Active Distribution Grid Management based on Robust AC Optimal Power Flow

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***Abstract*—The further integration of distributed renewable energy sources in distribution systems requires a paradigm change for the grid management by the distribution system operators (DSO). Recently, DSOs are moving to an operational planning approach based on the activation of flexibility from distributed energy resources in day/hour-ahead stages. This paper follows the DSO trends by proposing a methodology for active grid management, where robust optimization is applied to accommodate spatial-temporal uncertainty. The proposed method implies the use of a multi-period AC-OPF ensuring a reliable solution to the DSO. Wind and PV uncertainty is modeled based on spatial-temporal trajectories, while a convex hull technique to define uncertainty sets for the model is used. A case-study based on real generation data allows illustrating and discussing the properties of the model. An important conclusion is that the method allows the DSO increasing the system reliability in the real-time operation. However, the computational effort grows with the increase of the system robustness.**

***Index Terms* — Decision-making; uncertainty; distribution system operator; robust optimization; solar power; wind power.**

I. Introduction

THE continuous integration of distributed energy resources (DER) [1], specially renewable energy resources (RES), at the distribution grid level will lead to the development of new models and methodologies to deal with the uncertainty of these resources [2]. Hence, traditional methodologies for operation and management of the distribution grid must be replaced by new active management methodologies, where distribution system operator (DSO) can contract/control power generation/consumption (flexibility) from DER to solve congestion/voltage problems in the distribution network [3].

Presently, DSOs in most European countries employ a reactive approach for grid management imposing limits in terms of DER (mainly, RES) integration in MV and LV levels. For instance, a survey applied under an EU project in several European countries showed that a very few DSOs use the forecasts for operational purposes, as well as, contracting services to handle network constraints [4]. Furthermore, the degree of coordination between DER and the DSO control centers is very limited or inexistent in almost all countries.

The flexibility potential from DER (including flexible operation/bids from RES) requires a change of the present paradigm. The trend is to implement proactive and preventive grid management functions based on forecasts with the possibility of reserving/controlling DER connected to the distribution grid. The goal of the DSO remains the same, i.e. to ensure that congestion, voltage and energy delivery problems are solved, as well as, maintaining the proper operation of the system with adequate levels of safety, reliability and power quality. Under high RES integration levels, this goal can be fulfilled by combining multi-period optimal power flow (OPF) with uncertainty forecasts.

Most of the literature proposals for the distribution grid management problem are based on stochastic methods with relaxation approaches on the OPF. However, DSO usually operates under the premise of procuring a solution or scenario that can ensures higher robustness to the system. The regulatory framework “induces” risk aversion to both DSO and TSO.

In this context, several methodologies have been emerging for distribution grid management considering RES uncertainty. These methodologies are most often based on stochastic programming and robust optimization [5], [6]. A decentralized stochastic approach to manage a distribution network with PV production is proposed [7]. The model assumes the control of active and reactive power generation from PVs, while validated through a convex stochastic OPF. However, the model only ensures effectiveness under radial networks. In [8], a stochastic method based on chance-constrained optimization for voltage control under PV uncertainty production is proposed, however, using a probabilistic load flow to analyses the injection of PV in the system. The authors in [9] consider a point estimate method to deal with wind uncertainty and a probabilistic OPF, where power flow outputs are given by a certain distribution. However, the output from the stochastic OPF is a distribution of the decision variables. Nevertheless, for a DSO, a more seemly output would be a single solution that is robust in all or a pre-defined percentage of the scenarios.

In contrast with literature, this paper contributes with a new methodology (based on a robust approach) for solving technical problems in the distribution network under RES forecast uncertainty ensuring a single and safe solution more reliable than traditional approaches. The model minimizes the operational costs (flexibility activation) of the DSO, without relaxing any network constraints under a set of spatial-temporal trajectories (or scenarios). The methodology is proposed for a paradigm where the DSO preventively manages the distribution grid (recent trend in the scientific community [10]–[12]) by contracting flexibility from DER in advance based on forecasted information. Thus, the DSO will have more flexibility capacity to use in real-time operation, thereby, increasing the safety and reliability of his system. This work has two major contributions to the state of art: (a) integration of spatial-temporal trajectories [13] to model RES, while using convex hull based techniques to model the uncertainty set; (b) active distribution grid management in a multi-period AC OPF, which is able to ensure the most reliable solution for the distribution grid.

The paper is structured as follows. Section II describes the DSO management problem with a perspective on current and future trends. Section III presents the detailed formulation of the robust approach for the DSO problem on energy resources management under uncertainty. Section IV describes our empirical investigation based on a case study with real data. Section V gathers the most important conclusions.

II. Framework for Distribution Network Management

A. Current Management

The mission of a DSO is to ensure the quality and continuity of supply levels imposed by the regulatory framework. In the past, technical problems such as overcurrent and voltage limit violation were mitigated by planning network investments and changing the network configuration to meet the loads. Presently, DSOs have additional flexibility in the network that allows solving the local technical problems in the operational domain, instead of solving them in a planning phase. The main benefits are investment deferral and reduced curtailment of DER.

In the operating domain, the typical control actions are network reconfiguration, control of capacitor banks and activation, though on a very limited way, of non-firm connection contracts associated to industrial loads and some DER. Information about forecasts and corresponding uncertainty is not embedded in the current grid management functions.

B. Future Management

With the continuous introduction of DER, the DSO has been changing his operation and control paradigm. Thus, a full proactive grid management, where DERs are part of the solution by its ability of controlling its output power is crucial for the proper operation of the distribution system. However, RES have uncertain generation, thereby, they can increase the system operation uncertainties. In this way, future DSO management should integrate new methodologies to deal with RES uncertainty, as well as consider energy storage systems to help the DSO to solve congestion problems and efficiently deliver the energy [4].

A new structure for solving technical problems in the distribution grid (see diagram in Fig.1) divided into two phases is used: (*i*) definition of upward and downward flexibility bids and operating point from DER aggregator in the day-ahead market; and (*ii*) robust management of distribution grid considering internal resources, flexibility bids from DER and RES uncertainty.

