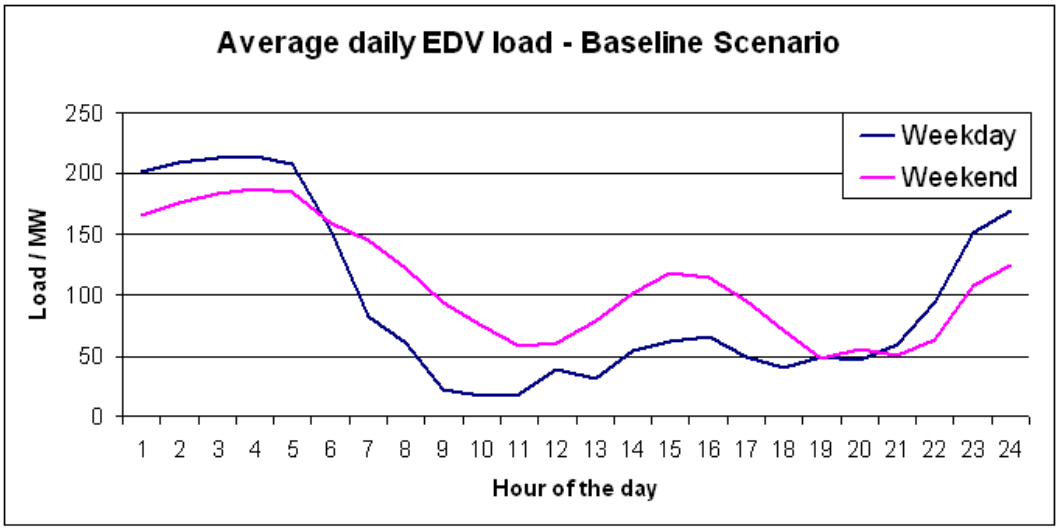


Figure 28: Baseline scenario: Average daily load on weekdays and weekends.

Charging in daytime takes place due to two effects. Firstly, because some of the EDVs (in particular the PHEVs) drive more than one long trip in a day and are assumed plugged in to recharge after the first trip. In order to minimize liquid fuel consumption charging is carried out almost as much as possible. This is due to the difference in efficiencies (90 % compared to 39 %) causing electricity to run the vehicle more than twice as far per energy unit[11]. Liquid fuel and electricity are equally attractive from a economic point of view at an electricity price of

$$P_{user} = \frac{\eta_C}{\eta_{ICE}} P_{fuel} = \frac{0.9}{0.39} \cdot 1000\text{DKK/MWh} = 2308\text{DKK/MWh} \tag{60}$$

which translates into a spot price of 965 DKK/MWh under the current tax system. A spot price higher than this occurred in only 13 of the 13008 hours simulated and only in 8 hours after the introduction of EDVs because of V2G reducing the price spikes. All of these hours had adjacent hours with lower prices.

Secondly, day time charging is also an effect of the fleet operator seeking to limit night time prices where most of the vehicles charge as pointed out in section 4.2. This is a consequence of the qudratic term in the objective function. The result is that the optimal solution for the fleet operator is inoptimal for society since it gives a worse utilization of base load capacity.

10.3 Examples of results for a specific week

Examples of load curves and spot-prices of a week (Tuesday-Monday) in December are given in figure 29 and 30. This week is not representative, but has been picked because it more visibly features many of the effects that can be observed in the results. The load pattern is typical for the cold season with two daily peaks on weekdays (morning and early evening)

[11] Three times as far is probably a more reasonable estimate since 39 % is a very high efficiency for an ICE.

and one evening peak in the weekend. As mentioned earlier weekend consumption is lower primarily due to industry being closed. Wind power production is very high in the week shown and varying substantially from day to day.

The corresponding spot prices seen in figure 30 are quite volatile. In the two nights preceding the Saturday and Sunday prices are at or close to zero due to the very high winds and low demand. Three price spikes occur during the week the first lasting several hours at about 650 DKK/MWh the second and third being single hour spikes at 761 and 820 DKK/MWh all three occurring at high demand and low winds.

By far the most charging takes place at night adding about 250 MW load to the valleys on the system load curve. This has a stabilizing effect on the prices. Especially in the two nights with extremely low prices price periods where power price is increased by about 150 DKK/MWh. In the weekdays the PHEVs charge and add up to 50 MW to the daytime load. Even during the periods of the day with high prices PHEV charging takes place. Sunday both types of vehicles are charged during the day. This is due to the low prices caused by the strong winds, but also in anticipation of the high prices Monday. None of the peak price hours have high enough prices for V2G to become economically feasible.

It is worth noting that about the same amount of charging takes place every day. This is to a large extent an effect of the contractual obligation imposed. This cause EDVs to have limited potential to balance wind power production. An example of this is the difference in load of the first two nights and the second two nights in figure 29. Although winds are very strong on the second two nights and prices are very low EDVs do not take up considerably more power in these two nights. In fact less power is provided in the second two nights. Unless battery capacity is increased, contractual obligations are softened and forecast horizon is expanded to cover several days, it seems likely that EDVs will mainly act as wind power balancing by adding the about 200 MW to base load.

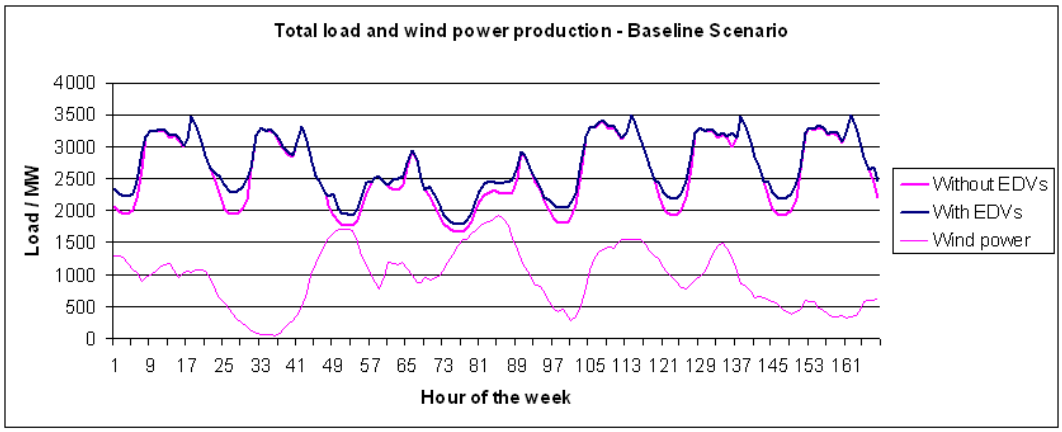


Figure 29: Baseline scenario: Example of load curve for one week (Thursday-Wednesday) with and without EDV load. Actual wind power production has been added to the graph. Historical data of 9-15 November has been used

10.4 Examples of results for specific vehicles

Turning to vehicle specific data may give a more clear idea of what is behind the aggregated numbers in the above shown figures. Three example vehicles have been picked. In figure 31, 32 and 33 for each vehicle the SOC is shown as a curve along with the corresponding driving/charging pattern shown as bars for clarity. These results only deal with the battery.

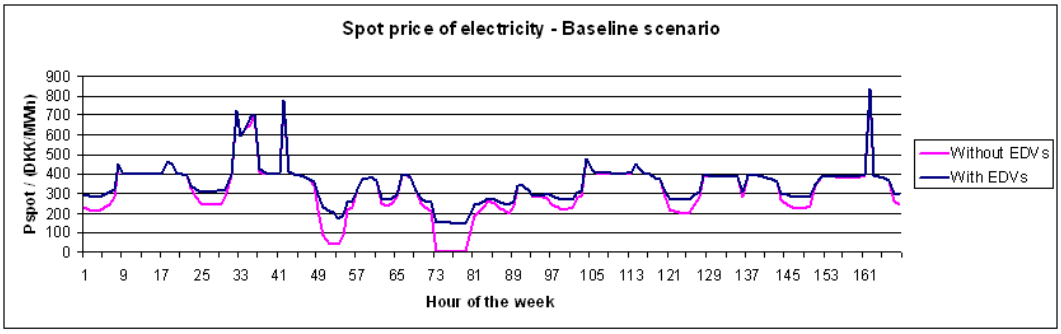


Figure 30: Baseline scenario: Price curve corresponding to the load curve in figure 29.

