The Cobweb Effect in Balancing Markets with Demand Response

Emil M. Larsen, Pierre Pinson, Jianhui Wang, Yi Ding and Jacob Østergaard

***Abstract*—Integration of renewable energy sources like wind into the power system is a high priority in many countries, but it becomes increasingly difficult as renewables reach a significant share of generation. Demand response (DR) can potentially mitigate some of these difficulties. However, activating DR in existing electricity markets has been observed to be unstable, resulting in oscillations in supply and demand. This so-called Cobweb effect is presented here using a Scandinavian real-time market structure that is adapted in a novel way to consider cross-time elastic terms. A demand profile based on real measurements from the EcoGrid EU demonstration is used, where five-minute electricity pricing is sent to 1900 houses. Volatility is measured for 1900 houses in the experiment and through further simulation, which demonstrates increased volatility leads to lower social welfare. A key outcome of this research shows that increases in social welfare due to DR appear to be limited by the cost of volatility in existing market structures.**

***Index Terms*—Demand response (DR), Cobweb effect, real-time pricing, volatility, smart grid.**

Nomenclature

$t \in T$	Index for time
$g \in G$	Index for conventional generation
$s \in S$	Index for scenario
$q_{t,s}$	Scenario probability
$c_{t,s}$	Real-time demand
α	Price-elasticity ratio
c_t^{D}	Day-ahead demand forecast
$c_{t,t',s}^{\lambda}$	Demand response with t' cross elasticities
$c_{t,s}^{\mathrm{shed}}$	Load shedding
λ_t^{D}	Day-ahead price
λ_t^{R}	Real-time price
λ^{shed}	Price for load shedding
λ^{spill}	Price for wind spillage
$B_{t,s}$	System imbalance
$p_{g,t}^{\mathrm{D}}$	Conventional generation scheduled day-ahead
w_t^{D}	Wind power day-ahead forecast
$w_{t,s}^{\mathrm{spill}}$	Wind power spillage
$\lambda_{g,t}^{\uparrow}, \lambda_{g,t}^{\downarrow}$	Price for up/down regulation
$p_{g,t,s}^{\uparrow}, p_{g,t,s}^{\downarrow}$	Up/down regulation delivered
$P_{g,t}^{\uparrow}, P_{g,t}^{\downarrow}$	Up/down regulation bid into market
$n_{g,t,s}, m_{g,t,s}$	Up/down regulation on/off

Manuscript submitted 29/01/2016. This work was partly supportly by the European Commission through the project EcoGrid EU (grant ENER/FP7/268199) and by Mogens Balslev's Foundation.

E. M. Larsen is at the Danish Energy Agency, Denmark (email: eml@danskenergi.dk). P. Pinson and J. Østergaard are with the Centre for Electric Power and Energy, Technical University of Denmark, Kgs. Lyngby, Denmark (email: {ppin,joe}@elektro.dtu.dk). J. Wang is at Argonne National Laboratory, USA (email: jianhui.wang@anl.gov). Y. Ding is at Zhejiang University, China (email: yiding@zju.edu.cn).

I. Introduction

Demand response (DR) is being strongly pursued because it increases the value of renewable energy sources (RES) when they are available, provides some additional capacity when renewables are not available, and balances the system when renewables do not behave as predicted [1]. In Denmark, the shift to RES meant that wind power met 43% of national electricity consumption in 2015, and is well on its way to hitting goals of 50% electricity consumption from wind power in 2020, and 100% of all energy consumption from renewable energy in 2050 [2].

There are many dynamic and static electricity price tariffs that can be used to activate DR, but two methods in particular have gained traction in recent years due to their fast activation characteristics that complement the uncertainty in RES generation. These are direct control, where utilities turn devices on and off remotely, and indirect control, where an incentive signal, e.g., an electricity price, is used to influence the load to change its consumption. Direct control is typically targeted at medium and large commercial and industrial loads and has the challenges of requiring reliable communication equipment, while indirect control is aimed at a large number of small-scale loads and has challenges of predictability [3]. Key benefits of indirect control include lower equipment costs and, when a price-based mechanism is used, there can be a clear value attributed to the resource. When traded in a power pool, DR has the additional benefit of improving liquidity and lowering the cost of supply, since it reduces the market power of price-maker generators. However, indirect control by means of true market-based pricing sent to supply and demand has long been associated with unstable behavior, as first identified in [4], where it was named the Cobweb effect due to the spider web-like back-and-forth oscillations that occur when a stable market equilibrium cannot be achieved.

The Cobweb effect has traditionally been studied in markets where demand for a commodity, for example apples, was higher or lower than supply had expected. The following season, apple growers then change their production level, but the market becomes over- or under-supplied and an overshoot causes demand to behave in a seemingly opposite fashion to what had been experienced the previous season. If every market participant has a perfect forecast of supply and demand, then the Cobweb effect should not happen, but uncertainty is usually present in markets. This is true in a modern power system and especially true for DR [5]. Electricity market clearing algorithms must also make assumptions about demand, including linearising non-linear behavior, in order to find a feasible and timely solution in an optimization framework. This can result in power and prices being more volatile than is optimal, as seen in Fig. 1. Here, an imbalance that exists only at the first time step where supply and demand intersect, causes the market clearing price to oscillate outwards as the initial decision leads to a greater imbalance, via feedback, in subsequent steps.