In the first phase, a day-ahead market with participation of DER aggregators (such as CHP, wind, PV and demand response) is accomplished, thereby, obtaining the operating point of each aggregator. Then, aggregators define their flexibility bids of changing the operating point of their own resources for upward and downward power. The flexibility bids are defined based on the strategy of each aggregator to provide flexibility to the DSO. Wind and PV aggregators can define their bids based on expectation profit for supplying this upward and downward flexibility to the DSO, accounting for the costs for changing their operating point [14].

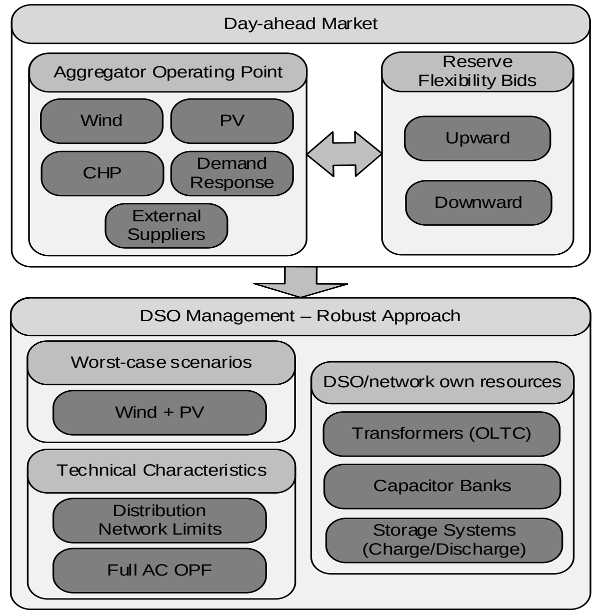

Fig. 1. Diagram of the developed methodology.

The model from the second stage, lies on the distribution grid management considering the upward $P_{dg(dg,t)}^{bid\_up}$ and downward $P_{dg(dg,t)}^{bid\_dw}$ flexibility bids from the aggregators, and the internal flexibility (static equipment's – transformers with on-load tap-changing (OLTC) ability, capacitor banks and storage systems) owned by the DSO under the limitations of the technical characteristics of the grid. The DSO contracts the flexibility to the DER aggregators based on capacity payments. Storage systems owned or managed by DSO, helps in the system management providing additional multi-period flexibility and avoiding constrained situations. Additionally, the storage system contributes to face with high uncertainty production in the distribution system. Besides that, it is noteworthy that wind and PV aggregators should guarantee the provision of the submitted upward and downward flexibility (for instance, by contracting generators or demand response electrically close to their point of power production).

Nevertheless, the DSO may contract all the flexibility needed to cover foreseen distribution network problems, such as energy congestion and voltage problems, considering RES uncertainty. Moreover, the core of the methodology lies on the use of a robust optimization approach to accommodate RES uncertainty, while providing solutions with high reliability levels.

III. Methodology

The methodology is based on robust optimization to model RES uncertainty and solve the DSO management problem.

A. Uncertainty Set Definition

Robust optimization requires the definition of uncertainty sets, e.g., vertices representative of the worst-case solution, as explained in [15]. Uncertainty sets can take different forms, for instance, polyhedral and ellipsoid are the most common in literature.

In this methodology, uncertainty sets for wind and PV generation are constructed using a scenario set. The scenario set J is obtained through the generation of spatial-temporal trajectories (or scenarios). For each period, the deviation between the scenario set and the conditional mean forecast create a cloud of N_J points representative of the uncertainty space. Then, the uncertainty set W is defined as the convex hull of these points constructed through the quickhull algorithm [16]. The vertices of the uncertainty set that are selected for the optimization process are represented by $\Delta P_{w(w,t,s)}$ in the formulation. In addition, the number of vertices of the convex hull can increase significantly, when considering a large amount of intermittent resources, which can be intractable in the time frame of the DSO to solve the problem. Thus, algorithms to reduce the number of vertices can be considered. The recursive Douglas-Peucker algorithm [17] is based on polyline simplifications and can reduce the number of vertices that characterize the uncertainty set. An improved and speeding up version of the algorithm [18] can be used to significantly reduce the vertices of the uncertainty set.

B. Mathematical Formulation

The objective function to be optimized directly relates to the minimization of the operational costs of the DSO. It leads to a multi-period robust optimization problem, which is expressed as

$$\underset{x}{Min}\, C_t^{DA}\left(x\right)+\underset{u}{max}\ \underset{y\in B(x,u)}{min}\ C_t^{RT}\left(x,y\right) \tag{1}$$

$$s.t.\ A_t^{DA}\left(x\right)-b\le 0, \tag{2}$$

$$h_t^{RT}\left(x,y\right)=0, \tag{3}$$

$$g_t^{RT}\left(x,y\right)\le 0, \tag{4}$$

where the vector x includes the day-ahead decision vectors for contracting flexibility, while real-time adjustments with respect to the contracted flexibility are included in the vector of recourse variables y, accounting for the vertices u of the uncertainty set. We grouped all the constraints involving only first-stage variables into (2), while the ones including recourse variables are divided into equalities (3) and inequalities (4).

