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Stochastic Modelling of Supermarket Refrigeration for Demand Response

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Abstract—**This paper presents a method for identifying and validating a model of the heat dynamics of a supermarket refrigeration display case for the purpose of intelligent control in the smart grid. The model is established to facilitate the development of novel local control techniques for individual display units in a supermarket refrigeration system. This will consequently facilitate the development of advanced supervisory control capabilities, allowing the entire refrigeration system to participate in the smart grid, including for the optimisation of power consumption for cost or energy efficiency. The grey-box modelling approach is adopted, using stochastic differential equations to define the dynamics of the model, combining prior knowledge of the physical system with data-driven modelling. Model identification is performed using the forward selection method, and the performance of candidate models is evaluated through cross-validation, using independent fitting, validation and testing datasets. The model developed in this work uses operational data from a small Danish supermarket. A three-state model is determined to be most appropriate for describing the dynamics of this system, with potential applications in one-step prediction or control of supermarket refrigeration display units.**

Index Terms—**Grey-Box Modelling, System Identification, Demand Response, Flexible Demand, Smart Grid, Semi-Physical Modelling, Refrigeration, Time Series Analysis**

I. Introduction

RENEWABLE power production and energy efficiency feature high on the priority list of policy makers and power system engineers and operators around the globe [1], [2]. Of central concern is the need for flexibility in the power system to facilitate higher penetrations of variable renewable power supply. This can be achieved through energy storage, activation of demand for flexibility in power consumption, and interconnection to neighbouring systems, among others.

The supermarket sector represents a potential 'low-hanging fruit' in the transition to flexible power consumption. Supermarkets account for 4% of national energy consumption in both France and the USA [3]. Supermarkets in the USA have an average energy intensity of 565kWh/m^2/year [4], while their counterparts in Sweden have a more modest intensity of 471kWh/m^2/year [5], which still results in a total consumption of 1.8TWh/year for the entire supermarket sector, or 3% of Swedish electricity consumption. Of this electricity consumption, the refrigeration system accounts for a share of up to 47% [6]. Electricity costs make up only 1% of costs for the typical supermarket, however with a typical profit margin of only 3%, any savings in energy costs would translate to an increase in profit for the operator [5]. Supermarket refrigeration systems, and the food stored within them, represent a large thermal mass, capable of storing thermal energy over time. This capability, combined with intelligent control, permits the optimised scheduling of power consumption for energy or cost efficiency, while still maintaining acceptable temperature limits within the refrigeration system.

The combination of scale, incentive and capability make supermarkets an ideal candidate for activation of demand response. In order to achieve this, it must be possible to control the refrigeration system with respect to an external signal. Controlling the system requires knowledge about system states (e.g. temperatures) that may not be directly measured, but which can be estimated by accurate models, at both the supervisory and local control levels. Supervisory control concerns the operation of the compressor banks and centralised system components, whereas local control concerns the temperatures in individual display units. This paper develops and validates a grey-box model of a refrigeration display unit in a supermarket in Denmark, for the purpose of local control. Typically, the temperature within individual display units is controlled using simple hysteresis or threshold control, where the temperature continually varies between a lower and upper threshold. The model presented in this work, and the method employed to reveal it, are intended to facilitate the development of advanced local control frameworks to optimise the temperature control in display units and consequently extend the capabilities of the supervisory controller for the provision of demand response in the smart grid.

This is an emerging research area, and there are limited works focussing on the identification of models of refrigeration systems for demand response purposes. The grey-box modelling approach is employed in [7] for the estimation of models for household refrigerators. Non-convex model predictive control of a supermarket refrigeration system for the provision of demand response is presented in detail in [8], using a model

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derived from first principles, rather than a data-driven model as presented in this work.

A simplified model of a supermarket with flexible power consumption is considered in [9], and a more advanced ordinary differential equation (ODE) model for supervisory control of the supermarket refrigeration system is provided in [10]. A modular modelling approach was adopted in that work, which included a model of the display units, as well as models of the compressors and other system elements. In contrast with the work of [10], the work presented in this paper focusses solely on the display units, but considers the additional complexity of both the inherent stochasticity of the system, and changing system conditions. The complexities of defrosting operations, changing thermal characteristics between opening and closing hours, and the stochasticity in the system due to the opening and closing of refrigerator doors and the removal and addition of foodstuffs by both consumers and employees, are all considered in the stochastic differential equation (SDE) models developed in this work. The developed models are intended for implementation within an adaptive control framework, where the model parameters can be re-estimated in response to slowly changing system conditions.

Detailed models of electrical loads are a timely contribution to this field. The advent of demand response as a power system service warrants in-depth modelling of potentially flexible demands. Such detailed modelling and the establishment of standard models of traditional power system components has long been argued for [11]. Both the IEEE and CIGRÉ have proposed standard models for system components. Such standards and models retain their relevance in a smart grid environment. Newly active elements of the power system, such as intelligent refrigeration loads, must be modelled to the same level of detail as other components which were once considered novel, such as wind turbines [12].

