Optimal Coupling of Heat and Electricity Systems: A Stochastic Hierarchical Approach

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***Abstract*—The large penetration of renewables in the power system increases the need for flexibility. A gain in flexibility and a reduction of wind curtailment problems can be achieved through a better coordination with other energy systems, in particular with district heating. Loose interactions between these two systems already exist due to the participation of CHPs in both markets. New market structures must be developed in order to exploit these synergies. Recognizing the abovementioned challenges this paper proposes a stochastic hierarchical formulation of the heat economic dispatch problem in a system with high penetration of CHPs and wind. The objective of this optimization problem is to minimize the heat production cost, subject to constraints describing day-ahead electricity market clearing scenarios. Uncertainties concerning wind power production, electricity demand and rival participants offers are efficiently modelled using a finite set of scenarios. This model takes advantage of existing market structures and provides a decision-making tool for heat system operators. The proposed model is implemented in a case study and results are discussed to show the benefits and applicability of this approach.**

***Index Terms*—Integrated energy systems, district heating, hierarchical optimization, MPEC, stochastic programming.**

NOMENCLATURE

	Indexes
t	Index for hours
g	Index for electricity generation units
w	Index for wind turbines
c	Index for CHP units
h	Index for heat-only units
ν	Index for day-ahead electricity market scenarios
	Variables
Q^0	Initial heat dispatch [MWh]
Q^+, Q^-	Redispatch of heat [MWh]
P	Power dispatch [MWh]
λ^{spot}	Spot price in the day-ahead electricity market [$/MWh]
μ	Dual variables in the day-ahead electricity market [-]
	Parameters
α	Marginal cost of thermal plants [$/MWh]
$\overline{F}$	Maximum hourly fuel input of thermal plants [MWh]
$\overline{Q}$	Maximum hourly heat production [MWh]
$\overline{P}$	Maximum hourly electricity production [MWh]
$\overline{W}$	Maximum hourly wind production [MWh]
R^D	Ramp-down limit [MWh/h]
R^U	Ramp-up limit [MWh/h]
r	Electricity/heat production ratio [-]
ρ^H	Fuel per heat unit [-]
ρ^E	Fuel per electricity unit [-]

I. INTRODUCTION

Due to growing environmental concerns and economic incentives renewable energy sources such as wind power have known a rapid growth. This large share of stochastic and non-dispatchable energy sources increases the need for flexibility in the power system. This issue can be addressed by achieving closer interaction with other energy systems, such as district heating [1], [2].

Combined heat and power units (CHPs) participate in both heat and electricity markets. In systems with high penetration of CHPs, the linkage between heat and power outputs imposes strong constraints to the electricity system. As a matter of fact CHPs' dispatch is traditionally driven by heat demand, while electricity is managed as a byproduct. This limited flexibility of CHPs can be a barrier for a large-scale penetration of stochastic generation as wind and solar power [3], [4].

These studies show that it is fundamental to develop new market structures in order to achieve an optimal coupling of heat and electricity systems and increase the flexibility of the overall system. [5] proposes a joint heat and power dispatch exploiting the existing storage capacity of district heating networks to increase the flexibility of CHPs. While [6] studies the optimal joint operation of heat storage and CHPs with a Nordic electricity market structure.

Moreover the large share of renewable energy sources increases the uncertainty and volatility in the electricity market. In order to deal with this issue, [7], [8] study the optimal dispatch of CHPs under uncertainty, using stochastic programming and robust optimization. These papers introduce uncertainty in the electricity market using electricity prices as exogenous uncertain parameters. However this approach doesn't allow us to model explicitly the influence of the participation of CHPs on both markets.

We propose a hierarchical optimization approach, where the heat dispatch is the upper-level optimization problem. The competition in the day-ahead electricity market is explicitly modelled in the lower-level problems. Hierarchical optimization, also referred to as mathematical problem with equilibrium constraints (MPEC), has found various applications in energy systems [9], [10]. Additionally we characterize the uncertainty in the day-ahead electricity market using uncertain wind production, electricity demand and rival participants bids.