The *here-and-now* decisions (stage 1) comprise the flexibility contracted by the DSO in the day-ahead market, while the wait-and-see decision (stage 2), uses the flexibility to solve network constraints, which adapts to the realization of the set of vertices (i.e., the vertices are taken as potential realizations of the "true values". Thus, $C_t^{DA}(x)$ is modeled as

$$\begin{aligned}&\sum_{dg=1}^{N_{DG}}\left(C_{dg(dg,t)}^{up}P_{dg(dg,t)}^{up}+C_{dg(dg,t)}^{dw}P_{dg(dg,t)}^{dw}\right)+\\&\sum_{w=1}^{N_W}\left(C_{w(w,t)}^{up}P_{w(w,t)}^{up}+C_{w(w,t)}^{dw}P_{w(w,t)}^{dw}\right)+\\&\sum_{pv=1}^{N_{PV}}\left(C_{pv(pv,t)}^{up}P_{pv(pv,t)}^{up}+C_{pv(pv,t)}^{dw}P_{pv(pv,t)}^{dw}\right)+\\&\sum_{l=1}^{N_L}\left(C_{dr(l,t)}^{up}P_{dr(l,t)}^{up}+C_{dr(l,t)}^{dw}P_{dr(l,t)}^{dw}\right)\qquad,\forall t\in\{1,...,T\}\end{aligned} \tag{1a}$$

where the decision variable vector relates to $\left\{P_{dg}^{up},P_{dg}^{dw},P_{w}^{up},P_{w}^{dw},P_{pv}^{up},P_{pv}^{dw},P_{dr}^{up},P_{dr}^{dw}\right\}$

On the other hand, the inner max min problem given by $C_t^{RT}(x,y)$ can be replaced by the auxiliary variable β representing the worst-case recourse. The first-stage constraints consider the upper and lower bound of flexibility offers for DG units (2a) and (2b), respectively, i.e.,

$$P_{dg(dg,t)}^{up}\le P_{dg(dg,t)}^{bid\_up} \tag{2a}$$

$$P_{dg(dg,t)}^{down}\le P_{dg(dg,t)}^{bid\_dw}\qquad \forall t\in\{1,...,T\},\forall dg\in\{1,...,N_{DG}\} \tag{2b}$$

In addition, wind and PV aggregators are modeled with the ability of up and down flexibility. The wind power for down flexibility is constrained by the operating point of the wind power aggregator $P_{w(w,t)}^{op}$, as in (2c), while the wind power for up flexibility is constrained by the wind bid for reserving power, as in (2d).

$$P_{w(w,t)}^{down}\le P_{w(w,t)}^{op} \tag{2c}$$

$$P_{w(w,t)}^{up}\le P_{w(w,t)}^{bid\_up}\qquad \forall t\in\{1,...,T\},\forall w\in\{1,...,N_W\} \tag{2d}$$

These constraints are also applied to the PV aggregators. Similarly, the upper and lower bounds for the DR aggregators are given by

$$P_{dr(l,t)}^{up}\le P_{dr(l,t)}^{bid\_up} \tag{2e}$$

$$P_{dr(l,t)}^{down}\le P_{dr(l,t)}^{bid\_dw}\qquad \forall t\in\{1,...,T\},\forall l\in\{1,...,N_L\} \tag{2f}$$

where $P_{DR(l,t)}^{bid\_up}$ is the maximum amount of load that can be reduced (offer).

In parallel, looking at equalities constraints from the recourse function (related to $h_t^{RT}(x,y)$ in eq. (3)), the decision variable vector y relates to

$$\left\{\begin{matrix}r_{dg}^{up},r_{dg}^{dw},P_{dg}^{cut},r_{w}^{up},r_{w}^{dw},\Delta P_w,P_w^{spill},r_{pv}^{up},r_{pv}^{dw},\Delta P_{pv},P_{pv}^{spill},r_{dr}^{up},r_{dr}^{dw},\\P_l^{shed},P_{Dch},P_{Ch},Q_{dg},Q_l,Q_{cb},E_{stored},X_{cb},X_{trf},V_i,V_{sb},\Delta V_{trf},\theta_{ij}\end{matrix}\right\},$$

including active and reactive power balance, reactive power consumption, capacitor banks tap changing, transformers with on-load tap-changing, and energy storage balance. Thus, the active power balance in each bus yields,

$$\begin{aligned}&\sum_{dg=1}^{N_{DG}}\left(P_{dg(dg,t)}^{op,i}+r_{dg(dg,t,s)}^{up,i}-r_{dg(dg,t,s)}^{dw,i}-P_{dg(dg,t,s)}^{cut}\right)+\\&\sum_{su=1}^{N_{SU}}P_{su(su,t,s)}^{i}+\sum_{st=1}^{N_{ST}}\left(P_{Dch(st,t,s)}^{i}-P_{Ch(st,t,s)}^{i}\right)+\\&\sum_{w=1}^{N_W}\left(P_{w(w,t)}^{op,i}+\Delta P_{w(w,t,s)}^{i}+r_{w(w,t,s)}^{up,i}-r_{w(w,t,s)}^{dw,i}-P_{w(w,t,s)}^{spill,i}\right)+\\&\sum_{pv=1}^{N_{PV}}\left(P_{pv(pv,t)}^{op,i}+\Delta P_{pv(pv,t,s)}^{i}+r_{pv(pv,t,s)}^{up,i}-r_{pv(pv,t,s)}^{dw,i}-P_{pv(pv,t,s)}^{spill,i}\right)+\\&\sum_{l=1}^{N_L}\left(P_{dr(l,t)}^{op,i}+r_{dr(l,t,s)}^{up,i}-r_{dr(l,t,s)}^{dw,i}+P_{l(l,t,s)}^{shed,i}-P_{l(l,t)}^{i}\right)=\\&G_{ii}V_{i(t,s)}^2+V_{i(t,s)}\sum_{j\in TL^i}V_{j(t,s)}\left(G_{ij}\cos\theta_{ij(t,s)}+B_{ij}\sin\theta_{ij(t,s)}\right)\\&\forall t\in\{1,...,T\},\forall i\in\{1,...,N_{Bus}\},\forall s\in\{1,...,N_S\},\theta_{ij(t,s)}=\theta_{i(t,s)}-\theta_{j(t,s)}\end{aligned} \tag{3a}$$

where $P_{w(w,t)}^{op}$ is the conditional mean forecast of wind producer scheduled at day-ahead market and $\Delta P_{w(w,t,s)}$ is the deviation of wind power production in the vertices from the conditional mean forecast that models the uncertainty set. In parallel, it is assumed that the reactive power balance only considers reactive power provided by DG (CHP) units, external suppliers and capacitor banks, formulated as