The grey-box modelling approach has previously been employed to develop models of building heat dynamics [13], [14], household refrigerators [7], photovoltaic modules [15], and mobile communication channels [16] among many other varied systems and fields.

In Section II, the grey-box modelling method is outlined, and the model development technique is explained. In Section III, the supermarket refrigeration system and available system data are described. Section IV details the model development process, and model fitting and evaluation are addressed in Section V. Conclusions and final remarks are presented in Section VI.

II. Grey-Box Modelling

A. Grey-Box Modelling Theory

Grey-box modelling facilitates the identification of models of dynamical systems using a combination of prior physical knowledge of the system under examination, and information revealed by observed time series data. The grey-box modelling method benefits from the advantages of both white- and black-box modelling; capturing potential non-linear behaviour typically considered with white-box models derived from first principles, and accounting for both process and measurement noise, which are considered in black-box models [17]. A grey-box model consists of a set of stochastic differential equations (SDEs) that describe a dynamical system, and a measurement equation. Together these form a continuous-discrete time, stochastic state-space model, where the discrete measurement equation describes how the measured data relates to the states of the system.

An initial, simplified, model of a display case system is presented here as an illustration of the format of a grey-box model. Fig. 1 shows the electric-thermal equivalent RC-representation of a first order model of the system. All of the models explored in this work consider the system as a set of lumped thermal bodies, such models are frequently called lumped parameter models. T_r and T_a are measured inputs to the model, the refrigerant temperature and the ambient temperature respectively. C_i is the thermal capacity of the interior of the display case, which in this case includes its structure, the food being displayed and the air circulating in the interior of the case. R_{ri} and R_{ia} represent the thermal resistance between the interior and the refrigerant, and the interior and the supermarket ambient air respectively. AKV is the opening degree of the expansion valve, a system input given as a percentage, and α is a scaling parameter. This model assumes that all heat exchange occurs through the internal air, T_i, with heat exchange between the refrigerant and the interior, and the interior and the ambient, but no direct exchange of heat between the refrigerant and the ambient. This can be represented in the form of a stochastic state-space model as:

$$dT_i = \Big(\frac{1}{C_i R_{ri}} (T_r - T_i) + \frac{1}{C_i R_{ia}} (T_a - T_i) + \frac{1}{C_i} AKV\alpha \Big) dt + \sigma \, d\omega \tag{1}$$

$$Y_t = T_{i,t} + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right) \tag{2}$$

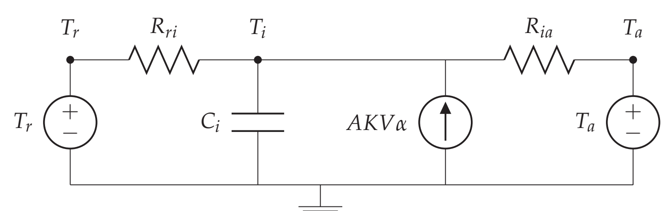


Fig. 1: RC-Representation of the initial display case model (T_i)

The $\sigma d\omega$ term in (1) represents the process noise, where ω is a Wiener process, and σ^2 is the incremental variance of this process. This representation of the process noise is referred to as the diffusion term, while the representation of the dynamics of the system (the remainder of (1)) is the drift term. The measurements as represented by (2), are encumbered with measurement noise, ε_t, which is assumed to be a Gaussian distributed white noise process, which is independent of the process noise.

The $R_{xx'}$, C_x, α, and noise parameters of the model must be determined by fitting the described model to the data. The maximum likelihood method is employed to fit the parameters. Maximum Likelihood Estimates (MLE) of the parameters are found by maximising the likelihood function of the parameters given the provided measurements. The likelihood function is:

$$L(\boldsymbol{\theta}, \boldsymbol{Y}) = p(\boldsymbol{Y}|\boldsymbol{\theta}) \tag{3}$$

where $\boldsymbol{\theta}$ is a vector containing the model parameters, and $\boldsymbol{Y}$ is a vector of the measured data. By finding the parameters that maximise this expression, we find the parameters for the model described by (1) which are most likely to generate the observed data, including process and measurement noise.