This paper is organized as follows. Section II describes the proposed approach. Section III details the mathematical formulation consisting in a stochastic hierarchical model. Section IV provides the results of a case study. Finally, Section V concludes the paper with some relevant remarks.

II. Problem Description

A. Heat and Electricity Systems Framework

In this work we consider independent heat and electricity systems with high penetration of CHPs and wind production. In the Copenhagen area the day-ahead heat dispatch is performed by Varmelast.dk. Although the heat price is fixed based on long term contracts, the heat system operators prepares the heat dispatch trying to minimize heat production costs. Yet due to the linkage between heat and electricity outputs, the heat production cost of CHPs is calculated as the total production cost minus the expected revenues from electricity sales in the day-ahead market

$$\text{Cost}\left(P_{ct}, Q_{ct}\right) - \lambda_t^{spot} P_{ct}. \tag{1}$$

After the day-ahead heat dispatch CHPs can participate on the Nordpool electricity market. Yet their power output is constrained by the scheduled heat production. [11]. In this work we consider two types of CHPs. Back-pressure units produce heat and electricity at a fixed ratio r_c and their FOR can be modelled as

$$P_{ct} = r_c Q_{ct} \qquad \forall c \in BP, t \tag{2a}$$
$$0 \leq Q_{ct} \leq \overline{Q_c} \quad \forall c \in BP, t. \tag{2b}$$

Extraction units allow more flexibility for heat and electricity production. The joint feasible operating region (FOR) of an extraction unit can be non-convex and complex to model. We approximate it using a convex set of inequalities

$$0 \leq P_{ct} \geq r_c Q_{ct} \qquad \forall c \in E, t \tag{3a}$$
$$0 \leq \rho_c^H Q_{ct} + \rho_c^E P_{ct} \leq \overline{F_c} \quad \forall c \in E, t \tag{3b}$$
$$0 \leq Q_{ct} \leq \overline{Q_c} \qquad \forall c \in E, t \tag{3c}$$

where Equation (3a) represents the minimum heat/power ratio, (3b) the maximum fuel intake and (3c) the maximum heat output. Figure 1 shows the simplified FOR of an extraction unit.

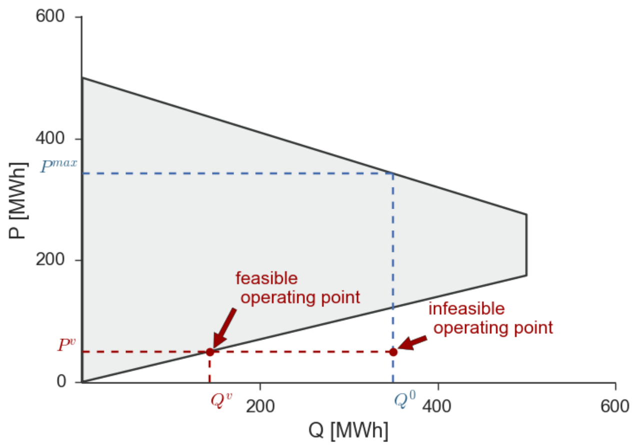

Fig. 1. Simplified FOR of an extraction unit

B. Hierarchical Setup

Our aim is to propose a decision-making tool for an independent heat system operator. The primary objective of heat system operators is to minimize the heat production cost, considering future revenues from the electricity market. This is incompatible with co-clearing approaches proposed in the literature [5], aiming at minimizing the overall operation cost.

In order to compute CHPs' heat cost, the heat system operator takes into account the expected revenues from electricity sales in the day-head market. In the current market framework these variables are unknown because the heat dispatch is performed before the day-ahead electricity market clearing. An approach that has been used in the literature is to model the electricity spot price as an exogenous uncertain parameter [7], [8]. In this work we consider a system with high penetration of CHPs. The influence of heat dispatch on the day-ahead electricity market clearing can not be neglected due to the linkage between heat and power output of CHPs. That is why it is essential to model explicitly the participation of CHPs in the electricity market in order to achieve informed decisions in the heat dispatch.

