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# Virtual Bidding for Coordination of Power and Natural Gas Markets Under Uncertainty

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**Abstract** The current design of energy markets is based on a sequential clearing of trading floors, e.g., day-ahead (DA) and real-time (RT), with deterministic description of uncertain supply. This setup is being challenged by the increasing share of renewables, since their inaccurate forecast in DA may result in high balancing costs and cause market inefficiencies. Besides, electricity and natural gas markets are cleared sequentially and separately. However, high integration of renewables increases their interactions as more operational flexibility is needed from gas-fired power units. These challenges require improving the coordination between power and gas markets (*sectoral coordination*) as well as the coordination between DA and RT markets in each of the energy sectors (*temporal coordination*). We explore virtual bidding (VB) as a potential market-based solution for improving both sectoral and temporal coordination, while preserving the sequences of current market clearings. Two types of VB, i.e., explicit and implicit, are investigated. The explicit VB by purely financial players can enhance temporal coordination, while the implicit one by phys-

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ical players (e.g., gas-fired power units) can improve sectoral coordination. Under the assumption of perfect competition, we formulate an equilibrium model including the deterministic market-clearing problems by the electricity and natural gas market operators in DA and RT stages as well as the self-scheduling problem by each virtual bidder who solves a stochastic program to maximize her expected profit. The resulting model is a generalized Nash equilibrium problem. With a fully stochastic co-optimization model as an ideal benchmark, we show in a case study the increase of market efficiency in terms of total system cost using VB, since flexible resources are dispatched more efficiently.

**Keywords** Electricity and natural gas markets · multi-energy system · generalized Nash equilibrium · operational flexibility · self-scheduling · virtual bidding.

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## 1 Introduction

High share of power production from stochastic renewable sources (e.g., wind and solar units) increases the need for operational flexibility to deal with their variability and uncertainty. By operational flexibility, we refer to the capability of a power system to modify its output or state in response to a change in renewable power production (Zhao et al., 2016). Natural gas-fired power plants are usually flexible units and compensate for the production variability and uncertainty caused by stochastic renewable sources (Gil et al., 2014). These gas-fired units operate at the interface of the electricity and the natural gas systems, yielding both physical and economic interactions. The natural gas system is crucial for ensuring fuel availability and technical feasibility, while it can also provide flexibility for power systems through stored gas in the pipelines (Ordoudis et al., 2019; Correa-Posada and Sánchez-Martín, 2015; Yang et al., 2018). An increasingly volatile dispatch of gas-fired power plants to offset wind intermittency introduces demand fluctuations and uncertainty into the gas market (Heinen et al., 2017; Dall'Anese et al., 2017; Nicholson and Quinn, 2019). The subsequent trend towards increasing volumes in gas trading in short-term spot markets like Gaspoint Nordic (Hibbard and Schatzki, 2012; Pinson et al., 2017) will become more important as natural gas demand profiles become more uncertain.

The current power markets are relied upon a *sequential* clearing of trading floors, e.g., day-ahead (DA) and real-time (RT), each with *deterministic* description of uncertain supply. There is a similar sequential deterministic market design for natural gas systems. Despite the recent advances in forecasting tools, the deterministic forecast of renewable sources in DA can be erroneous, which may cause wrong unit commitment and dispatch decisions, resulting in market inefficiency, i.e., higher system cost. This challenge requires *temporal coordination* between DA and RT markets in both power and gas systems. In addition, by increasing the interaction of power and natural gas systems, *sectoral coordination* between power and natural gas markets is crucial (Meibom et al., 2013), though they are cleared separately and sequentially in most countries (Hibbard and Schatzki, 2012; Tabors et al., 2012).

The market-based mechanisms for improving both temporal and sectoral coordination of power and natural gas systems range from an extremely disruptive choice of designing a fully stochastic integrated energy market (Correa-Posada and Sánchez-Martín, 2015; Zlotnik et al., 2016) to less-disruptive solutions that preserve the current regulatory framework with separate and sequential clearing of markets. The latter (i.e., less disruptive market mechanisms) is the focus of this paper, while the former (i.e., the fully stochastic integrated energy market) is used as an *ideal benchmark* to assess the performance of the proposed mechanisms. The less-disruptive (or "soft") market-based mechanisms for power and gas coordination can be realized for instance through more awareness and information exchange among the markets (Byeon and Van Hentenryck, 2019), introducing new market products (Chen et al., 2017; Warrington et al., 2013; Wang and Hobbs, 2016) and bidding formats (Liu et al., 2015; O'Connell et al., 2016), and allowing new market players.

In this paper, we explore the effect of *virtual bidding* (VB) (Hogan, 2016), also called "convergence bidding" (Li et al., 2015) in the literature, as a soft market-based mechanism for improving both temporal and sectoral coordination in power and natural gas systems. VB exists today in U.S. markets, and refers to the financial arbitrage between two trading floors in an energy market, e.g., between DA and RT electricity markets. The virtual bidder may earn profit due to price difference in DA and RT markets by performing arbitrage. This virtual bidder can be a purely financial player who has no physical asset (the so-called explicit VB), or she can be one of the existing physical market players (implicit VB), e.g., a generator, who does arbitrage between DA and RT markets by selling power in DA more than her capacity (Isemonger, 2006; Mather et al., 2017). It is known today that VB can improve the market efficiency of two-settlement deterministic electricity markets by enhancing the temporal coordination (Kazempour and Hobbs, 2018; Morales and Pineda, 2017) thanks to increasing market liquidity and bringing additional information to the markets. Though this improved market efficiency may not be realized under some circumstances (Parsons et al., 2015) or may have some limits (Birge et al., 2018; Ito and Reguant, 2016), e.g., when virtual bidders behave strategically (Prete et al., 2019).

As the core contribution of this paper, we first extend the application of VB to natural gas markets, yielding temporal coordination between DA and RT natural gas markets. Then, we investigate the possibility of gas-fired power plants who are at the interface of power and gas systems to behave as implicit virtual bidders. We underline the ability of these gas-fired power plants to submit virtual bids (by acting as self-scheduling units) to enhance both temporal and sectoral coordination between existing sequential power and gas markets.

Under the assumption of perfect competition, we formulate an equilibrium model including the deterministic market-clearing problems of the electricity and natural gas market operators in DA and RT stages as well as the self-scheduling problem by each virtual bidder (either explicit or implicit) who solves a stochastic program to maximize her expected profit. The resulting model is a generalized Nash equilibrium (GNE) problem, whose solution existence can be mathematically ensured under some assumptions. We also provide analytic insights for the comparison of the GNE problem and the ideal benchmark of the two-stage stochastic optimization problem of a fully integrated energy system.

The manuscript is organized as follows. In Section 2 we provide more details about temporal and sectoral market coordination, the concept of VB and our modeling assumptions. Sections 3 and 4 contain the mathematical formulations of the GNE model with explicit and implicit VB, respectively. The formulation of the ideal benchmark model is included in Section 5. In Section 6, we show the numerical results for a case study, and finally Section 7 concludes the paper. For clarity purposes, we stick to the generic representation of optimization problems throughout the paper, and include their detailed representations in the online appendix (Schwele et al., 2019b).

## 2 Preliminaries

This section first highlights the temporal and sectoral coordination of power and natural gas markets under uncertainty. Then, it further describes both types of VB (explicit and implicit). Finally, it summarizes the modeling assumptions made in this paper.

### 2.1 Two-dimensional Coordination: Temporal and Sectoral

The independent market operators clear each trading stage (DA and RT) separately and sequentially for electricity and natural gas markets. The current market-clearing framework for electricity and natural gas systems is illustrated in Fig. 1, including four market-clearing sequences. First, the electricity market is cleared in a day-ahead auction 12-36 hours before actual energy delivery using a deterministic forecast of uncertain parameters, e.g., renewable power generation and natural gas prices. Note that future natural gas prices directly impact the marginal production cost of gas-fired power plants and consequently the merit order in the electricity market. Second, the natural gas day-ahead market is cleared for given gas demand of gas-fired power plants

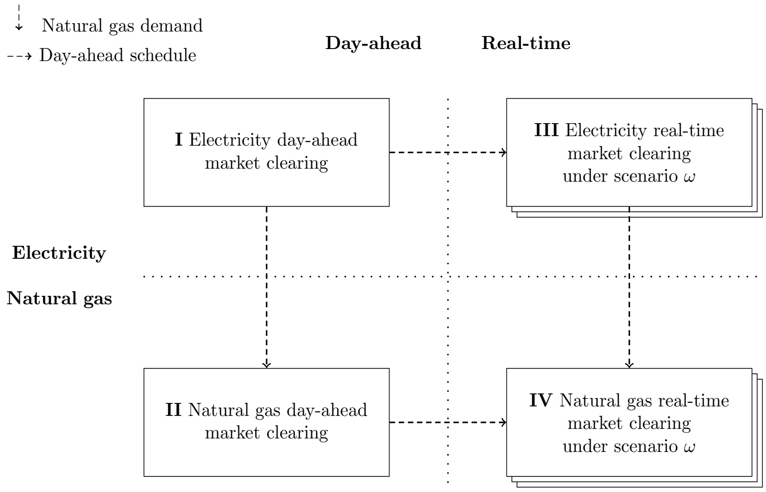


**Fig. 1** Sequential setup of electricity and natural gas markets, including four market-clearing sequences I to IV.

determined by their dispatch in the electricity market. Third, once the uncertainty is realized (e.g., scenario $\omega$ occurs), the real-time electricity market is cleared to adjust imbalances under fixed day-ahead unit commitment and schedule decisions. Fourth, the natural gas market is cleared in real-time, while the dispatch of gas suppliers in DA and the demand of gas-fired power plants in RT are given.

The sequential setup in Fig. 1, even though aligned with current practice, is totally uncoordinated in both temporal and sectoral dimensions. This setup is temporally uncoordinated since both electricity and gas markets in DA are cleared based on the available deterministic forecast in that stage, without foresight into the potential scenarios that may realize in RT. It is also sectorally uncoordinated because the electricity market is cleared based on an estimation of natural gas price, and the gas market is cleared afterwards. As common in practice, note that operating reserve is able to potentially enhance the temporal coordination between DA and RT markets under renewables uncertainty, however it may bring extra inefficiencies if the value assigned for the minimum reserve requirement in DA market is not properly selected (Doherty and O'Malley, 2005; Papavasiliou et al., 2011). This can be a more challenging issue in European markets, where energy and reserve markets are cleared sequentially (Dominguez et al., 2019). Note that the reserve market is not the focus of this paper.

While the share of stochastic renewable energy sources is growing, the lack of temporal and sectoral coordination in electricity and natural gas markets causes increasing market inefficiency (i.e., higher system cost) by potentially making faulty DA decisions. In other words, flexible sources dispatched in DA wrongly might not be available in RT for coping imbalances, and thereby, more expensive actions (e.g., load curtailment) might be required. Therefore, it is desirable to dispatch the flexible sources in DA in an efficient manner while preserving the current sequential market-clearing framework. This requires soft market-based mechanisms for enhancing the temporal and sectoral coordination of power and natural gas markets, which is the focus of this paper.

## 2.2 Virtual Bidding

Virtual bidding is a purely financial instrument, existing in the U.S. electricity markets, e.g., CAISO, PJM, and MISO (Li et al., 2015; Hogan, 2016; Birge et al., 2018). It allows market players to profit from anticipated price differences between the DA and RT markets by performing arbitrage. We explain below both explicit and implicit VB (Isemonger, 2006; Mather et al., 2017).