$$\sum_{dg=1}^{N_{DG}}\left(Q^{i}_{dg(dg,t,s)}\right)-\sum_{l=1}^{N_L}Q^{i}_{l(l,t)}+\sum_{su=1}^{N_{SU}}Q^{i}_{su(su,t,s)}+\sum_{cb=1}^{N_{CB}}\sum_{lv=1}^{N_{levels}}Q^{i}_{cb(cb,t,s,lv)}= \\ V_{i(t,s)}\sum_{j\in TL^i}V_{j(t,s)}\left(G_{ij}\sin\theta_{ij(t,s)}+B_{ij}\cos\theta_{ij(t,s)}\right)-B_{ii}V^2_{i(t,s)} \\ \forall t\in\{1,...,T\},\forall i\in\{1,...,N_{Bus}\},\forall s\in\{1,...,N_S\},\theta_{ij(t,s)}=\theta_{i(t,s)}-\theta_{j(t,s)} \tag{3b}$$

where the reactive power production for DG (CHP) units takes into account the active power production from the day-ahead market, as well as the up and down active power flexibility scheduled by the DSO with a fixed tan Φ = 0.3 [19].

$$Q_{dg(dg,t,s)}=\left(P^{op,i}_{dg(dg,t)}+r^{up,i}_{dg(dg,t,s)}-r^{dw,i}_{dg(dg,t,s)}-P^{cut}_{dg(dg,t,s)}\right)\tan\phi \\ \forall t\in\{1,...,T\},\forall dg\in\{1,...,N_{DG}\},\forall s\in\{1,...,N_S\} \tag{3c}$$

Besides the reactive power consumption in the system is modeled based on the relation between the total active power consumption and the power factor for each load l. $F_{power(l,t)}$ is the power factor in load l, usually assumed as 0.3 [20].

$$Q_{l(l,t,s)}=\left(P_{l(l,t)}-P^{op}_{dr(l,t)}+r^{dw}_{dr(l,t,s)}-r^{up}_{dr(l,t,s)}-P^{shed}_{l(l,t,s)}\right)F_{power(l,t)} \\ \forall t\in\{1,...,T\},\forall l\in\{1,...,N_L\},\forall s\in\{1,...,N_S\} \tag{3d}$$

Furthermore, the reactive power consumption is partly provided by generators and static equipment in the network. Capacitor banks are used to provide reactive power to the transformer, located at the substation. It is assumed that these equipment's are owned by the DSO. Traditionally, capacitor banks have levels of reactive power production, and can be modeled as

$$Q_{cb(cb,t,s,lv)}=Q^{levels}_{cb(cb,t,lv)}X_{cb(cb,t,s,lv)} \tag{3e}$$

$$\sum_{lv=1}^{N_{levels}}X_{cb(cb,t,s,lv)}=1 \\ \forall t\in\{1,...,T\},\forall cb\in\{1,...,N_{CB}\},\forall s\in\{1,...,N_S\},\forall lv\in\{1,...,N_{levels}\} \tag{3f}$$

In addition, transformers with OLTC ability are used to ensure voltage control in the substation. It is assumed that the transformers are owned by the DSO. Thus, it is known the voltage impact of each tap changing level in the secondary bus of the transformer. The tap changing constraints can be modeled as

$$\Delta V_{trf(trf,t,s,lv)}=V^{levels}_{trf(trf,t,lv)}X_{trf(trf,t,s,lv)} \tag{3g}$$

$$\sum_{lv=1}^{N_{levels}}X_{trf(trf,t,s,lv)}=1 \tag{3h}$$

$$V_{sb(t,s)}=V^{ref}_{sb(t,s)}+\sum_{lv=1}^{N_{levels}}\Delta V_{trf(trf,t,s,lv)} \\ \forall t\in\{1,...,T\},\forall s\in\{1,...,N_S\},\forall trf\in\{1,...,N_{TRF}\} \tag{3i}$$

where $\Delta V_{trf(trf,t,s,lv)}$ is the voltage level that will be activated by the DSO in the transformer unit at slack bus. In parallel $V^{levels}_{trf(trf,t,lv)}$ depicts all the possible levels available on the OLTC ability of the transformer. Finally, $X_{trf(trf,t,s,lv)}$ is a binary decision variable which define the activation of the chosen level. Moreover, the battery balance of storage units follows

$$E_{stored(st,t,s)}=E_{stored(st,t-1,s)}+\eta_{Ch(st)}P_{Ch(st,t,s)}-\frac{1}{\eta_{Dch(st)}}P_{Dch(st,t,s)} \\ \forall t\in\{1,...,T\},\forall st\in\{1,...,N_{ST}\},\forall s\in\{1,...,N_S\} \tag{3j}$$

where energy from previous period and charge and discharge ability is considered.

In parallel, inequalities constraints (related to $g^{RT}_t(x,y)$ in (4)), the decision variable vector y relates to

$$\left\{\begin{matrix} r^{up}_{dg},r^{dw}_{dg},P^{cut}_{dg},r^{up}_{w},r^{dw}_{w},\Delta P_w,r^{up}_{pv},r^{down}_{pv},\Delta P_{pv},r^{up}_{dr}, \\ r^{down}_{dr},P^{shed}_{l},P_{Dch},P_{Ch},E_{stored},X_{Dch},X_{Ch},V_i,\theta_{ij},\beta \end{matrix}\right\},$$

and includes operational costs for balancing the system, upper and lower bounds of active and reactive power to the up and down flexibility of all energy resources (such as DG, wind, PV, CHP, DR and storage devices), as well as non-simultaneity of storage devices, transformers and lines capacity, upper and lower bounds of voltage angles and magnitude, and declaration of non-negative variables.