We formulate the decision-making problem of the heat system operator as a hierarchical optimization problem, in which the day-ahead heat dispatch is the upper-level optimization problem and the scenarios for day-ahead electricity market clearing are explicitly modelled in the lower-level optimization problems. The heat system operator seeks to minimize the cost of heat production, considering the day-ahead electricity market clearing as a constraint of its problem. A hierarchical structure is relevant, in order to make informed decisions, when the decisions in the upper-level problem strongly influence the lower-level problems. The hierarchical setup of this problem is outlined in Figure 2.

Additionally the uncertainty on the electricity market clearing is introduced through three exogenous parameters: wind production, electricity demand and other participants bids. We model this uncertainty using a finite set of scenarios

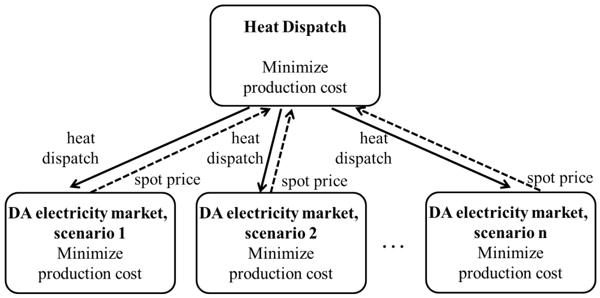

Fig. 2. Sketch of the hierarchical optimization setup

and use stochastic programming to solve the heat dispatch problem. For simplicity we denote ν the set of scenarios for all parameters and π_ν their probability.

III. Mathematical Formulation

A. Standard Stochastic Hierarchical Formulation

The standard form of the two-stage stochastic MPEC introduced above can be expressed as

$$\min_{\Theta} \; f^0\left(Q^0\right) + \mathbb{E}\left(f\left(Q_\nu^+, Q_\nu^-, P_\nu, \lambda_\nu\right)\right) \tag{4a}$$

$$s.t. \quad h^0\left(Q^0\right) \le 0 \tag{4b}$$

$$h\left(Q^0, Q_\nu^+, Q_\nu^-, P_\nu\right) \le 0 \qquad \forall \nu \tag{4c}$$

$$\left(P_\nu, \lambda_\nu, \mu_\nu\right) \in \underset{P_\nu \in H(Q^0)}{\arg\min} \left\{F\left(P_\nu\right)\right\} \quad \forall \nu \tag{4d}$$

where $\Theta = \{Q^0, Q_\nu^+, Q_\nu^-\} \bigcup \{P_\nu, \lambda_\nu\}$.

The upper-level problem consists in the minimization of the objective function (4a), subject to constraints (4b) and (4c) and the optimality of the lower level problems (4d). The decision variables of the upper-level problem Q^0 represent the initial heat dispatch. Additionally we consider that CHPs are allowed to default on the initial heat dispatch if it is not compatible with the electricity production dispatched in the day-ahead electricity market, as shown in Figure 1. The recourse variables Q_ν^+ and Q_ν^- represent the redispatch of CHPs and heat-only units. The upper-level objective function also depends on the lower-level variables P_ν and λ_ν of the lower-level problems (4d). This is due to the dependency of the heat cost on the revenues from electricity sales in the day-ahead electricity market as expressed in (1). Additionally the upper-level constraints (4c) link the upper-level variables Q^0, Q_ν^+, Q_ν^- to the lower-level variables P_ν, due to the linkage between heat and electricity outputs of CHPs (2)-(3).

The lower-level optimization problems (4d) represent the day-ahead electricity market clearing for each scenario ν. P_ν represents the primal variables of the lower-level problems and λ_ν, μ_ν represent their dual variables. The structure of the upper and lower-level problems are further detailed in section III-B.