In this paper, we investigate the Cobweb effect, which is discussed in more detail in section II, using the market structure and data collected from the EcoGrid EU project [6], which is DR demonstration on the Danish island of Bornholm. The EcoGrid EU experiment has 1900 residential households with a peak load of 5MW. Houses are equipped with smart meters and a range of distributed energy resources (DERs) with automated controllers that receive a new electricity price every five minutes and optimize consumption levels accordingly. DR from these customers is scheduled optimally with conventional generation in a market structure to meet the imbalance caused by wind power.

The contribution of this work lies in developing a real-time market that considers cross-time elasticity. We identify the different causes of volatility and investigate the impact the Cobweb effect has on social welfare, and how market recommitment frequency changes volatility and social welfare. We believe the latter to be important as system operators move to shorter settlement periods.

The paper is structured with section II presenting existing knowledge of the Cobweb effect. Section III defines price- and cross-time elasticity and illustrates the corresponding demand model. Section IV presents the market structure. Section V presents results for social welfare and volatility from simulations and the real experiment. The final section concludes.

II. The Cobweb Effect

Since initially investigated in 1938, the Cobweb effect was expanded to markets with non-linear supply and demand curves in [7], where it was also shown that the Cobweb effect happens with monotonic demand and supply curves, as is the case in electricity markets. In [8], the impact of demand expectation using auto-regressive methods on the Cobweb effect was identified. Traditional economics literature has been more focused on identifying the problem and improving the expectation of demand, including considering larger forecast horizons, leading to more stable market outcomes [9]. Solutions other than a better demand forecast have not been explored. Recent economics research on the Cobweb effect has moved to analyzing games between different players, the result of which is an equilibrium with the lowest forecast error on both the supply and demand side [10].

In the field of power system research, market-based volatility due to real-time pricing was first identified in [11], where it was noted that there is an upper limit on the market clearing time and the delay of the price signal beyond which the system becomes unstable. Here it was shown that delaying communication of the price sent to customers increased system stability greatly, while increasing the gate-closure time led to fragile system behavior. Cobweb-like volatility has been

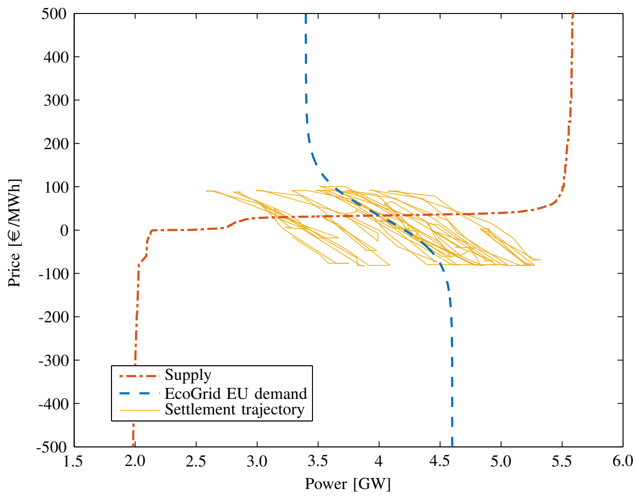

Fig. 1. The Cobweb effect in the EcoGrid EU market, where the settlement trajectory starts at the intersection of the initial supply (Nord Pool bid data) and demand (EcoGrid EU demand model) curves shown.

particularly problematic when using models from the New York ISO power system [12], however, the authors there used a mirror image of supply to represent demand in the absence of reliable information about its actual shape. In addition, authors there also assumed demand would only be non-linear with respect to time, but not conditional on past and future prices.

Recently, [13] identified the boundaries for volatility when closed-loop real-time pricing structures are used without an appropriate feedback law. No remedy was offered for the closed-loop instabilities simulated in this research, but it was noted that price volatility increases as the price-elasticity of consumers increases with respect to the price-elasticity of suppliers, indicating that volatility will vary from case to case. Real data was not used in [13], highlighted by a demand profile with eight peaks per day, rather than the archetypal one or two daily peaks. Consequently, there remains a lack of evidence about how much volatility will truly be observed in a power system with DR and real-time pricing, hence our curiosity as to whether the Cobweb effect is observable or significant in a realistic market setup.

III. Price and Cross-Time Elasticity Ratios

In this section, a definition of price and cross-time elasticity is given and the demand model is illustrated.

Price elasticity, also called self or own elasticity [14], describes how sensitive a load is to a change in price [15]. It is traditionally dimensionless and is often used to describe how consumption responds to a 1% change in price. For the purposes of operating a real-time market, however, we define price elasticity in absolute terms as a linear function of observed changes in load, ΔC, i.e.

$$\alpha_t^{DA} = \frac{\lambda_t^{DA}}{\Delta C_t^{DA}} \tag{1}$$

where λ_t^{DA} is day-ahead price. Real-time price elasticity is defined with respect to the day-ahead state, i.e.

$$\alpha_t = \frac{\lambda_t^{RT} - \lambda_t^{DA}}{\Delta C_t} \tag{2}$$

where λ_t^{RT} is the real-time price. α_t^{DA} and α_t are the ratios of price to change in consumption with units of €/MW^{2}h.

In addition to regular price elasticity, there also exists cross-time elasticity. This is the influence of past and future prices on the current demand, often occurring due to some overriding comfort boundaries that must be adhered to. When applied to electric heating, cross-time elasticity occurs because the heating device cannot stay on or off forever. It will have to stay within a comfortable temperature range and cross-time elasticities can describe how long the device can be perturbed for.

