Price-Taker Offering Strategy in Electricity Pay-as-Bid Markets

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Abstract—The recent increase in the deployment of renewable energy sources may affect the offering strategy of conventional producers, mainly in the balancing market. The topics of optimal offering strategy and self-scheduling of thermal units have been extensively addressed in the literature. The feasible operating region of such units can be modeled using a mixed-integer linear programming approach, and the trading problem as a linear programming problem. However, the existing models mostly assume a uniform pricing scheme in all market stages, while several European balancing markets (e.g., in Germany and Italy) are settled under a pay-as-bid pricing scheme. The existing tools for solving the trading problem in pay-as-bid electricity markets rely on non-linear optimization models, which, combined with the unit commitment constraints, result in a mixed-integer non-linear programming problem. In contrast, we provide a linear formulation for that trading problem. Then, we extend the proposed approach by formulating a two-stage stochastic problem for optimal offering in a two-settlement electricity market with a pay-as-bid pricing scheme at the balancing stage. The resulting model is mixed-integer and linear. The proposed model is tested on a realistic case study against a sequential offering approach, showing the capability of increasing profits in expectation.

Index Terms—Pay-as-bid, offering strategy, stochastic programming, mixed-integer linear program, thermal unit.

Nomenclature

Indices and Sets

$i, i'\,(I)$	Indices of day-ahead market price scenarios
$j, j'\,(J)$	Indices of balancing market price scenarios
$s\,(S)$	Index of generation blocks
$k\,(K)$	Index of time intervals
Π	Feasible region of the unit's offer curves
Ω	Feasible region of the unit's operation

Parameters

$\lambda_{ik}^{\mathrm{DA}}$	Day-ahead market price [€/MWh]
$\lambda_{ijk}^{\mathrm{BA}}$	Balancing market price [€/MWh]
$M_{jj'k}^{\mathrm{UP}}$	Acceptance matrix for up-regulation offers
$M_{jj'k}^{\mathrm{DW}}$	Acceptance matrix for down-regulation offers
$\overline{E}$	Capacity of the unit [MW]
$\underline{E}$	Minimum power limit of the unit [MW]
$R^{\mathrm{UP}}, R^{\mathrm{DW}}$	Ramp-up and down limits [MW/h]

E_s	Size of generation block s [MW]
$C^{\mathrm{UP}}, C^{\mathrm{DW}}$	Start-up and shut-down costs [€]
C_0	Cost at the minimum production level [€]
C_s	Marginal cost of generation block s [€/MWh]
π_i^{DA}	Probability of day-ahead price scenario i
π_{ij}^{BA}	Probability of balancing price scenario j, provided that day-ahead price scenario i realizes

Variables

q_{ik}^{DA}	Quantity offer at day-ahead market [MWh]
$o_{ijk}^{\mathrm{UP}}, o_{ijk}^{\mathrm{DW}}$	Up/down regulation incremental offer [MWh]
$q_{ijk}^{\mathrm{UP}}, q_{ijk}^{\mathrm{DW}}$	Up/down regulation quantity offer [MWh]
q_{ijk}^{A}	Total production quantity [MWh]
c_{ijk}	Operational cost [€/MWh]
$\rho_{ijk}^{\mathrm{UP}}, \rho_{ijk}^{\mathrm{DW}}$	Up/down regulation profit [€]
y_{ijk}, z_{ijk}	Start-up/shut-down (binary) status of the unit
u_{ijk}	Commitment (binary) status of the unit

Functions

$g(\cdot), h(\cdot)$	Operational cost functions

I. Introduction

IN recent years the power sector has experienced a significant increase in the deployment of renewable energy sources, such as wind and solar power. These sources are usually traded at zero marginal cost and their growing penetration is leading to a decrease of the prices in the day-ahead market [1]-[2]. Moreover, they can only be predicted with a limited accuracy, thus leading to real-time imbalances and increasing the need for balancing energy. These changes may affect the strategy of the conventional producers in both day-ahead and balancing markets.

A. Literature Review

The optimal offering strategy and self-scheduling of conventional thermal units have already been widely studied in the literature. Ref. [3] addresses the optimal response of a thermal generator to a given set of electricity market prices in terms of both energy and reserve. A mixed-integer linear programming (MILP) problem is developed considering a non-convex cost function, as well as its start-up costs, ramp rates and minimum-up and -down constraints. The same authors in [4] propose a detailed formulation to model start-up and shut-down characteristics of a thermal generator. Other works, such as [5] and [6], include risk measures while optimizing

the self-scheduling problem of thermal units. References [3]-[6] demonstrate that a detailed modeling of the generator's feasibility region and its production cost function may be essential for deriving its optimal self-scheduling. Indeed, the inter-temporal constraints (e.g., ramping constraints) and non-convex costs (e.g., start-up and shut-down costs) may affect the optimal solution. In this context, the pioneering paper [7] presents an offering strategy for a price-taker producer under price uncertainty. It develops a set of rules that aim to translate the results of a self-scheduling problem into market offers. Ref. [8] presents an algorithm for offering and self-scheduling of a unit including risk management. Ref. [9] presents an offering strategy for a price-taker power producer that aims to maximize profit expectation while hedging against possible infeasible schedules. Other works relax the price-taker assumption and develop tools for strategic offering considering the impact of power producer's decisions on market prices. This can be done through a residual demand model [10] or a bilevel optimization setup [11]-[12].

By analyzing the optimization models in [3]-[12], we can identify two different sets of variables and constraints. The first set defines the feasibility region and the cost function of the production unit. For instance, references [3]-[6] show how to successfully model it as a MILP problem. The second set simulates the trading problem, i.e., how the power producer participates in the market (e.g., through non-decreasing step-wise offering curves), while considering the market clearing mechanism (endogenously or exogenously) and the pricing scheme (e.g., uniform or pay-as-bid). The trading problem can be modeled using a linear programming (LP) approach [13], under price-taker assumptions and uniform pricing scheme. However, even though European day-ahead electricity markets are mostly settled under a uniform pricing scheme, several balancing markets (e.g., in Germany and Italy [14]) are settled under a pay-as-bid pricing scheme.