B. Lower-Level Problems

For a given scenario ν, the lower-level problem expressed in (4d) solves the following simplified day-ahead electricity market clearing

$$\min_{\Gamma} \sum_{gt} \alpha_{gt\nu} P_{gt\nu} + \sum_{wt} \alpha_{wt\nu} P_{wt\nu} + \sum_{ct} \alpha_{ct}^0 \rho_c^E P_{ct\nu} \tag{5a}$$

$$\text{s.t.} \quad L_{t\nu}^{elec} = \sum_g P_{gt\nu} + \sum_c P_{ct\nu} + \sum_w P_{wt\nu} \qquad \forall t \quad : \lambda_{t\nu}^{spot} \tag{5b}$$

$$0 \le \; P_{gt\nu} \le \overline{P_{gt}} \qquad \forall g,t \; : \underline{\mu_{gt\nu}}, \overline{\mu_{gt\nu}} \tag{5c}$$

$$0 \le \; P_{ct\nu} \le \overline{P\left(Q_{ct}^0\right)} \qquad \forall c,t \; : \underline{\mu_{ct\nu}}, \overline{\mu_{ct\nu}} \tag{5d}$$

$$0 \le \; P_{wt\nu} \le \overline{P_{wt\nu}} \qquad \forall w,t : \underline{\mu_{wt\nu}}, \overline{\mu_{wt\nu}} \tag{5e}$$

where $\Gamma = \{P_{ct\nu}, P_{gt\nu}, P_{wt\nu}\}$ and

$$\overline{P\left(Q_{ct}^0\right)} = \begin{cases} \dfrac{\overline{F_h} - \rho_c^H Q_{ct}^0}{\rho_{ct}^E} & \forall c \in E \\ r_c Q_{ct}^0 & \forall c \in BP. \end{cases}$$

The lower-level problem for scenario ν consists in minimizing the electricity production cost (5a), while enforcing power balance (5b) and constrained by the generators maximum power output of thermal power generators, CHPs and wind turbines (5c)-(5e). Additionally the maximum power output of CHPs (5d) depends on the first-stage heat dispatch variable Q^0, which is considered as a fixed parameter in the lower-level problem.

Since the lower-level optimization problem expressed above is linear in the continuous variables $P_{gt\nu}$, $P_{wt\nu}$, and $P_{ct\nu}$, its Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient. And it can be reformulated as

$$\alpha_{gt\nu} - \lambda_{t\nu}^{spot} + \overline{\mu_{gt\nu}} - \underline{\mu_{gt\nu}} = 0 \quad \forall g,t \tag{6a}$$

$$\alpha_{ct}\rho_c^E - \lambda_{t\nu}^{spot} + \overline{\mu_{ct\nu}} - \underline{\mu_{ct\nu}} = 0 \quad \forall c,t \tag{6b}$$

$$\alpha_{wt\nu} - \lambda_{t\nu}^{spot} + \overline{\mu_{wt}} - \underline{\mu_{wt}} = 0 \quad \forall w,t \tag{6c}$$

$$0 \le \overline{\mu_{gt\nu}} \perp \left(P_{gt\nu} - \overline{P_{gt}}\right) \le 0 \quad \forall g,t \tag{6d}$$

$$0 \le \underline{\mu_{gt\nu}} \perp \left(-P_{gt\nu}\right) \le 0 \quad \forall g,t \tag{6e}$$

$$0 \le \overline{\mu_{ct\nu}} \perp \left(P_{ct\nu} - \overline{P\left(Q_{ct}^0\right)}\right) \le 0 \quad \forall c,t \tag{6f}$$

$$0 \le \underline{\mu_{ct\nu}} \perp \left(-P_{ct\nu}\right) \le 0 \quad \forall c,t \tag{6g}$$