Usage of the ICE in the PHEV is not shown, but the ICE limits ΔE to -10.8 kWh and takes over after complete battery depletion[12]. This happens twice a day when the PHEV drives long distances.

EV #2 (figure 31) drives a long 3-hour trip (consisting of smaller trips) once a day on weekdays and two trips shortly after one another on weekends. Charging usually takes place during the night, but also in between trips in the weekend. Whether this would is realistic could be questioned. EDV users may not want to plug in after returning home if they are sure to depart an hour later and have sufficient energy in the battery for the next trip.

EV #14 (figure 32) shows a commuter pattern on weekdays and have a single trip Saturday and Sunday. Since the driver arrives early at work electricity prices are such that charging is commenced upon arrival. This is the case for all weekdays. SOC never goes below 60 % indicating that a battery half the size would suffice and give the same flexibility. It should be noted that this observation is only based on the week in question, but could be true for all days of the year since driving demand of this standard vehicle is modest.

PHEV #2 (figure 33) drives far twice a day. Since electricity is cheaper than fuel the battery is depleted in every journey. The battery completes 14 full cycles per week. Night time charging is distributed well among the hours whereas day time charging is focused on 1-3 hours giving fairly high charge rates. Power line capacity sets the limit of 11.1 kW, which is slightly less than the 12 kW it would take to charge the 10.8 kWh battery in a hour given the efficiency of 90 %.

11 Other scenarios

11.1 The VAT scenario

This scenario tests the implications of changing the entire electricity tax to a VAT as discussed in section 3.2.

Contrary to what could be expected giving tax incentives to use flexible demand only shifts consumption patterns slightly. The greatest effect is on V2G, which becomes far more economically favourable. V2G provides 14.1 GWh at an average spot price of 494 DKK/MWh. This implies that 1.3 % of the charging is used for V2G. The lowest spot price at which V2G is provided is 342 DKK/MWh (P_{user} = 1738 DKK/MWh). The reason for the modest change in consumption patterns is that economic incentives to carry out night charging are already sufficient under the current tax system given the assumptions on user behaviour.

[12]Since this is the optimal solution economically in this case. Should it be more optimal, the ICE will take over before battery depletion.

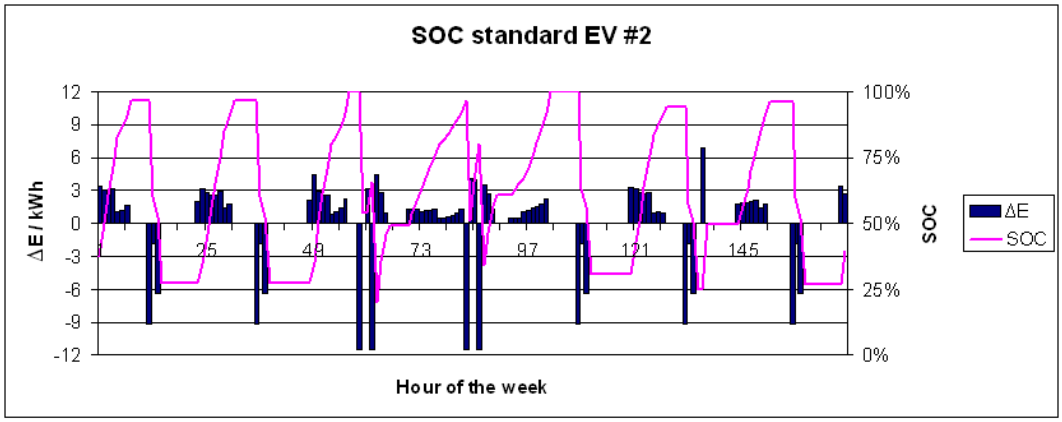


Figure 31: Baseline scenario: SOC for a standard EV shown with charging/Driving patterns. Week is the same as in figure 29.

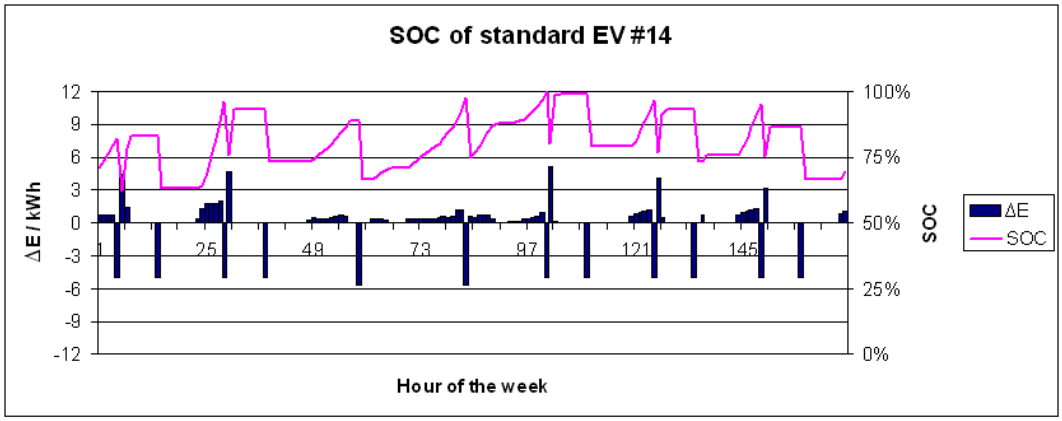


Figure 32: Baseline scenario: SOC for another standard EV shown with charging/Driving patterns. Week is the same as in figure 29.

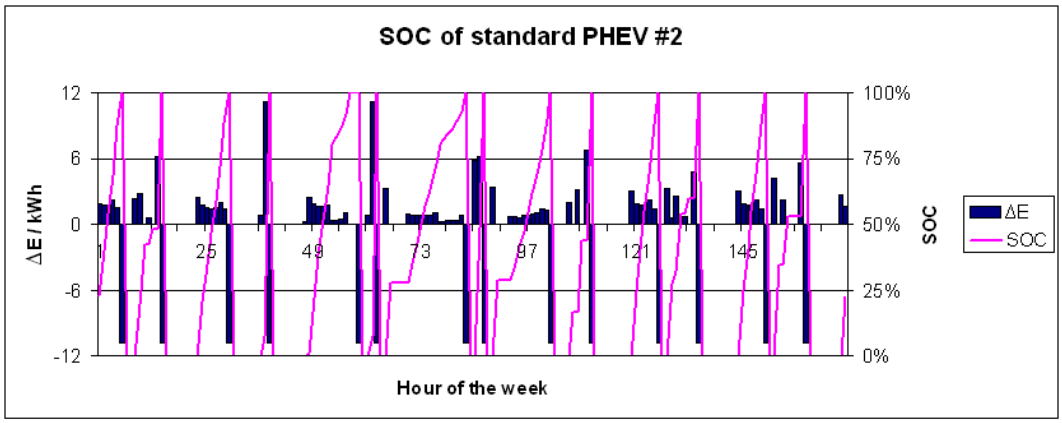


Figure 33: Baseline scenario: SOC for a standard PHEV shown with charging/Driving patterns. Week is the same as in figure 29.

Another effect of introducing the extra VAT is a fuel consumption increase. Fuel consump-

tion rise from 521 GWh in the baseline scenario to 540 GWh in the VAT scenario. Day time charging is needed to limit use of liquid fuels in PHEVs. When exacerbating high mid day prices with a VAT the ICE becomes more competitive. This side effect should be addressed if VAT-taxation of electricity is proposed.