The topic of trading under a pay-as-bid scheme and price uncertainty has not been extensively addressed in the literature. An analysis on optimal offering under pay-as-bid and uniform pricing schemes is presented in [15] and [16]. They obtain profit expectation and variance for both pricing schemes, while assuming that the market price follows a uniform distribution. Ref. [17] proposes a methodology that aims to maximize the profit expectation in a day-ahead pay-as-bid auction for power system reserve. Offering strategies for a joint energy and spinning reserve market under pay-as-bid pricing are presented in [18] and [19]. Risk aversion is introduced in [20]. References [15]-[20] show how to model the trading problem under pay-as-bid pricing scheme using a non-linear programming (NLP) approach. However, they do not consider an accurate modeling of production unit's operational constraints. Introducing the feasibility region would result in a mixed-integer non-linear problem (MINLP), which may have high computational cost and, generally, do not guarantee the optimality of the solution. It is worth mentioning that [19] proposes to solve the trading problem in pay-as-bid markets under price uncertainty with a two-step approach, obtaining the expected profit as a linear function of the quantity offer. However, this approach is not applicable in case of problems with inter-temporal constraints

or with more complex cost functions.

B. Approach and Contributions

Compared to the available literature, this paper provides a novel approach that allows to cast the optimal price-taker trading problem in pay-as-bid markets under price uncertainty as an LP problem. For that purpose, continuous random variables (i.e., market-clearing prices) are represented as discrete variables. We test our formulation against the existing continuous NLP alternative. In a simple setup, these two models bring similar optimal solutions, while our LP approach drastically reduces the computational cost. Hence, we demonstrate that our LP model is a good approximation of the NLP one. However, the value of our LP formulation, with respect to the NLP one, arises when including the feasible operating region of the unit.

Then, we use the proposed LP approach to build a multi-stage stochastic programming problem with recourse. This efficient decision-making tool could be used by a price-taker conventional producer to derive its best day-ahead market offer curves. In line with current practice in several European electricity markets, we consider a two-settlement market framework, in which the day-ahead market is cleared based on a uniform pricing scheme, while a pay-as-bid pricing scheme is used in the balancing stage. The forecast market prices in both stages are given but uncertain. This uncertainty is properly characterized by generating a set of foreseen scenarios. The resulting model is a stochastic MILP problem, where non-convexities (i.e., binary variables) arise from the unit commitment constraints. To the best of our knowledge, this kind of stochastic MILP optimization model for obtaining the offering strategy of a price-taker thermal producer in a two-settlement electricity market with a pay-as-bid pricing scheme in the balancing stage is not available in the literature. It is worth mentioning that [12] provides a formulation for obtaining optimal offering curves in markets settled under a pay-as-bid pricing scheme for a price-maker producer. However, market problems with equilibrium constraints may have high computational cost and rely on strong assumptions on opponents' behavior. Hence, when the production unit has a negligible impact on the market, a price-maker setup may not be the preferable choice.

C. Paper Organization

The remaining of the paper is organized as follows. Section II presents the electricity market framework, the modeling assumptions, and the methodology for generating market price scenarios. Section III provides an overview of the existing NLP setup as well as the proposed LP setup for deriving the offering strategy of a price-taker producer under a pay-as-bid pricing scheme. Then, Section IV extends the proposed approach and develops a two-stage model for a price-taker producer to derive its best offering curves in the day-ahead market, considering a pay-as-bid pricing scheme in the balancing stage. Section V presents a verification test to assess the performance of the proposed LP trading model, as well as an application of the two-stage model using a realistic test case. Finally, the conclusions are drawn in Section VI.

II. MODELING ASSUMPTIONS AND MARKET PRICE SCENARIOS

We consider a single conventional producer that trades in a two-settlement electricity market framework. The day-ahead market is cleared once a day, at noon, simultaneously for the whole 24 hourly trading periods of the following day. Generators are remunerated under a uniform pricing scheme in the day-ahead market. Then, a balancing market is cleared separately per each hourly interval, one hour prior to real-time operation. The provision of balancing energy is remunerated under a pay-as-bid pricing scheme. The intra-day trading floor is neglected for the sake of simplicity.

The power producer is assumed to be price-taker in both day-ahead and balancing markets. Hence, the market prices within the offering strategy problem of that producer are exogenous, but still uncertain. We model those uncertainties using a set of scenarios. Uncertainty characterization is a critical input to stochastic optimization. The quality of the solution of a stochastic optimization model is indeed strongly influenced by the quality of the scenarios provided as input. Given that the purpose of this paper is to analyze and test an optimization model, we exploit a fundamental model for generating market prices, instead of using real market data. This fundamental model generates a set of electricity market price forecasts, which is required as an input to our proposed offering strategy.

In the fundamental market model we assume, for the sake of simplicity, that the only stochastic generation is wind power generation. A dataset of wind power forecasts for a wind farm located in Denmark is used. The wind power forecasts are re-scaled and assumed representative of the aggregated wind power production in the market area. At the day-ahead stage, we assume that the demand curve is linear, known, and different per each hourly interval. Conversely, the supply curve of conventional producers is quadratic and uncertain. To model this uncertainty, we consider the coefficient of the second-degree term (i.e., γ_k, where k is the index of time interval) as a random variable with known marginal distribution. The methodology for fitting such distribution is beyond the scope of the paper. The coefficient of the first-degree term is also considered known to simplify the process of scenario generation. Then, we assume that the stochastic generation is offered in the day-ahead market at its mean forecast and at zero price.

At the balancing stage, the supply curve is assumed known but different from the day-ahead one. Indeed, the participants in the balancing market (under pay-as-bid pricing scheme) do not offer their marginal cost, since they have to internalize the expected revenues into their market offers [15]. Therefore, we fix a negative price floor λ_0 and impose $\gamma_k^{\text{BA}} = \eta \, \gamma_k$ ($\eta > 1$), where γ_k^{BA} is referred to the supply curve in the balancing market. Several factors may cause the real-time power imbalance in the system, e.g., errors in load and wind forecasts. For the sake of simplicity, we consider the wind stochasticity as the only source of uncertainty at the balancing stage. This simplifies the scenario generation process.