An *explicit* virtual bidder is a purely financial player who does not own any physical asset, and thereby, her positions in DA and RT need to even out to zero. For example, an explicit virtual bidder may buy 10 MW in the DA electricity market in a specific hour at the DA market price in that hour, and then sells the same 10 MW back in the RT electricity market at the same hour but at the price of the RT market. Therefore, her payoff is equal to the difference between the DA and RT prices times the amount of virtually traded power. Assuming that this virtual bidder is a price-taker with perfect foresight into the distribution of DA and RT prices, she is supposed to enhance informational and productive efficiency of the two-settlement market by bringing more competitiveness, liquidity and transparency to wholesale energy markets. Fig. 2 illustrates how such an explicit VB is integrated into the two-settlement market-clearing setup. While DA and RT energy markets are cleared deterministically and sequentially, the explicit virtual bidder solves a stochastic program maximizing her expected profit. The outcomes of the stochastic program of virtual bidders, i.e., virtual trades, are exogenous in DA and RT markets. It is demonstrated in (Kazempour and Hobbs, 2018) that this setup can bring temporal coordination. This is an interesting insight for market operators since they can keep the market clearing deterministic, while leaving the correction of market inefficiency to virtual bidders. However, VB may not always work in such a desirable way, as discussed in (Birge et al., 2018; Parsons et al., 2015).

Unlike the explicit VB, the *implicit* virtual bidder is a physical market player, e.g., a gas-fired power plant who is at the interface of power and natural gas systems, as illustrated in Fig. 3. Virtual bidding gas-fired units can bring temporal and sectoral coordination. Though the presence of explicit VB may eliminate the motivation of physical players to perform arbitrage, physical players may still find *self-scheduling* profitable to forgo the market and

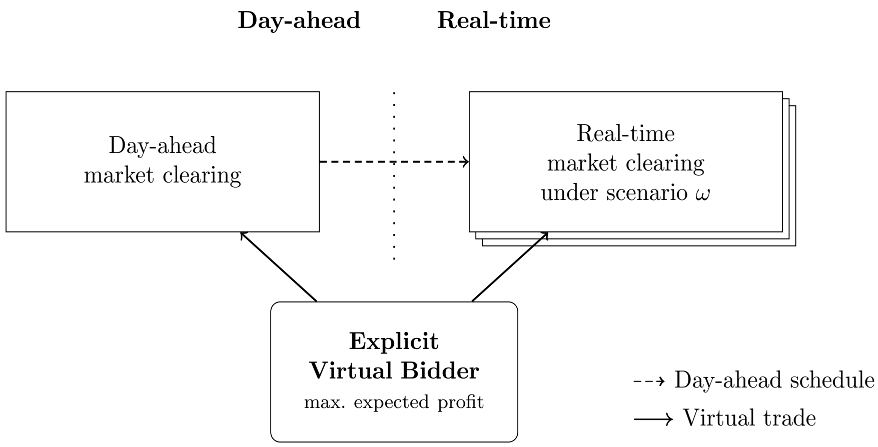


**Fig. 2** Explicit virtual bidding by arbitraging electricity between the DA and RT electricity markets (or by arbitraging gas between DA and RT natural gas markets). This type of virtual bidding can enhance temporal coordination between DA and RT markets.

dispatch their productions/consumptions themselves outside the market. For example, assume a gas-fired power plant that has perfect foresight into future DA and RT power and gas prices, and realizes that her profit is not maximized when she participates in the deterministic electricity and natural gas markets. In other words, she has the opportunity to gain a higher expected profit by self-scheduling outside the market (Sioshansi et al., 2010). Note that the power production and gas consumption of this unit are exogenous in the market-clearing problems, while she still pays/is paid based on the market-clearing prices (Jha and Wolak, 2015; Papavasiliou et al., 2015). An implicit virtual bidder may benefit from self-scheduling by solving her own stochastic program with better representation of uncertainty and technical constraints for a longer time horizon. However, these self-schedulers take on the full risk of RT price uncertainty. The influence of risk aversion and price volatility on the decision of generators to do self-scheduling is discussed in (Papavasiliou et al., 2015; Conejo et al., 2004).

### 2.3 Modeling Framework and Assumptions

The market-clearing problems in this paper are sequential and deterministic, while markets allow the participation of stochastic decision-makers who make their own dispatch decisions outside the market. Procurement of operating reserves is not enforced and there are no reserve products in the markets. Wind power production is assumed as the only source of uncertainty. Note that the wind power forecast in DA is a single point (deterministic), however different scenarios may occur in RT, i.e., we are not sure about the actual outcome of

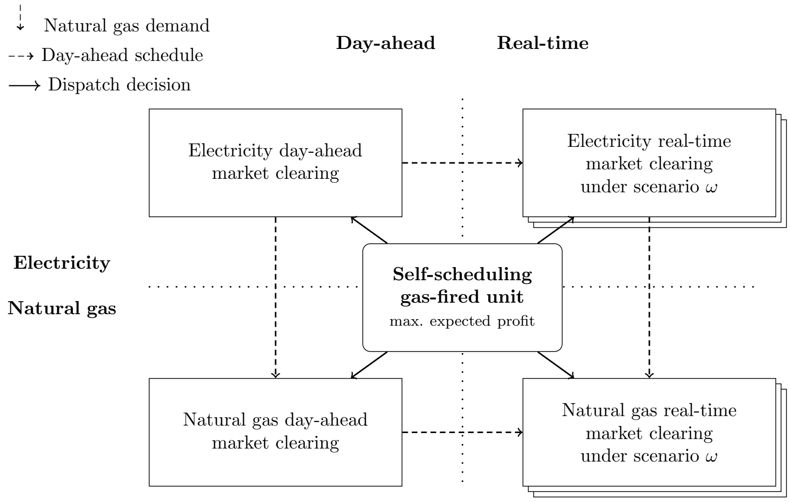


**Fig. 3** Implicit virtual bidding by a gas-fired power plant, who is on the interface of electricity and gas systems, and self-schedules her power productions and gas consumptions in DA and RT electricity and natural gas markets. This type of virtual bidding can enhance temporal and sectoral coordination between DA and RT electricity and gas markets.

uncertain parameter. We consider two trading floors (DA and RT) only, and other potential floors, e.g., intra-day adjustment markets, are excluded. Wind power uncertainty is represented using a finite set of scenarios. The uncertain parameter is characterized accurately by proper modeling of a realistic range of potential outcomes and their probability distribution (in the form of scenarios). The wind power production cost is zero, and can be spilled at zero cost. The linking parameters between the electricity and natural gas markets are natural gas spot price and fuel consumption of gas-fired power plants. Both electricity and natural gas demands are inelastic to price. All demand and supply in both energy sectors are assumed to be located at a single node, neglecting the transmission systems. Trading of natural gas takes place according to the entry-exit model (Vazquez and Hallack, 2015). On the power side, a multi-period unit commitment scheduling model is used. We relax the binary nature of commitment status of conventional generators to lie within zero and one, but in a tight manner (Hua and Baldick, 2017). This relaxation ensures convexity, which is required to solve the equilibrium model as a mixed complementarity problem. The consequences of this kind of relaxation on pricing are discussed in (Chao, 2019). The production cost of generators is assumed to be a linear function. We assume all markets players including virtual bidders (either explicit or implicit) to act competitively, non-strategically, and in a risk-neutral manner when participating in the markets, so they offer at prices identical to their marginal costs.

**Notation:** We denote by $\mathbb{R}$ real numbers and by $\mathbb{R}_+$ non-negative real numbers. We use upper case letters for matrices and lower case letters for vectors where $e$ is the vector of ones. Bold lower case letters denote vectors of variables. Note that $(.)^\top$ is the transpose operator. Hereafter, we use functions $h(.)$ and $g(.)$ to show equality and inequality constraints in every optimization problem, but note that these constraints for different optimization problems are not necessarily the same.

## 3 Temporal Coordination

In this section, we first explore temporal coordination between electricity DA and RT markets via explicit VB. We then argue such a coordination in DA and RT natural gas markets. Note that this section ignores self-scheduling (implicit VB) which brings sectoral coordination – it will be discussed later in Section 4.

### 3.1 Temporal Coordination Between DA and RT Electricity Markets

We present below the optimization problems for explicit electricity virtual bidder, DA electricity market and RT electricity market, which construct an equilibrium problem together.

#### *3.1.1 Explicit Electricity Virtual Bidder*

The expected profit-maximization problem of each explicit electricity virtual bidder $r \in \mathcal{R}$ over the time horizon $\mathcal{T}$ writes as

$$\left\{\begin{aligned} &\max_{\mathbf{v}_r^{\mathrm{E}}, \boldsymbol{\Delta}\mathbf{v}_r^{\mathrm{E}}} \ \boldsymbol{\lambda}^{\mathrm{DA,E}^\top} \ \mathbf{v}_r^{\mathrm{E}} + \left(\sum_{\omega} \pi_\omega \ \boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,E}}\right)^\top \boldsymbol{\Delta}\mathbf{v}_r^{\mathrm{E}} && \text{(1a)}\\ &\text{subject to } \ \mathbf{v}_r^{\mathrm{E}} + \boldsymbol{\Delta}\mathbf{v}_r^{\mathrm{E}} = 0 \end{aligned}\right\} \ \forall r, \qquad \text{(1b)}$$

i.e., as a linear stochastic program. The virtual bidder decides her DA position $\mathbf{v}_r^{\mathrm{E}} \in \mathbb{R}^{\mathcal{T}}$ in the electricity markets given the DA electricity prices $\boldsymbol{\lambda}^{\mathrm{DA,E}} \in \mathbb{R}^{\mathcal{T}}$ as well as the distribution of RT electricity prices $\boldsymbol{\lambda}_\omega^{\mathrm{RT,E}} \in \mathbb{R}^{\mathcal{T}}$ $\forall \omega$ weighted by probability $\pi_\omega$ over the set of scenarios $\omega \in \Omega$. This virtual bidder is a purely financial player without physical assets, and therefore is obliged to offset her DA position by her RT position $\boldsymbol{\Delta}\mathbf{v}_r^{\mathrm{E}} \in \mathbb{R}^{\mathcal{T}}$ in each scenario. Objective function (1a) maximizes the expected profit of explicit virtual bidder who arbitrages between the DA and RT electricity markets. Equation (1b) ensures that the virtual bidder sells (buys) the same amount back in the RT market that was bought (sold) in the DA market. One important observation about these explicit virtual bidders is that they yield the convergence of DA and expected RT electricity prices (Kazempour and Hobbs, 2018). Derived from optimality conditions of (1), the virtual bidder enforces the DA and the expected RT electricity prices to be equal, i.e., $\boldsymbol{\lambda}^{\mathrm{DA,E}} = \sum_\omega \pi_\omega \, \boldsymbol{\lambda}_\omega^{\mathrm{RT,E}}$ (see online appendix (Schwele et al., 2019b)). Note that the market operators treat the dispatch decision of the virtual bidders as fixed input into the market clearing problem presented in the following section.

### *3.1.2 DA Electricity Market*

Consider $\mathcal{G}$ gas-fired generators and $\mathcal{C}$ non gas-fired generators, such that $\mathcal{G} \cup \mathcal{C} = \mathcal{I}$. Besides, consider $\mathcal{J}$ wind power units. For given production cost of non gas-fired generators $\mathbf{C}^{\mathrm{E}} \in \mathbb{R}_+^{\mathcal{C}}$, estimation of natural gas prices $\tilde{\boldsymbol{\lambda}}^{\mathrm{G}} \in \mathbb{R}^{\mathcal{T}}$ to compute the production cost $C(\tilde{\boldsymbol{\lambda}}^{\mathrm{G}}) \in \mathbb{R}^{\mathcal{G} \times \mathcal{T}}$ for gas-fired generators, and fixed dispatch of virtual bidders $\mathbf{v}_r^{\mathrm{E}}$ obtained from (1), the electricity market operator clears the market in DA to minimize system costs as

$$\min_{\mathbf{p},\mathbf{u},\mathbf{s},\mathbf{w}} \quad e^\top \mathbf{p}^{\mathrm{C}} \; \mathbf{C}^{\mathrm{E}} + e^\top \mathbf{p}^{\mathrm{G}} \; C(\tilde{\boldsymbol{\lambda}}^{\mathrm{G}}) \; e + e^\top \mathbf{s} \; e \tag{2a}$$

$$\text{subject to} \quad h(\mathbf{p}, \mathbf{w}, \mathbf{v}_r^{\mathrm{E}}) = 0 : \boldsymbol{\lambda}^{\mathrm{DA,E}}, \tag{2b}$$

$$g(\mathbf{p}, \mathbf{w}, \mathbf{u}, \mathbf{s}) \leq 0, \tag{2c}$$

where (2) is a deterministic linear program, and variables $\mathbf{p}, \mathbf{u}, \mathbf{s} \in \mathbb{R}_+^{\mathcal{T} \times \mathcal{I}}$ are the dispatch, commitment status, and start-up cost of conventional generators in DA, respectively. Note that $\mathbf{p}^{\mathrm{C}} \in \mathbb{R}_+^{\mathcal{T} \times \mathcal{C}}$ and $\mathbf{p}^{\mathrm{G}} \in \mathbb{R}_+^{\mathcal{T} \times \mathcal{G}}$. The commitment status $\mathbf{u}$ is relaxed to lie within zero and one. These variables associated with gas-fired and non gas-fired generators are specified by superscripts G and C, respectively. Besides, $\mathbf{w} \in \mathbb{R}_+^{\mathcal{T} \times \mathcal{J}}$ refers to the DA dispatch of wind farms, which is limited by the available deterministic wind power forecast in DA.