11.2 The 7 am scenario

Suppose the EDV fleet contract was such that vehicles had to be fully charged every morning before departure. This reduce the optimization problem to only deal with 12-43 hours ahead since a setpoint of 100 % SOC at 7 am will be placed. A few vehicles drive in that hour and the hours before. These are dealt with by setting the constraint as:

$$E(v, 31) = Emax(v) - Dcons \sum_{t=28}^{30} D(v, t) \tag{61}$$

Adding this constraint ensures that the battery is fully charged at departure or at 7 am depending on which is the earliest. No major changes in costs are observed when placing a contractual obligation of 100 % SOC in the morning and day time charging only becomes slightly less used (24.8 % vs. 25.6 % in the baseline scenario). This small change is mainly a consequence of the fact that PHEVs and BEV needing to drive far are also fully charged in the morning in the baseline scenario. The average price at which electricity is bought increases by 2.7 DKK/MWh resulting in an additional average annual cost of 12 DKK per vehicle. Given the extra convenience obtained from having a fully charged vehicle in the morning it seems reasonable that users will choose a contract of this type instead of sometimes finding their vehicle at e.g. 80 % charge in the morning.

11.3 The All EDV scenario

In this scenario the number of vehicles was set to 1.19 million. This corresponds to all cars in Western Denmark being EDVs.

With about four times as many vehicles charging demand increases by a factor of four (to 5154 GWh). V2G, however, is almost at the same insignificant level as in the baseline scenario (0.319 GWh). The constraint on V2G is economic, not technical and therefore adding more vehicles does not raise it considerably. Adding as many EDVs as done in this scenario cause the model to yield unrealistic results as can be seen from figure 34. 42 % of charging takes place in day-time. The reason for this is the price dependency on demand used. EDV loads are of a magnitude where the assumption of dP/dQ being constant over the range of Q considered does no longer hold. This was discussed in section 4.5. Looking at figure 11 it can be seen that for all but very extreme prices the assumption is reasonable when loads are a few MW. When increasing total electricity demand four-fold EDV loads become several MW. The effect of this is that day-time charging is used on the expense of night time charging which the model predicts become expensive when many vehicles charge at the same time.

Summing the total driven distance of all vehicles one arrives at $22 \cdot 10^9$ km/year. This should be compared to the total annual Danish driving demand of $35 \cdot 10^9$ km/year, which also includes Eastern Denmark. Driving demand is probably higher in Western Denmark with less urbanization, but 69 % higher seems too much. This indicates that driving demand is exaggerated which probably goes back to the assumption of all vehicles driving every day.

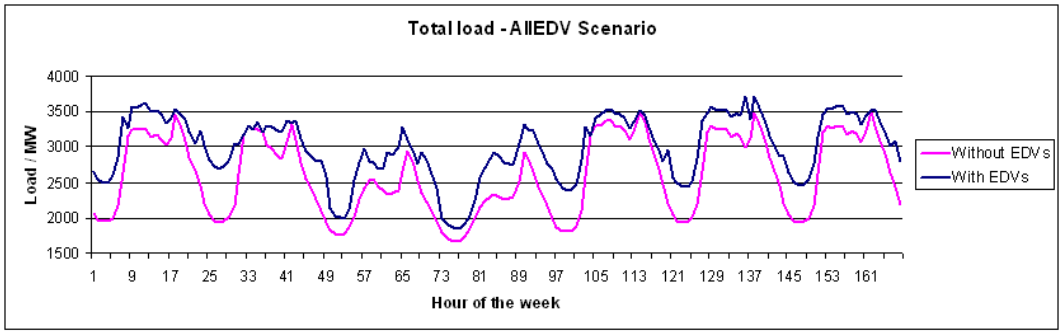


Figure 34: All EDV Scenario: Load with and without electric vehicles. The example week is the same as in figure 29.

11.4 The 1 % EDVs scenario

Having only 1 % of the car fleet changed to EDVs the EDVs have little impact on the electricity system. Maximum charging is at 132 MW corresponding to almost all vehicles charging at full power. Interestingly more V2G power is provided in this scenario than in the baseline scenario, and more per vehicle than in the VAT scenario (0.605 GWh in total). This is due to night time prices not being raised significantly by night time charging. The effect of this is a larger price difference from which profits can be earned. 10.7 % of all charging takes place at day-time (only 8.4 % on weekdays), which represents the charging needed to limit the use of liquid fuels in PHEVs. This should be seen as the minimum daytime charging level.

11.5 The Constant dP/dQ scenario

Changing the way price dependency of demand is modeled to one fixed value of dP/dQ for all (P, Q) of 0.1174 DKK/MWh^2 cause the model to yield unrealistic results. As was mentioned in section 4.4 the approach of using a single value for dP/dQ works particularly bad when extreme prices are considered. A total of 12.9 GWh of V2G is provided. 2092 MWh of these in a *single* peak price hour only reducing the price from 1419 DKK/MWh to 1245 DKK/MWh. At low prices too much charging takes place due to the price not rising enough. In general using a constant dP/dQ increases ramping rates dramatically compared to the baseline scenario.

11.6 Using the iterative approach

The iterative approach to improve the non-linear function method was applied to the baseline scenario and AllEDV scenario. As expected, in both scenarios the effect was lower charging costs, a slight increase in the use of V2G and less day time charging. The AllEDV scenario went from 41.7 % to 35.7 % of charging taking place at day-time. The resulting load curve for the example week can be seen in figure 35. Comparing this figure to figure 34 it is clearly evident that far more charging is allocated in hours with low prices and that EDVs contributes to a considerable smoothing of the load curve.

The example week has fairly volatile prices which make the non-linear function model work particularly well here. In other weeks the price is very stable although demand varies substantially. In these weeks charging is allocated fairly evenly over the course of the day causing mid-day loads to increase to levels that would normallly trigger a price spike. This highlights a shortcoming of the version of the non-linear approach used in this thesis where dP/dQ is

found on the basis of $\overline{P}$. The price can be a poor indicator of how far the system is from its limits.

It should be noted that the above is a critique of the non-linear function used and not the method in general. If the correct non-linear function is known the non-linear function approach with iterations yields the correct result, while keeping the problem as a quadratic programming one.

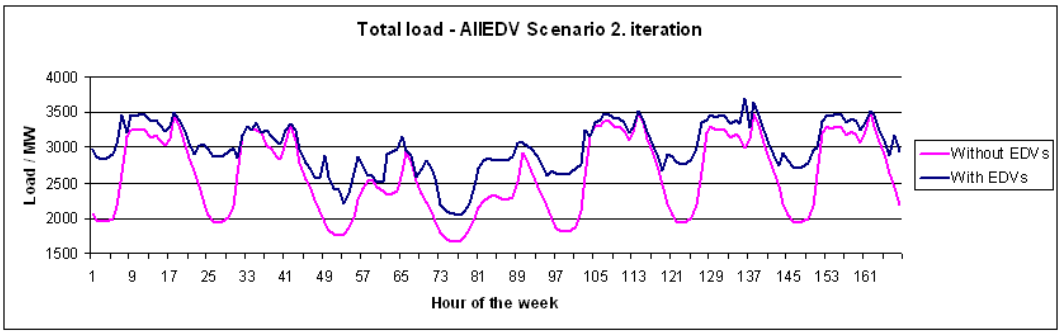


Figure 35: All EDV Scenario with 2 iterations: Load with and without electric vehicles. Compare this figure to 34.

11.7 Comparison of scenarios

Table 7 sums up the aggregated results of the various scenarios for comparison. Per vehicle results are shown in table 8 and effects on prices can be found in table 9.

Table 7: System level results of the various scenarios. E and P denote the total energy amount and the average spot price at which charge power and V2G power is traded. Max ramp denotes the maximum ramping rate. Scenarios marked with a star are calculated using the iterative approach.