Conceptually, the impact of cross-time elasticity can be explained using the relationship

$$\Delta C_t = \sum_{t'=T_a}^{T_b} \frac{\lambda_{t'}}{\alpha_{t,t'}}, \qquad T_a \leq t \leq T_b \tag{3}$$

where $\lambda_{t'}$ can be a day-ahead price or the difference between the real-time and day-ahead prices, as in equation (2). To clarify the definition given in (3), first consider that we have a time-series of length T. We then place a sliding window around each time-step with the index t'. The sliding window starts at T_a and goes on to T_b and DR (ΔC_t) is a function of all the prices valid from T_a to T_b. This process describes the energy shifted from each slice of time, past and future, to the present time-step, t. In other words, the sum over the whole sliding window describes the energy shifted to the current time-step.

Fig. 2 encapsulates price and cross-time elasticity for the EcoGrid EU load, when given a step increase in price, in the form of a finite impulse response (FIR). The change at t_0 represents the self elasticity, while any differences between this consumption and previous and future changes represent cross-time elasticity. A full description of this model is described in [16], which also describes a congestion management experiment using the EcoGrid EU market framework. The model was based on consumption during the coldest six months of the year. DR peaks 15 minutes after the price change and subsequently fades away.

The elasticity matrix is derived from the FIR by laying it out in a matrix with the structure

$$\theta_{t,t'} = \begin{bmatrix} \theta_1 & \theta_0 & \theta_{-1} & \vdots \\ \theta_2 & \theta_1 & \theta_0 & \theta_{-1} \\ \theta_3 & \theta_2 & \theta_1 & \theta_0 \\ \vdots & \theta_3 & \theta_2 & \theta_1 \end{bmatrix} \tag{4}$$

Each column has the FIR according to external conditions such as ambient temperature, with the diagonal containing new time steps [17]. The element-wise reciprocal of $\theta_{t,t'}$ gives the elasticity matrix, $\alpha_{t,t'}$.

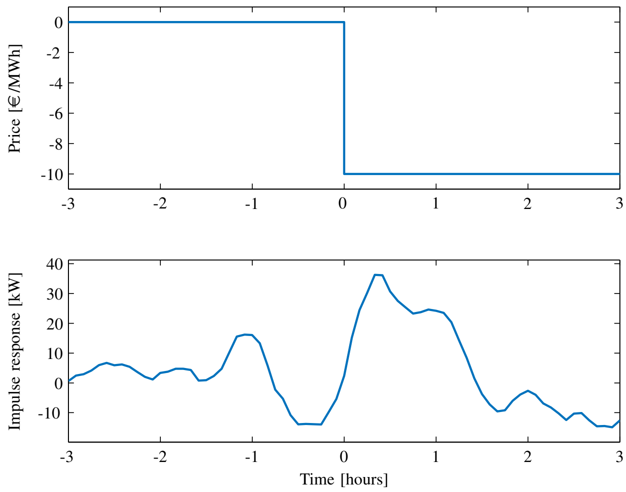

Fig. 2. Finite impulse response (FIR) of the EcoGrid EU demand in early 2014, peaking at 36kW when given a 10€/MWh price decrease at t_0.

IV. MARKET STRUCTURE

This section presents the market structure used experimentally and simulated for different cases. The EcoGrid EU demonstration has two hardware in-the-loop market steps, as shown in Fig. 3, which were used to generate five minute electricity pricing for 1900 houses in 2014 and 2015. Generator bids are based on historical Nord Pool bid data, as shown in Fig. 1, while inflexible demand and wind power injection comes from commercial real-time observations. The imbalance signal is derived from the day-ahead wind power forecast error, scaled by the Danish nominal capacity, which is around 5%.

The timeline for market operation is shown in Fig. 4. The day-ahead market price is given at 13:00 the day before operation. Real-time prices are then revealed one minute before each five minute settlement period they are valid for. In the experiment, an hour-ahead price forecast was sent to demand half past every hour. This forecast comes naturally from the market clearing, so long as the market clearing has a long enough forecast horizon. The forecast helps DERs schedule their consumption optimally, but the forecast itself is optional.

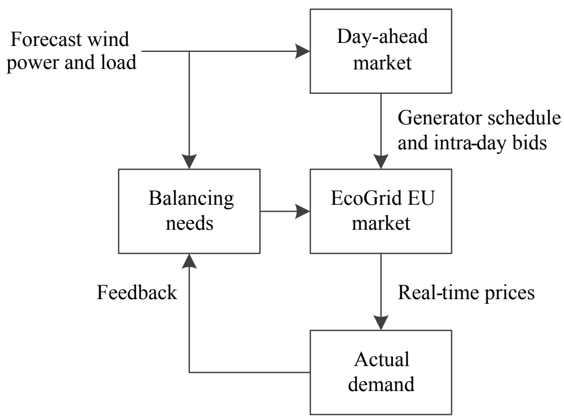

Fig. 3. Hardware-in-the-loop market structure of the EcoGrid EU demonstration.

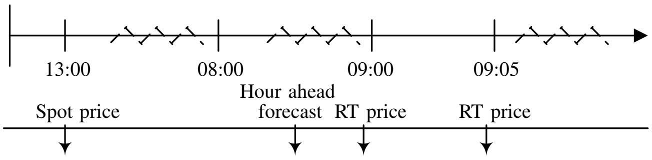

Fig. 4. Timing of EcoGrid EU real-time market.

Without it, the elasticity matrix will become more reactive, with fewer terms before the diagonal values [17].