Denoting the observations as:

$$\mathcal{Y}_{\mathbf{N}} = [Y_N, Y_{N-1}, \ldots, Y_1, Y_0] \tag{4}$$

The likelihood can be expressed as a product of conditional densities:

$$L(\boldsymbol{\theta}; \mathcal{Y}_{\mathbf{N}}) = \left(\prod_{k=1}^{N} p(Y_k|\mathcal{Y}_{k-1}, \boldsymbol{\theta})\right) p(Y_0|\theta) \tag{5}$$

where $p(Y_k|\mathcal{Y}_{k-1}, \boldsymbol{\theta})$ is the conditional density describing the probability of generating the current observation, given the previous observations and the parameters set θ. The initial conditional densities, $p(Y_0|\theta)$, are evaluated based on the estimation of the initial state of the system. The conditional densities in (5) are Gaussian, following from the fact that the noise processes are Gaussian and the system equations are linear. Due to this, an ordinary Kalman filter can be used to evaluate the likelihood function and consequently find the MLE, $\hat{\theta}$, of the unknown parameters. An optimisation function can then be used in conjunction with the Kalman filter to determine the parameter values that maximise the likelihood function. In this work, the software tool CTSM-R (Continuous Time Stochastic Modelling in R) [18] is employed to estimate these parameters. Detailed discussion on the mathematics behind this tool, and the maximum likelihood method, can be found in [17] and [19].

B. Model Development Process

A forward selection approach (see [19] for detailed discussion) is adopted in this work, similar to the approach used in [13]. Modelling commences with the simplest feasible model, and additional complexity is introduced until the point where no significant improvement is found. The improvement is defined in terms of selected error metrics; the mean absolute error (MAE), root mean squared error (RMSE), and model bias. These fit statistics consider the one-step prediction error of each of the models, in line with the focus of this work on models for control rather than longer-term forecasting.

The log-likelihood approach of comparing models based on their log-likelihood value is not adopted in this work for two reasons; first, sequential iterations of model development do not always involve nested models, rendering the likelihood ratio test inapplicable; secondly, not all models explored here are structurally identifiable, meaning that the parameter values found by the CTSM-R solver are locally optimal, but not necessarily globally optimal. Non-identifiability is not a major issue for models for control as the physical meaning of the parameters is not relevant. The determined parameter values and model structure govern the dynamics of the system, and it is not necessary for them to reflect the actual physical construction of the system.

The model selection process is conducted in three stages: fitting, validation and testing. A separate dataset is employed for each stage, where the data used for model fitting contains twice as many observations as each of the sets used for validation and testing, as recommended by [20]. The fitting step involves the fitting of parameters to a large number of models. In the validation step only a subset of the models are considered; the parameters found in the fitting step are retained and the performance of the selected models is compared using the error metrics detailed above. Finally, the model with the best performance on the validation dataset is tested using the testing dataset; the purpose of this test is to assess the generalization error of the model. In all cases (fitting, validation and testing) the model performance is defined by the one step prediction error. As the models are being employed for control purposes, it is intended that the selected model will adapt with the system, so it is not a requirement that a particular model should perform as well in the testing step as in the fitting step, however it is a useful indicator of the general performance of the model.

III. Description of System and Data

A. System Description

The system considered in this paper is a small supermarket located on the island of Funen, in Denmark. The refrigeration system consists of 7 medium temperature (above freezing) display cases and 4 low temperature (below freezing) units. Two compressor banks power the refrigeration system, and are arranged in a booster configuration.

The refrigeration system operates in a hierarchical control structure. Supervisory control determines the operation of the larger system, consisting of the compressor banks, fans, the condenser and the suction manifold, such that a desired refrigerant temperature at the input to the medium and low temperature display units can be achieved. The food display units are mounted in parallel, each of the medium temperature units and each of the low temperature units have the same refrigerant temperature at the input to their (individual) evaporators. Local control at each display unit ensures that temperature limits are respected by modulating the degree to which the expansion valves in the individual evaporators are open. Each display unit has distinct characteristics and thermal interactions, as a result of differing temperature bounds and different foodstuffs being stored. The local control at the level of the display case is the focus of this paper; currently it resembles thermostatic control, where the temperature continually oscillates between upper and lower limits. This is a simplistic approach that is acceptable in the current, passive, systems, however as the overall system becomes activated through intelligence, it will be advantageous to have more precise temperature control within the display cases, such that the system as a whole can operate optimally and participate fully in a smart grid framework. Fig. 2 provides a simplified schematic of the display case system, the distinct control regions, and the link to the power system.

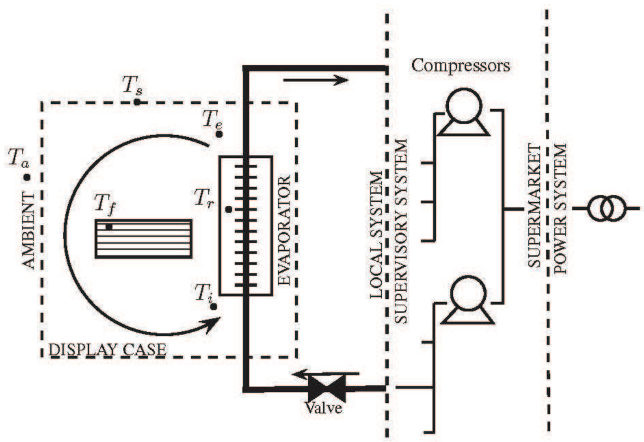


Fig. 2: Simplified graphical representation of the display case system

There are a number of complexities in the operation of the system that complicate the task of modelling its thermal behaviour; each of the display units undergoes numerous defrosting operations during the day, where the temperature within the unit is raised above the typical operating range to allow any accumulated ice to melt. Furthermore, there is an observable difference in how the system operates during shop opening hours compared to the night-time period. This regime change can be explained by the insulating covers placed on display units outside of opening hours.