Objective function (2a) minimizes the total system cost in DA stemming from the operational and start-up costs of conventional generators. Equality constraint (2b) enforces the balance between power production and consumption in DA with inelastic demand treating the virtual DA positions $\mathbf{v}_r^{\mathrm{E}}$ as given inputs. The dual variable associated with power balance (2b), i.e., $\boldsymbol{\lambda}^{\mathrm{DA,E}} \in \mathbb{R}^{\mathcal{T}}$, provides the DA electricity price. Recall that this vector of dual variables was treated as exogenous values in the problem of virtual bidders (1). Inequality constraints (2c) enforce lower and upper bounds on the DA dispatch of wind and conventional generation, impose ramping limits of conventional generators, represent the tight relaxation of unit commitment, and compute the start-up cost of each generator. The detailed representation of all equality and inequality constraints is given in the online appendix (Schwele et al., 2019b).

### *3.1.3 RT Electricity Market*

The actual wind power production is realized in RT, which might not be necessarily identical to the deterministic wind power forecast in DA. Therefore, the electricity market operator clears the RT market to make the necessary adjustments in order to keep the system balanced. The balancing actions are the power adjustment of generators and the two extreme actions, i.e., wind spillage and load shedding. The (relaxed) commitment status of fast-starting conventional generators $\mathcal{F} \subset \mathcal{I}$ and therefore their start-up cost can be updated in RT, while that is not the case for the slow-starting generators $\mathcal{S} \subset \mathcal{I}$. Note that $\mathcal{F} \cup \mathcal{S} = \mathcal{I}$. For given production costs of non gas-fired and gas-fired generators $\mathbf{C}^{\mathrm{E}} \in \mathbb{R}_{+}^{\mathcal{C}}$ and $C(\tilde{\boldsymbol{\lambda}}^{\mathrm{G}}) \in \mathbb{R}^{\mathcal{G}\times\mathcal{T}}$, load shedding cost $\mathbf{C}^{\mathrm{sh,E}} \in \mathbb{R}_{+}^{\mathcal{T}}$, fixed dispatch of explicit virtual bidders $\boldsymbol{\Delta}\mathbf{v}_r^{\mathrm{E}}$ achieved from (1) and fixed DA electricity market-clearing outcomes $\mathbf{p}$ and $\mathbf{u}$ obtained from (2), the RT electricity market clearing under scenario $\omega \in \Omega$ writes as

$$\left\{\begin{aligned} &\min_{\substack{\boldsymbol{\Delta}\mathbf{p}_\omega,\boldsymbol{\Delta}\mathbf{u}_\omega,\boldsymbol{\Delta}\mathbf{s}_\omega,\\ \boldsymbol{\Delta}\mathbf{w}_\omega,\boldsymbol{\Delta}\mathbf{d}_\omega^{\mathrm{E}}}} \quad e^\top \boldsymbol{\Delta}\mathbf{p}_\omega^{\mathrm{C}}\ \mathbf{C}^{\mathrm{E}} + e^\top \boldsymbol{\Delta}\mathbf{p}_\omega^{\mathrm{G}}\ C(\tilde{\boldsymbol{\lambda}}^{\mathrm{G}})\ e + e^\top \boldsymbol{\Delta}\mathbf{s}_\omega\ e + {\mathbf{C}^{\mathrm{sh,E}}}^\top\ \boldsymbol{\Delta}\mathbf{d}_\omega^{\mathrm{E}} && \text{(3a)}\\ &\text{subject to}\ \ h(\boldsymbol{\Delta}\mathbf{p}_\omega, \boldsymbol{\Delta}\mathbf{w}_\omega, \boldsymbol{\Delta}\mathbf{d}_\omega^{\mathrm{E}}, \boldsymbol{\Delta}\mathbf{v}_r^{\mathrm{E}}) = 0 : \boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,E}}, && \text{(3b)}\\ &\qquad g(\boldsymbol{\Delta}\mathbf{p}_\omega, \boldsymbol{\Delta}\mathbf{w}_\omega, \boldsymbol{\Delta}\mathbf{d}_\omega^{\mathrm{E}}, \boldsymbol{\Delta}\mathbf{u}_\omega, \boldsymbol{\Delta}\mathbf{s}_\omega, \mathbf{p}, \mathbf{u}) \leq 0, \Bigg\}\ \forall\omega, && \text{(3c)} \end{aligned}\right.$$

where (3), one per scenario, is a deterministic linear program. We denote $\boldsymbol{\Delta}\mathbf{p}_\omega \in \mathbb{R}^{\mathcal{T}\times\mathcal{I}}$ the power adjustment of conventional generators while by $\boldsymbol{\Delta}\mathbf{u}_\omega \in \mathbb{R}^{\mathcal{T}\times\mathcal{F}}$ and $\boldsymbol{\Delta}\mathbf{s}_\omega \in \mathbb{R}^{\mathcal{T}\times\mathcal{F}}$ we denote the adjusted relaxed commitment decision and the adjusted start-up cost of fast-starting units. Also, wind spillage and load shedding actions are denoted as $\boldsymbol{\Delta}\mathbf{w}_\omega \in \mathbb{R}_{+}^{\mathcal{T}\times\mathcal{J}}$ and $\boldsymbol{\Delta}\mathbf{d}_\omega^{\mathrm{E}} \in \mathbb{R}_{+}^{\mathcal{T}}$, respectively.

Objective function (3a) minimizes balancing costs for underlying scenario $\omega$. Equality constraint (3b) balances the wind power deviations in RT from the DA schedule with the position of virtual bidders $\boldsymbol{\Delta}\mathbf{v}_r^{\mathrm{E}}$ as fixed input. The dual variable vector $\boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,E}} \in \mathbb{R}^{\mathcal{T}}$ represents the RT electricity prices under scenario $\omega$. Recall that this vector was exogenous in the problem of virtual bidders (1). Inequality constraints (3c) enforce lower and upper bounds on the load shedding and power adjustment of wind farms, conventional slow- and fast-starting generators, restrict the ramp-rate limits of generators, enforce the adjusted unit commitment, and calculate the start-up cost for fast-starting units.

## 3.2 Temporal Coordination within DA and RT Natural Gas Markets

Similar to Section 3.1, we present here the optimization problems for explicit natural gas virtual bidder, and DA and RT natural gas markets, which define an equilibrium problem together.

#### 3.2.1 Explicit Natural Gas Virtual Bidder

Similarly to the electricity VB, the profit-maximization problem of each explicit natural gas virtual bidder $q \in \mathcal{Q}$ participating in the natural gas DA and RT markets is given as

$$\left\{ \max_{\mathbf{v}_q^{\mathrm{G}}, \boldsymbol{\Delta}\mathbf{v}_q^{\mathrm{G}}} \ \boldsymbol{\lambda}^{\mathrm{DA,G}\top} \ \mathbf{v}_q^{\mathrm{G}} + \left( \sum_{\omega} \pi_{\omega} \ \boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,G}} \right)^{\top} \boldsymbol{\Delta}\mathbf{v}_q^{\mathrm{G}} \right. \tag{4a}$$

$$\left. \text{subject to } \ \mathbf{v}_q^{\mathrm{G}} + \boldsymbol{\Delta}\mathbf{v}_q^{\mathrm{G}} = 0 \right\} \forall q. \tag{4b}$$

For given DA and RT natural gas market prices $\boldsymbol{\lambda}^{\mathrm{DA,G}} \in \mathbb{R}^{\mathcal{T}}$ and $\boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,G}} \in \mathbb{R}^{\mathcal{T}}$ $\forall \omega$, the virtual bidder solves stochastic linear program (4), deciding her positions in DA, i.e., $\mathbf{v}_q^{\mathrm{G}} \in \mathbb{R}^{\mathcal{T}}$ and in RT, i.e., $\boldsymbol{\Delta}\mathbf{v}_q^{\mathrm{G}} \in \mathbb{R}^{\mathcal{T}}$, to maximize her expected profit stemming from the price differences of the two settlements. Recall that we assume that the virtual bidder has a perfect foresight into future DA and distribution of RT prices over scenarios. Equality constraint (4b) zeros out the DA and RT trades of the explicit virtual bidder. As an important observation, this explicit virtual bidder enforces the DA and the expected natural gas prices to be equal. This observation can be derived by the KKT optimality conditions of (4) similar to explicit electricity VB.

#### 3.2.2 DA Natural Gas Market

For given scheduled gas consumption of gas-fired generators as a function of $\mathbf{p}^{\mathrm{G}}$ obtained from the DA electricity market (2) and the DA trade of virtual bidders $\mathbf{v}_q^{\mathrm{G}}$ determined in (4), the natural gas market operator clears the DA market with $\mathcal{K}$ gas suppliers as

$$\min_{\mathbf{g}} \ e^{\top} \mathbf{C}^{\mathrm{G}} \mathbf{g} \ e \tag{5a}$$

$$\text{subject to } \ h(\mathbf{g}, \mathbf{p}^{\mathrm{G}}, \mathbf{v}_q^{\mathrm{G}}) = 0 : \boldsymbol{\lambda}^{\mathrm{DA,G}} \tag{5b}$$

$$g(\mathbf{g}) \leq 0, \tag{5c}$$

where (5) is a deterministic linear program, and parameters $\mathbf{C}^{\mathrm{G}} \in \mathbb{R}_{+}^{\mathcal{K}}$ represent the supply cost of gas suppliers, and variables $\mathbf{g} \in \mathbb{R}_{+}^{\mathcal{T} \times \mathcal{K}}$ are the DA schedule of those suppliers. Objective function (5a) minimizes the total gas supply cost. Equality constraint (5b) represents the DA natural gas supply balance with inelastic demand including given gas demand for power production and virtual trade $\mathbf{v}_q^{\mathrm{G}}$. The “actual” natural gas prices are derived through dual variables $\boldsymbol{\lambda}^{\mathrm{DA,G}} \in \mathbb{R}^{\mathcal{T}}$, which are not necessarily identical to the estimated prices $\tilde{\boldsymbol{\lambda}}^{\mathrm{G}}$ used in the electricity market-clearing problems (2) and (3). Constraint (5c) enforces the lower and upper bounds on the gas supply.

