Accommodating Bounded Rationality in Pricing Demand Response

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Abstract—Demand response is considered an effective mechanism to deal with the intermittency and stochasticity of renewable sources in power systems. These mechanisms assume that participants behave as rational decision-makers. However, behavioural economists commonly agree that decision-makers are rather bounded rational and deviate from the optimal response. The purpose of this paper is to generalize the existing mechanisms to account for bounded rationality. This is done in the framework of the coupon incentive-based demand response program, by formulating a Stackelberg game between the load serving entity and the consumers. The game is recast as a bilevel optimization problem that can account or not for bounded rationality to study its effects on social welfare and revenue streams. Our analysis of a case study shows that bounded rational consumers lead to undesirable loss of revenue for the load serving entity that can be reduced by accounting for bounded rationality in pricing demand response.

Index Terms—Bounded rationality, demand response, game theory, bilevel programming

I. INTRODUCTION

Demand response (DR) has proven to be a powerful mechanism to support the operation of power systems with high penetration of renewable sources. DR programs unlock the existing flexibility on the demand side through different forms of financial incentives. They improve the competitiveness of electricity markets and the security of power systems [1], [2].

Traditionally, the design of DR programs has relied on micro economic theories of utility maximization and consumer rationality. These theories have been used to estimate the financial incentives that can mobilize the desired demand response [3], i.e., rational consumers are expected to optimally respond to price signals allowing to fully exploit the demand-side flexibility of the system. However, recent studies have noted a gap between the technological potentials and the actual outcomes [4]. Explanations for this gap are: limited access to capital, lack of relevant information on available technologies, imperfect markets, misaligned incentives and organisational barriers [5]. Many of these failures are closely related to how individuals make decisions [6] and, therefore, the ability to overcome them rely on a better understanding of humans’ models of choice and their integration in the design of DR programs [3], [5], [7], [8].

Behavioural economists commonly agree that decision-makers are rather bounded rational; due to cognitive limits and lack of knowledge they deviate from the optimal response assumed by traditional economic theories. They proposed several models of bounded rationality (BR) that captures the psychological properties of humans. Such models have been investigated in energy applications, e.g., demand response [9], microgrids [10] and energy storage [11], as well as non-energy related fields, such as security applications [12] and wireless networks [13]. These studies gain new insights on the system analysis and more accurate models when dealing with human interactions.

In this paper we investigate how BR can be modelled in a demand response environment. Instead of modeling BR with a suboptimality factor as in [9], we chose the sparsity-based model of bounded rationality [14] and prospect theory [15] as inspiration for building two models of consumers’ response to price signals that we termed limited attention and valuation, respectively. The first model reflects the inability of decision-makers to take into account all the variables affecting their decisions while the second one focuses on how asymmetric perception leads to inconsistent decisions. We study the impact of modeling these two aspects of BR decision-makers in the individual and aggregated payoffs of the Coupon Incentive-based Demand Response (CIDR) program [16]. CIDR program offers a simple setup to mimic the degree of participation of consumers and focus on a single-stage market settlement. Nevertheless this formulation can be easily generalized to more complex setups like the model of real time pricing proposed in [17]. We finally integrate these models of BR in the design of the CIDR by formulating a Stackelberg game between the load serving entity (LSE) and the consumers. The Stackelberg game is modelled as a bilevel optimization program which is then reformulated as single-level mixed-integer linear program (MILP). This solution method assures finding the Nash equilibrium, similarly to the iterative approach used in [16] under smooth and convex objective functions. The numerical results show that modeling consumers as BR decision-makers lead to undesirable loss of revenue for the LSE that can be minimized by accounting for BR models in pricing DR. It is worth mentioning that our working framework relies on price signals as the only tool to achieved the desired response. Nonetheless, other type of interventions or tools might be needed to fully exploit the existing demand-side flexibility.