$$0 \le \overline{\mu_{wt\nu}} \perp \left(P_{wt\nu} - \overline{P_{wt\nu}}\right) \le 0 \quad \forall w,t \tag{6h}$$

$$0 \le \underline{\mu_{wt\nu}} \perp \left(-P_{wt\nu}\right) \le 0 \quad \forall w,t \tag{6i}$$

$$\text{(5b)} \qquad \forall t \tag{6j}$$

$$\lambda_{t\nu}^{spot} \text{ free} \qquad \forall c,t. \tag{6k}$$

Equations (6a)-(6c) correspond to the gradient of the Lagrangian function $\vec{\triangle}\mathcal{L} = 0$. Equations (6d)-(6i) are the complementarity conditions $0 \le \mu \perp g \le 0$ corresponding to the inequality constraints (5c)-(5e). The bilinear complementarity conditions can be linearised using binary variables [12] or relaxed using an SOS1-based approach [13].

C. Upper-Level Problem

The upper-level optimization problem is the optimal heat dispatch. We consider a system with extraction and back-pressure CHPs and heat-only units. We neglect transmission constraints as well as start-up and shut-down costs. This problem can be formulated as follows

$$\begin{aligned}\min_{\Theta,\Gamma,\Lambda} & \sum_{ht} \alpha^0_{ht} Q^0_{ht} + \sum_{ct} \alpha^0_{ct} \rho^H_c Q^0_{ct} \\ & + \sum_{\nu} \pi_\nu \big[\sum_{ct} \left(\alpha^+_{ct} Q^+_{ct\nu} - \alpha^-_{ct} Q^-_{ct\nu} \right) \\ & \quad + \sum_{ht} \left(\alpha^+_{ht} Q^+_{ht\nu} - \alpha^-_{ht} Q^-_{ht\nu} \right) \\ & \quad + \sum_{ct} \left(\alpha^0_{ct} \rho^E_c P_{ct\nu} - \lambda^{spot}_{t\nu} P_{ct\nu} \right) \big] \end{aligned} \tag{7a}$$

$$\text{s.t.} \quad L^{heat}_t = \sum_h Q^0_{ht} + \sum_c Q^0_{ct} \qquad \forall t \tag{7b}$$

$$0 \le Q^0_{ct} \le \overline{Q_c} \qquad \forall c, t \tag{7c}$$

$$0 \le Q^0_{ht} \le \overline{Q_h} \qquad \forall h, t \tag{7d}$$

$$-R^D_c \le Q^0_{c(t+1)} - Q^0_{ct} \le R^U_c \qquad \forall c, t \tag{7e}$$

$$-R^D_h \le Q^0_{h(t+1)} - Q^0_{ht} \le R^U_h \qquad \forall h, t \tag{7f}$$

$$L^{heat}_t = \sum_h Q_{ht\nu} + \sum_c Q_{ct\nu} \qquad \forall t, \nu \tag{7g}$$

$$Q_{c,t,\nu} \in \mathcal{D}\left(P_{ct\nu}\right) \qquad \forall c, t, \nu \tag{7h}$$

$$0 \le Q_{h,t,\nu} \le \overline{Q_h} \qquad \forall h, t, \nu \tag{7i}$$

$$-R^D_c \le Q_{c(t+1)\nu} - Q_{ct\nu} \le R^U_c \qquad \forall c, t, \nu \tag{7j}$$

$$-R^D_h \le Q_{h(t+1)\nu} - Q_{ht\nu} \le R^U_h \qquad \forall h, t, \nu \tag{7k}$$

$$0 \le Q^+_{ct\nu}, Q^-_{ct\nu}, Q^+_{ht\nu}, Q^-_{ht\nu} \qquad \forall h, c, t, \nu \tag{7l}$$

$$(6a) - (6k) \qquad \forall \nu \tag{7m}$$

where

$$\begin{aligned} \Theta &= \left\{ Q^0_{ct}, Q^0_{ht}, Q^+_{ct\nu}, Q^-_{ct\nu}, Q^+_{ht\nu}, Q^-_{ht\nu} \right\} \\ Q_{ht\nu} &= Q^0_{ht} + Q^+_{ht\nu} - Q^-_{ht\nu} \\ Q_{ct\nu} &= Q^0_{ct} + Q^+_{ct\nu} - Q^-_{ct\nu} \end{aligned}$$

and $\mathcal{D}\left(P_{ct\nu}\right)$ represents the FOR of CHPs, as defined in (2) and (3).