Scenario	Ntot	E_{charge}	P_{charge}	E_{V2G}	P_{V2G}	Day	E_{Fuel}	Max ramp
	1000	GWh	DKK/MWh	GWh	DKK/MWh	%	GWh	MWh/h
Baseline	300	1299	249	0.246	857	25.8	521	679
Baseline*	300	1299	240	0.316	853	22.7	521	853
VAT	300	1308	250	14.1	494	24.7	540	646
7 am	300	1300	251	0.226	864	24.8	521	696
AllEDV	1190	5154	319	0.320	878	41.7	2066	864
AllEDV*	1190	5153	300	0.438	855	35.7	2067	1059
1pct	11.9	52	183	0.605	753	10.7	21	661
Constdpdq	300	1314	222	12.9	910	16.9	521	2157

Most of the results have already been discussed in the previous subsections. Overall charging demand and fuel use is fairly scenario independent and almost proportional to the number of vehicles. However, at what time charging takes place is highly dependent on the number of vehicles (since more vehicles raise night time consumption, and thereby prices, making it less attractive to charge). Higher overall consumption also result in higher average charging prices as can be seen from the spot-price figures.

The same result goes for V2G where more vehicles diminishes the price differences from which profits can be made. Note that V2G provided under the 1pct scenario is more than in the AllEDV scenario. When V2G is used more in the AllEDV scenario compared to the baseline scenario it is due to PHEV midday charging pushing prices up causing V2G to become feasible such that the BEVs indirectly charge the PHEVs. The Constdpdq scenario has extreme ramping rates compared to the other scenarios.

From a government point of view the spot price of 274.6 DKK/MWh from which the special VAT was calculated is more than the spot price at which the vehicles charge causing a loss of tax revenues of 73 MDKK on electricty bought for charging and additional 17 MDKK on electricity sold as V2G in the VAT scenario compared to the Baseline scenario. These 90 MDKK corresponds to a vehicle average of 300 DKK during the period simulated or 201 DKK/year. The increased tax revenues of extra sales of 19 GWh of liquid fuels (9.5 MDKK - assuming half the fuel price is tax) should be taken into account as well.

Table 8 shows the vehicle averages. Showing the results on a per vehicle basis makes some of the points stand out much clearer. The only scenario actually increasing fuel consumption is the VAT scenario. The most V2G per vehicle is provided in the 1 pct scenario. The MaxC and MaxV2G figures shows that an hour exists in which almost all EDVs charge at full power (11.1 kW) this is not the case when increasing the number of EDVs. The effect is even more pronounced for V2G and could lead one to doubt if the extra cost of adding V2G ability to all vehicles would pay off in a future system with a high penetration of EDVs[13]. Less than 1 % of the vehicle fleet is capable of providing the V2G needed for the day ahead market. A possible future scenario would be that the fleet operator decides on which vehicles should have V2G capability and then carries out the investment to install V2G capability. This could be modeled by making the η_{V2G} vehicle dependent and setting it close to zero for vehicles not having V2G capability.

The 7 am scenario raises night time peak consumption somewhat leading to a higher maximum charge rate as would be expected.

Using the iterative approach higher maximum charging and V2G rates occur. This is expected since the dP/dQ overestimation effect is removed and more charging/discharging hence can be carried out in a low/high-price hour at the same price increase/decrease.

Table 8: Per vehicle results of the various scenarios. e denotes the average energy per vehicle, whereas MaxC and MaxV2G denote the average per vehicle charging and V2G power in the two hours where each of them peaks.

Scenario	e_{charge} kWh	e_{v2g} kWh	e_{Fuel} kWh	MaxC kW	MaxV2G kW
Baseline	4330	0.8	1736	1.66	0.08
Baseline*	4330	1.1	1736	2.24	0.11
VAT	4359	47.1	1801	1.54	0.35
7am	4333	0.8	1736	1.97	0.08
AllEDV	4331	0.3	1737	0.89	0.05
AllEDV*	4330	0.4	1737	0.99	0.06
1pct	4384	50.9	1736	11.05	2.10
Constdpdq	4380	43.1	1736	3.52	6.97

[13]much revenue from V2G can be earned from ancillary services (e.g. regulation power) which is not discussed in this thesis. However, the competition between EDVs will make the marginal value of V2G ability decrease with the number of vehicles.

Table 9 compares the scenarios with the 'No EDV' scenario (the historic no EDV case) by the average resulting price (non-weighted) and the standard deviation. As expected all scenarios show higher prices and lower standard deviation. The 1pct scenario has a very small effect, but all other scenarios show an appreciable price increase and price variance decrease. The VAT scenario has the largest effect on the price variance due to the extra strong incentives to shift consumption to hours where the electricity price is low. The 7 am scenario follows, since night time consumption with low prices is forced. In the Constdpdq scenario price increases are underestimated and therefore resulting prices are too low. Especially low prices are not increased enough by extra demand. This shows in the variance which is larger and the average price which is lower for this scenario than the three other (Baseline, VAT and 7am) also having 300,000 EDVs. The AllEDV scenario increases prices more than what is reasonable due to the breakdown of the non-linear price model as discussed in setion 11.3. Using the iterative approach corrects this error yielding somewhat smaller price increases.

Table 9: Average prices (non-weighted) and standard deviation in the various scenarios. The No EDVs scenario corresponds to historic data.

Scenario	P_{avg}	$\sigma(P)$
No EDVs	265	117
Baseline	285	105
Baseline*	282	106
VAT	284	100
7am	284	102
AllEDV	333	103
AllEDV*	322	103
1pct	266	114
Constdpdq	276	109

12 Conclusion

A model developed to optimize charging patterns of EDVs from a fleet operators point of view was implemented and applied to various scenarios. The objective of the fleet operator was defined as minimizing total costs of operating the vehicle fleet given the nature of the day-ahead spot market and the physical limitations set by the vehicles and the constraints of user behaviour – in particular drive patterns. The scenarios investigated various penetrations of EDVs and different models for the correlation between price and demand (dP/dQ). Driving demand was modeled using 30 standard vehicles found by clustering surveyed driving patterns. 20 of these based on patterns with a total daily driving demand not exceeding 150 km. 10 based on those driving more than 150 km. The 20 standard vehicles were assumed to be BEVs, the 10 PHEVs.

The outcome of the modeled scenarios supports a series of findings and conclusions:

As expected, an optimal charging scheme places most of the charging at night, where prices are generally low. The more EDVs in the system the more the night time prices are increased. This price stabilization has two clear effects

1. Less incentive to charge at night - making day-time charging relatively more common.
2. Less incentive to provide V2G - due to the price difference from which profits can be made is diminished.

Furthermore, the monopolist status of the fleet operator cause the optimal solution to limit charging at night in order to keep prices from rising too much. This is a consequence of the quadratic term in the objective function. The result is a charging pattern which does not maximize social welfare.

Very little V2G was provided in all of the scenarios. Price differences are too small and the battery wear cost and incurred losses limit the profitability considerably. Therefore V2G as a means of large scale electricity storage seems infeasible.

The most V2G per vehicle was provided in the 1 % scenario where only 1 % of the vehicles are EDVs. A first mover advantage for V2G exists, but as penetration levels grow this advantage will diminish.

For very large penetration levels of EDVs V2G becomes slightly more used as BEVs indirectly charge PHEVs at mid-day in order to limit liquid fuel consumption, which is relatively very expensive.

For all scenarios the V2G power could be supplied by only a fraction of the fleet. If additional costs are associated with installing V2G capability this should lead to considerations about limiting investments in V2G capability to be used for the day-ahead market. In the baseline scenario with 300.000 EDV (∼25 % of the car fleet) no extra generation and transmission capacity was required although about 25 % of charging takes place between 9 am and 9 pm. Mid-day charging is partly due to PHEVs seeking to avoid liquid fuel consumption on their second or third trip in the day. Baseload consumption increased by about 200 MW.

Due to the imposed constraint requiring the EDVs to be at minimum 80 % battery charge level in the morning, it is difficult to shift consumption between days. Therefore EDV load is not as good at balancing wind power as one might expect. Even if electricity was free it is impossible to allocate much more charging in a single day since the EDVs have a limited charging demand.