#### *3.2.3 RT Natural Gas Market*

The natural gas operator clears the RT natural gas market to offset the change in fuel consumption of gas-fired generators $\mathbf{\Delta p}_{\omega}^{\mathrm{G}}$ occurred under scenario $\omega$. This deterministic linear problem writes as

$$\left\{\begin{aligned} &\min_{\mathbf{\Delta g}_{\omega},\mathbf{\Delta d}_{\omega}^{\mathrm{G}}} \; e^{\top}\mathbf{C}^{\mathrm{G}}\mathbf{\Delta g}_{\omega}\; e + \mathbf{C}^{\mathrm{sh,G}\top}\;\mathbf{\Delta d}_{\omega}^{\mathrm{G}} && \text{(6a)}\\ &\text{subject to } \; h(\mathbf{\Delta g}_{\omega}, \mathbf{\Delta p}_{\omega}^{\mathrm{G}}, \mathbf{\Delta d}_{\omega}^{\mathrm{G}}, \mathbf{\Delta v}_{q}^{\mathrm{G}}) = 0 : \boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,G}} && \text{(6b)}\\ &\qquad\qquad g(\mathbf{\Delta g}_{\omega}, \mathbf{\Delta d}_{\omega}^{\mathrm{G}}, \mathbf{g}) \leq 0 \Big\}\; \forall \omega, && \text{(6c)} \end{aligned}\right.$$

where objective function (6a) minimizes the balancing costs. The first balancing action is gas supply adjustment $\mathbf{\Delta g}_{\omega} \in \mathbb{R}^{\mathcal{T}\times\mathcal{K}}$ whose cost is $\mathbf{C}^{\mathrm{G}} \in \mathbb{R}_{+}^{\mathcal{K}\times\mathcal{T}}$. The second but extreme balancing action is the natural gas load shedding $\mathbf{\Delta d}_{\omega}^{\mathrm{G}} \in \mathbb{R}_{+}^{\mathcal{T}}$ at the comparatively high cost of $\mathbf{C}^{\mathrm{sh,G}} \in \mathbb{R}_{+}^{\mathcal{T}}$. Equality constraint (6b) balances the gas supply adjustments in RT. The actual natural gas RT prices under scenario $\omega$ are the vector of dual variables $\boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,G}} \in \mathbb{R}^{\mathcal{T}}$. Constraints (6c) enforce the lower and upper bounds on gas supply, gas adjustments and gas load shedding.

### 3.3 Analysis of Equilibrium Problems

To achieve temporal coordination in the models in Sections 3.1 and 3.2, the inclusion of explicit virtual bidders with perfect foresight into DA and distribution of RT prices over scenarios requires solving the DA and RT market-clearing optimization problems together. However, the virtual bidders do not link the electricity and natural gas markets – they will be linked later in Section 4 with implicit VB. So far, we can identify the following two equilibrium problems for each sector: (1) $\forall r$, (2), and (3) $\forall \omega$ represent the equilibrium of electricity markets and (4) $\forall q$, (5), and (6) $\forall \omega$ of natural gas markets.

Note that these two equilibrium problems should be solved sequentially, i.e., one should first solve (1)-(3), and then for given gas demands, (4)-(6) can be solved.

*Remark 1* Each linear optimization problem (2), (3), (5), and (6) can be equivalently reformulated as a pure Nash equilibrium problem of profit maximizing price-taking agents in a perfectly competitive market.

The Karush-Kuhn-Tucker (KKT) optimality conditions of each optimization problem (2), (3), (5), and (6) and pure Nash equilibrium problem are identical – See Appendix A for more details. As explained in Remark 1 and illustrated in Appendix A, each optimization problem (2), (3), (5), and (6) can be replaced by a set of optimization problems that constitute the corresponding Nash equilibrium problem. However, solving these problems simultaneously as the equilibrium problems (1)-(3) and (4)-(6) leads to coupled strategy sets and destroys integrability of the equilibrium (Facchinei and Pang, 2007).

*Remark 2* The two equilibrium problems (1) $\forall r$, (2), and (3) $\forall \omega$, and (4) $\forall q$, (5), and (6) $\forall \omega$ are Generalized Nash Equilibrium (GNE) problems.

The feasible set of players depends on the decision of the other players. For example, decisions of virtual bidders in (1), i.e., $\mathbf{v}_r^{\mathrm{E}}$ and $\boldsymbol{\Delta}\mathbf{v}_r^{\mathrm{E}}$, appear within the power balance constraints in (2) and (3). Replacing (2) and (3) with their equivalent Nash equilibrium problems will not change the GNE nature of the overall problem, as the DA power schedule of generators affects the feasible set of those generators in their RT problem. This is a challenging issue, because a GNE problem is formulated as a quasi-variational inequality (QVI), which is hard to solve and generally admits multiple (or even infinite) solutions that arise from coupling constraints (Facchinei and Kanzow, 2007). References (Facchinei and Kanzow, 2007; Harker and Pang, 1990; Harker, 1991; Schiro et al., 2013; Kulkarni and Shanbhag, 2012) explore a specific class of GNE problems with *shared constraints*. However, the coupling constraints in our proposed equilibrium problems, i.e., (1)-(3), and (4)-(6), are not shared constraints.

*Remark 3* Existence of a solution to the GNE problems can be mathematically proven using (Harker, 1991, Theorem 1) and (Harker, 1991, Theorem 2) when the feasible set of every agent in the GNE is non-empty, convex and compact. In our case, this condition will be fulfilled only if we assume bounds on market prices (i.e., by imposing price floors and caps) and bounds on virtual trades (e.g., by imposing a budget constraint for each virtual bidder). The investigation of solution uniqueness for these GNE problems is not straightforward (Harker, 1991).

## 4 Sectoral and Temporal Coordination

In the previous section, we described two equilibrium models (one for the electricity and another for the natural gas sector) with explicit VB for temporal coordination only. For improving the sectoral coordination between electricity and natural gas markets, this section extends the model in Section 3 and allows natural gas-fired generators to act as implicit virtual bidders. In other words, they are allowed to self-schedule outside the markets to optimally allocate their operational flexibility in the power market and their fuel consumption in the natural gas market. Each self-scheduler (i.e., implicit virtual bidder) maximizes her own expected profit. Similar to the explicit virtual bidders, we assume that each self-scheduler has a perfect foresight into DA and distribution of RT prices over scenarios in both electricity and natural gas markets. Note that having these self-schedulers in the model link the power and natural gas markets, so that a single equilibrium model is required to solve both electricity and natural gas markets.

We consider both slow- and fast-starting types of gas-fired generators as potential self-schedulers. The difference between these two types of generators is that the slow-starting gas-fired units fix their unit commitment status in DA and cannot change it in the RT, while the fast-start units can.

The expected-profit maximization problem of each self-scheduling slow-starting gas-fired unit $\mathcal{G} \cap \mathcal{S}$ participating in both electricity and natural gas markets is

$$
\begin{aligned}
\max_{\mathbf{p},\mathbf{u},\mathbf{s},\boldsymbol{\Delta}\mathbf{p}_\omega} \quad & \left(\boldsymbol{\lambda}^{\mathrm{DA,E}} - C(\boldsymbol{\lambda}^{\mathrm{DA,G}})\right)^\top \mathbf{p} - e^\top \mathbf{s} \\
& + \sum_\omega \pi_\omega \Big[ \left(\boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,E}} - C(\boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,G}})\right) \Big]^\top \boldsymbol{\Delta}\mathbf{p}_\omega && \text{(7a)} \\
\text{subject to} \quad & g(\mathbf{p},\mathbf{u},\mathbf{s}) \leq 0 : \boldsymbol{\mu}, && \text{(7b)} \\
& g(\boldsymbol{\Delta}\mathbf{p}_\omega, \mathbf{p}, \mathbf{u}) \leq 0 : \boldsymbol{\nu}_\omega, \ \ \forall \omega, && \text{(7c)}
\end{aligned}
$$

where (7) is a two-stage stochastic linear program, whose objective function (7a) maximizes the expected profit of self-scheduling gas-fired generators. Note that this objective function includes the actual DA and RT gas prices $\boldsymbol{\lambda}^{\mathrm{DA,G}}$ and $\boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,G}}$ from models (5) and (6), and not the estimated gas price $\tilde{\boldsymbol{\lambda}}^{\mathrm{G}}$. This problem is subject to the DA (7b) and RT operational constraints (7c), so that the final production of gas-fired units in RT have to lie within their feasible operational limits.

Similarly, each fast-start self-scheduling gas-fired unit $\mathcal{G} \cap \mathcal{F}$ solves a two-stage stochastic linear program to maximize her expected profit as

$$
\begin{aligned}
\max_{\substack{\mathbf{p},\mathbf{u},\mathbf{s},\boldsymbol{\Delta}\mathbf{p}_\omega, \\ \boldsymbol{\Delta}\mathbf{u}_\omega,\boldsymbol{\Delta}\mathbf{s}_\omega}} \quad & \left(\boldsymbol{\lambda}^{\mathrm{DA,E}} - C(\boldsymbol{\lambda}^{\mathrm{DA,G}})\right)^\top \mathbf{p} - e^\top \mathbf{s} \\
& + \sum_\omega \pi_\omega \Big[ \left(\boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,E}} - C(\boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,G}})\right)^\top \boldsymbol{\Delta}\mathbf{p}_\omega + e^\top \boldsymbol{\Delta}\mathbf{s}_\omega \Big] && \text{(8a)} \\
\text{subject to} \quad & g(\mathbf{p},\mathbf{u},\mathbf{s}) \leq 0 : \boldsymbol{\mu}, && \text{(8b)} \\
& g(\boldsymbol{\Delta}\mathbf{p}_\omega, \boldsymbol{\Delta}\mathbf{u}_\omega, \boldsymbol{\Delta}\mathbf{s}_\omega, \mathbf{p}, \mathbf{u}) \leq 0 : \boldsymbol{\nu}_\omega, \ \ \forall \omega. && \text{(8c)}
\end{aligned}
$$

The resulting GNE problem is (2), (3) $\forall\omega$, (5), (6) $\forall\omega$, (7) and (8). Note that in this equilibrium problem, the decisions of self-schedulers $\mathbf{p}$, and $\boldsymbol{\Delta}\mathbf{p}_\omega$ in (7) and (8) are exogenous values within the market-clearing problems (2), (3), (5) and (6).

*Remark 4* In a case including both implicit and explicit VB, if the dispatch of self-schedulers in DA is restricted by constraint (7b) or (8b), then the equilibrium problem will be feasible only and only if such DA constraints are inactive. Any non-zero dual variable corresponding to the DA constraints of self-schedulers will make the equilibrium problem infeasible.

**Proposition 1** *Self-scheduling in* (7) *and* (8) *respecting both DA and RT operational constraints and explicit VB according to* (1) *and* (4) *are mutually exclusive, i.e. cannot coincide or exist together.*

*Proof.* The KKT optimality conditions of the problem of each virtual bidder (1) and (4) enforce DA and expected RT prices to be equal, i.e., $\boldsymbol{\lambda}^{\mathrm{DA,E}} = \sum_{\omega} \pi_{\omega}\ \boldsymbol{\lambda}_{\omega}^{\mathrm{RT,E}}$ and $\boldsymbol{\lambda}^{\mathrm{DA,G}} = \sum_{\omega} \pi_{\omega}\ \boldsymbol{\lambda}_{\omega}^{\mathrm{RT,G}}$. The KKT optimality conditions of each self-scheduler if her DA dispatch is restricted enforce $C(\boldsymbol{\lambda}^{\mathrm{DA,G}}) - \boldsymbol{\lambda}^{\mathrm{DA,E}} + \boldsymbol{\mu} + \sum_{\omega} \boldsymbol{\nu}_{\omega} = 0$ and $C(\boldsymbol{\lambda}_{\omega}^{\mathrm{RT,G}}) - \boldsymbol{\lambda}_{\omega}^{\mathrm{RT,E}} + \boldsymbol{\nu}_{\omega} = 0, \forall \omega$. Then these KKTs enforce that $\boldsymbol{\lambda}^{\mathrm{DA,E}} + \boldsymbol{\mu} = \sum_{\omega} \boldsymbol{\lambda}_{\omega}^{\mathrm{RT,E}}$, so that if both explicit virtual bidders and self-schedulers are included at the same time, the problem is feasible only if $\boldsymbol{\mu} = 0$, which means that DA constraints of self-schedulers are inactive (see online appendix (Schwele et al., 2019b)). □

Including explicit and implicit VB requires solving (1)-(8) as a GNE problem neglecting the operational bounds of self-schedulers in DA (7b) and (8b). Thus, self-schedulers can submit physical and virtual bids as long as their positions in RT adhere to their feasible operational limits, thus acting as implicit virtual bidders and not as self-schedulers.