B. System Data

There is a wealth of relevant data available for the supermarket refrigeration system considered here. Temperature sensors have been placed at a number of locations within each of the display units in the supermarket; the refrigerant temperature, the temperatures of the air at the inlet and outlet of the evaporator, and the opening degree of the expansion valves are reported for each display unit. General system data is also available, including the ambient air temperature within the supermarket, the external air temperature, refrigerant mass flows throughout the system, and the power consumption of each of the compressor banks. All of the data series are recorded at a resolution of one minute. Data was recorded over a period of 11 months, however due to large amounts of missing data observations, consideration in this work is limited to 24 hour periods in October 2010. The three datasets used for fitting, validation and testing correspond to consecutive Wednesdays. These dates were selected to ensure reasonably similar system conditions in all datasets. Considering, for example, a weekend day for one of the datasets would not have been appropriate as it is expected that a different control model may be necessary due to significantly different system conditions, for example supermarket opening hours. Fig. 3 shows the data employed to establish a grey-box model of the open medium temperature display unit considered in this paper. The uppermost plot shows the development of three key temperatures in the display unit over 24 hours on the 10th of October 2010, this is the training dataset. The internal temperature represents the temperature to be controlled within the display unit; this is the temperature most commonly monitored by supermarket operators and food regulatory bodies, and is consequently required to remain strictly within set limits. The evaporator temperature is the temperature of the air exiting the evaporator. The dynamics of these two temperatures are very similar, with the evaporator exhibiting more extreme temperature variations, as is expected as the evaporator temperature drives the internal temperature in the display unit, and is subject to thermal losses to a number of sources. The blue trace shows the temperature of the refrigerant in the system. This is common for all of the medium temperature display units, and is a fixed input, as determined by the supervisory control of the system. The second plot in the figure indicates the instances of defrost operation for the considered display case, and the hours during which the supermarket is closed. The impact of these factors on the temperatures in the display unit can be observed in the first plot, with spikes in temperatures during defrost operations and distinctly different temperature trends when the supermarket is closed. This day/night difference can also be observed in the expansion valve time series displayed in the third plot. During the night, when the unit benefits from increased insulation from an insulating cover as well as decreased ambient air temperature, the expansion valve is not opened as frequently or for as long as occurs during the day. This change is also reflected in the temperature plots, where the dynamics are notably slower outside of the supermarket opening hours. The degree to which the expansion valve is open determines the temperature within the display unit (subject to the refrigerant and ambient temperatures), it is the control variable in the local control system.

IV. Model Development

The derivation of a grey-box model of the display case system is detailed in this section. The system temperatures considered in this work include that of the

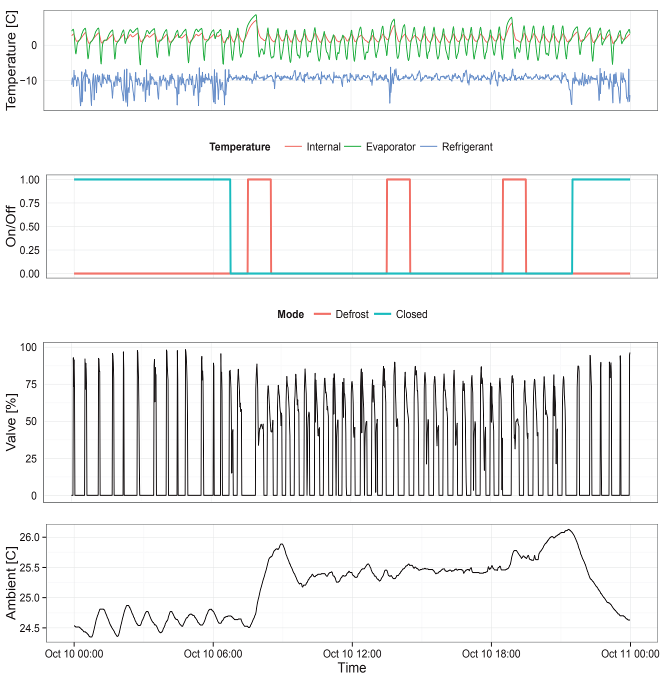


Fig. 3: Temperature, environmental (open/closed status, defrost status, ambient temperature) and control input (valve) data for an open medium temperature display case in a supermarket in Funen, Denmark

refrigerant (T_r), the ambient air in the supermarket (T_a) (both known model inputs) and four temperatures in the interior of the fridge; the air at the evaporator outlet (T_e), the food (T_f), the air within the display case (T_i), and the temperature of the display structure (T_s). Only the model inputs and the air temperature within the display case are known when fitting the models, the other temperatures are modelled as hidden states.