The first step is a day-ahead market that minimizes the cost of conventional generation, considering the day-ahead wind power production forecast, the known Nord Pool spot price, and the demand forecast including price-response to spot prices. If there is no imbalance, then the spot price is sent to consumers and demand. However, the day-ahead market is not the main area of focus, because it does not generate the day-ahead price and demand is a fixed input. This step is only needed to find reasonable and feasible starting points for generators participating in the real-time market. The overall problem reads

$$\min_{\Theta} \sum_{g,t} \lambda_{g,t} p^{\mathrm{D}}_{g,t} + \sum_{t} \lambda^{\mathrm{shed}} c^{\mathrm{shed}}_{t} - \sum_{t} \lambda^{\mathrm{spill}} w^{\mathrm{spill}}_{t} \tag{5a}$$

subject to

$$\sum_{g} p^{\mathrm{D}}_{g,t} + w^{\mathrm{D}}_{t} - w^{\mathrm{spill}}_{t} = c^{\mathrm{D}}_{t} - c^{\mathrm{shed}}_{t} \qquad \forall t \tag{5b}$$

$$p^{\mathrm{D}}_{g,t} \geq P^{\min}_{g}; \quad p^{\mathrm{D}}_{g,t} \leq P^{\max}_{g} \qquad \forall g,t \tag{5c}$$

$$p^{\mathrm{D}}_{g,t} - p^{\mathrm{D}}_{g,t-1} \leq r_{g,t}; \quad p^{\mathrm{D}}_{g,t-1} - p^{\mathrm{D}}_{g,t} \leq r_{g,t} \qquad \forall g,t \tag{5d}$$

where the set of decision variables, $\Theta = p^{\mathrm{D}}_{g,t}, c^{\mathrm{shed}}_{t}, w^{\mathrm{spill}}_{t}$, are generation, load shedding and wind spillage. These variables are balanced with day-ahead wind power and load forecasts in (5b). Wind power injection is a parameter that is treated as a negative load (minus wind spillage). Minimum and maximum generation is constrained in (5c), while up and down ramp rates are bound by the ramp rate parameter $r_{g,t}$ in (5d).

The second market step is the EcoGrid EU market, where social welfare is maximized with respect to the day-ahead market outcome. The market schedules an optimal amount of manual reserves and flexible demand, and is formulated as a stochastic optimization problem that commits bids for conventional generation and creates real-time prices (RTP) for demand until the market is cleared again. Unscheduled and scheduled generation from the first market step is used as the up and down regulation bids respectively in the real-time market, i.e. $p^{\uparrow}_{g,t,s} = P^{\max}_{g} - p^{\mathrm{D}}_{g,t}$ and $p^{\downarrow}_{g,t,s} = p^{\mathrm{D}}_{g,t} - P^{\min}_{g}$. This yields

$$\max_{\Theta} \sum_{t} \sum_{s} q_{t,s} \Bigg\{ \lambda^{DA}_{t} \sum_{t'} c^{\lambda}_{t,t',s} + \frac{1}{2} \sum_{t'} \alpha_{t,t',s} c^{\lambda}_{t,t',s} \sum_{t'} c^{\lambda}_{t,t',s} - \sum_{g} \beta_{g,t,s} - \lambda^{\mathrm{spill}} w^{\mathrm{spill}}_{t,s} - \lambda^{\mathrm{shed}} c^{\mathrm{shed}}_{t,s} \Bigg\} \tag{6a}$$

subject to

$$c^{\lambda}_{t,t',s} = c^{\lambda}_{t-1,t',s} \frac{\theta_{t,t',s}}{\theta_{t-1,t',s}} \qquad t \neq t', \theta_{t,t'} \neq 0 \tag{6b}$$

$$c^{\lambda}_{t,t',s} = 0 \qquad \forall t,t',s, \alpha_{t,t',s} = 0 \tag{6c}$$

$$\sum_{t'} c^{\lambda}_{t,t',s} = 0 \qquad \forall s, t > 2h \tag{6d}$$

$$c^{\lambda,\min}_{t'} \leq c^{\lambda}_{t,t',s} \leq c^{\lambda,\max}_{t'} \qquad \forall t,t',s \tag{6e}$$

$$c_{t,s} = c^{\mathrm{D}}_{t} - c^{\mathrm{shed}}_{t,s} + \sum_{t'} c^{\lambda}_{t,t',s} \qquad \forall t,s \tag{6f}$$

$$w_{t} = w^{\mathrm{D}}_{t} - w^{\mathrm{spill}}_{t,s} \qquad \forall t,s \tag{6g}$$

$$\beta_{g,t,s} = \lambda^{\uparrow}_{g,t} p^{\uparrow}_{g,t,s} - \lambda^{\downarrow}_{g,t} p^{\downarrow}_{g,t,s} \qquad \forall g,t,s \tag{6h}$$

$$p_{g,t,s} = \sum_{g} p^{\mathrm{D}}_{g,t,s} + p^{\uparrow}_{g,t,s} - p^{\downarrow}_{g,t,s} \qquad \forall g,t,s \tag{6i}$$

$$c_{t,s} = p_{g,t,s} + w_{t,s} - B_{t,s} - e_{t} \qquad \forall t,s \tag{6j}$$

$$\lambda^{R}_{t,s} = \sum_{t'} \alpha_{t,t',s} c^{\lambda}_{t,t',s} + \lambda^{D}_{t} \qquad \forall t,s \tag{6k}$$

$$\Delta p^{\uparrow}_{g,t,s} = p^{\uparrow}_{g,t,s} - p^{\uparrow}_{g,t-1,s} \qquad \forall g,t,s \tag{6l}$$

$$\Delta p^{\downarrow}_{g,t,s} = p^{\downarrow}_{g,t,s} - p^{\downarrow}_{g,t-1,s} \qquad \forall g,t,s \tag{6m}$$