Further restricting the optimal charging pattern so that the battery must be fully charged in the morning only increases the users electricity costs by 12 DKK/year, which makes it likely that users will demand such contracts. Changing the fixed electricity taxes to an all VAT of 246 % had a quite modest effect on charging patterns. The most notable effect was an

increase in liquid fuels consumption in PHEVs due to day-time charging being very expensive when taxed severely.

Two models for the correlation between demand and price were derived and tested in the scenarios: A linear and a non-linear relationship. Both models were based on historical data from which an attempt was made to remove the effect of long term price driving factors. Using a linear regression model to do this was found to give a fairly poor fit, which was expected since many effects in electricity markets are non-linear. Interestingly, wind power was found to have only 57 % of the effect on prices compared to that of demand. This is expected to mainly be caused by demand coinciding with demand in neighbouring power market areas, whereas wind power does can only conincide to a very limited extent due to little wind power capacity in neighbouring regions and geographical dispersion of winds.

The linear price demand relationship gave a dP/dQ value of 0.1174 DKK/MWh^2. The linear model performs badly at extreme loads and prices. It does not account for the fact that prices increases rapidly at very high loads and goes to zero or negative values at very low loads. To account for this, a non-linear model was proposed which defined dP/dQ as a function of the squared deviation of the price from a mean price. The parameters were estimated by fitting the solution of the differential equation (a tan-function) visually to the data since the nature of the data did not allow for the use of statistical methods. This non-linear model was found to overestimate dP/dQ for large demand changes towards moderate load levels and underestimate dP/dQ at demand changes to more extreme overall levels. This problem could be solved by using an iterative approach. The convergence of the method is very fast only requiring a few iterations.

The model using the non-linear function approach for price dependency on demand produces reasonable results in the scenarios with 300,000 EDVs and below, but starts breaking down when a large number of vehicles are added. This is due to the assumption of constant dP/dQ no longer being true at very large ΔQ. Using the iterative approach this problem can be alleviated. However, the validity of the dependency of dP/dQ on P used in this thesis shows to be questionable.

A novel approach of obtaining drive patterns from statistical survey data was used. EDVs were succesfully modeled as a set of standard vehicles. Each standard vehicle represented a number of real world vehicles with similar driving patterns, which allowed for a realistic treatment of battery charge levels. The driving patterns of the standard vehicles were obtained by clustering using a slightly modified version of the k-means algorithm and a distance measure developed by the author accounting for adjacent hours being more similar than non-adjacent hours. Due to the nature of drive patterns, using standard k-means and euclidean distance measure gave very poor results.

12.1 Further work

To improve the validity of the model in pricing extremes, which is particular important as EDV penetration grows, further delevelopment should be put into developing more advanced price/demand models. Any non-linear monotonic continuous function could be used to describe the relationship between price and demand in the non-linear function approach. Using numerical methods these non-linear functions need not be solutions to differential equations. Also the non-linear function does not need to be the same for all hours. If price forecasting is used the non-linear function could be forecasted as well as the price without EDV load.

In order to completely avoid the problem of estimating how prices and demand correlates part of the model and most of the data found in the other sections could be used to create an add-on to an existing system-model (e.g. Balmorel). The effect on prices from EDV load

would then be accounted for by activation of power plants at successively higher marginal costs within the model.

A more advanced clustering algorithm could be used and in particular a better distance measure than the one used in this thesis. Since using standard vehicles gives a far better representation of reality than the methods applied in other EDV research, more effort should be put into how the standard vehicles are obtained. Very little research has been carried out in this field.

A major shortcoming of the current model is that vehicles are always assumed plugged in when not driving. This would not be the case in reality. The model could easily be expanded to include this if a plug-in pattern was available for each standard vehicle. Keeping the model as a quadratic programming problem requires that this pattern is exogenously given. In reality the users plug-in preferences would probably depend on the battery state of charge (SOC). The same problem arises from the other major shortcoming: the assumption that the user has no control of how the vehicle is charged besides the contractual obligations. Including the possibility of forced charging upon return from driving would also require an exogenously given pattern. This could be dealt with by using the time-dependent minimum SOC constraint. For the hours where forced charging is used the minimum SOC level is set to a high value.

Some vehicles with certain purposes are well suited to be EDVs (e.g. City Taxis, elder care vehicles, postal service vehicles). These will likely be relatively more prevalent than privately owned EDVs for personal transportation. Therefore special driving pattern for these types of vehicles should be obtained. In general, ensuring good quality of data describing vehicles and users is a large but important task.

The ability of EDVs to provide ancillary services (regulation power, reserve capacity, etc.) have not been addressed in this thesis. A large revenue potential lies here [2] and the possibilities and consequences of providing these services are interlinked with the charging patterns obtained from the day-ahead market optimization. A complete model to optimize charging patterns should include both the day-ahead markets and the regulating power market while taking possible contractual obligations to be able to provide other ancillary services into account.

Network losses accounted for about 4.5 % of the Danish electricity consumption in 2007 [14]. A very large fraction of these losses takes place in the low-voltage distribution network supplying electricity to households. Since losses in a wire are dependent on the power flowing through the wire the losses are not distributed evenly over the course of the day. These electricity losses have to be bought on the market as well and therefore should be taken into account when optimizing charging patterns. Also V2G delivered at a building where consumption takes place could reduce network losses for all users on the line supplying the building. Including network losses in this project was initially planned, but was dropped due to lack of data. With the emergence of hourly metering of households data to create a simple model for network losses could be available in the near future.

The current model assumes that a single fleet operator is in control of all vehicles. This will likely not be the case in the future. Investigating how several market players, each in control of their vehicle fleet would interact is almost an entire research field in itself, but the issue should be addressed. Especially since regulation of the fleet operators could be needed in order to avoid the use of market power on the expense of social welfare.

Market players must place their bids based on predicted prices and driving patterns. An expansion of the model from having perfect foresight into using stochastic optimization of both would make the results more relevant since this is the problem a fleet operator faces: Minimizing costs in a market where uncertainties play a major role.

References

[1] Better Place Denmark. Better place, dong energy close 103m euro (770m danish kroner) investment for denmark electric car network. http://www.betterplace.com/presskit/denmark_20090127/DENRelease_ENGLISH.pdf, January 2009.

[2] W. Kempton and J. Tomić. Vehicle-to-grid power fundamentals: Calculating capacity and net revenue. *Journal of Power Sources*, 144:268–279, 2005.

[3] W. Kempton and J. Tomić. Vehicle-to-grid power implementation: From stabilizing the grid to supporting large-scale renewable energy. *Journal of Power Sources*, 144:280–294, 2005.

[4] L. Göransson. *Wind Power in Thermal Systems - Large-Scale Integration*. PhD thesis, Chalmers University of Technology, 2008.

[5] H. Lund and W. Kempton. Integration of renewable energy into the transport and electricity sectors through v2g. *Energy Policy*, 36:3578–3587, 2008.

[6] Nord pool. http://www.nordpool.com, April 2009.

[7] European market coupling company. http://www.emcc.com, April 2009.

[8] Nord Pool Spot. Market cross points. http://www.nordpoolspot.com/reports/systemprice/Marked-Cross-Points/?groupBy=200701, April 2009.

[9] Nord Pool Spot. Calculation of system- and area prices. http://www.nordpoolspot.com/Documents/Product%20Sheets/6689-01%20NP_prisberegn.pdf, April 2009.

[10] Energinet.dk. Udtræk af markedsdata. http://www.energinet.dk/da/menu/Marked/Udtræk+af+markedsdata/Udtræk+af+markedsdata.htm, March 2009.

[11] Danish Energy Association. Elforsyningens tariffer & elpriser pr. 1. januar 2008, 2008.

[12] ENV Net A/S. Prisoversigt pr. 1 april 2009. http://www.env.dk/media/ENV-_25515-v2-Elpriser_april_2009.pdf.