## 5 Ideal Benchmark

We compare the proposed "soft" market-based mechanisms for power and gas coordination with new market players including explicit and implicit virtual bidders to the ideal benchmark of a fully stochastic integrated energy market clearing. This ideal benchmark is indeed a disruptive solution to achieve a full temporal and sectoral coordination, which ignores the current market sequences. It is a single two-stage (DA and RT) stochastic linear optimization problem, including both power and natural gas systems. Assuming that the given set of scenarios represents well the probability distribution of uncertainty, the stochastic market clearing efficiently makes informed DA decisions by anticipating the potential recourse actions in RT (Pritchard et al., 2010). In this benchmark, the integrated power and gas system is co-optimized under complete exchange of operational information. This two-stage stochastic program aiming to minimize the total expected system cost writes as

$$\begin{aligned}
\min_{\substack{\mathbf{p},\mathbf{u},\mathbf{s},\mathbf{w},\mathbf{g},\boldsymbol{\Delta}\mathbf{p}_{\omega},\\ \boldsymbol{\Delta}\mathbf{u}_{\omega},\boldsymbol{\Delta}\mathbf{s}_{\omega},\boldsymbol{\Delta}\mathbf{w}_{\omega},\\ \boldsymbol{\Delta}\mathbf{d}_{\omega}^{\mathrm{E}},\boldsymbol{\Delta}\mathbf{g}_{\omega},\boldsymbol{\Delta}\mathbf{d}_{\omega}^{\mathrm{G}}}} \quad & e^{\top}\mathbf{p}^{\mathrm{C}}\mathbf{C}^{\mathrm{E}} + e^{\top}\mathbf{s}\ e + e^{\top}\mathbf{C}^{\mathrm{G}}\mathbf{g}\ e + \sum_{\omega} \pi_{\omega} \Big( e^{\top}\boldsymbol{\Delta}\mathbf{p}_{\omega}\mathbf{C}^{\mathrm{E}} + e^{\top}\boldsymbol{\Delta}\mathbf{s}_{\omega}\ e \\
& + \mathbf{C}^{\mathrm{sh,E}\top}\ \boldsymbol{\Delta}\mathbf{d}_{\omega}^{\mathrm{E}} + e^{\top}\mathbf{C}^{\mathrm{G}}\boldsymbol{\Delta}\mathbf{g}_{\omega}\ e + \mathbf{C}^{\mathrm{sh,G}\top}\ \boldsymbol{\Delta}\mathbf{d}_{\omega}^{\mathrm{G}} \Big) \Big] && \text{(9a)} \\
\text{subject to} \quad & (2b), (2c), (5b), (5c), && \text{(9b)} \\
& (3b), (3c), (6b), (6c), \quad \forall \omega. && \text{(9c)}
\end{aligned}$$

Objective function (9a) minimizes the total DA system cost for power production and gas supply as well as the expected RT balancing costs in both sectors, while respecting the operational constraints in DA (9b) and in RT (9c) for each scenario. The stochastic optimization problem (9) can be equivalently reformulated as a pure Nash equilibrium problem, in which each market player is a stochastic decision-maker, who maximizes her expected profit with respect to DA and RT operational constraints with perfect information regarding prices and uncertainty about both sectors.

*Remark 5* The GNE problem (1)-(8) defined in Section 4 including explicit and implicit VB is not necessarily equal to the ideal benchmark (9), since their KKTs are different.

Recall that the GNE problem enforces convergence of DA and expected RT prices in both power and gas sectors through the optimality conditions of explicit virtual bidders. On contrary, in the stochastic market clearing problem (9), the DA and RT prices converge in expectation *only if* all DA operational inequalities are non-binding, i.e., every market player acts as an unrestrained arbitrager between DA and RT markets. This can be easily explored by checking the KKT optimality conditions of (9).

The co-optimization of power and gas system correctly accounts for the impact of natural gas prices on the electricity merit order, i.e., placing the power plants with an ascending order of marginal costs. Allowing all gas-fired units to self-schedule in the sequential setup with perfect knowledge over both gas and electricity prices approximates system integration. This is further explored in the following proposition.

**Proposition 2** *If DA operational bounds on* $\mathbf{p}, \mathbf{u}, \mathbf{w}, \mathbf{g}$ *in the stochastic optimization problem* (9) *are non-binding, the DA and the RT prices converge in expectation (i.e.,* $\boldsymbol{\lambda}^{E,DA} = \sum_{\omega} \pi_{\omega} \boldsymbol{\lambda}_{\boldsymbol{\omega}}^{E,RT}$ *and* $\boldsymbol{\lambda}^{G,DA} = \sum_{\omega} \pi_{\omega} \boldsymbol{\lambda}_{\boldsymbol{\omega}}^{G,RT}$*) and the outcomes of* (9) *are equal to the GNE problem* (1)-(8) *when all gas-fired units are implicit virtual bidders.*

*Proof.* This is proven by demonstrating that the KKT optimality conditions of the two problems above under the conditions mentioned are identical – See online appendix (Schwele et al., 2019b) for more details. □

Table 1 summarizes all models introduced. Note that while sequential and ideal benchmark can be solved as linear programs (LP), all other models are recast as mixed complementarity problems (MCP) by collecting the KKT conditions of the respective optimization models.

## 6 Numerical Results

This section provides a case study to analyze and compare the proposed market setups presented in Sections 3, 4 and 5, which are summarized in Table 1. We solve all models using an Intel CoreTM i7-7820HQ with four processors clocking at 2.70 GHz and 16 GB of RAM in GAMS using PATH and CPLEX solver for MCP and LP models, respectively. The CPU time for LP models is below 1 second, while that time for different MCPs varies between 1 and 500 seconds. See online appendix (Schwele et al., 2019b) for further details.

| Market setup | Name | Model | Optimization | Equilibrium | Model type |
|---|---|---|---|---|---|
| Sequential | *Seq* | (2)<br>(5)<br>(3) $\forall\omega$<br>(6) $\forall\omega$ | ✓<br>✓<br>✓<br>✓ | $-^*$<br>$-^*$<br>$-^*$<br>$-^*$ | LP<br>LP<br>LPs<br>LPs |
| Sequential with explicit virtual bidding | *Seq+eVB* | (1) $\forall r$, (2), (3) $\forall\omega$<br>(4) $\forall q$, (5), (6) $\forall\omega$ | –<br>– | GNE<br>GNE | MCP<br>MCP |
| Sequential with self-scheduling | *Seq+SS* | (2), (3) $\forall\omega$,<br>(5), (6) $\forall\omega$, (7), (8) | – | GNE | MCP |
| Sequential with both explicit and implicit virtual bidding | *Seq+VB* | (1) $\forall r$, (2), (3) $\forall\omega$,<br>(4) $\forall q$, (5), (6) $\forall\omega$,<br>(7a), (7c) , (8a), (8c) | – | GNE | MCP |
| Ideal benchmark | *Ideal* | (9) | ✓ | $-^*$ | LP |

**Table 1** Summary of market setup models.
*There exists a pure Nash equilibrium (NE) which is equivalent to the optimization problem, see Appendix A.

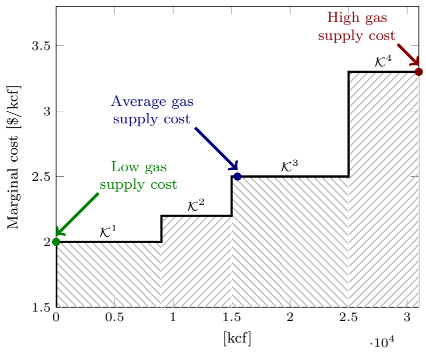


**Fig. 4** Natural gas supply function

## 6.1 Input Data

This case study contains a power system with 6 non gas-fired generators (namely, $\mathcal{C}^1$ to $\mathcal{C}^6$) and 4 gas-fired generators (namely, $\mathcal{G}^1$ to $\mathcal{G}^4$). These gas-fired generators connect the power system to a natural gas system with four gas suppliers, namely $\mathcal{K}^1$ to $\mathcal{K}^4$. We consider a 24-hour time horizon. All technical details of generators and gas suppliers are provided in Appendix B. The natural gas supply curve is shown in Fig. 4, which is the same throughout all 24 hours. Fig. 5 illustrates the shifting of the electricity merit order curve due to a potential change in the natural gas price. The reason for this shift is that the gas price affects the marginal production cost of the gas-fired generators. Since in both DA and RT stages, the electricity market is cleared before the natural gas market, the electricity market operator needs an estimation of the gas price. In the following, we assume that the electricity market operator uses the average gas supply cost, i.e., $2.5/kcf, as a deterministic and static estimation of the natural gas prices in both DA and RT. The value of lost load in the electricity and natural gas sectors are set to $600/MWh and $100/kcf, respectively.

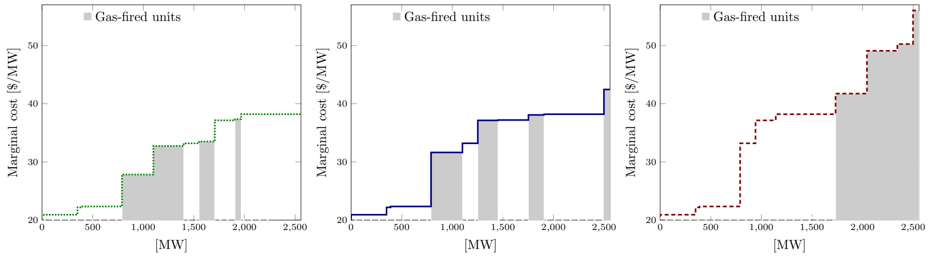


**Fig. 5** Electricity merit order depending on natural gas price. The plots on the left-hand, middle, and right-hand sides show the merit order corresponding to the low, average and high prices for natural gas (as illustrated in Fig. 4), respectively.

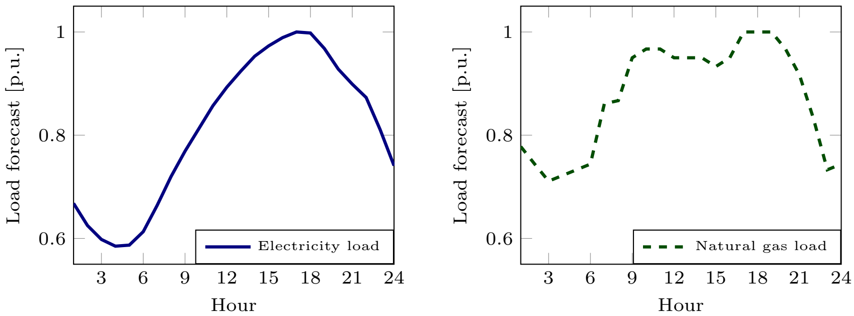


**Fig. 6** Electricity and natural gas demand. The plots on the left- and right-hand sides show the total hourly demand for power and natural gas, respectively.

The total hourly demand in both power and natural gas sectors is shown in Fig. 6. Note that the demand in both sectors is certain, and the only source of uncertainty is assumed to be the wind power. The profile of deterministic wind power forecast (in per-unit) in DA is illustrated by a solid curve in the upper plot of Fig. 7, while the lower plot provides the five equiprobable wind scenarios that may realize in RT. Due to potential forecast error in DA, observe that the DA deterministic forecast (solid curve in the upper plot) is not necessarily identical to the expected wind power realization in RT (dashed curve in the same plot). In this case, the DA wind forecast underestimates the available wind power production during hours 1 to 6 and 19 to 23, while overestimates it from hour 7 to 18. The wind power penetration, i.e., total wind power capacity installed divided by the total electricity demand, is 34%. The next subsections provide the market outcomes obtained from different setups.