$$\Delta p^{\uparrow}_{g,t,s} = \Delta p^{\uparrow}_{g,t-1,s} \qquad \forall g,t,s, \gamma_{t} = 0 \tag{6n}$$

$$\Delta p^{\downarrow}_{g,t,s} = \Delta p^{\downarrow}_{g,t-1,s} \qquad \forall g,t,s, \gamma_{t} = 0 \tag{6o}$$

$$\Delta p^{\uparrow}_{g,t,s} = 0; \quad \Delta p^{\downarrow}_{g,t,s} = 0 \qquad \forall g,t,s, \gamma_{t} = 2 \tag{6p}$$

$$p^{\uparrow}_{g,t,s} \leq n_{g,t,s} p^{\uparrow}_{g,t,s} \qquad \forall g,t,s \tag{6q}$$

$$p^{\downarrow}_{g,t,s} \leq m_{g,t,s} p^{\downarrow}_{g,t,s} \qquad \forall g,t,s \tag{6r}$$

$$p^{\uparrow}_{g,t,s} \geq n_{g,t,s} \chi_{g} p^{\uparrow}_{g,t,s} \qquad \forall g,t,s, \gamma_{t} = 2 \tag{6s}$$

$$p^{\downarrow}_{g,t,s} \geq m_{g,t,s} \chi_{g} p^{\downarrow}_{g,t,s} \qquad \forall g,t,s, \gamma_{t} = 2 \tag{6t}$$

$$n_{g,t,s} = 0; \quad m_{g,t,s} = 0 \qquad \forall t,s, P^{\uparrow}_{g,t} = 0 \tag{6u}$$

$$p^{\uparrow}_{g,t,s} \leq d_{t,s} p^{\uparrow}_{g,t,s}; \quad p^{\downarrow}_{g,t,s} \leq l_{t,s} p^{\downarrow}_{g,t,s} \qquad \forall g,t,s \tag{6v}$$

$$d_{t,s} + l_{t,s} \leq 1 \qquad \forall g,t,s, \gamma_{t} = 2 \tag{6w}$$

where the set of decision variables, $\Theta = \{c^{\lambda}_{t,t',s}, \beta_{g,t,s}, w^{\mathrm{spill}}_{t,s}, c^{\mathrm{shed}}_{t,s}, \lambda^{R}_{t,s}, c_{t,s}, p^{\uparrow}_{g,t,s}, p^{\downarrow}_{g,t,s}, n_{g,t,s}, m_{g,t,s}, d_{t,s}, l_{t,s}\}$, contains DR due to self and cross-time elasticities, generator cost, wind spillage, load shedding, real-time price, aggregated demand, up regulation, down regulation, on/off status of up regulation bid, on/off status of down regulating bid, on/off status of global up regulation, and on/off status of global down regulation for all $g \in G$, $t \in T$, $t' \in T$ and $s \in S$ respectively.

The objective function (6a) maximises social welfare, where the first term is customer utility and the last terms are cost of generation and slack variables.

When cross-time elasticity is ignored, then only the diagonal term in $\alpha_{t,t'}$ is non-zero, and is fixed to the average price and cross-time elasticity until the market is cleared again, i.e. if the market is run hourly, then the first 12 values of price elasticity are used to determine price elasticity. Clearing an electricity market considering cross-time elasticity is not needed in today's deregulated power systems because the loads that participate in existing DR schemes have a cross-time elasticity that is longer than the re-commitment time of the system they participate in. For example, a factory that reduces its consumption for an hour to meet the terms of a DR contract will not compensate for this reduction in the following hour, since it will cause an imbalance and be penalised as a result. Instead, its cross-time elasticity depends on long term planning ranging from days to years, far slower than day-ahead and real-time markets recommit bids today. This is highlighted in [18], where the average rebound effect of an office building, a furniture store and a bakery was 15.5%. Small-scale DR, which indirect control leans towards, usually has time-constant of just a few minutes, which means the response does not last very long, and with a rebound effect of 42% as seen in Fig. 2. This time-constant is a similar order of magnitude to the re-commitment frequency in real-time markets today, which suggests that cross-time elasticities must be fully incorporated into the market to obtain an economically efficient and controllable outcome.

Methods for clearing day-ahead markets considering cross-time elasticities have previously been proposed in [14]. However, existing algorithms do not converge on a solution if the demand's self elasticity is smaller than its cross-time elasticity. This solution may work well in an hourly market, where the demand characteristics are likely to lead to a solution, but in a balancing market, the cross-time terms will often be larger than the self elastic terms. In addition, the previously proposed market structure requires the elasticity matrix to be symmetrical. In reality, demand does not prepare for an event in exactly the same way as it behaves after the event has happened. I.e. load shifting is not symmetrical and therefore not controllable when the wrong (symmetrical) elasticity matrix is used. Previous market structures with cross-time elasticities do not consider generation constraints, which necessitate additional constraints on the supply side to reach a controllable outcome. Constraints (6b)-(6d) are the most important in this respect. Constraint (6b) ties DR together in an auto regressive fashion, so that a full model of the FIR for price is included in the market formulation. Equation (6c) ensures that the resulting FIR is zero when price-elasticity is zero. Constraint (6d) sets DR to zero for twice the FIR length, h, so that market outcomes do not create infeasible starting points for subsequent re-commitments.