The objective of this optimization problem is to minimize the expected heat production cost (7a), calculated as initial heat dispatch, plus the potential redispatch cost, minus the revenues from the day-ahead electricity market. We neglect fixed, start-up and shut-down costs, and approximate the production cost of heat-only units and CHPs as piecewise linear functions, proportional to the total fuel consumption. The first stage constraints represent the initial heat balance (7b), heat production limits (7c)-(7d), and ramping constraints (7e)-(7f). Equations (7g)-(7k) enforce the same constraints for the recourse variables in each scenario. In particular Equation (7h) represents the FOR of the CHPs and also depends on the lower-level variables $P_{ct\nu}$. Finally Equation (7m) represents the lower-level optimization problems, formulated as their equivalent KKT conditions.

Due to the bilinear terms $\lambda^{spot}_t P_{ct\nu}$ the upper-level objective function is non-linear. We provide below an exact linearisation of the objective function based on the expression of the KKT conditions (6b)-(6h) and the strong duality theorem, as explained in [14]:

$$\begin{aligned} & \sum_{ct} \lambda^{spot}_{t\nu} P_{ct\nu} \\ & = \sum_t \lambda^{spot}_{t\nu} L^E_{t\nu} - \sum_{gt} \left(\alpha_{gt\nu} P_{gt\nu} + \overline{\mu_{gt\nu}} \overline{P_{gt}} \right) \\ & \quad - \sum_{wt} \left(\alpha_{wt\nu} P_{wt\nu} + \overline{\mu_{wt\nu}} \overline{P_{wt\nu}} \right). \end{aligned} \tag{8}$$

IV. Case Study

A. Case Study Setup

In this case study we consider a heat system comprising one extraction unit, one back-pressure unit and one heat-only unit. The back-pressure unit is cheaper than the extraction unit but it is less flexible due to its fixed electricity/heat ratio and ramping limits. The heat-only unit is more expensive than both CHPs and completely flexible. The technical specifications of these units are listed in Table I. These parameters are loosely derived from values for existing units of the Copenhagen area as reported in [7]. For heat demand we use a modified version of data from the western Copenhagen area provided in [15].

TABLE I
Heat system parameters

	CHP_1	CHP_2	H_1
Type	*Extraction*	*Back-pressure*	*Heat-only*
$\overline{F}$	500	500	-
$\overline{Q}$	250	500	1000
R^U	150	150	1000
R^D	150	150	1000
Q^{init}	0	400	0
ρ^H	0.25	0.4	-
ρ^E	2.4	2.4	-
r	0.6	0.25	-
α^0	25	12.5	100
α^+	50	25	125
α^-	10	5	100

For the day-ahead electricity market we consider a system with two thermal power plants and a wind power producer. The technical specifications of these units are listed in Table II.

A major challenge in stochastic programming is to model accurately the probability density of the uncertain parameters in order to generate appropriate scenarios. The uncertain parameters of this model, namely wind production, electricity demand and rival participants' bids, are assumed independent. In addition we assume for simplification that the electricity

TABLE II
POWER SYSTEM PARAMETERS

	W_1	G_1	G_2
Type	*Wind*	*Thermal*	*Thermal*
$\overline{P}$	400	500	500
α	0	45	65
σ^2	-	5	10

demand is perfectly known. Figure 4 shows the heat and electricity loads used for this simulation. We use a set of wind production scenarios provided at [16]. Figure 3 shows these scenarios. For simplicity we assume mutually independent normal distributions for each producer's marginal cost and we draw five independent scenarios from these normal distributions.