## 6.2 Main Results: Total Expected System Cost

The total expected cost of electricity and natural gas systems achieved under different market setups is shown in Fig. 8. As expected, the highest system cost corresponds to the sequential setup *Seq* (first bar in Fig. 8), which is a fully uncoordinated model. On the other hand, the fully coordinated ideal

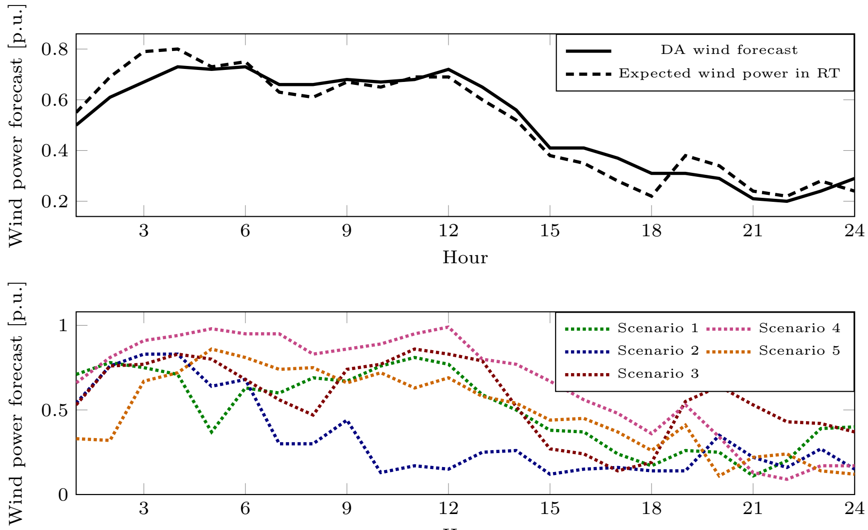


**Fig. 7** Wind power forecast in DA and potential scenarios in RT: The upper plot shows the deterministic wind power forecast in DA and the expected value of five wind power scenarios in RT. These five equiprobable scenarios in RT are depicted in the lower plot.

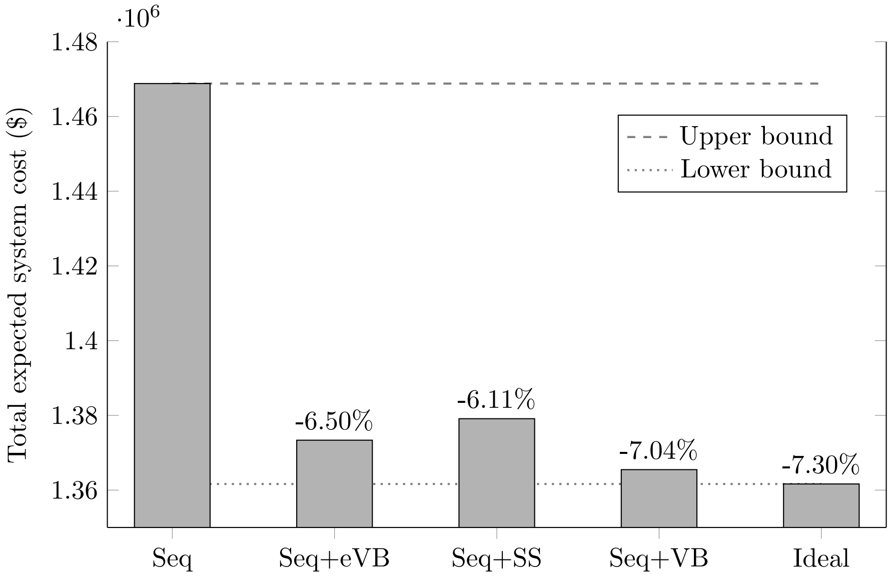


**Fig. 8** Total expected cost of the electricity and natural gas systems calculated by (9a) under different market setups. The percentages show the reduction in the total expected system cost compared to that cost in the fully uncoordinated sequential setup (first bar).

model (i.e., last bar in Fig. 8) yields the lowest cost. In this case study, the full temporal and sectoral coordination results in a 7.30% cost reduction. The

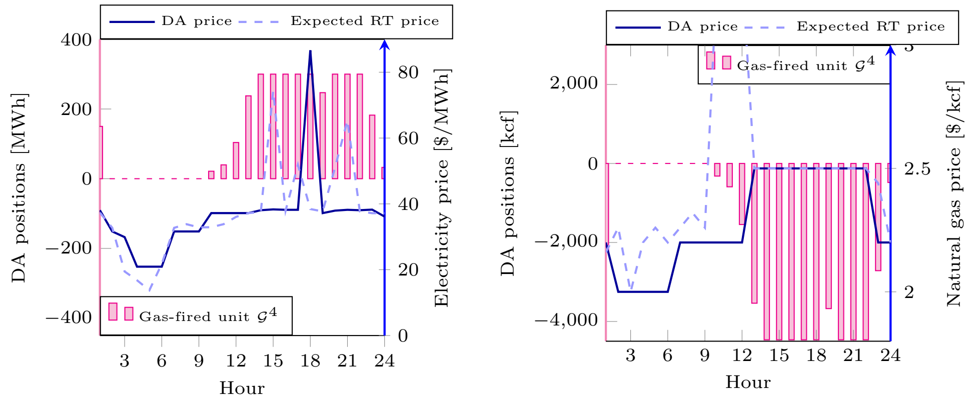


**Fig. 9** Hourly DA schedule of slow-start gas-fired generator $\mathcal{G}^4$ as well as DA and expected RT market-clearing prices obtained from fully uncoordinated sequential market setup *Seq*. The left- and right-hand side plots correspond to the electricity and natural gas market outcomes, respectively.

three proposed setups *Seq+eVB*, *Seq+SS* and *Seq+VB* provide partial coordination, and therefore, the system cost achieved in those setups is between the upper and lower bounds. Among these three market setups, *Seq+VB* with both implicit and explicit VB yields the highest cost saving, which is 7.04% (fourth bar in Fig. 8). In the following three subsections, we discuss in details how each market setup impacts the DA schedules. For clarity, we focus on DA dispatch of one of the slow-start gas-fired generators, i.e., $\mathcal{G}^4$, and analyze how each market setup affects her dispatch, and therefore her individual expected profit.

### 6.3 Upper Bound: Sequential Market Setup (*Seq*)

The corresponding market-clearing outcomes of the fully uncoordinated sequential market setup *Seq* are given in Fig. 9. The DA schedules in this setup have no foresight into uncertainty in RT and sectoral interactions. Thus, the DA and expected RT prices can largely differ – see, for example, the electricity market prices during hours 14 to 21 in the left-hand side plot and the natural gas market prices during hours 4 to 12 in the right-hand side plot of Fig. 9. The slow-start gas-fired generator $\mathcal{G}^4$ is dispatched in the DA electricity market myopically, without considering the volatility of the actual hourly natural gas price and the need for flexibility provided by $\mathcal{G}^4$ in RT. This generator is scheduled in hours 10 to 13 relying on the comparatively low estimated gas price, while her real production cost is higher due to comparatively high natural gas market prices. When power system flexibility is required, which is evident from the high expected RT electricity prices in hours 14 and 20, generator $\mathcal{G}^4$ is unable to provide upward adjustment since she is already dispatched at full capacity in DA. Apart from the high expected system cost, this

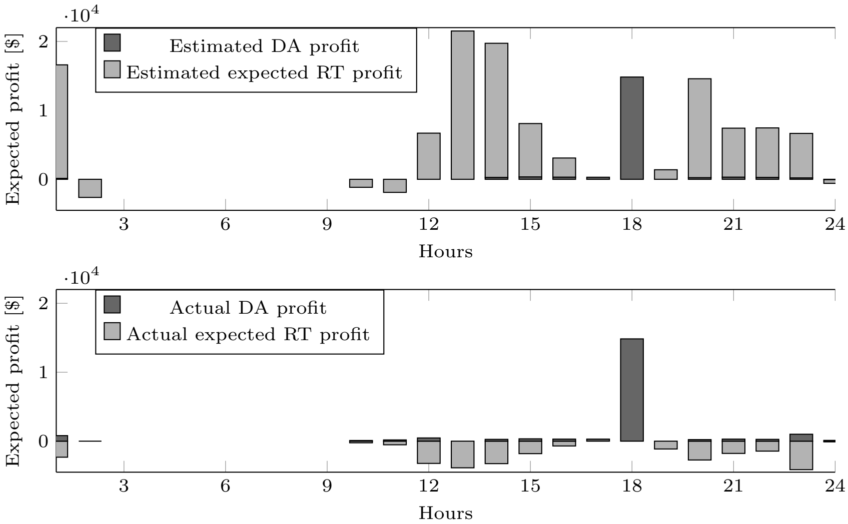


**Fig. 10** Hourly profit in DA and in expectation in RT of slow-start gas-fired generator $\mathcal{G}^4$ obtained from fully uncoordinated sequential market setup *Seq*. The upper plot shows the estimated profits using natural gas price estimations while the actual profits for realized natural gas prices are depicted in the lower plot.

| | *Seq* | *Seq+eVB* | *Seq+SS* (self-scheduling by $\mathcal{G}^4$) | *Seq+VB* (implicit VB by $\mathcal{G}^4$) | *Ideal* |
|---|---|---|---|---|---|
| $\mathcal{C}^1$ | 14,078 | 13,693 | 13,641 | 13,350 | 12,350 |
| $\mathcal{C}^2$ | 18,713 | 18,180 | 18,557 | 17,881 | 16,600 |
| $\mathcal{C}^3$ | 26,029 | 8,673 | 11,901 | 10,924 | 8,673 |
| $\mathcal{C}^4$ | 12,422 | 13,892 | 13,037 | 14,327 | 13,638 |
| $\mathcal{C}^5$ | 134,062 | 126,703 | 129,883 | 123,140 | 115,148 |
| $\mathcal{C}^6$ | 124,068 | 119,296 | 118,614 | 116,865 | 110,248 |
| $\mathcal{G}^1$ | -2,003 | -38,601 | 14,882 | 9,327 | 8,745 |
| $\mathcal{G}^2$ | 1,317 | 0 | 130 | 0 | 0 |
| $\mathcal{G}^3$ | 48,826 | 22,945 | 59,707 | 42,625 | 46,876 |
| $\mathcal{G}^4$ | -7,801 | -70,093 | 33,186 | 10,911 | 18,643 |

**Table 2** Expected profit of each generator under different market setups

inefficient DA dispatch results in a negative expected profit (-$7,801) for $\mathcal{G}^4$, as given in Table 2. The faulty estimation of natural gas prices by the electricity markets leads to underrating power generation costs and overestimating the profits of $\mathcal{G}^4$ in RT, such that $\mathcal{G}^4$ actually operates at negative profits in RT, see Fig. 10. This illustrates the need for market coordination, and specifically the potential for this generator to be scheduled in DA more efficiently.

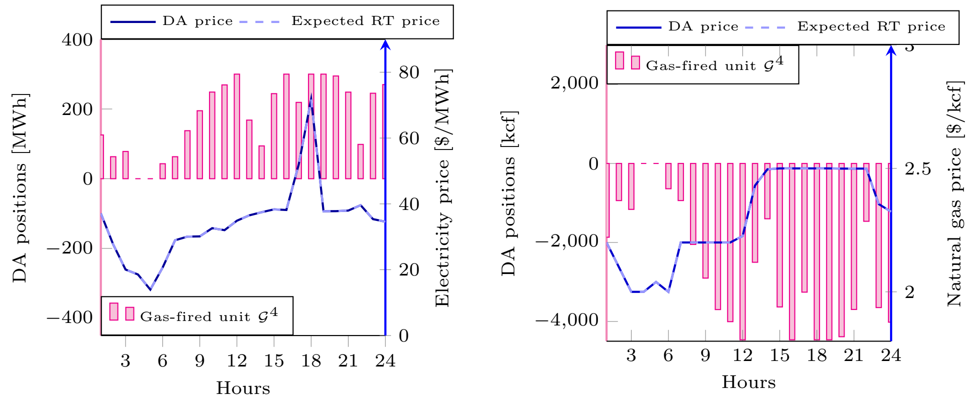


**Fig. 11** Hourly DA schedule of slow-start gas-fired generator $\mathcal{G}^4$ as well as DA and expected RT market-clearing prices obtained from fully coordinated market setup *Ideal*. The left- and right-hand side plots correspond to the electricity and natural gas market outcomes, respectively.