[13] Kurver 2007. http://www.elforbrugspanelerne.dk, April 2009.

[14] Danish Energy Association. Danish electricity supply - statistical survey 2007, 2008.

[15] M. Zugno and P. Giabardo. Competitive bidding and stability analysis in electricity markets using control theory. Master's thesis, Technical University of Denmark, 2008.

[16] T. Jónsson. Forecasting of electricity prices accounting for wind power predictions. Master's thesis, Technical University of Denmark, 2008.

[17] H. Ravn. Balmorel: A model for analyses of the electricity and chp markets in the baltic sea region. Technical report, Elkraft System, Denmark, 2001. www.balmorel.com.

[18] M. Barlow. Institute of electrical and electronics engineers inc. *Mathematical Finance*, 12(4):287–298, October 2002.

[19] V. Lee X. Lu, L. Sugianto. Test of asymmetry effect of demand on spot price using mcmc methods. *International Conference on Power System Technology, POWERCON2006*, 2006.

[20] EEX. Market data - coal futures. http://www.eex.com/en/Download/Market%20Data, March 2009.

[21] APX group. www.apxgroup.com.

[22] Nordpool Spot. Reservoir content for electrical exchange area. http://www.nordpoolspot.com/reports/reservoir/Reservoir-content-Elspot-exchange-area/, March 2009.

[23] Nord Pool ASA. Green market data. ftp://194.19.110.71/Green_markets/co2_allowances/Daily_key_figures, March 2009.

[24] Nina Detlefsen. Energinet.dk. Personal correspondence, March 2009.

[25] Miljørapport 2008. Technical report, Energinet.dk, 2008.

[26] C. Mehlsen. Enstedværket - grønt regnskab 2007. Technical report, DONG Energy, 2007.

[27] S. Chakraborty B. Kramer and B. Kroposki. A review of plug-in vehicles and vehicle-to-grid capability. In *Industrial Electronics, 2008. IECON 2008. 34th Annual Conference of IEEE*.

[28] J. Romm. The car and fuel of the future. *Energy Policy*, 34:2609–2614, 2006.

[29] S. Smets F. Badin A. Brouwer M. Alaküla G. Passier, F. Conte and D. Santini. Status overview of hybrid and electric vehicle technology (2007). Technical report, IEA, December 2007.

[30] T.H. Bradley and A.A. Frank. Design, demonstrations and sustainability impact assessments for plug-in hybrid electric vehicles. *Renewable and Sustainable Energy Reviews*, 13:115–128, 2009.

[31] X. Yu. Impacts assessment of phev charge profiles on generation expansion using national energy modeling system. In *Power & Energy Society General Meeting*, pages 1–5. IEEE, 2008.

[32] G.J. Suppes. Roles of plug-in hybrid electric vehicles in the transition to the hydrogen economy. *International Journal of Hydrogen Energy*, 31(3):353–360, March 2006.

[33] AC Propulsion. Ac-150 gen-2 ev power system - integrated drive and charging for electric vehicles. http://www.acpropulsion.com/tzero/AC150_Gen2_specs.pdf, June 2009.

[34] DTU Transport. Transportvaneundersøgelsen. http://www.dtu.dk/centre/modelcenter/TU.aspx, March 2009.

[35] N. J. Andersen and P. Meibom. Optimal configuration of future energy systems including road transport and vehicle-to-grid capabilities. In *Scientific Proceedings - European Wind Energy Conference and Exibition*.

[36] P. Hansen and B. Jaumard. Cluster analysis and mathematical programming. *Mathematical programming*, 79:191–215, 1997.

[37] S. Doki S. Okuma t. Naitou T. Shiimado N. Miki S. Ichikawa, Y. Yokoi. Novel energy management system for hybrid electric vehicles utilizing car navigation over a commuting route. In *2004 IEEE Intelligent Vehicles Symposium*, pages 161–166, University of Parma, June 2004.

[38] J. MacQueen. Some methods for classification and analysis of multivariate observations. In *Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability*, volume 1, pages 281–297. Univ. of Calif. Press, 1967.

[39] V. I. Levenshtein. Binary codes capable of correcting deletions, insertions, and reversals. *Soviet Physics Doklady*, 1966.

[40] F. Hillier and G. Lieberman. *Introduction to Operations Research*. Number 0073017795. McGraw-Hill, 8th edition, 2005.

[41] W. Karush. Minima of functions of several variables with inequalities as side constraints. Master's thesis, University of Chicago, Illinois, 1939.

[42] H. W. Kuhn and A.W. Tucker. Nonlinear programming. *Proceedings of the second Berkeley symposium*, pages 481–492, 1951.

[43] StatBank Denmark. Bil707: Stock of vehicles per 1 january by region and type of vehicle. http://www.statbank.dk/, June 2006.

Appendix

A Solving an example problem with the simplex method

The example problem shown in figure 24 is

$$\begin{aligned} &\text{Maximize} \\ &Z = 4x_1 + 3x_2 \\ &\text{Subject to the constraints} \\ &x_1 \leq 6 \\ &x_2 \leq 5 \\ &\frac{1}{2}x_1 + x_2 \leq 7 \end{aligned} \tag{62}$$

Introducing the slack variables s_1,s_2 and s_3 changes the constraints to

$$\begin{aligned} x_1 + s_1 &= 6 \\ x_2 + s_2 &= 5 \\ \frac{1}{2}x_1 + x_2 + s_3 &= 7 \end{aligned} \tag{63}$$

Picking $x_1 = x_2 = 0$ as the starting point allows for the problem to be written in the following matrix form

$$\begin{bmatrix} 1 & -4 & -3 & 0 & 0 & 0 \\ 0 & \boxed{1} & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 5 \\ 7 \end{bmatrix} \tag{64}$$

Noting that -4 is the most negative coefficient in the first row x_1 is picked as entering variable. This makes the second column the so-called pivot column, which is to be cleared but for one value. Row 2 and 4 both have a coefficient for x_1. The ratios of the right hand side to the coefficient are $6/1 = 6$ and $7/\frac{1}{2} = 14$. With the lowest ratio (6) row 2 is picked. This makes the 1, which is boxed, the pivot in the current operation. Row 2 is added four times to the first row and subtracted one half time from the fourth row. This yields the result:

$$\begin{bmatrix} 1 & 0 & -3 & 4 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & \boxed{1} & -\frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} Z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 6 \\ 5 \\ 4 \end{bmatrix} \tag{65}$$

-3 is the most (and only) negative coefficient in the first row, which is why x_2 is picked as the next entering variable. Row 4 has the lowest ratio (4 vs. 5 in row 3) and is added three

times to the first row and subtracted once from the fourth row yielding

$$\begin{bmatrix} 1 & 0 & 0 & \frac{5}{2} & 0 & 3 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} Z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 36 \\ 6 \\ 1 \\ 4 \end{bmatrix} \tag{66}$$

No more negative coefficients appear in the first row and the optimal solution has been found, which is:

$$\boxed{Z = 36 \qquad x_1 = 6 \qquad x_2 = 4 \qquad s_1 = 0 \qquad s_2 = 1 \qquad s_3 = 0} \tag{67}$$

Note that the CPF solution found in equation 65 is

$$Z = 24 \qquad x_1 = 6 \qquad x_2 = 0 \qquad s_1 = 0 \qquad s_2 = 5 \qquad s_3 = 4 \tag{68}$$

Which is the CPF in the bottom right of figure 24. Had x_2 been picked as entering variable in the first step finding the optimal solution would have gone through the points $(0, 0) \rightarrow (0, 5) \rightarrow (4, 5) \rightarrow (6, 4)$ instead.