A. Market Model

The demand curve of the day-ahead market at hourly interval k is

$$p_k^{\text{DA,d}} = \alpha_k + \delta \, e_k^{\text{DA,d}}, \tag{1}$$

where $e_k^{\text{DA,d}}$ is the amount of energy demand at price $p_k^{\text{DA,d}}$. The parameters α_k and δ control the shape of the demand curve. For the same interval k, the supply curve is

$$p_k^{\text{DA},s} = \begin{cases} 0, & \text{if } \Delta_k^{\text{DA,s}} \leq 0 \\ \beta \, \Delta_k^{\text{DA,s}} + \gamma_k \left(\Delta_k^{\text{DA,s}} \right)^2, & \text{otherwise} \end{cases} \tag{2}$$

where $\Delta_k^{\text{DA,s}} = e_k^{\text{DA},s} - W_k^{\text{DA}}$. Note that $p_k^{\text{DA},s}$ is the price for scheduling the quantity $e_k^{\text{DA},s}$, and W_k^{DA} is the amount of wind power production offered in the day-ahead market. The parameters β and γ_k control the shape of the supply curve. The value of W_k^{DA} is computed as

$$W_k^{\text{DA}} = \mathbb{E}\left[w_k \right] \overline{W}, \tag{3}$$

where w_k is the normalized value ($w_k \in [0, 1]$) of wind power production, and $\overline{W}$ is the total installed wind capacity. The uncertain parameter γ_k follows a Normal distribution, i.e.,

$$\gamma_k \sim \mathcal{N}\left(\mu_\gamma, \sigma_\gamma^2 \right), \tag{4}$$

where μ_γ and σ_γ^2 are the mean value and variance of γ_k, respectively. In the balancing market, the supply curve at time interval k is

$$p_k^{\text{BA},s} = \begin{cases} \lambda^0, & \text{if } \Delta_k^{\text{BA},s} \leq 0 \\ \beta_k^{\text{BA}} \Delta_k^{\text{BA},s} + \gamma_k^{\text{BA}} \left(\Delta_k^{\text{BA},s} \right)^2 + \lambda^0, & \text{otherwise} \end{cases} \tag{5}$$

where $\Delta_k^{\text{BA},s} = e_k^{\text{BA},s} - e_k^0$. The variables $p_k^{\text{BA},s}$ and $e_k^{\text{BA},s}$ are the price and quantity of the balancing market supply curve, respectively. The term $e_k^{\text{BA},s}$ is obtained as the difference between e_k^{DA} and the imbalance generated by the stochastic generation, i.e.,

$$e_k^{\text{BA},s} = e_k^{\text{DA}} - \left(w_k \overline{W} - W_k^{\text{DA}} \right). \tag{6}$$

Parameters β_k^{BA} and e_k^0 are evaluated by imposing that $p_k^{\text{BA},s} = \lambda_k^{\text{DA}}$ when $e_k^{\text{BA},s} = e_k^{\text{DA}}$. This ensures that the day-ahead and the balancing market prices coincide when no balancing power is required.

B. Scenario Generation

We generate scenarios following a methodology presented in [21] and [22]. These papers propose a method for generating trajectories of a stochastic process when predictive distributions are available. The idea is to convert series of forecast errors into a multivariate Gaussian random variable and use a unique covariance matrix to describe its interdependence structure. This covariance matrix can be modeled through an exponential covariance function [22], where an exponential parameter ($\nu > 1$) controls the correlation between different lead times.

First, a set of scenarios $\{\gamma_{ik} : i \in I, k \in K\}$ is generated while fixing the exponential parameter ν to 5. Then, a set of market price trajectories $\{\lambda_{ik}^{\text{DA}} : i \in I, k \in K\}$ is generated,

where the market-clearing price λ_{ik}^{DA} is obtained from the intersection between the demand and the supply curves at interval k under scenario i.

For the balancing stage, we generate a set of wind power production trajectories $\{w_{jk} : j \in J, k \in K\}$ by fixing the exponential parameter ν to 7 as suggested in [22]. The probabilistic forecasts of wind power production are available in form of 19 quantiles (from 0.05 to 0.95). To fit the cumulative distribution function we follow the approach of [23]. Then, a set of market price scenarios $\{\lambda_{ijk}^{\text{BA}} : i \in I, j \in J, k \in K\}$ is generated by clearing the balancing market model.

III. TRADING PROBLEM UNDER A PAY-AS-BID PRICING SCHEME

When trading in an electricity market, power producers can usually submit price-quantity offers. The quantity identifies the amount of energy they are willing to produce, and the price is the minimum price for which they are willing to produce that energy. Then, the offer is accepted only when the market price is higher than or equal to the offered one. Since the given producer is assumed to be price-taker, the market price in each future time interval is necessarily treated as an exogenous but uncertain parameter [4]-[5]. In this paper, the market price λ is considered as a random variable following the density function $f^{\lambda} : \mathbb{R} \mapsto \mathbb{R}^{+}$. Given a price-quantity offer of a producer, denoted as (p, q), the acceptance probability of the offer is

$$\mathbb{P}\left[\lambda \geq p\right] = \int_{p}^{\infty} f^{\lambda}(l) \, \mathrm{d}l, \tag{7}$$

where l is an auxiliary integration variable. Under a pay-as-bid pricing scheme, the expected remuneration price p^{*}, providing that the producer's offer is being accepted, is computed as

$$\mathbb{E}\left[p^{*}|\lambda \geq p\right] = p. \tag{8}$$

The expected return ρ of the producer, following [18], is

$$\mathbb{E}\left[\rho\right] = \mathbb{P}\left[\lambda \geq p\right] \mathbb{E}\left[p^{*}|\lambda \geq p\right] q. \tag{9}$$

By replacing (7) and (8) in (9) we obtain

$$\mathbb{E}\left[\rho\right] = q \, p \int_{p}^{\infty} f^{\lambda}(l) \, \mathrm{d}l. \tag{10}$$

Notice that the expected return in (10) is non-linear. The offers in generation-side of real-world electricity market are generally non-decreasing step-wise functions, which can be modeled through a set of B price-quantity offers $\{(p_b, q_b), b = 1, .., B\}$. The formulation in (10) is extended to the generic model (11) below including the multiple offer blocks, i.e.,

$$\max_{q_b, p_b} \quad \sum_{b} \mathbb{E}\left[\rho_b\right] - \mathbb{E}\left[c_b\right] \tag{11a}$$

$$\text{s.t.} \quad \mathbb{E}\left[\rho_b\right] = p_b \, q_b \int_{p_b}^{p_{b+1}} f^{\lambda}(l) \, \mathrm{d}l, \; \forall b \tag{11b}$$

$$\mathbb{E}\left[c_b\right] = h(q_b) \int_{p_b}^{p_{b+1}} f^{\lambda}(l) \, \mathrm{d}l, \; \forall b \tag{11c}$$

$$q_b \geq q_{b-1}, \; \forall b \tag{11d}$$

$$p_b \geq p_{b-1}, \; \forall b \tag{11e}$$

$$\underline{E} \leq q_b \leq \overline{E}, \; \forall b \tag{11f}$$

where $p_{B+1} = \infty$. The parameter c_b is the operational cost for producing the quantity q_b, whose value is computed through the function $h(\cdot)$. Constraints (11b) and (11c) compute the expected return ρ_b and the operational cost of the offering block (p_b, q_b), respectively. Constraints (11d) and (11e) impose the non-decreasing condition of the offering curve. Finally, (11f) imposes the minimum and maximum production levels of the unit.