## 6.4 Lower Bound: Ideal Benchmark (*Ideal*)

In this ideal stochastic co-optimization model, the DA decisions are made while perfectly foreseeing uncertainty in RT as well as the sectoral interdependencies. As given in Fig. 11, the DA and expected RT prices are converged in both power and natural gas sectors. The fully efficient DA dispatch in this ideal market setup ends up to a non-negative expected profit for all generators (see Table 2), including $\mathcal{G}^4$ whose expected profit is \$18,643.

## 6.5 Temporal Coordination: *Seq+eVB*

Recall that the market setup *Seq+eVB* provides the DA-RT temporal (but not sectoral) coordination by allowing explicit VB in both electricity and natural gas markets. Note that it is sufficient to consider a single explicit virtual bidder only in each sector since the transmission network is not considered. The hourly amount of DA virtual bids in both sectors is shown in Fig. 12. The virtual bidders act as either buyers or sellers over the 24 hours in the DA market. For example, the virtual bidder in DA electricity market acts as a seller in hours 2-5, 10, 11 and 20, while as a buyer in the rest of hours (the left-hand plot of Fig. 12). The DA positions of this player are going to be zeroed out by her RT actions: every MW the virtual bidder sells in DA in hours 2-5, 10, 11 and 20 will be bought back in the same hours in RT. The right-hand plot of Fig. 12 shows that in the DA natural gas market, the virtual bidder acts as a supplier in most of hours. She behaves as a natural gas consumer in hours 5, 10, 11 and 24 only. Note that allowing explicit VB achieves full convergence of DA and expected RT prices in both power and gas markets. Explicit VB also impacts the DA dispatch of generators. For example, the

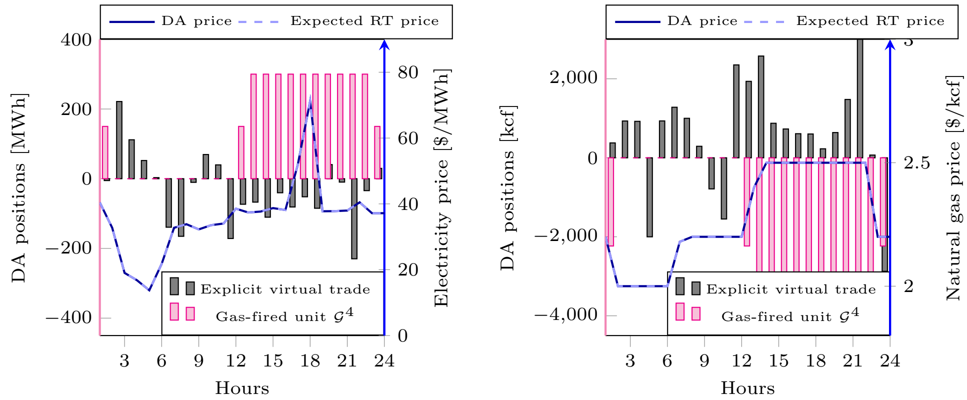


**Fig. 12** Hourly DA schedule of explicit virtual bidder (i.e., the purely financial player) and slow-start gas-fired generator $\mathcal{G}^4$ as well as DA and expected RT market-clearing prices obtained from market setup *Seq+eVB*. The left- and right-hand side plots correspond to the electricity and natural gas market outcomes, respectively.

slow-start gas-fired generator $\mathcal{G}^4$ is no longer dispatched between hours 2 and 11, while she is fully dispatched in hours 13 to 22. Explicit VB alone decreases the total expected system cost, but to the disadvantage of several individual generators. For example, the expected profit of $\mathcal{G}^4$ is -$70,093, which is even worse than her expected profit in the fully uncoordinated sequential model (-$7,801). Fig. 13 shows the discrepancy between $\mathcal{G}^4$'s estimated (upper plot) and actual (lower plot) profit. The misestimation of natural gas prices leads to negative profits in RT. In hour 12 gas load shedding occurs in one scenario and the actual expected natural gas price in RT is higher than the estimated price, resulting in expected RT loss for $\mathcal{G}^4$.

## 6.6 Temporal and Sectoral Coordination: *Seq+SS* and *Seq+VB*

The efficient dispatch of market players operating on the interface of electricity and natural gas sectors can enhance the sectoral coordination. A foresighted schedule of gas-fired generators in the DA electricity market may improve not only the temporal coordination with the RT electricity market, but also the sectoral coordination with the DA natural gas market. We analyze below the two market setups *Seq+SS* and *Seq+VB* separately.

### *6.6.1 Self-scheduling Gas-fired Generators: Seq+SS*

As realized in the previous subsections, the DA dispatch of gas-fired generator $\mathcal{G}^4$ in both setups *Seq* and *Seq+eVB* is inefficient, such that she ends up to a negative expected profit. This shows the significant potential for this generator to do self-schedule, rather than participating in the markets relied upon a deterministic sequential clearing procedure. Fig. 14 shows the DA dispatch

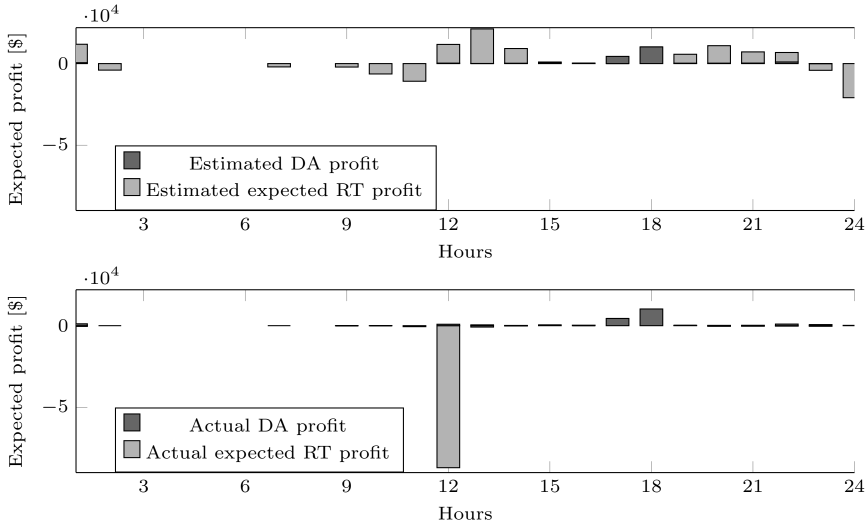


**Fig. 13** Hourly profit in DA and in expectation in RT of slow-start gas-fired generator $\mathcal{G}^4$ obtained from temporally coordinated sequential market setup *Seq+eVB* including explicit VB. The upper plot shows the estimated profits using natural gas price estimations while the actual profits for realized natural gas prices are depicted in the lower plot.

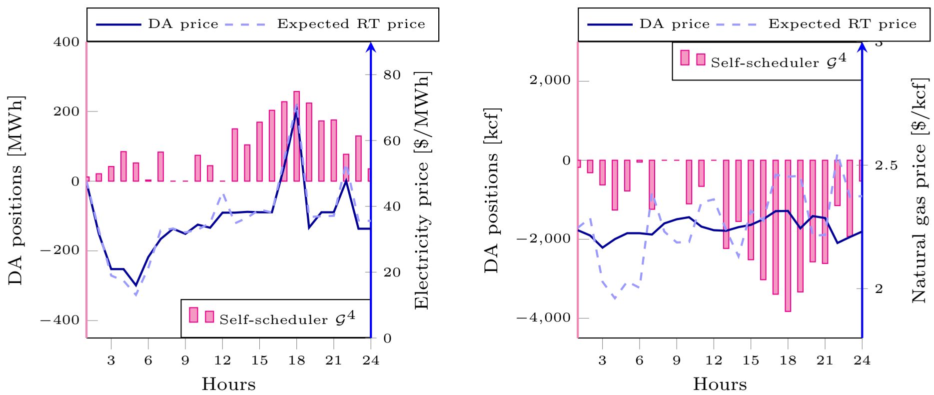


**Fig. 14** Hourly DA schedule of slow-start gas-fired generator $\mathcal{G}^4$ as well as DA and expected RT market-clearing prices obtained from market setup *Seq+SS*. Generator $\mathcal{G}^4$ does self-scheduling. The left- and right-hand side plots correspond to the electricity and natural gas market outcomes, respectively.

and market outcomes when generator $\mathcal{G}^4$ does self-scheduling. Note that in this setup, the self-scheduling generator has to still respect her operational constraints in both DA and RT stages, i.e., she does not behave as a temporal arbitrager between DA and RT markets. This restriction will be relaxed later in setup *Seq+VB*. According to Fig. 14, generator $\mathcal{G}^4$ increases her production during hours 1 to 13 when the actual natural gas price is comparatively low, whereas she reduces her power production and consequently natural gas con-

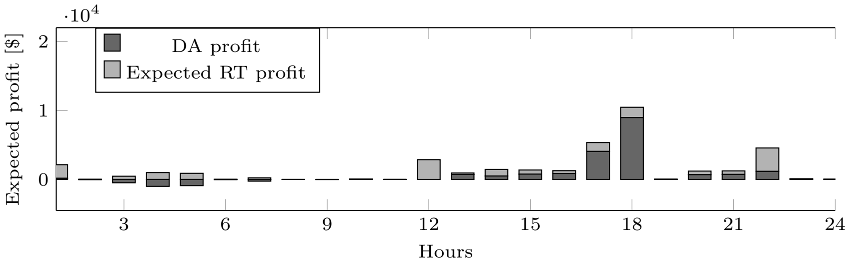


**Fig. 15** Hourly profit in DA and in expectation in RT of slow-start gas-fired generator $\mathcal{G}^4$ self-scheduling in the sequential market setup *Seq+SS*.

sumption when the gas price is comparatively high in hours 14 to 24. Allowing this gas-fired generator to self-schedule alone increases not only her expected profit to \$33,186 (see Fig. 15), but also improves the total social welfare in terms of non-negative expected profit for other generators and 6.11% cost reduction of total expected system cost (third bar in Fig. 8). Another important observation is that the self-scheduling by $\mathcal{G}^4$ causes shrinking the price spread between DA and expected RT prices in both power and gas sectors.

### *6.6.2 Explicit and Implicit Virtual Bidding: Seq+VB*

This setup allows explicit VB by purely financial players and implicit VB by gas-fired generator $\mathcal{G}^4$. Fig. 16 shows that the explicit and implicit VBs together achieve full price convergence in expectation in both power and natural gas markets. When generator $\mathcal{G}^4$ is allowed to submit virtual bids in electricity and natural gas markets, the amount of explicit virtual trade decreases in the power market and drastically in the natural gas market compared to Fig. 12. Note that $\mathcal{G}^4$ extends her bidding behaviour in the DA electricity and natural gas markets beyond her operational constraints acting as an implicit virtual bidder. This generator submits virtual bids to act as an electricity consumer and natural gas producer in the DA markets, e.g., in hours 7 and 8. She bids in DA below her operational capacity in hours 7 and 8 and above her capacity in hours 14-16, 18, 20 and 21. The convergence of DA and expected RT prices indicates the full temporal coordination. Besides, the additional system cost reduction compared to the case with explicit VB only (see second and fourth bars in Fig. 8) suggests improved sectoral coordination. All generators can expect a non-negative expected profit in this market setup with both implicit and explicit VB. The implicit virtual bidder $\mathcal{G}^4$ expects to earn \$10,911. Although this generator can extend her bidding activity beyond her operational constraints in DA, her expected profit is lower than that in a case when $\mathcal{G}^4$ is the only self-scheduler in the market setup without explicit VB (*Seq+SS*). However, when explicit VB is allowed (*Seq+SS* and *Seq+VB*), generator $\mathcal{G}^4$ is better off submitting virtual bids, see Table 2.