B GAMS-code

```
$title Optimized charging of electric drive vehicles in a market environment
$offupper
$eolcom #

*=== Sets and scalars used for optimization
Scalar Ntot Total number of EDVs / 300000 /;

Set d all days (max = 543) /d1*d543/;
Set t hours over which optimization is carried out /h1*h48/;
Set dh set of hours used for drive patterns /h1*h24/;
Scalar Dcons All-electric drive consumption (MWh per km) / 0.0001667 /;
Scalar EffCharge charging efficiency / 0.9 /;
Scalar EffV2G V2G efficiency / 0.93 /;
Scalar ChargeMax maximum charge in MWh per h / 0.0111 /;
Scalar V2GMax maximum charge in MWh per h / 0.0111 /;
Scalar V2Gcost price of doing V2G due to battery wear in DKK per MWh / 394 /;
Scalar ICEcost price of using liquid fuels in DKK per MWh primary energy / 1000 /;

* The following paramters are taken from the section "Taxes and charges"
Scalar fixedtax fixed taxes in DKK per MWh {880.4 or 665.4 or 205.4} / 880.4 /;
Scalar regvat Regular VAT of 25 % / 1.25 /;
Scalar exvat  Extra VAT {1 or 1.82 or 3.46} / 1 /;
Scalar setSOC SOC at setpoint / 0.7 /;
Scalar contSOC SOC at contract point / 0.8 /;

*=== Import from Excel using GDX utilities
*=== First unload to GDX file (occurs during compilation phase)
$CALL GDXXRW.EXE Input.xls Index=Index!a1
*=== Now import data from GDX
$GDXIN Input.gdx
$LOAD
Sets v,h all vehicles and hours;
$LOAD v h
Parameter allDriveWeekday(v,dh) drive patterns for weekdays;
$LOAD allDriveWeekday
Parameter allDriveWeekend(v,dh) drive patterns for weekend days;
$LOAD allDriveWeekend
Parameter f(v) Fraction of vehicles in each group
$LOAD f
Parameter allpspot(h) all price parameters;
$LOAD allpspot
Parameter alldpdq(h) all price-demand influence parameters;
$LOAD alldpdq
Parameters Einit(v),Emin(v),Emax(v), EffICE(v) vehicle and battery data;
$LOAD Einit Emin Emax EffICE
Parameter daytype(d) type of day;
$LOAD daytype
$GDXIN

Parameters dpdq(t),pspot(t) Price parameters used for one optimization step;
Parameter Drive(v,t) drive pattern used for one optimization step;
Parameter Drivesum(t) sum of all driving in hour t;
Parameter Esum(t) sum of all energy stored in batteries;
Parameters Ekwh(v,t), Dkwh(v,t), Ckwh(v,t), V2Gkwh(v,t), ICEkwh(v,t) used for print;
*==== defining output file
file out;
out.nr = 2 ; #'rounding option' used to force e format
out.nd = 8 ; # number of digits
out.nw = 0 ; # width as required

file vehout;

Variables
Charge(v,t)     Charge of vehicle v at time t
V2G(v,t)        V2G from vehicle v at time t
E(v,t)          SOC
ICE(v,t)        ICE fuel usage in vehicle v at time t
p(t)            Price
TotCharge(t)    Total charge in hour t
TotV2G(t)       Total energy from vehicle to grid in hour t
TotICE(t)       Total ICE use in hour t
Totload(t)      Total ev load at time t
Totcost         total costs in DKK;

*=== Defining lower bounds on specific variables
```

```
Positive variable Charge(v,t);
Positive variable V2G(v,t);
Positive variable E(v,t);
Positive variable p(t);
Positive variable ICE(v,t);
Positive variable TotCharge(t);
Positive variable TotV2G(t);
Positive variable TotICE(t);

Equations
Chargesum(t) summing all charge values
V2Gsum(t) summing all V2G values
load(t) determine overall edv load
ICEsum(t) summing all ICE values
cost define objective function
nexte(v,t) propagates state of charge
price(t) price of electricity
select1(v,t) } restrict action to drive or charge or V2G
select2(v,t) }
maxcharge(v,t) observe maximum charge limit
maxV2G(v,t) observe maximum V2G limit
mine(v,t) observe minimum SOC
maxe(v,t) observe maximum SOC
contract(v,t) minimum charge level in the morning
setpoint(v) end SOC setpoint;

Chargesum(t) .. TotCharge(t) =e= Ntot*sum(v,f(v)*Charge(v,t));
V2Gsum(t) .. TotV2G(t) =e= Ntot*sum(v, f(v)*V2G(v,t));
load(t) .. Totload(t)  =e= TotCharge(t) - TotV2G(t);
ICEsum(t) .. TotICE(t) =e= Ntot*sum(v,f(v)*ICE(v,t));
price(t) .. p(t) =e= Totload(t)*dpdq(t) + pspot(t);
cost .. Totcost =e= sum(t,regvat*exvat*Totload(t)*Totload(t)*dpdq(t) + regvat*Totload(t)*(exvat
     *pspot(t) + fixedtax) +  TotV2G(t)*V2Gcost + TotICE(t)*ICEcost);
nexte(v,t)$(ord(t) ne card(t)) .. E(v,t+1) =e= E(v,t) + Charge(v,t)*EffCharge - V2G(v,t)/EffV2G
      - Drive(v,t)*Dcons + ICE(v,t)*EffICE(v);

select1(v,t) .. Charge(v,t)*Drive(v,t) =e= 0;
select2(v,t) .. Drive(v,t)*V2G(v,t) =e= 0;

maxcharge(v,t) .. Charge(v,t) =l= ChargeMax;
maxV2G(v,t) .. V2G(v,t) =l= V2GMax;
mine(v,t) .. E(v,t) =g= Emin(v);
maxe(v,t) .. E(v,t) =l= Emax(v);
setpoint(v) .. E(v,'h48') =e= setSOC*Emax(v) - Dcons*Drive(v,'h47');
contract(v,t)$(ord(t)=6 or ord(t)=30) .. E(v,t) =g= contSOC*Emax(v) - Drive(v,t-1)*Dcons;

Model optcharge /all/;

*===Initial values for SOC
loop(v,E.fx(v,'h1') = Einit(v));
*===Assigning drive patterns
loop(v,
loop(t,
loop(dh$(ord(dh) + 24 = ord(t)),
if(daytype('d1')=1,
Drive(v,t) = allDriveWeekday(v,dh);
elseif(daytype('d1')=2),
Drive(v,t) = allDriveWeekend(v,dh);
else abort "weekday not specified";
);
);
);
);

*=== Primary simulation loop
loop(d,
*=== Extracting paramters to be used in 36-hour optimization
loop(h$(ord(h) <= (ord(d)*24+24) and ord(h) > (ord(d)-1)*24),
loop(t$(ord(t)+24*(ord(d)-1) = ord(h)),
pspot(t) = allpspot(h);
dpdq(t) = alldpdq(h);
);
);

loop(v,
*===copying yesterday's next-day drive patterns to current day
loop(t$(ord(t)<25),
```

```
Drive(v,t) = Drive(v,t+24);
);
*===copying new values to next-day drive patterns
loop(t,
loop(dh$(ord(dh) + 24 = ord(t)),
if(daytype(d+1)=1,
Drive(v,t) = allDriveWeekday(v,dh);
elseif (daytype(d+1)=2),
Drive(v,t) = allDriveWeekend(v,dh);
else abort "weekday not specified";
);
);
);
);
*=== Solving the model for 36 hours
solve optcharge using qcp minimizing Totcost;

*=== Output file with aggregated system data
put out;
Esum(t) = Ntot*sum(v,f(v)*E.l(v,t));
Drivesum(t) = Ntot*sum(v,f(v)*Drive(v,t));
loop(t$(ord(t)<25),put Esum(t) Drivesum(t) Totload.l(t) TotV2G.l(t) TotICE.l(t) pspot(t) dpdq(t
     ) p.l(t)/;);

*=== Output file with vehicle specific data
put vehout;
loop(v,loop(t,
Ekwh(v,t) = 1000*E.l(v,t);
Ckwh(v,t) = 1000*Charge.l(v,t);
V2Gkwh(v,t) = 1000*V2G.l(v,t);
ICEkwh(v,t) = 1000*ICE.l(v,t);
););

loop(t$(ord(t)<25),
loop(v$(ord(v)=2 or ord(v)=14 or ord(v)=22),put Ekwh(v,t) Drive(v,t) Ckwh(v,t) V2Gkwh(v,t)
     ICEkwh(v,t););
put pspot(t) dpdq(t) p.l(t)/;);

*=== Transferring SOC of all vehicles to next optimization
loop(v,E.fx(v,'h1') = E.l(v,'h25'));

*=== End of primary simulation loop
);
```