The rest of the paper is organized as follows: Section II introduces the CIDR program and the modeling approach. Section III describes the two models of BR and how to account
for them in pricing the CIDR program. Section IV presents a case study of the CIDR program under rational and BR assumptions. Finally, Section V concludes on the results.

II. COUPON INCENTIVE-BASED DEMAND RESPONSE

We start by describing the mechanism of the CIDR program, time-line and actors involved, continued by the corresponding modeling framework.

A. Description of the CIDR program

The central agent in the CIDR program is the load serving entity (LSE) that supplies retail customers, e.g. residential and industrial consumers. At the day-ahead stage, prior to the CIDR activation, the LSE estimates the load of the consumers and contracts the estimated energy quantities at the day-ahead market at corresponding prices. The contracted energy quantities are offered to the consumers at the flat rate that is estimated ex-ante, generally higher than the day-ahead prices, and decoupled from the actual price formation in the system. Thus, given that the load forecast is accurate enough, the LSE generates strictly positive profits. In a more realistic scenario, however, the load of consumers may significantly vary closer to the real-time, and the LSE has to participate in the real-time market to compensate for forecast errors. The CIDR considers the scenario, when the day-ahead contracts of the LSE are lower than the actual load of the consumers, and the LSE needs to purchase extra energy at the real-time market. The goal of the CIDR program is to reduce the cost of the LSE associated with the purchase in real-time by incentivizing the consumers to reduce their consumption.

Under the CIDR program, the LSE interacts with the independent system operator (ISO) on the one side and with retail customers on the other, as depicted in Fig. 1. Shortly before the real-time operations, the ISO announces the estimated real-time price \(\lambda_{RT}\), and the LSE computes the cost of real-time purchase given the actual mismatch between the day-ahead contracts and actual consumption \(\Delta_{RT}\). To reduce the purchase cost, the LSE broadcast a coupon \(\pi\) to all consumers. The coupon is seen as a premium for each unit of consumption the consumers are willing to reduce. By re-optimizing their utility with respect to a given \(\pi\), consumers reply with the energy quantities \(\Delta_{\Sigma} = \{\Delta_1, \Delta_2, \ldots, \Delta_l\}\) that they are willing to subtract from their current consumption. Once all consumer’s responses are obtained, the LSE offers the aggregated energy reduction \(\Delta_{RT} - \Delta_{\Sigma}\) to the ISO.

As a result, the CIDR benefits the system overall. The LSE reduces the cost of real-time purchase by reducing the purchased amount. The consumers are compensated for reduction in their consumption with a premium \(\pi\) per energy unit, and the system imbalance seen by the ISO is reduced by the mobilized flexibility \(\Delta_{\Sigma}\).

B. Modeling CIDR

We formulate the interdependence between the LSE and the consumers as a Stackelberg game where the LSE acts as a leader and the consumers are considered as followers.
the final consumption \( \{l_i\}_{\forall i} \) in the lower-level problems that, in turn, influences the optimal coupon price.

Previous work on CIDR models solved the bilevel problem between the LSE (2) and the consumers (1) with an iterative solution method [16]. This method only assures finding global maxima when the objective functions of the lower-level problems are smooth and convex. In this work, we propose to reformulate the bilevel optimization problem into a single-level MILP considering the following procedure. First, we replace the set of lower-level problems with their corresponding Karush–Kuhn–Tucker (KKT) conditions. Due to the convexity of the lower-level problems, the following KKT conditions are necessary and sufficient for optimality

\[
\begin{aligned}
\alpha_i - \beta_i l_i - \lambda^{RR} - \pi + \rho_i &= 0, \quad \forall i \in \Omega, \\
0 &\leq \rho_i \perp l_i \geq 0, \quad \forall i \in \Omega,
\end{aligned}
\]  

(3a)