One of the main contributions of this work is to derive an alternative linear formulation to (11), which is non-linear. First, we discretize the continuous random variables. The uncertain market price can be represented using a set of possible scenarios $\{\lambda_i, i = 1, .., N\}$, where each price scenario λ_i is associated with a probability π_i such that $\sum_i \pi_i = 1$. We consider each price scenario λ_i as the potential offer price of the price-taker producer, and obtain the optimal quantity offer q_i corresponding to each offer price λ_i. A collection of all price-quantity offers, i.e., (λ_i, q_i), builds the offer curve of the producer, providing that the following conditions are enforced to ensure that the offer curve is non-decreasing [13]:

$$q_i \geq q_{i'} \; \text{if} \; \lambda_i \geq \lambda_{i'} , \; \forall i, \forall i', \tag{12a}$$

$$q_i = q_{i'} \; \text{if} \; \lambda_i = \lambda_{i'} , \; \forall i, \forall i', \tag{12b}$$

where i and i' are indices of the market price scenarios. Note that this offer curve is now scenario-independent, i.e., it is adapted to all scenarios, though it is built based on scenario-dependent price-quantity offers. Under a uniform pricing scheme, the expected market return ρ can be computed as

$$\mathbb{E}[\rho] = \sum_{i} \pi_i \lambda_i q_i, \tag{13}$$

However, the market return formulation (13) needs to be changed under the pay-as-bid scheme, since each block offer is remunerated at its corresponding offer price. Therefore, we introduce variable o_i, which represents the additional quantity offered at price λ_i. Figure 1 illustrates an example offer curve with three offer blocks. In this curve, λ_1, λ_2 and λ_3 are not only the three price scenarios, but also they are price offers of the price-taker producer. For example, the producer offers its q_1 MWh at price λ_1 (as the first offer block), and then the additional o_2 MWh (i.e., q_2 - q_1) at price λ_2 (as the second offer block). The total quantity q_i corresponding to price λ_i can be computed as

$$q_i = \sum_{i'} M_{ii'} \, o_{i'}, \tag{14}$$

where the acceptance matrix M is defined as

$$M_{ii'} = \begin{cases} 1, & \text{if} \; \lambda_i \geq \lambda_{i'} \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

Matrix $M_{ii'}$ indicates whether the offer block $(\lambda_{i'}, o_{i'})$ is accepted in the market, providing that the market price realization is λ_i. The total expected return is computed as the sum of the expected returns for each offer block $(\lambda_{i'}, o_{i'})$, denoted as $\rho_{i'}$, i.e.,

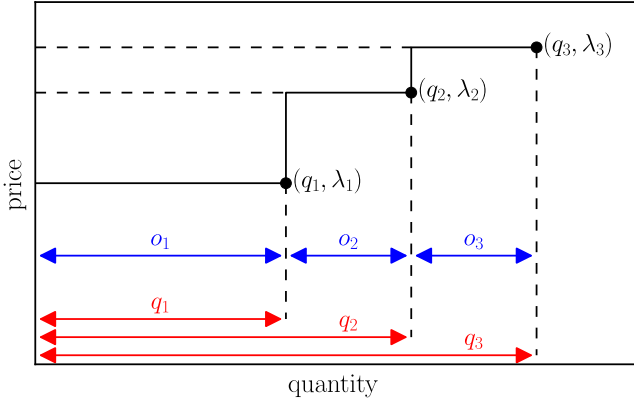$$\mathbb{E}[\rho] = \sum_{i'} \mathbb{E}[\rho_{i'}]. \tag{16}$$

Fig. 1. An example offer curve with multiple blocks, which shows how market price scenarios are used as price offers, building an offer curve.

In line with (9), we compute the expectation of $\rho_{i'}$ as

$$\mathbb{E}\left[\rho_{i'}\right] = \mathbb{P}\left[\lambda \geq \lambda_{i'}\right] \mathbb{E}\left[p^*|\lambda \geq \lambda_{i'}\right] o_{i'}, \quad (17)$$

where the acceptance probability of each block offer is

$$\mathbb{P}\left[\lambda \geq \lambda_{i'}\right] = \sum_i M_{ii'}\pi_i, \quad (18)$$

and the expected remuneration price is

$$\mathbb{E}\left[p^*|\lambda \geq \lambda_{i'}\right] = \lambda_{i'}. \quad (19)$$

Substituting (18) and (19) in (17) renders

$$\mathbb{E}\left[\rho_{i'}\right] = o_{i'}\lambda_{i'} \sum_i M_{ii'}\pi_i. \quad (20)$$

The total expected return can thus be computed as

$$\mathbb{E}\left[\rho\right] = \sum_{i'} o_{i'}\lambda_{i'} \sum_i M_{ii'}\pi_i. \quad (21)$$

Note that (21) is linear. The expected profit $\mathbb{E}\left[\rho\right]$ can also be seen as $\sum_i \mathbb{E}\left[\rho_i\right]$, where ρ_i is the return when scenario i realizes. Therefore, we rewrite (21) as

$$\mathbb{E}\left[\rho\right] = \sum_i \pi_i\rho_i = \sum_i \pi_i \sum_{i'} M_{ii'}\lambda_{i'}o_{i'}. \quad (22)$$

Given the linear formulation in (22), we rewrite the generic nonlinear model (11) in a linear manner, i.e.,

$$\underset{\Theta}{\text{Max}} \quad \sum_i \pi_i\left[\rho_i - c_i\right] \quad (23a)$$

$$\text{s.t.} \quad q_i = \sum_{i'} M_{ii'}\, o_{i'}, \ \forall i \quad (23b)$$

$$\rho_i = \sum_{i'} M_{ii'}\, \lambda_{i'}\, o_{i'}, \ \forall i \quad (23c)$$

$$c_i = h(q_i), \ \forall i \quad (23d)$$

$$\underline{E} \leq q_i \leq \overline{E}, \ \forall i \quad (23e)$$

where $\Theta = \{q_i, o_i, c_i, \rho_i, \forall i\}$. Constraints (23d) compute the production cost through the function $h(\cdot)$. It is worth mentioning that the non-decreasing conditions are not necessary in (23) since they are inherently included in the way we compute the accepted quantity q_i in (23b). The performance of both models, i.e., (11) and (23), is analyzed and compared using a simple case study in Section V-A.