The models are presented in order of increasing complexity. Models of two, three and four states are presented in the following sub-sections. A number of different configurations of the described models were tested, with the most successful presented in detail. The performance of the alternative models is given in tabular form in the results section which follows, with the electric-thermal equivalent RC-representations provided in Appendix A.

A. Model T_iT_e

The two state (T_i,T_e) model shown in Fig. 4 considers that the dynamics of the system are governed by the thermal masses in the interior of the unit (T_i) and at the outlet of the evaporator (T_e). In this model the impact of the expansion valve operation is considered to directly affect the temperature at the evaporator outlet. Heat exchange from the refrigerant to the ambient occurs through the evaporator and the interior of the display unit, in sequence, with no direct heat exchange between the refrigerant or the evaporator outlet with the ambient.

The two-state stochastic model of this system is given by:

$$dT_i = \left(\frac{1}{C_i R_{ei}} (T_e - T_i) + \frac{1}{C_i R_{ia}} (T_a - T_i) \right) dt + \sigma_1 \, d\omega_1 \tag{6}$$

$$\begin{aligned} dT_e = &\left(\frac{1}{C_e R_{re}} (T_r - T_e) + \frac{1}{C_e R_{ei}} (T_i - T_e) \right. \\ &\left. + \frac{1}{C_e} AKV\alpha \right) dt + \sigma_2 \, d\omega_2 \end{aligned} \tag{7}$$

$$Y_t = T_{i,t} + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right) \tag{8}$$

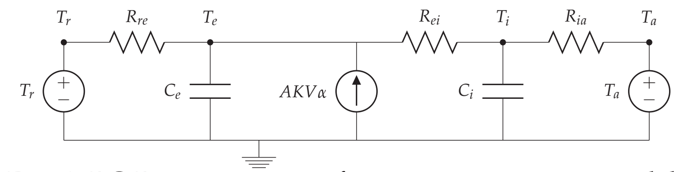


Fig. 4: RC-Representation of a two time constant model (T_iT_e)

B. Model $T_iT_eT_f$

Additional complexity is added to the model from Subsection IV-A by considering the thermal capacitance of the food, C_f. The simplest three state (T_i,T_e,T_f) model is shown in Fig. 5, and comprises of thermal links between the refrigerant and the internal air, the internal air and the food, and the food and the ambient supermarket air; no other direct heat exchanges are considered in this simple model. Alternative considered configurations of this three state model include the addition of a thermal link between the internal air and the supermarket ambient. This is an intuitive development as not all thermal energy can be expected to flow through the foodstuffs to the ambient, and a degree of direct leakage of energy from the interior to the ambient is expected. Three further model configurations consider inputs indicating the open/closed state of the supermarket and the instances of defrosting operations. Full details of these alternative configurations are provided in Appendix A. Fig. 5 illustrates the model in its electric-thermal equivalent circuit, and the three-state stochastic model of the system is as follows.

$$dT_i = \left(\frac{1}{C_i R_{ei}} (T_e - T_i) + \frac{1}{C_i R_{if}} \left(T_f - T_i\right) \right) dt + \sigma_1 \, d\omega_1 \tag{9}$$

$$\begin{aligned} dT_e = &\left(\frac{1}{C_e R_{re}} (T_r - T_e) + \frac{1}{C_e R_{ei}} (T_i - T_e) \right. \\ &\left. + \frac{1}{C_e} AKV\alpha \right) dt + \sigma_2 \, d\omega_2 \end{aligned} \tag{10}$$

$$\begin{aligned} dT_f = &\left(\frac{1}{C_f R_{if}} \left(T_i - T_f\right) + \frac{1}{C_f R_{fa}} \left(T_a - T_f\right) \right) dt \\ &+ \sigma_3 \, d\omega_3 \end{aligned} \tag{11}$$

$$Y_t = T_{i,t} + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right) \tag{12}$$

C. Model $T_iT_eT_fT_s$

Finally, additional complexity is added by considering that the dynamics of the system may not be fully