C C♯-code for clustering

```
ï»¿/*
 * The following C# code is the implementation of the k-means algorithm.
 * The C++ code found at http://www.programmersheaven.com/download/40276/0/ZipView.aspx
 * was used as a starting point. The code has been adapted to the problem of clustering
 * drive patterns and has been expanded with a function to find the pattern most similar
 * to all others in the same cluster.
 *
 * Karsten Capion, M.Sc. Student, Technical Univeristy of Denmark, June 2009
 */

using System;

namespace Calculations
{
    public class ClusterCalculator
    {
        public static int[] k_means(double[][] data, int n, int m, int k, int xmax, double t,
             double[][] centroids, double[][] pattern)
        {
            int[] labels = new int[n];
            int[] labels_opt = new int[n];

            Random r = new Random();
            int h, i, j; // Loop counters
            int[] counts = new int[k]; // Number of patterns in each cluster
            int[] counts_opt= new int[k]; // Number of patterns in each cluster
            double old_error, error = double.MaxValue; // Error measure
            double[][] c_opt = centroids;
            double[][] c = new double[k][];
            double[][] c1 = new double[k][]; // Temp centroids (used for spatial mean)

            System.Diagnostics.Debug.Assert(data != null && k > 0 && k <= n && m > 0 && t >= 0)
                 ; /* for debugging */
            double min_error = double.MaxValue;

            for (int x=0;x<xmax; x++)
            {
                /* pick k patterns as initial centroids */
                int[] initcent = new int[k]; // Array holding pattern-ids for initial centroids
                i=0; // Counter
                do
                {
                    h = r.Next(0, n);           // Pick random h
                    int check = 0;
                    for (j = 0; j < i; j++)
                    {
                        if (initcent[j] == h)   // Has h already been picked?
                            check++;            // if so check becomes non-zero
                    }
                    if (check == 0)
                    {
                        initcent[i] = h;
                        i++;
                    }
                } while (i < k);

                for (i = 0; i < k; i++)
                {
                    c1[i] = new double[m];
                    c[i] = new double[m];
                    for (j = 0; j < m; j++)
                    {
                        c[i][j] = data[initcent[i]][j];
                    }
                }

                /* main loop */

                //int iter = 0;
                do
                {
                    //Console.WriteLine("iteration: {0}\terror={1}\n", iter++, error);
                    /* save error from last step */
                    old_error = error;
```

```
                    error = 0;

                    /* clear old counts and temp centroids */
                    for (i = 0; i < k; counts[i++] = 0)
                    {
                        for (j = 0; j < m; c1[i][j++] = 0) ;
                    }

                    for (h = 0; h < n; h++)
                    {
                        /* identify the closest cluster */
                        double min_distance = double.MaxValue;
                        for (i = 0; i < k; i++)
                        {
                            double distance = Distance(data[h], m, c[i]); // Distance measure

                            if (distance < min_distance)
                            {
                                labels[h] = i;              // Labeling each pattern with a cluster number
                                min_distance = distance;    // Setting distance for pattern to centroid
                            }
                        }


                        /* update size and temp centroid of the destination cluster */
                        for (j = 0; j < m; j++)
                        {
                            c1[labels[h]][j] += data[h][j]; // Sum to be used for spatial mean
                        }
                        counts[labels[h]]++;                // Counting number of patterns in cluster
                        /* update standard error */
                        error += min_distance;              // Summing errors
                    }

                    /* find most typical pattern in each cluster */

                    int a, b;                                   // counter variables
                    for (i = 0; i < k; i++)     // Loop on all clusters
                    {
                        double[][] dist = new double[counts[i]][];      // Inter-pattern distance matrix
                        for (j = 0; j < counts[i]; j++)             // Size is counts[i] x counts[i]
                        {                                           // where counts[i] is number of
                            dist[j] = new double[counts[i]];        // patterns in cluster i.
                        }
                        int[] cmembers = new int[counts[i]];            // Member patterns of cluster i
                        a = 0;
                        for (j = 0; j < n; j++)     // Searching through all patterns
                        {
                            if (labels[j] == i)     // Finding those in cluster i
                            {
                                cmembers[a] = j;
                                b = 0;

                                for (h = 0; h < j; h++) // Searching for other pattern
                                {
                                    if (labels[h] == i) // If also member of cluster i
                                    {
                                        dist[a][b] = Distance(data[h], m, data[j]); // Calculating distance
                                        b++;
                                    }
                                }
                                a++;
                            }
                        }


                        double[] distsum = new double[counts[i]];       // sum of distances from a to other members of cluster
                        double min_distsum = double.MaxValue;       // Minimum sum of distances
```

```
                    int min_a = 0;// -1; // Pattern resulting in min_distsum. Initialized
                         to -1 in order to get error if unassigned.

                    /* Summation of all distances from a */
                    for (a = 0; a < counts[i]; a++)
                    {
                        distsum[a] = 0;
                        for (b = 0; b < a; b++)
                        {
                            distsum[a] += dist[a][b];
                        }

                        for (b = a + 1; b < counts[i]; b++)
                        {
                            distsum[a] += dist[b][a];
                        }
                        if (distsum[a] < min_distsum)
                        {
                            min_distsum = distsum[a];   // Saving minimum distance found
                            min_a = a;                  // Saving corresponding value of a
                        }
                    }
                    for (int s = 0; s < m; s++)
                    {
                        c[i][s] = data[cmembers[min_a]][s]; // Assigning cluster center
                    }
                //  Console.WriteLine("Cluster {0} has {1} members", i, counts[i]);
                }

                /* update all centroids *** Use this code when using spatial mean centroids
                for (i = 0; i < k; i++)
                {
                    System.Diagnostics.Debug.Assert(counts[i] > 0); // for debugging
                    Console.WriteLine("Cluster {0} has {1} members", i, counts[i]);
                    for (j = 0; j < m; j++)
                    {
                        if (counts[i] != 0)
                            c[i][j] = c1[i][j] / counts[i];
                        else c[i][j] = c1[i][j];
                    }
                } */

            } while (Math.Abs(error - old_error) > t); // Loop exit condition

            if (error < min_error)
            {
                min_error = error;
                Console.Write("\nAttempt {0} gave Min_error = {1}",x, error);
                for (i = 0; i < n; i++)
                {
                    labels_opt[i] = labels[i];
                }
                for (i = 0; i < k; i++)
                {
                    for (j = 0; j < m; j++)
                    {
                        c_opt[i][j] = c[i][j];
                    }
                    counts_opt[i] = counts[i];
                }

            }
        }

        /* Summing all patterns in each cluster */
        for (i = 0; i < k; i++)
        {
            for (j = 0; j < m; j++)
            {
                pattern[i][j] = 0.0;
            }
        }
        for (i = 0; i < n; i++)
        {
            for (j = 0; j < m; j++)
            {
                pattern[labels_opt[i]][j] += data[i][j];
```

```
                }
            }
            return counts_opt; // Return number of patterns to main() function.
        }

        private static double Distance(double[] pat1, int m, double[] pat2)
        {
            double result = 0;
            for (int j = 0; j < m-1; j++)
            {
                if (pat1[j] != 0 || pat2[j] != 0)
                {
                    result += Math.Abs(pat1[j] + pat1[j + 1] - pat2[j + 1] - pat2[j]);
                    j++;
                }
            }
            //return Math.Sqrt(result);
            return result;
        }
    }
}
```