IV. OFFERING STRATEGY AS A TWO-STAGE STOCHASTIC OPTIMIZATION MODEL

Given the linear offering strategy setup under a pay-as-bid pricing scheme proposed in Section III, we derive the optimal offering curves of a conventional producer in the two-settlement electricity market described in Section II. In this model, we also consider the unit commitment constraints of that producer. At noon, the power producer has to submit its offering curves for the day-ahead market of the following day, based on the price scenarios for both day-ahead and balancing markets. In this two-stage setup, the producer maximizes its expected profit from both markets simultaneously, in the sense that it endogenously determines its future balancing actions while solving its offering problem in the day-ahead. Accordingly, we model the day-ahead production level in time interval k, i.e., q_k^{DA}, as first-stage (here-and-now) decision, and the up and down production adjustments in the real-time stage, i.e., q_{ik}^{UP} and q_{ik}^{DW}, as second-stage (wait-and-see) variables. Following the approach described in Section III for building the producer's offer curve, we now relax the day-ahead production variable to be scenario-dependent (i.e., $q_k^{\text{DA}} \to q_{ik}^{\text{DA}}$). Similarly, the real-time production adjustments q_{ik}^{UP} and q_{ik}^{DW} are made scenario-dependent (i.e., $q_{ik}^{\text{UP}} \to q_{ijk}^{\text{UP}}$, $q_{ik}^{\text{DW}} \to q_{ijk}^{\text{DW}}$) in order to build the producer's offer curve in the balancing stage. In addition, we add a detailed representation of the feasible operating region of the thermal unit. The optimization model that the power producer solves to decide the day-ahead market offers reads as follows

$$\underset{\Xi}{\text{Max}} \quad \sum_{ijk} \pi_i^{\text{DA}}\pi_{ij}^{\text{BA}}\left[\lambda_{ik}^{\text{DA}}q_{ik}^{\text{DA}} + \rho_{ijk}^{\text{UP}} - \rho_{ijk}^{\text{DW}} - c_{ijk}\right] \quad (24a)$$

$$\text{s.t.} \quad q_{ijk}^{\text{A}} = q_{ik}^{\text{DA}} + q_{ijk}^{\text{UP}} - q_{ijk}^{\text{DW}}, \ \forall i, \forall j, \forall k \quad (24b)$$

$$q_{ijk}^{\text{UP}} = \sum_{j'} M_{ijj'k}^{\text{UP}}\, o_{ij'k}^{\text{UP}}, \ \forall i, \forall j, \forall k \quad (24c)$$

$$q_{ijk}^{\text{DW}} = \sum_{j'} M_{ijj'k}^{\text{DW}}\, o_{ij'k}^{\text{DW}}, \ \forall i, \forall j, \forall k \quad (24d)$$

$$\rho_{ijk}^{\text{UP}} = \sum_{j'} \lambda_{ij'k}^{\text{BA}}\, M_{ijj'k}^{\text{UP}}\, o_{ij'k}^{\text{UP}}, \ \forall i, \forall j, \forall k \quad (24e)$$

$$\rho_{ijk}^{\text{DW}} = \sum_{j'} \lambda_{ij'k}^{\text{BA}}\, M_{ijj'k}^{\text{DW}}\, o_{ij'k}^{\text{DW}}, \ \forall i, \forall j, \forall k \quad (24f)$$

$$q_{ik}^{\text{DA}}, o_{ijk}^{\text{UP}}, o_{ijk}^{\text{DW}} \in \Pi, \ \forall i, \forall j, \forall k \quad (24g)$$

$$q_{ijk}^{\text{A}}, u_{ijk}, y_{ijk}, z_{ijk} \in \Omega, \forall i, \forall j, \forall k \quad (24h)$$

$$c_{ijk} = \text{g}(q_{ijk}^{\text{A}}, u_{ijk}, y_{ijk}, z_{ijk}), \ \forall i, \forall j, \forall k \quad (24i)$$

$$q_{ik}^{\text{DA}}, o_{ijk}^{\text{UP}}, o_{ijk}^{\text{DW}} \geq 0, \ \forall i, \forall j, \forall k \quad (24j)$$

$$u_{ijk}, y_{ijk}, z_{ijk} \in \{0, 1\}, \ \forall i, \forall j, \forall k \quad (24k)$$

where

$$\Xi = \{q_{ik}^{\text{DA}}, o_{ijk}^{\text{UP}}, o_{ijk}^{\text{DW}}, q_{ijk}^{\text{UP}}, q_{ijk}^{\text{DW}}, \rho_{ijk}^{\text{UP}}, \rho_{ijk}^{\text{DW}}, q_{ijk}^{\text{A}},$$
$$u_{ijk}, y_{ijk}, z_{ijk}, x_{ijks}, c_{ijk} : \forall i, \forall j, \forall j', \forall s, \forall k\}.$$

The acceptance matrices $M_{jj'k}^{\mathrm{UP}}$ and $M_{jj'k}^{\mathrm{DW}}$ are defined as

$$
M_{ijj'k}^{\mathrm{UP}} = \begin{cases} 1, & \text{if } \lambda_{ijk}^{\mathrm{BA}} \geq \lambda_{ij'k}^{\mathrm{BA}} \text{ and } \lambda_{ijk}^{\mathrm{BA}} > \lambda_{ik}^{\mathrm{DA}} \\ 0, & \text{otherwise}, \end{cases} \quad (25a)
$$

$$
M_{ijj'k}^{\mathrm{DW}} = \begin{cases} 1, & \text{if } \lambda_{ijk}^{\mathrm{BA}} \leq \lambda_{ij'k}^{\mathrm{BA}} \text{ and } \lambda_{ijk}^{\mathrm{BA}} < \lambda_{ik}^{\mathrm{DA}} \\ 0, & \text{otherwise}. \end{cases} \quad (25b)
$$

The objective function (24a) maximizes the expected profit of the producer from selling energy in both day-ahead and balancing markets. Constraints (24b) yield the total power production q_{ijk}^{A} when both day-ahead price scenario i and balancing price scenario j realize at time interval k. For the same scenario realization, constraints (24c) compute the level of up-regulation energy q_{ijk}^{UP} scheduled. Similarly, constraints (24d) obtain the level of down-regulation energy q_{ijk}^{DW}. Constraints (24e) and (24f) give the expected revenues from selling regulation energy in the balancing market under a pay-as-bid pricing scheme. Constraints (24g) include a set of constraints associated with the offer curves, which is represented later in Section IV-A. Constraints (24h) force the power producer to operate in its feasible operating region, which is provided in Section IV-B. Constraints (24i) compute the operational costs for given schedule, whose formulation is provided in Section IV-C.