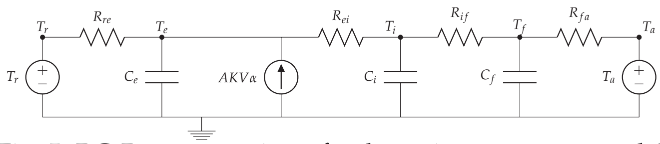


Fig. 5: RC-Representation of a three time constant model ($T_iT_eT_f$)

accounted for by the three states used in the previous models. A further state, T_s, is added. There is no intuitive interpretation of this state as any particular component of the display case unit, however it could be considered to be any number of possible components, including the structure of the unit. Furthermore, as the model is based on a lumped parameter approximation of a distributed system, the addition of an additional state could correspond to the better representation of an aspect of the model that was previously represented by a single state; for example, the food may be better represented by individual states for the inner and outer sections of the foodstuff. Fig. 6 shows the configuration of this model in its electric-thermal equivalent circuit form. The four-state stochastic model of the system is:

$$dT_i = \Big(\frac{1}{C_i R_{si}} (T_s - T_i) + \frac{1}{C_i R_{if}} \left(T_f - T_i \right) + \frac{1}{C_i R_{ia}} (T_a - T_i) \Big) dt + \sigma_1 \ d\omega_1 \tag{13}$$

$$dT_e = \Big(\frac{1}{C_e R_{re}} (T_r - T_e) + \frac{1}{C_e R_{es}} (T_s - T_e) + \frac{1}{C_e} AKV\alpha \Big) dt + \sigma_2 \ d\omega_2 \tag{14}$$

$$dT_f = \Big(\frac{1}{C_f R_{if}} \left(T_i - T_f \right) + \frac{1}{C_f R_{fa}} \left(T_a - T_f \right) \Big) dt + \sigma_3 \ d\omega_3 \tag{15}$$

$$dT_s = \Big(\frac{1}{C_s R_{es}} (T_e - T_s) + \frac{1}{C_s R_{si}} (T_i - T_s) \Big) dt + \sigma_4 \ d\omega_4 \tag{16}$$

$$Y_t = T_{i,t} + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right) \tag{17}$$

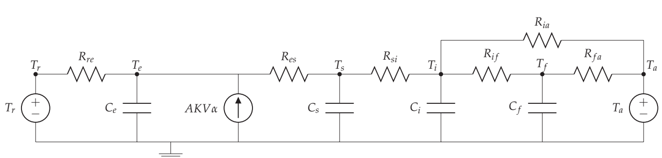


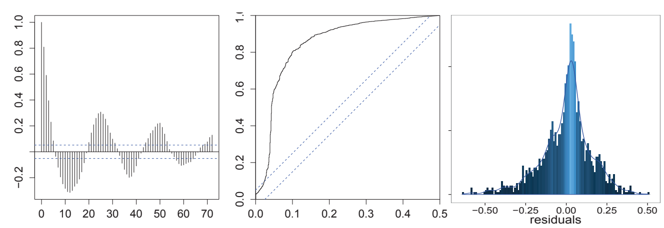
Fig. 6: RC-Representation of a four time constant model ($T_iT_eT_fT_s$)

No further complexity is considered in the modelling process as the four-state model is found to provide no significant improvement over the three-state model, as discussed in the following section. Furthermore, the residuals of the three-state model show no systematic pattern that would have called for a further state or time-constant.

V. Results

If the one-step ahead prediction errors for a particular model show a systematic pattern, this indicates an extension of the model is necessary. If, on the other hand, the residuals resembles white-noise, it can be concluded that there is no information in the data than contradicts the conclusion that the particular model adequately describes the system. This criterion for model adequacy is checked by examining the histogram, auto-correlation function (ACF) and the cumulative periodogram for each of the models in the fitting stage.

Fig. 7 shows the ACF for the simplest, single state, stochastic model, as presented in Section II. It is very clear that the residuals of this model are not Gaussian distributed, and thus the model is inadequate. The histogram reveals a positive bias in the model, and the behaviour of ACF indicates that there are additional time constants in the system that are not accounted for by the current model.

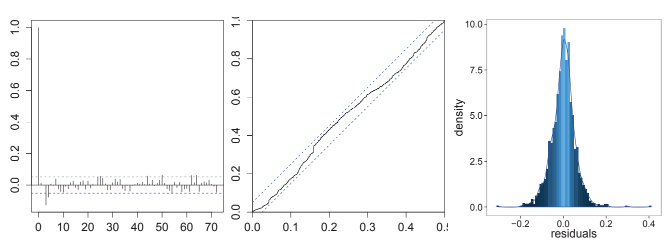


(a) ACF (b) Cum. Periodogram (c) Histogram

Fig. 7: Analysis of the residuals of a single state model (T_i)

A significant improvement is achieved by adding an additional time constant to the model (T_iT_e), however the resulting model residuals are not Gaussian distributed.

Fig. 8 shows that the residuals of the three state model, $T_iT_eT_f$, can be said to be normally distributed, indicating that this model represents the dynamics of the observed system adequately. Similar results are found for the residuals of the four state model, $T_iT_eT_fT_s$, thus the benefit of increasing the model complexity to four states must be evaluated by considering the error metrics.