Thus, the inequality constraint for the operational costs for up and down flexibility of different aggregators is considered (4a). Aggregators with distributed generation, wind and PV related with uncertainty and DR are modeled as external entities that provide flexibility and information about its resources electrical location to the DSO. On the other hand, it is assumed that storage units, capacitor banks and transformers with OLTC ability are owned by the DSO, thereby, these resources are modeled to balance the distribution system, with

$$\beta_t\geq\sum_{dg=1}^{N_{DG}}\left[C^{act}_{dg(dg,t)}\left(r^{up}_{dg(dg,t,s)}-r^{dw}_{dg(dg,t,s)}\right)+C^{cut}_{dg(dg,t)}P^{cut}_{dg(dg,t,s)}\right]+ \\ \sum_{w=1}^{N_W}\left[C^{act}_{w(w,t)}\left(r^{up}_{w(w,t,s)}-r^{dw}_{w(w,t,s)}\right)+C^{spill}_{w(w,t)}P^{spill}_{w(w,t,s)}\right]+ \\ \sum_{pv=1}^{N_{PV}}\left[C^{act}_{pv(pv,t)}\left(r^{up}_{pv(pv,t,s)}-r^{dw}_{pv(pv,t,s)}\right)+C^{spill}_{pv(pv,t)}P^{spill}_{pv(pv,t,s)}\right]+ \\ \sum_{l=1}^{N_L}\left[C^{act}_{dr(l,t)}\left(r^{up}_{dr(l,t,s)}-r^{dw}_{dr(l,t,s)}\right)+C^{shed}_{l(l,t,s)}P^{shed}_{l(l,t,s)}\right]+ \\ \sum_{st=1}^{N_{ST}}\left(C_{Dch(st,t)}P_{Dch(st,t,s)}-C_{Ch(st,t)}P_{Ch(st,t,s)}\right)+ \\ \sum_{cb=1}^{N_{CB}}\sum_{lv=1}^{N_{levels}}C_{cb(cb,t)}\left|X_{cb(cb,t-1,s,lv)}-X_{cb(cb,t,s,lv)}\right|+ \\ \sum_{trf=1}^{N_{TRF}}\sum_{lv=1}^{N_{levels}}C_{trf(trf,t)}\left|X_{trf(trf,t-1,s,lv)}-X_{trf(trf,t,s,lv)}\right| \\ \forall t\in\{1,...,T\},\forall s\in\{1,...,N_S\} \tag{4a}$$

where, the upper and lower bounds of the activation of active power for DG units considering the up and down flexibility is given by (4b) and (4c), respectively, while the generation curtailment power is expressed in (4d), where $P^{op}_{dg(dg,t)}$ is the operating point of the DG units. This gives

$$r^{up}_{dg(dg,t,s)}\leq P^{up}_{dg(dg,t)} \tag{4b}$$

$$r^{dw}_{dg(dg,t,s)}\leq P^{dw}_{dg(dg,t)} \tag{4c}$$

$$P^{cut}_{dg(dg,t,s)}\leq P^{op}_{dg(dg,t)}-r^{dw}_{dg(dg,t,s)} \\ \forall t\in\{1,...,T\},\forall dg\in\{1,...,N_{DG}\},\forall s\in\{1,...,N_S\} \tag{4d}$$

In addition, wind and PV aggregators are modeled with the ability of up and down flexibility. The wind power for activating up flexibility is constrained by the wind contracted in the first-stage decision for reserve power, as in (4e), while the wind power for activating down flexibility is constrained by the downward flexibility contracted in the first-stage decision, as in (4f). When the downward flexibility is insufficient to the meet the DSO requirement, wind spillage can be used being constrained by (4g).

$$r^{up}_{w(w,t,s)} \le P^{up}_{w(w,t)} \tag{4e}$$

$$r^{dw}_{w(w,t,s)} \le P^{dw}_{w(w,t)} \tag{4f}$$

$$\begin{gathered} P^{spill}_{w(w,t,s)} \le P^{op}_{w(w,t)} - r^{dw}_{w(w,t,s)} + \Delta P_{w(w,t,s)} \\ \forall t \in \{1,...,T\}, \forall w \in \{1,...,N_W\}, \forall s \in \{1,...,N_S\} \end{gathered} \tag{4g}$$

These constraints are also applied to the PV aggregators. In parallel, the upper and lower bounds for the activation of DR aggregators is given by

$$r^{up}_{dr(l,t,s)} \le P^{up}_{dr(l,t)} \tag{4h}$$

$$r^{dw}_{dr(l,t,s)} \le P^{dw}_{dr(l,t)} \tag{4i}$$

$$\begin{gathered} P^{shed}_{dr(l,t,s)} \le P_{l(l,t)} \\ \forall t \in \{1,...,T\}, \forall l \in \{1,...,N_L\}, \forall s \in \{1,...,N_S\} \end{gathered} \tag{4j}$$

where the load shedding is limited by the load in the system. Storage technical limits in each period t combine distinct inequalities constraints. Thus, the storage devices are used to reduce congestion when needed. Furthermore, the cost of using charge and discharge ability is modeled in eq. (4a). It is assumed that the cost for charge and discharge already consider the battery degradation over time [21]. Up and lower bounds for energy stored in the battery, as well as the charge and discharge limit per storage unit are modeled as

$$E_{Min(st,t)} \le E_{stored(st,t,s)} \le E_{BatCap(st,t)} \tag{4k}$$

$$P_{Ch(st,t,s)} \le P^{Max}_{Ch(st,t)} X_{Ch(st,t)} \tag{4l}$$

$$P_{Dch(st,t,s)} \le P^{Max}_{Dch(st,t)} X_{Dch(st,t)} \tag{4m}$$

$$\begin{gathered} X_{Ch(st,t)} + X_{Dch(st,t)} \le 1 \\ \forall t \in \{1,...,T\}, \forall st \in \{1,...,N_{ST}\}, \forall s \in \{1,...,N_S\} \end{gathered} \tag{4n}$$

where the charge and discharge ability of each storage unit cannot occur at the same time, as in (4n). Furthermore, the energy flow from upstream networks is limited through transformers that adapt the voltage level from high voltage to medium voltage. Therefore, the external supplier provide energy to the DSO through these transformers, which results in a constraint considering the upper limit of the transformers, such that

$$\begin{gathered} \left(\sum_{su=1}^{N_{SU}} P^i_{su(su,t,s)}\right)^2 + \left(\sum_{su=1}^{N_{SU}} Q^i_{su(su,t,s)}\right)^2 \le \left(S^{Max}_{trf}\right)^2, \forall t \in \{1,...,T\}, \\ \forall i \in \{1,...,N_{Bus}\}, \forall s \in \{1,...,N_S\}, \forall trf \in \{1,...,N_{TRF}\} \end{gathered} \tag{4o}$$

where $P_{su(su,t,s)}$ is the energy that the external supplier (managed by the TSO) provides to the DSO to balance the distribution system. Similarly, the thermal limit of distribution lines constrains the power flowing from bus i to bus j, and vice-versa, such as