Model (24) can be also used to compute the expected profit from a sequential offering approach. First, we solve the model considering the day-ahead scenarios only. To do that, we force the balancing variables to be null (i.e., $q_{ijk}^{\mathrm{UP}} = 0$ and $q_{ijk}^{\mathrm{DW}} = 0 \ \forall i,j,k$). The optimal solutions $\widetilde{q}_{ik}^{\mathrm{DA}*}$ represent the optimal market offers when considering the day-ahead market only. Then, we solve again the model while imposing $q_{ik}^{\mathrm{DA}} = \widetilde{q}_{ik}^{\mathrm{DA}*}$. The optimal solutions $\widetilde{q}_{ijk}^{\mathrm{UP}*}$ and $\widetilde{q}_{ijk}^{\mathrm{DW}*}$ are the balancing market offers that maximize the expected profit, provided that the day-ahead offers are $\widetilde{q}_{ik}^{\mathrm{DA}*}$.

A. Linear Expression of Π

The offer curve constraints (24g), denoted as Π, are

$$
q_{ik}^{\mathrm{DA}} \leq \overline{E}, \ \forall i, \forall k \quad (26a)
$$

$$
q_{ik}^{\mathrm{DA}} \leq q_{i'k}^{\mathrm{DA}} \text{ if } \lambda_{ik}^{\mathrm{DA}} \leq \lambda_{i'k}^{\mathrm{DA}}, \ \forall i, \forall i', \forall k \quad (26b)
$$

$$
q_{ik}^{\mathrm{DA}} = q_{i'k}^{\mathrm{DA}} \text{ if } \lambda_{ik}^{\mathrm{DA}} = \lambda_{i'k}^{\mathrm{DA}}, \ \forall i, \forall i', \forall k \quad (26c)
$$

$$
o_{ij'k}^{\mathrm{UP}} = 0 \text{ if } \lambda_{ij'k}^{\mathrm{BA}} \leq \lambda_{ik}^{\mathrm{DA}}, \ \forall i, \forall j', \forall k \quad (26d)
$$

$$
o_{ij'k}^{\mathrm{DW}} = 0 \text{ if } \lambda_{ij'k}^{\mathrm{BA}} \geq \lambda_{ik}^{\mathrm{DA}}, \ \forall i, \forall j', \forall k \quad (26e)
$$

Constraints (26a) restrict the day-ahead production quantity of the producer to its capacity. Constraints (26b) and (26c) enforce the non-decreasing and non-anticipativity conditions of the producer's offer curve in the day-ahead, respectively. These two conditions are required for offer curves to be submitted to markets settled under a uniform pricing scheme. Constraints (26d) and (26e) impose that no balancing energy is contracted when it is not required by the system.

B. Linear Expression of Ω

Constraints (24h) represent the feasible operating region of the producer and can be replaced by

$$
q_{ijk}^{\mathrm{A}} \leq u_{ijk}\overline{E}, \forall i, \forall j, \forall k \quad (27a)
$$

$$
q_{ijk}^{\mathrm{A}} \geq u_{ijk}\underline{E}, \forall i, \forall j, \forall k \quad (27b)
$$

$$
q_{ijk}^{\mathrm{A}} - q_{ij(k-1)}^{\mathrm{A}} \leq R^{\mathrm{UP}}, \ \forall i, \forall j, \forall k \quad (27c)
$$

$$
q_{ij(k-1)}^{\mathrm{A}} - q_{ijk}^{\mathrm{A}} \leq R^{\mathrm{DW}}, \ \forall i, \forall j, \forall k \quad (27d)
$$

$$
u_{ijk} - u_{ij(k-1)} \leq y_{ijk}, \ \forall i, \forall j, \forall k \quad (27e)
$$

$$
u_{ij(k-1)} - u_{ijk} \leq z_{ijk}, \ \forall i, \forall j, \forall k \quad (27f)
$$

Constraints (27a) and (27b) impose that the total power production lies between its minimum and maximum production levels. Constraints (27c) and (27d) enforce ramp-up and ramp-down limits [3]. Constraints (27e) and (27f) determine the start-up status y_{ijk} and the shut-down status z_{ijk}. Notice that constraints (27c)-(27f) require the initial production level and commitment status $\left(q_0^{\mathrm{A}}, u_0\right)$.

C. Linear Expression of Operational Cost

Constraints (24i) define the operational cost of the thermal unit and can be replaced by

$$
\begin{aligned} c_{ijk} = C_0 u_{ijk} + \sum_s C_s x_{ijks} + \\ + C^{\mathrm{UP}} y_{ijk} + C^{\mathrm{DW}} z_{ijk}, \ \forall i, \forall j, \forall k \end{aligned} \quad (28a)
$$

$$
0 \leq x_{ijks} \leq E_s, \ \forall i, \forall j, \forall k, \forall s \quad (28b)
$$

$$
\underline{E} u_{ijk} + \sum_s x_{ijks} = q_{ijk}^{\mathrm{A}}, \ \forall i, \forall j, \forall k. \quad (28c)
$$

Constraints (28a) compute the operational cost of the unit including its start-up and shut-down costs [4]. Constraints (28b) and (28c) are auxiliary constraints representing a linear formulation of the quadratic cost function through a piecewise approximation [3], where x_{ijks} is the amount of energy produced by the generation block s.

V. CASE STUDY

A. LP Trading Model Verification

In this section we test and compare the proposed LP model (23) and the existing NLP model (11) using a simple test case, considering a single time period. The market price λ follows a Normal distribution, i.e.

$$
\lambda \sim \mathcal{N}(50, 5). \quad (29)
$$

The production unit has a capacity $\overline{E}$ of 60 MW and a minimum production level $\underline{E}$ of 0 MW. The quadratic cost function $h(\cdot)$ is approximated with a two-step piecewise linear cost function, i.e.,

$$
h(q) = C_1 x_1 + C_2 x_2, \quad (30a)
$$

$$
q = x_1 + x_2, \quad (30b)
$$

$$
0 \leq x_1 \leq E_1, \quad (30c)
$$

$$
0 \leq x_2 \leq E_2, \quad (30d)
$$

where E_1 and E_2 are 30 MW, while $C_1 = €35$/MWh and $C_2 = €47$/MWh.