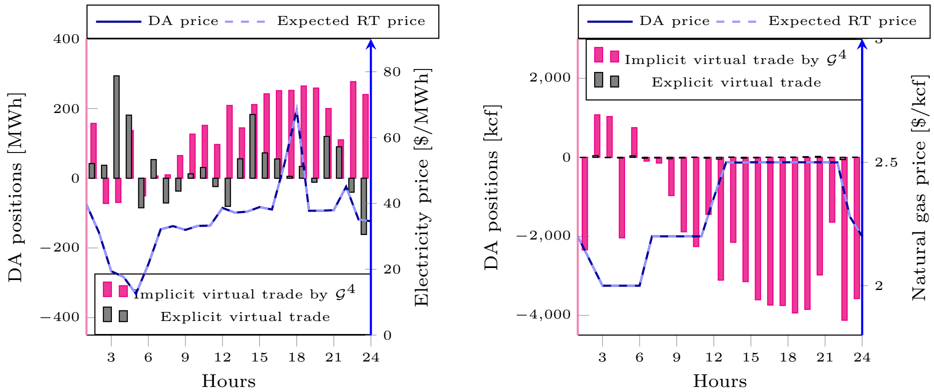


**Fig. 16** Hourly DA schedule of explicit (i.e., purely financial player) and implicit virtual bidder (i.e., generator $\mathcal{G}^4$) as well as DA and expected RT market-clearing prices obtained from market setup *Seq+VB*. The left- and right-hand side plots correspond to the electricity and natural gas market outcomes, respectively.

### 6.7 Main Observations

Based on the above results, allowing market players to arbitrage seems to enhance the coordination of sectors and trading floors. The explicit VB helps to better reflect the uncertainties inherent in the RT stages of the systems through better price signals. These price signals improve DA schedules so that the existing flexibility is allocated and utilized more efficiently. The VB improves the temporal coordination of the sequential DA and RT markets in the electricity and natural gas sectors. The self-scheduling gas-fired generator strengthens the temporal coordination of DA and RT markets by decreasing the price spread and the sectoral coordination by making use of her superior information of natural gas prices. In the same manner, the implicit VB by gas-fired generators helps sectoral coordination between the electricity and natural gas markets and improves the temporal coordination between DA and RT markets. The gas-fired generator is able to arbitrage not only over the trading floors but also between the sectors by submitting virtual bids in both electricity and natural gas markets. This updates the information exchange between the sectors and fosters coordination. Better price signals and improved DA schedules help allocate and utilize the existing flexibility more efficiently. The DA schedules are improved through bidding activities to better reflect the uncertainties inherent in the systems and account for the interactions of power and gas sectors.

## 7 Conclusion

This work explores the capability of VB either by purely financial players (explicit VB) or by physical players like gas-fired generators (implicit VB) in improving the temporal and sectoral coordination in two-stage (DA and

RT) electricity and natural gas markets under uncertainty. We use two models as benchmarks: a fully uncoordinated sequential model which achieves an upper bound for the total expected system cost, and a stochastic ideal co-optimization which provides a full temporal and sectoral coordination and yields a lower bound for the total expected system cost. The resulting models including VB are equilibrium problems, including deterministic market-clearing problems in DA and RT in both power and gas sectors, and two-stage stochastic optimization problems of virtual bidders, who maximize their expected profit.

Our results reveal that competitive virtual bidders who have prefect insight into the probability distribution of RT prices in power and natural gas markets increase the efficiency of deterministic sequential markets, such that the resulting total expected system cost is between the lower and upper bounds – it is even very close to the lower bound in our case study. In particular, the explicit VB provides DA-RT temporal coordination in power and natural gas markets. In addition, self-scheduling and implicit VB by gas-fired generators bring both temporal and sectoral coordination. This implies that the sequential market with VB may approximate the stochastic ideal integrated energy system, and help reveal and exploit the existing flexibility in the systems more efficiently.

The main policy implication is that a disruptive market re-design to a stochastic and integrated energy market might not be necessarily crucial for unlocking the existing flexibility. Instead, this can be done to some extent by allowing VB, while preserving the current sequential market-clearing setup.

As potential future works, it is of high interest to relax the assumption that self-schedulers as well as explicit and implicit virtual bidders have perfect knowledge of the probability distribution of real-time prices. This requires modeling the potential information asymmetry in the equilibrium model (Lo Prete et al., 2019). It is also important to analyze the cases where virtual bidders behave as strategic and/or risk-averse players. The proposed equilibrium may become computationally hard to solve if more players and scenarios are considered, and thus more efficient solution techniques might be required. One potential solution can be distributed optimization by solving the problem as an iterative Walrasian auction (e.g., similar to the methods used in (Mays et al., 2019; Höschle et al., 2018)), but the GNE nature of the model may bring some computational challenges. The other potential extension is to include network, especially in the natural gas sector as it allows modeling linepack (stored gas in the pipelines), however it will need either approximation (Correa-Posada and Sánchez-Martín, 2015; Ordoudis et al., 2019) or relaxation (Schwele et al., 2019a) methods to convexify the linepack model.

# A Appendix

The linear optimization problem (2) can be equivalently reformulated as a pure Nash equilibrium problem of profit maximizing agents, namely (10). For given market prices $\lambda^{\mathrm{DA,E}}$, each non gas-fired generator $\mathcal{C}$ maximizes her DA profit with respect to her operational constraints as

$$\max_{\mathbf{p}^{\mathrm{C}},\mathbf{u}^{\mathrm{C}},\mathbf{s}^{\mathrm{C}}} \quad \left(\boldsymbol{\lambda}^{\mathrm{DA,E}} - \boldsymbol{C}^{\mathrm{E}}\right)^{\top} \mathbf{p}^{\mathrm{C}} - e^{\top}\mathbf{s}^{\mathrm{C}} \tag{10a}$$

$$\text{subject to} \quad g(\mathbf{p}^{\mathrm{C}}, \mathbf{u}^{\mathrm{C}}, \mathbf{s}^{\mathrm{C}}) \leq 0. \tag{10b}$$

Similarly, each gas-fired generator $\mathcal{G}$ maximizes her DA profit as:

$$\max_{\mathbf{p}^{\mathrm{G}},\mathbf{u}^{\mathrm{G}},\mathbf{s}^{\mathrm{G}}} \quad \left(\boldsymbol{\lambda}^{\mathrm{DA,E}} - C(\tilde{\boldsymbol{\lambda}}^{\mathrm{G}})\right)^{\top} \mathbf{p}^{\mathrm{G}} - e^{\top}\mathbf{s}^{\mathrm{G}} \tag{10c}$$

$$\text{subject to} \quad g(\mathbf{p}^{\mathrm{G}}, \mathbf{u}^{\mathrm{G}}, \mathbf{s}^{\mathrm{G}}) \leq 0. \tag{10d}$$

Likewise, each wind farm $\mathcal{J}$ maximizes her DA profit limited by her deterministic wind forecast in DA as

$$\max_{\mathbf{w}} \ \boldsymbol{\lambda}^{\mathrm{DA,E}^{\top}} \mathbf{w} \tag{10e}$$

$$\text{subject to} \quad g(\mathbf{w}) \leq 0, \tag{10f}$$

and eventually, for given production decisions of conventional and wind generators and dispatch of virtual bidders, a price-setting agent determines the DA electricity price $\boldsymbol{\lambda}^{\mathrm{DA,E}}$ as

$$\min_{\boldsymbol{\lambda}^{\mathrm{DA,E}}} \ \boldsymbol{\lambda}^{\mathrm{DA,E}^{\top}} \ h(\mathbf{p}, \mathbf{w}, \mathbf{v}_r^{\mathrm{E}}). \tag{10g}$$

The Karush-Kuhn-Tucker (KKT) optimality conditions of optimization problem (2) and pure Nash equilibrium problem (10) are identical – See online appendix (Schwele et al., 2019b) for more details.

In the same manner, the RT market-clearing optimization problem (3) under scenario $\omega$ can be equivalently reformulated as a pure Nash equilibrium problem. Note that in such an equilibrium problem, each agent's DA schedule is fixed. For example, the slow-starting non gas-fired generators $\mathcal{C} \cap \mathcal{S}$ maximize their profit in RT with respect to their DA commitment decisions as

$$\left\{\begin{aligned} &\max_{\boldsymbol{\Delta}\mathbf{p}_{\omega}^{\mathrm{C}}} \quad \left(\boldsymbol{\lambda}_{\boldsymbol{\omega}}^{\mathrm{RT,E}} - \mathbf{C}^{\mathrm{E}}\right)^{\top} \boldsymbol{\Delta}\mathbf{p}_{\omega}^{\mathrm{C}} && \text{(11a)}\\ &\text{subject to} \quad g(\boldsymbol{\Delta}\mathbf{p}_{\omega}^{\mathrm{C}}, \mathbf{p}^{\mathrm{C}}, \mathbf{u}^{\mathrm{C}}) \leq 0, && \text{(11b)} \end{aligned}\right\} \forall \omega.$$

Optimization problems (5) and (6) can also be equivalently reformulated as pure Nash equilibrium problems, in which every agent maximizes her own profit and a price-setter agent determines the price, similar to Proposition 1 – see online appendix (Schwele et al., 2019b) for more details.

# B Appendix

Table 3 gives the technical characteristics of power generators, whose columns one to ten show the unit name, minimum power production ($P_i^{\min}$), capacity ($P_i^{\max}$), ramp rate ($R_i$), start-up cost ($C_i^{\mathrm{SU}}$), initial commitment status at the beginning of time horizon ($U^{\mathrm{ini}}$), initial dispatch ($P^{\mathrm{ini}}$), type, production cost for non gas-fired generators ($C_i^{\mathrm{E}}$), and gas-to-power conversion ratio for gas-fired generators ($\phi_i$), respectively. In addition, Table 4 provides the technical characteristics of four gas suppliers, including minimum and maximum gas capacity ($G_k^{\min}$ and $G_k^{\max}$), ramp rate ($G_k^{\mathrm{adj}}$), and supply cost ($C_k^{\mathrm{G}}$).

| Unit | $P_i^{\min}$ [MW] | $P_i^{\max}$ [MW] | $R_i$ [MW/h] | $C_i^{\mathrm{SU}}$ [$] | $U^{\mathrm{ini}}$ [0/1] | $P_i^{\mathrm{ini}}$ [MWh] | Type | $C_i^{\mathrm{E}}$ [$/MWh] | $\phi_i$ [kcf/MWh] |
|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{C}^1$ | 0 | 40 | 20 | 17,462 | 1 | 40 | non gas-fired | 22.18 | - |
| $\mathcal{C}^2$ | 0 | 152 | 50 | 13,207 | 1 | 100 | non gas-fired | 33.2 | - |
| $\mathcal{C}^3$ | 0 | 300 | 195 | 22,313 | 0 | 0 | non gas-fired | 37.14 | - |
| $\mathcal{C}^4$ | 100 | 591 | 230 | 28,272 | 0 | 0 | non gas-fired | 38.2 | - |
| $\mathcal{C}^5$ | 400 | 400 | 400 | 50,000 | 1 | 400 | non gas-fired | 22.34 | - |
| $\mathcal{C}^6$ | 0 | 350 | 80 | 33,921 | 0 | 0 | non gas-fired | 20.92 | - |
| $\mathcal{G}^1$ | 0 | 155 | 100 | 21,450 | 1 | 100 | gas-fired | - | 15.23 |
| $\mathcal{G}^2$ | 0 | 60 | 60 | 10,721 | 0 | 0 | gas-fired | - | 16.98 |
| $\mathcal{G}^3$ | 0 | 310 | 200 | 42,900 | 0 | 0 | gas-fired | - | 12.65 |
| $\mathcal{G}^4$ | 0 | 300 | 150 | 10,000 | 0 | 0 | gas-fired | - | 14.88 |

**Table 3** Technical characteristics of power generators

| Supplier | $G_k^{\min}$ [kcf] | $G_k^{\max}$ [kcf] | $G_k^{\mathrm{adj}}$ [kcf/h] | $C_k^{\mathrm{G}}$ [$/kcf] |
|---|---|---|---|---|
| $\mathcal{K}^1$ | 0 | 9,000 | 2,000 | 2 |
| $\mathcal{K}^2$ | 0 | 6,000 | 1,000 | 2.2 |
| $\mathcal{K}^3$ | 0 | 15,000 | 3,000 | 2.5 |
| $\mathcal{K}^4$ | 0 | 15,000 | 1,000 | 3.3 |

**Table 4** Technical characteristics of gas suppliers

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