Constraint (6e) determines the flexible demand limits for each price lag. The total demand and production from wind are defined in (6f) and (6g). The regulating cost from conventional generation is defined in (6h). The total power produced by conventional generation is stated in (6i). Constraint (6j) is the balance constraint, also considering the imbalance from wind and inflexible demand, B_t, and the error term e_t, which describes undesirable feedback that is caused when DR does not behave as scheduled. It is the dual variable of this constraint that gives the real-time price, $\lambda^R_{t,s}$, which can also be found in constraint (6k).

Constraints (6l) - (6w) dictate generator behaviour like minimum on-times and ramping characteristics that are in-line with the Scandinavian balancing market today. Constraints (6l) and (6m) define generator ramp rates. Constraints (6n) and (6o) keep ramping constant for 15 minutes. Constraints (6p) ensure that a generator is at a fixed set-point for at least 15 minutes. When used with the generator behavior of $\gamma_t = \{1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0...\}$, (6l)-(6p) result in a minimum on-time of 45 minutes. Constraints (6q) and (6r) limit maximum regulation. Constraints (6s) and (6t) are minimum generation constraints. In the Scandinavian regulating market, bids under 10MW must be activated in full, while bids above 10MW can be activated in part; the proportion of each bid to be activated is described by the parameter χ_g. Constraints (6u) ensure that a generator is off when it does not bid into the market. Constraints (6v) determine whether any up or down generation is active, according to binary variables $d_{t,s}$ and $l_{t,s}$ respectively, and (6w) prevents simultaneous up and down regulation.

For each scenario-based decision variable there exists a non-anticipativity constraint that ensures its outcome is identical across all scenarios in the first few time periods for which prices are fixed, for example $t = 1 \ldots 6$ if the market is cleared every half hour. Scenarios for imbalance, B, are generated using a non-parametric method. Bootstrapping is employed, where historical outcomes are sampled with replacement. Scenarios for price elasticity are normally distributed and scenario reduction is done using the Fast Forward method [19].

Any imbalance after the market cleared is penalised by a primary frequency reserve (PFR) energy cost, which is set to the highest and lowest electricity price observed in each hour ex-post (after delivery), as in Scandinavia today. All other prices in Nord Pool and EcoGrid EU are defined ex-ante (before delivery).

When only self-elasticity is considered, the market is formulated as a mixed integer quadratically constrained program (MIQCP) which is readily solved with CPLEX. When considering all cross-time elasticities that results in an asymmetrical elasticity matrix, the formulation becomes a mixed integer non-linear program (MINLP). Such problems are typically solved by decomposition into NLP and MIP subproblems. We do so by first relaxing the problem (i.e., no binary variables) and solving with CONOPT. Subsequently, DR ($c^{\lambda}_{t,t',s}$) is fixed, which removes all non-linearities, and bid commitment is finalised in the MIP subproblem using CPLEX. This approach led to solutions consistently being found in under 60 seconds.

V. Results

A. Quantifying Volatility

To measure volatility, we use a rainflow counting algorithm [20], which is traditionally used in material fatigue and battery ageing analysis. The rainflow counting algorithm is a simple but powerful tool and the result is intuitive; Whenever there is a change of sign in the signal of interest, a turning point is defined. The distance between turning points is measured and binned for similar distances to give the number of oscillations observed per day. In Fig. 5, trough half cycles are counted and the distance for each cycle is measured for a time-series of DR. The total number of full cycles (troughs plus peaks) in this example are 16, with an average amplitude of 80.6MW.

B. The cause of the Cobweb effect

The main cause for the Cobweb effect is uncertainty, but this can be further specified as structural uncertainty, that is

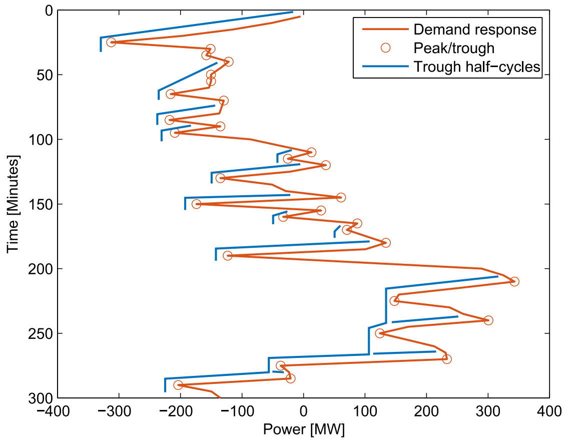

Fig. 5. Rainflow counter, where the broken blue lines are half-cycles used count the number and amplitude of oscillations.

the linearisation of demand characteristics to fit into a market structure, and aleatoric uncertainty, which is the intrinsic randomness in natural processes. Structural uncertainty in our system comes from the non-linear demand curve, assumed to be linear in the market, and ignoring cross-time elastic effects.

To understand which source of uncertainty causes the greatest volatility, we performed simulations for Denmark for one month for a range of different cases. The first case is one with no DR. The second case is an open loop system, where feedback is ignored by the market and left for faster moving reserves like PFR to remedy. An open loop cannot be operated in reality, because it requires no uncertainty in the source of an imbalance, but it is a useful benchmark since most academic electricity market studies are open loop. The next case is a closed loop system, where the market is run without cross-time elasticities, as in Scandinavia today, and feedback from an unexpected response creates a new imbalance in subsequent market re-commitments. In the fourth and fifth cases (Closed NL and CE), feedback from non-linear and cross-time elastic behaviour (based on demand models in [16]) is remedied by the market, one at a time, while the other is left as an open loop imbalance. This allows us to identify which is the bigger cause of the Cobweb effect. The sixth case (Closed M) simulates a full closed loop but with a modified market, where the full cross-time elastic effects are optimised for in the market. Finally, demonstration results are included. The demonstration cannot be directly compared to simulated cases because it uses a local imbalance signal derived from an island, and is a pseudo-closed loop where delayed meter data causes imbalance-feedback with a 15 minute delay.