The input scenarios for the LP model (23) are selected following the scenario reduction technique in [24]. We generate 1000 scenarios from (29) and then keep the 20 most representative ones. The LP model is implemented using GUROBI [25] in PYTHON, and it is solved in around 0.001s. The optimal solution obtained is given in Table I, where the expected profit $\mathbb{E}[\rho]$ is re-computed considering the continuous distribution of λ in (29). The NLP continuous model (11) is solved using COBYLA [26] algorithm in PYTHON in around 0.115s. Its optimal solution is also reported in Table I. The two models show similar optimal solutions, while the gap in the expected profit due to the discretization procedure is lower than 0.07%. The LP model is solved around 115 times faster than the NLP one. It is worth mentioning that the computational time of the NLP model increases to around 3.2s when the integrals in (11) are numerically computed, instead of using the cumulative distribution function of (29). However, the main advantage of the LP formulation is to be more suitable to be merged with the unit commitment constraints of the thermal unit. Therefore, a comparison between a MILP and MINLP offering model would be more appropriate to test the advantages of our LP formulation. Nevertheless, the solution of a MINLP problem is out of the scope of this paper.

This test case shows that the LP formulation, when provided with an accurate sampling of input scenarios, brings an optimal solution close to the continuous NLP alternative. In the following section, we extend the LP model by including the unit commitment constraints of the thermal unit in a multi-time period and multi-stage stochastic optimization problem.

B. Day-ahead Offering Model Test Case

We test the two-stage stochastic optimization model (24) on a realistic case study. We generate market price scenarios according to the methodology presented in Section II. The input parameters are shown in Tables II and III. First, we generate 300 scenarios for $\lambda_{ik}^{\mathrm{DA}}$ and we select the 20 most representative ones. Then, for each scenario $\lambda_{ik}^{\mathrm{DA}}$, we generate 300 scenarios of $\lambda_{ijk}^{\mathrm{BA}}$ and keep the 20 most representative ones. This procedure results in a scenario tree with 400 branches. We repeat this sampling procedure for different values of $\overline{W}$, i.e. 10, 20 and 30 GW. Figure 2 shows the 20 scenarios of $\lambda_{ik}^{\mathrm{DA}}$ (in blue) and the 20 scenarios of $\lambda_{ijk}^{\mathrm{BA}}$ (in green) for a given realization of λ_k^{DA} (in red), when $\overline{W}$ = 20 GW. We consider a thermal unit with a capacity of $\overline{E}$ = 120 MW and a minimum production level of $\underline{E}$ = 40

TABLE I
THE PRODUCER'S OPTIMAL MARKET OFFERS OBTAINED FROM THE LP AND NLP MODELS

		q_b (MWh)	λ_b (€/MWh)	$\mathbb{E}[\rho]$ (€)
NLP	b=1	30	46.6	313.4
	b=2	60	51.7	
LP	b=1	30	46.4	313.2
	b=2	60	51.8	

TABLE II
PARAMETERS OF THE MARKET PRICE GENERATION MODEL

δ (€/MWh^2)	β (€/MWh^2)	μ_γ (€/MWh^3)	σ_γ^2 (€/MWh^3)	λ^0 (€/MWh)
-6.67×10^{-3}	1×10^{-4}	2×10^{-8}	3×10^{-9}	-20

TABLE III
VALUES OF PARAMETER α_k

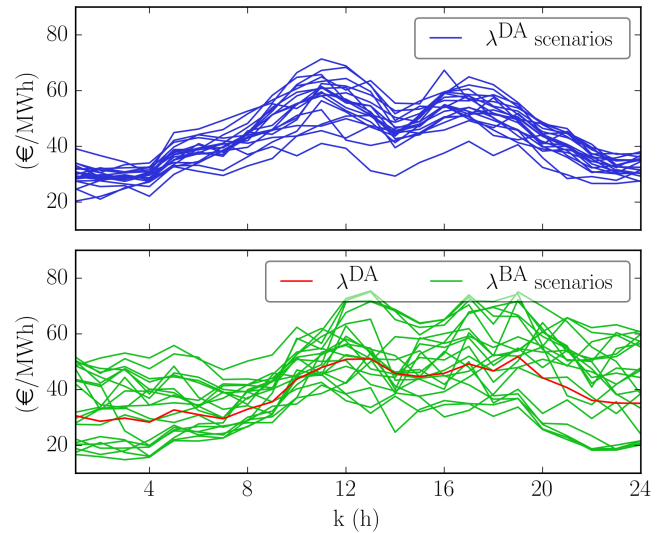| k	1	2	3	4	5	6	7	8
α_k (€/MWh)	322	312	315	317	340	349	353	369
k	9	10	11	12	13	14	15	16
α_k (€/MWh)	394	424	444	445	440	429	437	458
k	17	18	19	20	21	22	23	24
α_k (€/MWh)	446	423	408	383	373	346	331	332

Fig. 2. Day-ahead and balancing market price scenarios.

MW. Ramping limits are 40 MW for both R^{UP} and R^{DW}. The quadratic cost function is approximated by a piecewise linear function of four generation blocks of equal size, i.e. $E_s = 20$ MW $\forall s$. Table IV shows the marginal cost C_s of each block, the cost C_0, the start-up cost C^{UP} and the shut-down cost C^{DW}.

The optimization model is implemented using GUROBI in PYTHON environment. We compare the two-stage co-optimization model with a sequential offering approach. Indeed, by modeling balancing market variables as recourse decisions at the day-ahead stage, we co-optimize the offering strategy for the two markets.

As an example, Table V reports the optimal value of the day-ahead production variable q_{ik}^{DA} in time interval $k = 7$ obtained from both the co-optimized and sequential approaches. Note that $\lambda_{i7}^{\mathrm{DA}}$, $i = 1, ..., 20$, is the set of the day-ahead price scenarios, and each member of this set is viewed as a potential price offer. For the co-optimized approach, the results given

TABLE IV
PARAMETERS OF THE COST FUNCTION

C_0 (€)	C_{s_1} (€/MWh)	C_{s_2} (€/MWh)	C_{s_3} (€/MWh)	C_{s_4} (€/MWh)	C^{UP} (€)	C^{DW} (€)
2860	23.5	31.5	45.6	72.3	800	100

in Table V can be summarized as given in (31) below:

$$q_7^{\mathrm{DA}} = \begin{cases} 0, & \text{if } \lambda_7^{\mathrm{DA}} < 39.7 \\ 40, & \text{if } 39.7 \le \lambda_7^{\mathrm{DA}} < 42.0 \\ 80, & \text{if } 42.0 \le \lambda_7^{\mathrm{DA}} < 51.7 \\ 120, & \text{if } \lambda_7^{\mathrm{DA}} \ge 51.7, \end{cases} \quad (31)$$