(3b)

where \( \rho_i \) is the dual variable of the positivity constraint imposed on \( l_i \). By replacing consumers optimization with (3), we obtain a mathematical program with equilibrium constraints (MPEC) problem. The MPEC problem includes (3), we obtain a mathematical program with equilibrium constraints (MPEC) problem. The MPEC problem includes a set of bilinear terms due to the products of the coupon price and final consumption of each consumer in the last term of the LSE’s problem. By rearranging (3a) and multiplying both sides by \( l_i \), we have the following convex equivalents for the non-linear terms

\[
\pi l_i = \alpha_i l_i - \beta_i l_i^2 - \lambda^{RR} l_i, \quad \forall i \in \Omega.
\]  

(4)

Finally, we consider the linearization of (3b) by using auxiliary binary variables \( \{u_i\}_{\forall i} \) and some sufficiently large value \( M \), yielding the mixed-integer reformulation

\[
\max_{\pi, l_i, \rho_i \geq 0, \{u_i\}_{\forall i}} E^{DA} + \sum_{i \in \Omega} (l_i)\lambda^{RR} + \sum_{i \in \Omega} \lambda^{RR} l_i - \pi \sum_{i \in \Omega} (\tilde{l}_i - l_i),
\]  

(5a)

s.t. \( \alpha_i - \beta_i l_i - \lambda^{RR} - \pi + \rho_i = 0, \quad \forall i \in \Omega, \)  

(5b)

\( \rho_i \leq M u_i, \quad \forall i \in \Omega, \)  

(5c)

\( l_i \leq M (1 - u_i), \quad \forall i \in \Omega. \)  

(5d)

This formulation allows to calculate the optimal coupon price and load levels obtained with the CIDR program, assuming consumers are rational decision-makers who always choose the optimal load level. Nevertheless, the solutions obtained might not capture the actual market behaviour where consumers behave as BR decision-makers.

### III. Modeling bounded rationality

In this section, we describe how we can use the sparsity-based model of bounded rationality [14] and prospect theory as inspiration for building two models of consumers’ response termed limited attention and valuation, respectively. In addition, we reformulate the Stackelberg game between the LSE and the consumers to integrate the models of consumers’ response and find a new equilibrium point.

#### A. Limited attention

Recent psychological studies have shown that decision-makers pay limited attention to certain factors, replacing their actual values with defaults, and purposefully allocate attention to factors that seem more important to them. In this way, decision-mak...
The formulation of the consumers’ optimization problem is rewritten as
\[
\max_{l_i \geq 0} \alpha_i l_i - \frac{1}{2} \beta_i l_i^2 - \lambda^{RR} l_i + f(\pi)(\bar{l}_i - l_i), \quad (8)
\]
where \( f(\pi) \) is the value function, which assigns a subjective value to the coupon \( \pi \). In this formulation, the coupon is framed as a gain for the consumers, and will normally have a value function with a concave shape according to prospect theory. Even though we base the formulation in coupons framed as gains, this can be extended in future work to account for inconsistencies due to framing effects. We integrate this formulation into the bilevel problem to find a new equilibrium point as follows
\[
\max_{\pi \geq 0} \left( E^{DA} - \sum_{i \in \Omega} \gamma^{RT} l_i + \sum_{i \in \Omega} \lambda^{RR} l_i - \pi \sum_{i \in \Omega} (\bar{l}_i - l_i) \right), \quad (9a)
\]
\[
s.t. \ l_i \in \arg \{(8)\}, \forall i \in \Omega. \quad (9b)
\]
Due to the non-linearity of the value function, the lower-level problems can no longer be replaced by their KKT conditions to solve the bilevel optimization problem (9). For this specific problem we use the iterative algorithm proposed in [16] to find the optimal price signal and load levels and check whether a global optimum has been reached.

IV. NUMERICAL CASE STUDY

To show the impact of BR in the CIDR program we use three test cases. By first solving the CIDR model under rational assumptions (5), we obtain a benchmark of the desired outcomes of the CIDR. For each of the BR aspects, limited attention and valuation, we have an individual test case with two different LSE strategies. In the baseline strategy (9), the LSE does not acknowledge consumers’ BR and solves (5) while consumers follow (6) and (8), respectively. In the aware strategy (10), the LSE accommodates BR and solves (7) and (9), respectively.