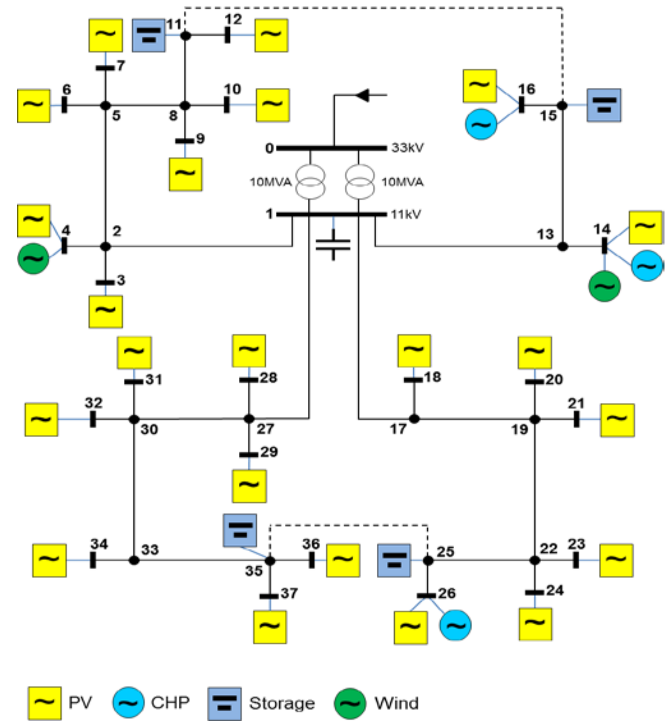

Fig. 2. 37-Bus distribution network (adapted from [23]).

$$\left|\overline{V_{i(t,s)}}\left[\overline{y_{ij}V_{ij(t,s)}} + \overline{y_{sh(i)}V_{i(t,s)}}\right]^*\right| \le S^{Max}_{TL} \quad , \overline{V_{ij(t,s)}} = \overline{V_{i(t,s)}} - \overline{V_{j(t,s)}} \tag{4p}$$

$$\begin{gathered} \left|\overline{V_{j(t,s)}}\left[\overline{y_{ij}V_{ji(t,s)}} + \overline{y_{sh(j)}V_{j(t,s)}}\right]^*\right| \le S^{Max}_{TL} \quad , \overline{V_{ji(t,s)}} = \overline{V_{j(t,s)}} - \overline{V_{i(t,s)}} \\ \forall t \in \{1,...,T\}, \forall i,j \in \{1,...,N_{Bus}\}, \forall s \in \{1,...,N_S\}, i \ne j \end{gathered} \tag{4q}$$

where the bus voltage magnitude limits is represented by

$$V^i_{Min} \le V_{i(t,s)} \le V^i_{Max}, \forall t \in \{1,...,T\}, \forall s \in \{1,...,N_S\} \tag{4r}$$

assuming that the voltage magnitude is fixed and defined by the DSO for the slack bus (upstream bus connection).

IV. Evaluation of Distribution Network Management

This section presents a case study that illustrates the application of the proposed models and its respective performance. The presented case study has been chosen to cover a diversity of uncertain situations, allowing demonstrating the proposed model. The simulation was performed with MATLAB and GAMS.

A. Outline

The case study is partially based on the case study presented in [22]. The original distribution network is presented in [23], while the energy mix in 2050 used for updating the network is proposed in [24]. Figure 2 shows an 11 kV distribution network with 37 buses, connected to the high voltage network through two power transformers with 10 MVA each one. The DERs are aggregated by technology, so one aggregator represents a specific type of DER technology. In each consumption point, the aggregation of the available DR is also considered. The distribution network supplies energy to 1908 consumers (185 domestic consumers, 2 industries, 50 commerce stores, and 6 service buildings) [23]. The consumption characteristics in each consumption bus, as well as the consumption profiles of each type of consumer are the same than those used in [22].

TABLE I: GENERAL CHARACTERISTICS AND OPERATING POINT FOR DER.

DER	Number of units	Total installed power	Operating point (MW)		
			Max	*Mean*	*Min*
CHP	3	2.5 (Mva)	1.5	1.15	1
External supplier	1	20 (Mva)	19.4	10.58	5.75
PV	22	7.74 (MWp)	5.55	1.96	0
Wind	2	2.5 (MW)	1.88	1.77	1.52
DR	22	4.65 (MW)	0.1	0.03	0

TABLE II: DER UP AND DOWN FLEXIBILITY AND COSTS.

DER	Up cost (m.u./kWh)			Down cost (m.u./kWh)		
	Max	*Mean*	*Min*	*Max*	*Mean*	*Min*
CHP	0.15	0.10	0.05	0.09	0.06	0.03
PV	-	0.11	-	-	0.06	-
Wind	-	0.10	-	-	0.05	-
DR	-	0.22	-	-	0.17	-

TABLE III: DAY-AHEAD TOTAL OPERATIONS COSTS, FLEXIBILITY AND LOAD SHEDDING FOR A 24-HOUR PERIOD SIMULATION.