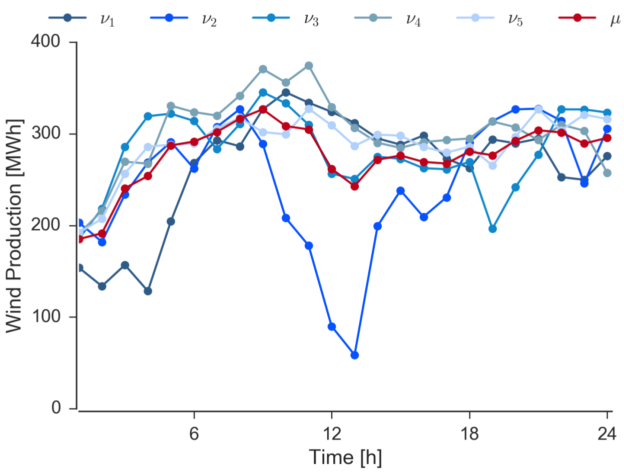

Fig. 3. Wind production scenarios

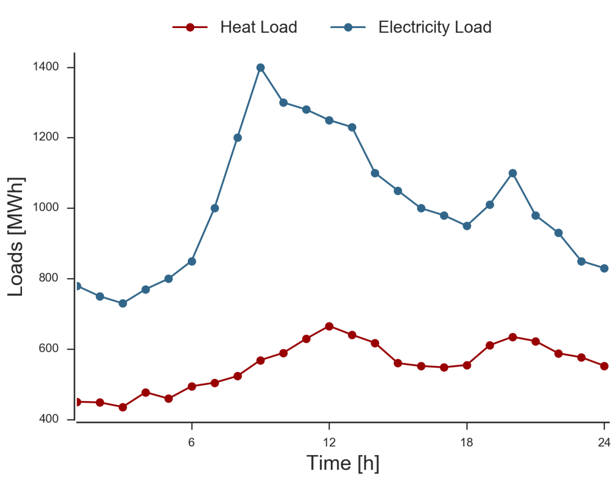

Fig. 4. Heat and electricity loads

B. Simulation Results

Figure 5 shows the results of the stochastic optimization model and compares them to the corresponding nominal model[1]. In the stochastic model the extraction CHP is less dispatched than in the deterministic model. This is due to the occurrence of certain scenarios where the extraction unit is not competitive in the day-ahead electricity market.

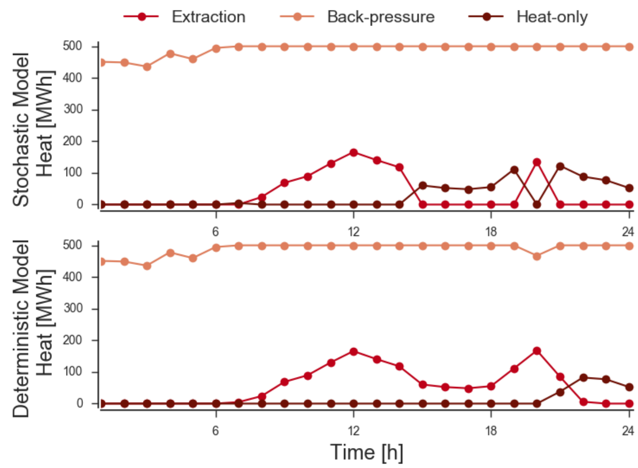

Fig. 5. Stochastic and deterministic models: Optimal heat dispatch

Figure 6 shows the heat and electricity dispatch in a specific scenario, where Generator 2 has a higher marginal cost than the extraction unit. The back-pressure unit is used to cover base-load, and the heat-only unit as a peaker. During time periods 7-10 the electricity output of the extraction unit is at its maximum and the electricity/heat ratio varies in order to adjust the heat production. This shows that the heat dispatch is performed optimally taking into account the electricity dispatch, contrary to heat-demand-driven dispatches.