We consider the scenario in which the contracted energy at the day-ahead market is 0.85 GWh. The contracted energy is lower than the actual load of consumers 1.13 GWh, and the LSE has to purchase energy in the real-time market at $60/MWh. The energy is supplied at a flat rate of $50/MWh to two groups of consumers characterized by different utility parameters \( \beta_i, \alpha_i \) and baseline load \( \bar{l}_i \), given in Table I.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>CONSUMER GROUPS AND THEIR PARAMETERS</th>
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<tbody>
<tr>
<td>( \beta_i ) ($/MWh²)</td>
<td>( \alpha_i ) ($/MWh)</td>
</tr>
<tr>
<td>( i_1 )</td>
<td>0.049</td>
</tr>
<tr>
<td>( i_2 )</td>
<td>0.026</td>
</tr>
</tbody>
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A. Rational case

When no coupon is broadcast, the LSE incurs a loss of k$16.99 in the real-time market. This loss is overcome thanks to the energy payments of consumers, making an overall profit of k$39.64. Even if the profit is positive, the LSE could improve it by sending a coupon in order to reduce the actual load. By solving (5) we found that the optimal coupon price is $3.25/MWh. Thanks to the broadcast of the coupon, the LSE can increase the profit by $930. The optimal response of consumers to that coupon is a load reduction of 66.3 MWh and 71.4 MWh for \( i_1, i_2 \) respectively. Due to the load reduction, the consumers increase their utility by $108 and $165, respectively.

The CIDR program benefits the system overall. The LSE increases its profits by reducing the cost of real-time purchase. The consumers are compensated for reducing their consumption with a premium of $3.25 per energy unit. The social welfare (SW) increases from k$67.31 to k$68.51 and the system imbalance seen by the ISO is reduced. Even if the program increases the SW, it is not equally distributed among the participants. The LSE benefits more from the CIDR program, with a share of 77.29% of the SW, while the consumers get a share of 8.96% and 13.75% for \( i_1 \) and \( i_2 \) respectively. These results are used as the benchmark for limited attention and valuation cases.

B. Limited attention

We study the effect of consumers showing no price-responsiveness for each of the consumers’ types participating in the CIDR program. The results obtained are presented in Table II for the different cases and strategies: the rational case in which both consumers are attentive, the case in which only \( i_1 \) is attentive and the case in which only \( i_2 \) is attentive; and for strategies: (9) and (10).

When the LSE is blind against consumers’ BR and follows the baseline strategy (9), it obtains a lower profit than the one expected under rational assumptions. This loss can be seen from Figure 2 where the profit is plotted against coupon prices for the different cases. The LSE expects to obtain a profit of k$40.57 by sending a coupon of $3.25/MWh, represented with a black circle in Figure 2. Since not all consumers are attentive to the coupon, the obtained profits are smaller in both cases, see purple and orange circles in Figure 2. In addition, this loss depends on the consumers characteristics, being more prejudicial when the more price-elastic consumer does not pay attention to the coupon. In this way, when \( i_2 \) does not participate the loss of profit attains $480, compared to $450 in the case of \( i_1 \) not participating.

This loss can be reduced by adapting the coupon prices to the consumers’ price responsiveness. When the LSE changes the strategy to (10), the optimal coupon amounts to $5/MWh when only \( i_1 \) is attentive, and $1.87/MWh when only \( i_2 \) is attentive. These points are plotted with squared markers in Figure 2, which correspond to the maximum profit that can be achieved in each case. By following strategy (10) the LSE can reduced the loss by $60 and $100 in each case. It is worth mentioning that the awareness of the LSE may reduce the welfare of rational consumers when accommodating BR of other consumers. In terms of social welfare distribution, we can observe that the LSE gains a larger share of SW in all the
cases. Moreover, this share increases if we compare strategy A to B. Meaning that being aware of consumers’ BR make SW distribution more uneven.