Model	Deterministic	Robust 3 vert.	Robust 4 vert.	Robust 6 vert.
DG flex (MW)	1.195	1.298	1.325	1.375
DR flex (MW)	2.633	3.645	3.814	4.108
Storage (MW)	0.511	0.450	0.455	0.461
Load shedding (MW)	0	0	0	0
Flex cost (m.u.)	23.722	23.95	24.01	24.063
Operational cost (m.u.)	23.722	23.95	24.01	24.063

1) DSO Internal Resources

The DSO is the owner of some equipment installed in the network that supports grid management. Thus transformers with OLTC, capacitor banks and energy storage systems are considered. The transformers installed in the network have 21 tap changers to adjust the voltage level in the secondary bus of the transformers. The tap changing can lead to a maximum deviation in the voltage level of 0.1 p.u. A cost for using tap changing ability is considered, since its use reduces the lifetime and increases the maintenance of the equipment. It is assumed a cost of 0.19 m.u. per change, which was determined based on the flexibility cost of the high voltage/medium voltage substation with OLTC proposed in [25]. Capacitor banks are installed in the network to allow the DSO to manage reactive power in the secondary of the transformer. The capacitor bank has 5 levels of tap changing of reactive power production reaching a maximum reactive power of 0.8 Mvar. As for the transformers, the capacity bank lifetime reduces with the number of changes of the tap position. A cost of 0.47 m.u. per change is assumed, based on the formula for capacity bank tap changing [25]. Throughout the network are installed energy storage systems with charging and discharging ability. All the ESS equipment have the same characteristics of charge rate (0.15 MW), discharge rate (0.20 MW), capacity (0.25 MWh), charge price (0.030 m.u./kWh) and discharge price (0.065 m.u./kWh). The discharge price incorporate a degradation cost of 0.03 m.u./kWh, based on the study in [21].

2) DER in the Network

The distribution network considers different aggregators of DER representing a different DER each one. Table I provides general information on the DER. In addition, the operating point of the DER is given by a previous dispatch from the market.

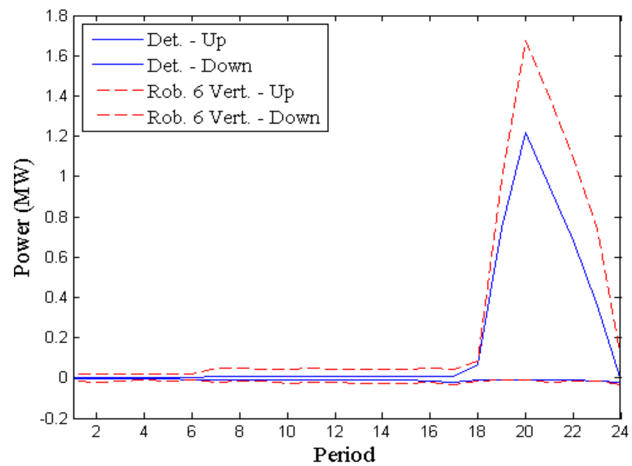

Fig. 3. Contracted flexibility by the DSO under deterministic and robust approach with 6 vertices.

All DER are able to provide flexibility based on their generation level. Table II shows the costs of flexibility (up and down) of the different aggregators. The up and down flexibility of CHP, external suppliers and DR goes from its level of operating point to its maximum and minimum level of output power, respectively. In addition, it is assumed in the validation stage (real-time simulation with measured data) of the robust solution that the up and down flexibility costs increase 20% when procured during the validation stage (assuming that the real-time activation of these resources is more expensive).

The PV and wind power from aggregators are modeled as random variables. Up and down flexibility is used according the offering bids that these aggregators submit to the DSO. The downward flexibility bid is equal to the energy operating point of these aggregators, previously scheduled in the market. The scenarios for wind power generation over the 24-periods can be found in [26], [27]. The offering bids were determined for 24-hour period based on [14]. The use of the constant strategy has been assumed. The constant strategy splits part of the available wind power for energy and up flexibility [14]. For PV aggregators, a scenario generation based on probability forecasts for short-term production has been performed. The probabilistic forecast was based on the quantile forecast from [28]. These quantiles were used to generate the scenarios and offering bids. It has been used the scenario generation process described in [29] to generate the spatial-temporal trajectories or scenarios. The offering bids were performed based on the constant approach shown in [14].

B. Results

1) Day-ahead Solution

The total operation costs for 24 hour period simulation considering a comparison between deterministic and robust approach are shown in Table III. It can be concluded that increasing the number of vertices of the robust optimization approach, will generate a more robust solution to the system, which results in a higher cost to the DSO. The high cost of the robust model with 6 vertices lies on contracting (or reserving) more flexibility in the system for some periods.

Fig. 3 presents an hourly comparison of the contracted flexibility for upward and downward for all resources (wind,

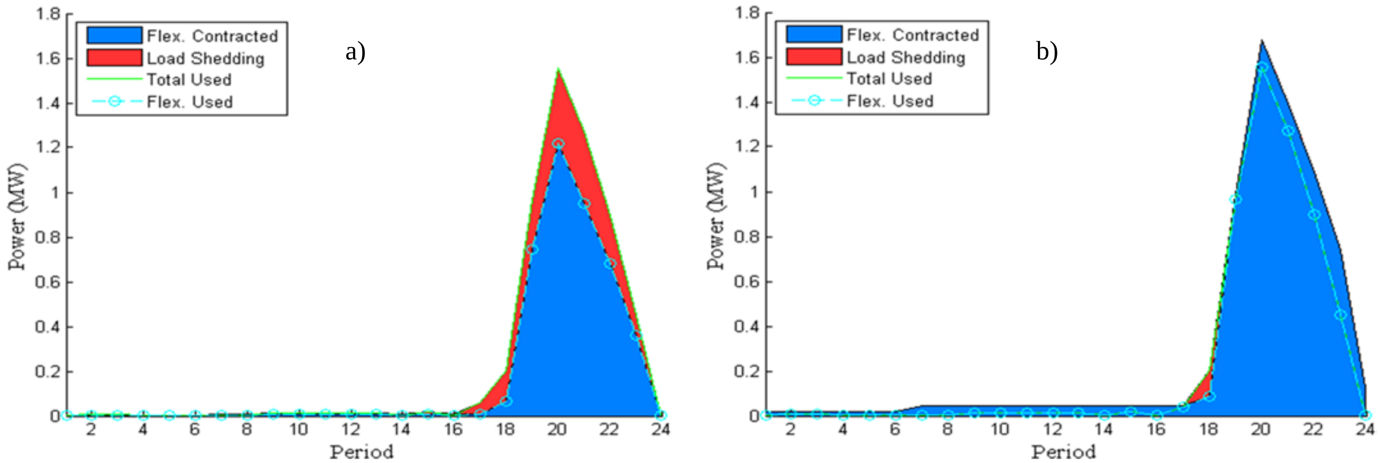

Fig. 4. Contracted and used flexibility, as well as, load shedding over 24-hour for a) deterministic and b) robust (6 vertices) approaches under real-time operation.