(a) ACF (b) Cum. Periodogram (c) Histogram

Fig. 8: Analysis of the residuals of a three state model ($T_iT_eT_f$)

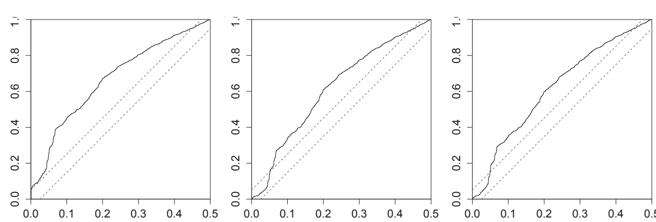
Table I provides the error metrics for each of the models described above, as well as some of the alternative configurations that were explored, as presented in Appendix A. The model selection process involves the addition of model complexity through alternative configurations and additional states (and correspondingly, time-constants). The models are grouped according to the number of time-constants, denoted i in Table I, and compared considering the number of parameters, m. Error metrics are presented in both absolute and relative terms, where the current model is compared to the most successful model with one less time constant (as highlighted in red); for example, model $T_iT_eT_f$, with three states, is compared to model T_iT_e, the best performing two-state model. The best performing models were selected by considering both their error metrics and the ACF, cumulative periodogram and histogram. These models (excluding model T_i) were retained for the validation stage, all other models were discarded.

TABLE I: Error metrics for all models at the model fitting stage, where m denotes the number of parameters

i	Model	m	Absolute			Relative [%]	
			RMSE	MAE	Bias	RMSE	MAE
1	T_i	7	0.1549	0.1138	7.03e-5	-	-
2	T_iT_e	11	0.0577	0.0424	-6.34e-4	62.74	62.74
	T_iT_e2	11	0.0727	0.0508	-1.69e-3	53.07	55.36
	T_iT_e3	12	0.0577	0.0424	-6.21e-4	62.76	62.76
3	$T_iT_eT_f$	15	0.0556	0.0407	2.47e-4	3.85	4.02
	$T_iT_eT_f2$	16	0.0556	0.0408	1.28e-4	3.69	3.63
	$T_iT_eT_fN$	16	0.0558	0.0407	-2.15e-4	3.23	4.01
	$T_iT_eT_fN2$	17	0.0559	0.0407	-2.12e-4	3.22	4.01
	$T_iT_eT_fND$	18	0.0558	0.0407	-2.02e-4	3.26	3.99
4	$T_iT_eT_fT_s$	20	0.0570	0.0412	-3.13e-4	-2.76	-1.32

A. Validation Stage

Fig. 9 shows the cumulative periodograms of the residuals of each of the models when validated using the validation dataset (retaining the model parameters determined in the model fitting stage). Table II provides the error metrics for this validation. By considering both the proximity of the residuals to white noise, and the error metrics, it can be seen that model $T_iT_eT_f$ is the best candidate. This model most closely represents the observed dynamics of the display unit system.



(a) Model T_iT_e (b) Model $T_iT_eT_f$ (c) Model $T_iT_eT_fT_s$

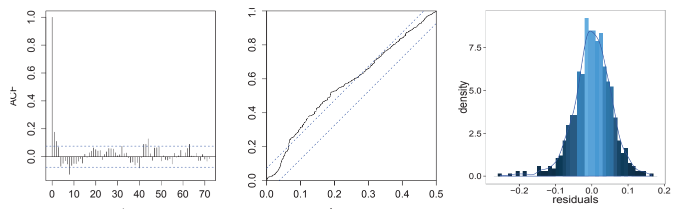
Fig. 9: Cumulative periodograms of the residual of the models selected for advancement to the validation stage, using the validation dataset

TABLE II: Error metrics for models at the model validation stage

Model	RMSE	MAE	Bias
T_iT_e	0.069	0.047	6e-3
$T_iT_eT_f$	0.064	0.043	1.9e-3
$T_iT_eT_fT_s$	0.069	0.044	2.1e-3

B. Testing Stage

Model $T_iT_eT_f$ is tested using testing dataset. Fig. 10 shows the performance of this model. Although the residuals are clearly not white noise, the performance is very satisfactory considering that the parameters were fit using a completely separate dataset. Most interestingly, the model performs better with the test dataset than with the validation dataset, implying that its generalisation error is very low. The MAE, RMSE and Bias in this case are 0.0664, 0.0424 and -3.5e-3 respectively.