In the original formulations, coupon price is identical for the whole set of consumers. However, we can show that LSE can hedge the risk associated with BR if it issues different coupons, whose values are specific for each consumer. We term this strategy as A. This strategy yields higher profit in case of rational consumers, and avoids the losses in case of non-responsive consumers reported under strategy B.

C. Valuation

This section investigates the impact of consumers that assign a subjective value to the coupons on the outcomes of the CIDR. We chose three value functions, $f_1$, $f_2$, $f_3$ depicted in Figure 3 to account for different perception patterns. They all have a concave shape in accordance with prospect theory, and show different slopes and dead bands. The dead band represents the values of the coupons that are treated as zeros by the consumers. The corresponding simulation results are summarized in Table III.

When the LSE follows strategy B and consumers value the coupon following $f_1$, the profit of LSE is slightly higher than the one expected under rational assumptions. This gain can be seen from Figure 4 where the profit is plotted against coupon prices for the different cases. We can observe that the coupon price obtained with strategy B, see orange circle, corresponds to a higher profit than the rational case, yet the LSE does not attain the maximum possible profit. In this case, the fact that the consumers have an asymmetric perception of the coupon results in a higher price elasticity. Consumer $i_1$ provides 2.36 MWh extra of demand response for the same coupon while $i_2$ is already providing its maximum demand response which is 71.40 MWh.

In the cases where consumers follow $f_2$ and $f_3$, and the LSE is not aware of it, it faces a loss of $600 and $930 respectively compared to the rational case. This is due to the fact that the consumers undervalue the coupon and demonstrate less price elasticity, thus reducing the demand response provided in the case of $f_2$ and even showing zero response in the case of $f_3$.

If the LSE is able to anticipate the value functions and follows strategy A, the coupon prices increase to $4.67/MWh, $5.42/MWh and $5.98/MWh for $f_1$, $f_2$ and $f_3$, respectively. This points are plotted with squared markers in Figure 4, which correspond to the maximum profit that can be achieved in each case. In all three cases the obtained profits are higher than the ones obtained in the rational case, meaning that it is beneficial for the LSE that consumers are BR if it is able to anticipate it. In that case, the elasticity of consumers increases, providing more demand response and flexibility to the grid.

In all cases and strategies, the social welfare is not evenly distributed, with the LSE having again a higher share. However, opposite to the results obtained in Section IV-B, this inequality is smoothen when the LSE is aware of consumers’ BR and follow strategy A. Among the consumers, $i_2$ is always
obtaining a higher share than \( i_1 \), benefiting more from the CIDR program.

V. CONCLUSIONS

This paper presents a generalization of the existing CIDR mechanism that accommodates bounded rational decision-makers. The Stackelberg game between the LSE and the consumers is recast as a bilevel optimization problem that can account for both rational and bounded rational consumers. The working framework relies on price signals to change the consumers’ response. The integration of models of BR explain the misinterpretation of those price signals and the suboptimal performance of the CIDR program.

A case study is used to derive the optimal strategy of the LSE under consumers’ rationality assumptions. The impact of human choice modeling is studied under two cases. The first case addresses limited attention aspects, following the sparsity-based model of bounded rationality. The results show that consumers’ attention to price signals limits the available profit of the LSE. The loss of profit can be minimized if the LSE is able to anticipate which group of consumers is price-attentive and tailor coupon prices to each group response. This strategy helps to hedge the risk associated with BR consumers. The second case analyzes the valuation of coupon signals in the sense of prospect theory. The results reveal that asymmetric perception of coupons can lead to suboptimal LSE decisions. The anticipation of the value functions leads to an increase of the consumers’ elasticity as well as higher profits for the LSE compared to the rational case.

Our research can be extended to other DR programs, e.g., models of real time pricing. In addition, new models of bounded rationality can bring interesting insights on the understanding of consumers’ response to price signals.

REFERENCES