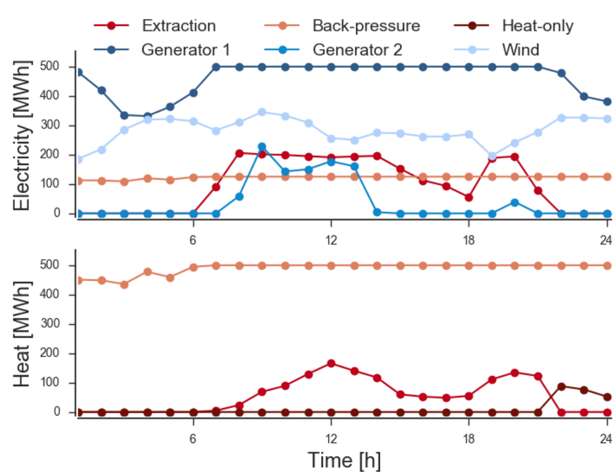

Fig. 6. Stochastic model: Heat and electricity dispatch for a specific scenario (ν_1, $\alpha_{g_1} = 37$ $\$/MWh$, $\alpha_{g_2} = 69$ $\$/MWh$)

[1]The code for this optimization problem is available at [17]

The value of stochastic solution (VSS) shows the benefit of including uncertainty in the decision-making, by comparing the optimal value of the stochastic problem to the average optimal value of the nominal problem over all scenarios. In this case study the VSS equals 2130 \$, i.e. a 4 % improvement compared to the average nominal optimal value. This indicator is calculated in-sample and a valuable addition for future works would be to compute this indicator out-of-sample.

V. Conclusion

The model proposed in this paper provides a method for optimal heat dispatch in the context of large penetration of wind power and CHPs. This problem is formulated as a stochastic MPEC where the heat system operator minimizes the heat production cost, subject to constraints representing various scenarios of day-ahead electricity market clearing. The uncertainties on the electricity market are modelled through a finite set of scenarios for wind power production, electricity demand and rival participants' offers.

The hierarchical structure allows the heat system operator to model explicitly the effect of the participation of CHPs on the day-ahead electricity market. This is crucial in order to make informed decisions. Additionally this coordination of heat and electricity systems allows the heat system to provide more flexibility to the power system by exploiting optimally the flexibility of extraction units. This approach can help mitigate the impact of inflexible CHP units in the power sector and reduce wind curtailment. Moreover, contrary to co-clearing of heat and electricity approaches proposed in the literature, this method provides a decision-making tool for heat system operators applicable in the current market framework.

This work opens up several directions for future research. First of all it is important to assess the benefits of this method in terms of increased flexibility and wind power integration. For that purpose we should compare it to other approaches, such as co-clearing, and existing decoupled deterministic market clearing. We should also asses out-of-sample the benefits of including uncertainty in the decision-making. Additionally the proposed model can be upgraded by introducing heat storage and heat pumps in order to allow inter-temporal and fuel arbitrages; thus providing more flexibility to the overall system. Including network constraints can be another natural extension. Furthermore MPECs are complex non-linear models that can become intractable as the size of the system and the number of scenarios increase. In order to implement this model on large-scale heat and power systems and integrate a larger number of scenarios that capture accurately the uncertainties we can use decomposition techniques as suggested in [18]. Finally including the balancing stage can be an interesting extension to this model in order to provide more flexibility to the power system.

Acknowledgement

This work was partly supported by the Danish Energy Development Programme (EUDP) through the EnergyLab Nordhavn project (Grant id: EUDP 64015-0055). The authors also wish to acknowledge Dr. Jalal Kazempour (DTU), Tue Vissing Jensen (DTU) and Nicolò Mazzi (Unipd) for discussion and feedback on previous versions of this work.

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