TABLE IV: TOTAL OPERATIONS COSTS; FLEXIBILITY AND LOAD SHEDDING OF 24 HOUR PERIOD SIMULATION AFTER THE VALIDATION PROCESS.

Model	Deterministic	Robust 3 vert.	Robust 4 vert.	Robust 6 vert.
DG flex (MW)	1.192	1.297	1.324	1.370
DR flex (MW)	2.632	3.002	3.093	3.209
Storage (MW)	0.303	0.435	0.450	0.460
Load shedding (MW)	1.030	0.423	0.290	0.118
Flex cost (m.u.)	23.696	23.894	23.929	23.982
Operation cost (m.u.)	24.627	24.249	24.124	24.059

PV, CHP and DR). In general, robust approach reserves a higher level of up and down flexibility, since the solution is based on the worst case solution. Periods 18 to 23 present higher contracted flexibility, since there are expected congestion situations in the power transformer and in the network branches.

2) Validation of the Day-ahead Solution

The validation stage consists on performing an hourly optimal power flow considering the real measurement data of wind and PV and the reserved/contracted flexibilities by the DSO. This means that only the flexibility contracted by the DSO to the aggregators can be used during the validation stage. Flexibility contracted is used to solve congestion in the system. In cases where contracted flexibility is not enough to solve congestion problem, wind and PV curtailment and load shedding are used to balance the system as last resort measures. From this, it is possible to evaluate the robustness of the proposed solution and compare with the traditional deterministic approach (where the traditional approach lies on an OPF scheduling based on point forecast information). Table IV shows the total operation costs of each approach after the validation process. One can see that robust approach ensures lower operational costs than traditional DSO approach (2.31% more efficient). This is due to the broad flexibility that is scheduled under the worst-case of the robust approach in each hour.

Comparing the results of the traditional and the robust approach under the day-ahead scheduling and during the validation process (Table III and Table IV), one can verify that robust approaches reserve more flexibility during the day-ahead scheduling that can be used during the validation process with measured data, resulting in less operation costs after validation stage. Thus, in this case study, from an economical point of view, the proposed approach is better than the traditional deterministic DSO approach (present-day practice). For instance, if the DSO choose the robust approach instead of the deterministic in the day-ahead market for more 0.341 m.u. (24.063-23.722, Table III), it would have a saving of 0.568 (24.627-24.059, Table IV) in the validation stage, which means 0.227 m.u. of net saving.

Although the cost saving are small since the case study is a daily analysis, a yearly analysis can represent a significant saving for the DSO. However, it is possible that a different case study could show distinct behavior. Additionally, the proposed approach also ensures higher reliability by requiring less load shedding than the traditional approach.

The behavior of the deterministic approach under the validation process for the 24-hour period is illustrated in Fig. 4 a). The blue area represents the flexibility contracted at day-ahead stage, while the red area shows the load shedding used by the deterministic approach during the validation. Green line represents the total power used by the DSO to manage the grid during the validation process, while the blue line shows the flexibility used by the deterministic approach during the validation process. One can see that the flexibility contracted in the day-ahead is not enough to solve the congestion problem that occurred during the real-time operation. Thus, load shedding is used by the DSO to manage this congestion.

Fig. 4 b) depicts the behavior of the proposed approach under the validation process. One can see that in most of the periods, the contracted flexibility is more than enough to solve congestion problems that occur during real-time operation. However, between 17 and 18 periods, there is a need for extra power to solve congestion. Thus, load shedding is used to manage the congestion, accounting for a high penalty.

Comparing the results of deterministic - Fig. 4 a) - and proposed - Fig. 4 b) - approaches, one can identify different behavior and portions of the scheduled and used flexibility. The proposed approach reserves more flexibility than deterministic approach, which is useful during the real-time operation. Thus, the proposed approach infers less operation cost than the equivalent deterministic approach after the validation process, due to the lower need of load shedding (extremely expensive) to manage congestion in the system.

3) Computational Performance

The computations were carried out with DICOPT [30] as an MINLP solver on an Intel Core i5 2.70 GHz processor with 8 GB RAM. All modeling has been performed in GAMS [31] modeling language. The deterministic approach has been performed in 8 minutes, while the robust 3, 4 and 6 vertices perform 2.5h, 6.4h and 16.2h, respectively. The robust approach takes a high computational time to converge, due to the complexity of the proposed formulation.

V. Conclusions

The increasing flexibility of DER will allow the DSO to reserve this flexibility to handle local technical problems in the distribution system, thereby, improving security of supply.

This work proposes a new method for the DSO distribution grid management under spatial-temporal uncertainty. It is assumed that the DSO applies a preventive approach on the grid management by reserving flexibility from DER at day-ahead stage. The results show that such approach is more expensive than present-day practice at day-ahead stage, but cheapest in the operating day, i.e. the robust approach provides savings to the DSO operation costs by reserving some flexibility at day-ahead stage to be used during real-time operation, avoiding extra penalties. In addition, results show that the level of robustness depends of the modeling of the uncertainty set, i.e. the number of vertices of the uncertainty set to use in the optimization process. However, the price of robustness is paid on the computational effort. An important conclusion from this work is that robust solutions increase reliability of the distribution system, representing a preventive approach for the grid management.

Nevertheless, the use of this methodology by the DSO requires a yearly evaluation between the costs provided by this approach and its usefulness (perhaps, measured by the number of events in a year where the method is useful). Thus, the future work should be focused in this trade-off, as well as, in improving the computational performance of the optimization algorithm, potentially by combining meta-heuristics with mathematical optimization techniques to assure tractable robust solutions.

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