Fig. 6 shows a simulation day with outcomes of market clearing price, demand and regulating power activated respectively. In a closed-loop system, oscillating behaviour is seen in both regulating power and demand. In an open loop system, similar volatility as the closed loop system can be seen in the first few hours of the system price. Increased volatility is therefore not a problem in itself from a market perspective - DR increases volatility of the demand even when expectation of demand is perfect, and this is to be cherished if DR is to help balance volatile renewable energy production. However, increased activation of regulating power is a clear indicator of the Cobweb effect in action.

Table I summarises the number of cycles counted by the rainflow counting algorithm for different cases. There is an increase in demand cycles across all cases with DR, which occurs naturally as the demand becomes dynamic. There is a reduction in supply cycles for all DR cases, which should be interpreted as fewer regulating bids being committed, which in turn means that DR has achieved its goal of reducing reliance on conventional power generation. The closed loop experiences the most volatile pricing, with the most price cycles.

Table II shows the cycle amplitude summed per day. For supply and demand, this describes the total amount of balancing power activated, and for prices, this represents the sum of price changes. Higher demand cycle amplitudes in all DR cases suggests load shifting is occurring. Price amplitudes are lowest in the open loop and the Closed NL case, suggesting they behave in a similar manner and that feedback from a non-linear demand curve is insignificant for volatility. In the basic closed loop setup and the Closed CE case, price amplitude is quadruple the open loop case. Price amplitudes paint a similar picture, with the greatest volatility in the cases with feedback from cross-time elasticity, and less volatility in the non-linear feedback case. The modified case should be directly compared

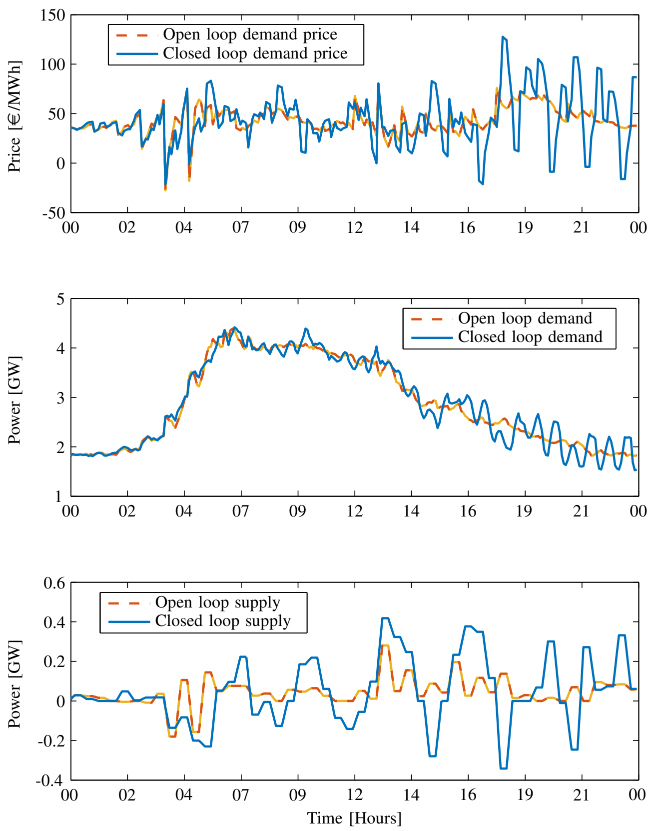

Fig. 6. Electricity prices, demand and supply for an open- and closed-loop simulation day.

TABLE I
AVERAGE CYCLES PER DAY

	Price [Cycles]	Demand [Cycles]	Supply [Cycles]
No DR	0	62.9	20.4
Open	56.8	58.6	15.9
Closed	38.9	39.6	18.1
Closed NL	56.1	56.1	15.9
Closed CE	39.4	36.7	16.9
Closed M	88.6	75.0	17.1
Demonstration	73.3	65.9	24.5

TABLE II
AVERAGE SUM OF CYCLE AMPLITUDES PER DAY

	Price [€/MWh]	Demand [GWh]	Supply [GWh]
No DR	0	1.9	1.4
Open	482.8	4.3	1.1
Closed	2013.8	16.1	4.7
Closed NL	507.7	4.7	1.2
Closed CE	2122.0	15.6	4.2
Closed M	1690.3	5.2	2.3
Demonstration	475.2	3.8	3.6

to the closed loop case, where it exhibits half the supply volatility and a reduction of 46% in supply price. Despite this, it exhibits higher supply amplitudes (but at a lower price) than the zero-DR case, suggesting the Cobweb effect is still present here, but the market is able to exploit lower generation costs in spite of volatility.