where q_7^{DA} is expressed in MWh and λ_7^{DA} in €/MWh. According to (31), a scenario-independent offer curve in day-ahead can be built using three price-quantity offer points, i.e., (€39.7/MWh, 40 MW), (€42.0/MWh, 80 MW) and (€51.7/MWh, 120 MW). A graphic representation of this curve is provided in Figure 3a, blue curve. Similarly, the day-ahead offer curve at time interval $k = 7$ for the sequential approach can be obtained (red curve). Note that in both approaches, the producer is not willing to produce if $\lambda_7^{\mathrm{DA}} \le 39.7$ while desires to operate at its full capacity if $\lambda_7^{\mathrm{DA}} \ge 51.7$. However, when $42.0 \le \lambda_7^{\mathrm{DA}} \le 48.2$, the co-optimized approach suggests to produce 80 MW, while the sequential approach does 40 MW only. In addition, Figure 3b shows the offering curve of the producer in the balancing market at time interval $k = 7$, provided that the realization of day-ahead price λ_7^{DA} is €44.1/MWh. Based on the co-optimized approach, the producer is scheduled to produce 80 MW and then to reduce its production level in the balancing stage to 40 MW if $\lambda_7^{\mathrm{BA}} \le 35.6$, or to increase it to 120 MW in case $\lambda_7^{\mathrm{BA}} \ge 55.7$. Unlike the co-optimized approach, the sequential one schedules the producer at 40 MW in the day-ahead market, and then provides the up-regulation service only in the balancing stage. For instance, its production increases by 40 MW if $\lambda_7^{\mathrm{BA}} \ge 56.0$ while that increase is even more (80 MW) in case $\lambda_7^{\mathrm{BA}} \ge 58.7$. The expected profits obtained from these two approaches under different conditions are shown in Table VI. In a case in which $\overline{W} = 10$ GW, the expected profit loss in the sequential approach is around 2%. The power producer gains a lower expected profit in the day-ahead market while earning more in the balancing stage, such that its total expected profit (including both stages) increases as well. This behavior is more observable in the cases with a higher value of installed wind capacity. For instance, the loss of profit is 22% and 91% in cases in which $\overline{W}$ is equal to 20 and 30 GW, respectively. The last case ($\overline{W}$=30 GW) gives more insight: in the sequential approach, the producer does not participate in the day-ahead market, and earns a low profit in the balancing stage only. In contrast, the producer gains a significant money in the co-optimized approach, though it loses money in the day-ahead stage. In fact, it takes such a loosing position in day-ahead market to be able to produce profitable regulation services in the balancing stage.

TABLE V
OPTIMAL VALUES OF q_{i7}^{DA} FOR THE SEQUENTIAL AND CO-OPTIMIZED APPROACHES

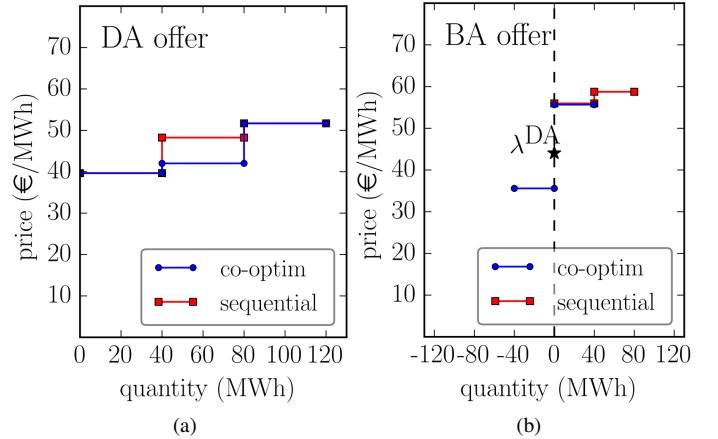| i	$\lambda_{i7}^{\mathrm{DA}}$ (€/MWh)	q_{i7}^{DA} (MWh) co-opt	seq	i	$\lambda_{i7}^{\mathrm{DA}}$ (€/MWh)	q_{i7}^{DA} (MWh) co-opt	seq
1	44.1	80	40	11	36.3	0	0
2	33.0	0	0	12	44.4	80	40
3	45.1	80	40	13	44.6	80	40
4	45.6	80	40	14	43.4	80	40
5	39.7	40	40	15	52.7	120	120
6	37.1	0	0	16	44.7	80	40
7	48.2	80	80	17	42.0	80	40
8	44.9	80	40	18	41.1	40	40
9	51.7	120	120	19	44.3	80	40
10	43.0	80	40	20	46.9	80	40

Fig. 3. The producer's optimal offer curve in (a) day-ahead market, (b) balancing market at time interval $k = 7$ ($\overline{W} = 20$ GW).

VI. CONCLUSIONS AND FURTHER WORKS

This paper presents a novel method for deriving optimal offering curves of a price-taker conventional producer in an electricity market under a pay-as-bid pricing scheme. The importance of this study is that several European balancing markets in the balancing stage are settled under a pay-as-bid pricing scheme. The main contribution of this paper is that we develop an LP approach. In contrast, the existing tools in the literature are mainly non-linear and less suitable to be merged to the MILP feasibility region of a thermal unit. Then, we extend our proposed approach to a two-stage market setup including day-ahead and balancing stages. In this setup, being consistent with the structure of several European electricity markets, the day-ahead market is cleared based on a uniform

TABLE VI
EXPECTED PROFIT OF THE PRODUCER

$\overline{W}$ (GW)	Model	Profit in DA (10^3€)	Profit in BA (10^3€)	Total (10^3€)
10	co-optimized	16.82	3.25	20.09
	sequential	18.08	1.59	19.68
20	co-optimized	4.45	8.27	12.27
	sequential	7.68	2.27	9.95
30	co-optimized	-8.45	18.44	9.99
	sequential	0.00	0.87	0.87

pricing scheme, while a pay-as-bid scheme is used in the balancing stage. To make our setup more realistic, we include the unit commitment constraints of the thermal units to our proposed setup, resulting in a MILP model.

We first test our proposed single-stage LP model against the existing non-linear models. Our LP formulation successes in well approximating the non-linear one. Then, we compare our proposed two-stage MILP model against a sequential offering model, which does not consider the balancing stage while offering in the day-ahead market. Our proposed approach shows a better performance in terms of expected profit achieved.

In future research it is of interest to test the proposed model using real market price data. Besides, intra-day markets could be included in the offering model. These additional trading floors may bring more business opportunities to the producer. Moreover, the proposed approach could be extended to derive the optimal offering strategy of different technologies, such as energy storage systems. For such facilities, the participation in the balancing market may be significantly important for exploiting their flexibility.

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