(a) ACF (b) Cum. Periodogram (c) Histogram

Fig. 10: Analysis of the residuals of the three state model ($T_iT_eT_f$) using the test dataset

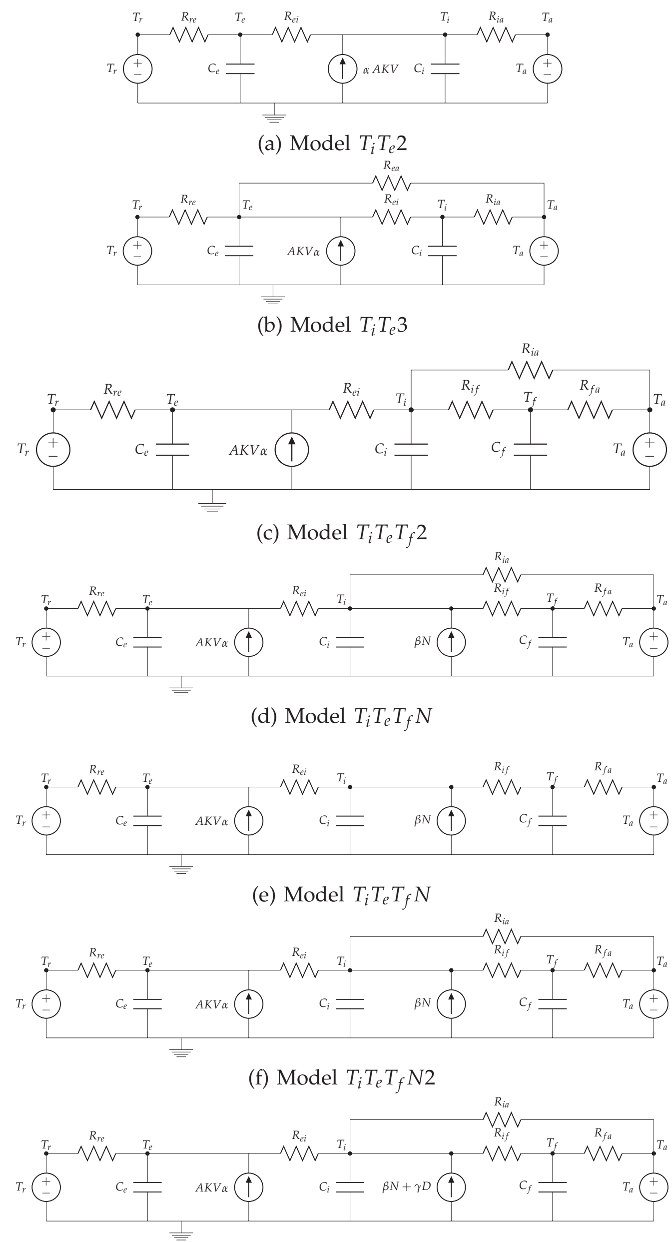
VI. Conclusions

A grey-box model describing the dynamics of a supermarket refrigeration display case has been developed in this work. The system is described by stochastic differential equations, reflecting the inherent stochasticity of a system with randomly occurring stimuli, including the addition and removal of foodstuffs, and the opening and closing of unit doors. The model selection procedure is described in detail, commencing with the simplest feasible model and continuing with increasing complexity until no further significant improvement occurs. Models are compared using a defined set of metrics describing the model residuals, and model performance and generalization error are evaluated by employing three independent datasets for model fitting, validation and testing. The results show that a three time constant model ($T_iT_eT_f$) is most appropriate for modelling the display case considered in this study. This model is specific to the display case considered here, and is not directly applicable to other display units, as the model is dependent on the foodstuff stored in the display case and the case structure, among other factors. Going forward, to facilitate participation in the smart grid, it would be advantageous for this modelling approach to be applied to a number of different display case units in different supermarkets. This will establish an overview of the performance and characteristics of a population of display units. The parameter values found in this work could then be used as a priori information to provide a starting point for further model development.

The model identified in this work has an important application in the development of novel local control strategies for supermarket refrigeration system. Advanced control strategies beyond the current hysteresis approach will enable greater flexibility throughout the refrigeration system. This is a key contribution towards achieving overall flexibility of power consumption of the supermarket refrigeration system. This has advantages for both the supermarket and power system operators; allowing the supermarket operator to optimise operations towards cost or energy efficiency, and facilitating the participation of demand in the electricity market and for the provision of power system services.

Appendix A
Equivalent RC-Networks of Models Investigated

The additional model configurations not detailed in the main body of this paper are presented here. Four further variables are introduced in the circuits below; βN introduces the impact of opening/closing hours of the supermarket into the model, where N is a binary input to the model indicating the regime (open or closed) and β is a parameter that is fit; γD introduces the impact of defrosting operations into the model, where D is a binary input to the model indicating the regime (defrost or regular operation) and γ is a parameter that is fit.



(a) Model T_iT_e2

(b) Model T_iT_e3

(c) Model $T_iT_eT_f2$

(d) Model $T_iT_eT_fN$

(e) Model $T_iT_eT_fN$

(f) Model $T_iT_eT_fN2$

(g) Model $T_iT_eT_fND$

Fig. 11: Alternative Model Configurations

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