C. Demand response penetration

To see if the Cobweb effect increases cost, cases were simulated for different levels of DR penetration. DR penetration was scaled from 0% to 100%, as shown in Fig. 7. The upper limit assumes that all of Denmark behaves like an EcoGrid EU load and represents a DR peak response about twice that of DR in the Nord Pool day-ahead market today, albeit with significantly more activations due to a lower price-elasticity characteristic (i.e. DR is cheaper to activate). DR penetration beyond 30% results in a reduction in social welfare in the closed-loop system, as the cost of volatility outweighs the benefit of DR. The case where feedback stems from cross-time elastic effects (Closed CE) results in equally low social welfare, while the case where only non-linear effects are feedback has a very similar result to the open loop case. As with the rainflow counting results, this confirms that cross-time elasticity is a bigger cause of the Cobweb effect than our approximation of a non-linear demand curve. Finally, the modified market, which is a full closed loop, successfully increases social welfare for all levels of DR penetration. At low levels of DR penetration, social welfare gains are very small compared to the other cases because the modified market treats DR far more rigidly with fewer activations when it knows that a rebound will occur after 90 minutes. Lower DR activations means that costly conventional generator bids are activated instead, when leaving residual imbalances to faster moving reserves might have been more cost efficient.

Fig. 7 shows that DR has the potential to significantly increase social welfare in a real-time market, equivalent to €3.5 per flexible house per day, but only when cross-time elasticity is explicitly optimized for in the market. However, this result should be moderated by the fact that revenue here is significantly smaller than in the day-ahead market. In addition, this result is only applicable to the winter months when DR from heating in Denmark is expected to be active, so the year-round gain will be lower. The results are also highly dependent on assumptions about the supply curve, which may change as DR schemes grow.

D. Market re-commitment frequency

System dynamics change as system operators move to shorter settlement periods, shorter gate-closure times, and more regular unit re-commitments to reduce the impact of RES uncertainty. We investigated re-commitment frequency by increasing how often the market was cleared from 15 minutes to 150 minutes in 15 minute intervals. The settlement period remains five minutes throughout (i.e. prices and set-points are valid for five minutes at a time), but new decisions are only taken every time the market is cleared. The theoretical benefit of using a higher re-commitment frequency is that newer forecasts with less uncertainty can be used, leading to lower costs and therefore higher social welfare. Fig. 8 shows the outcome of changing unit commitment frequency on generation volatility and social welfare. The cross-time elastic market exhibits similar behaviour for all timings, while the closed loop market actually exhibits lower volatility and higher social welfare for longer re-commitment intervals, which is the opposite to what would traditionally be expected. Volatility here translates to more generator bids being activated for more frequent re-commitments. The local peak in volatility at 45 minutes suggests also that the market clearing frequency

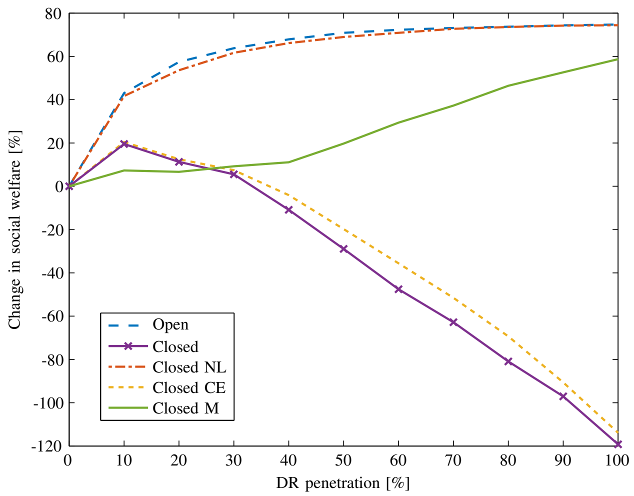

Fig. 7. Social welfare as a function of DR penetration. There is a reduction in social welfare as DR reaches significant proportions in closed-loop cases that do not account for a cross-time elastic response.

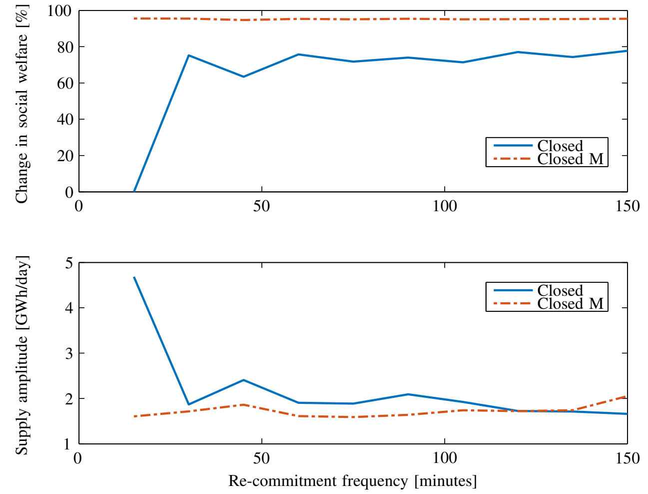

Fig. 8. Social welfare increases and volatility decreases as the re-commitment frequency is reduced in the standard closed-loop case.

is resonating with the minimum-on time for generation, and highlights additional market design challenges.

VI. Conclusion

We have provided evidence that the Cobweb effect impacts balancing markets, provoking costly oscillations on the supply side. Simulations, broadly in line with empirical results from the EcoGrid EU's experiment with 1900 households, show that the Cobweb effect causes three times more generator bids to be activated than in a market with no DR, leading to higher costs and lower social welfare. We observed that a non-linear demand curve does cause the Cobweb effect, but not enough to reduce social welfare. Aleatoric uncertainty appears to have no discernible impact on the Cobweb effect, contrary to previous studies. This means that the natural uncertainty from stochastic generation such as wind and solar power do not provoke the Cobweb effect. However, ignoring cross-time elasticities in a market does lead to significant volatility and reduced social welfare. To mitigate this, we have directly incorporated cross-time elasticity into the market clearing algorithm. Such a solution may appear obvious, yet new, DR-focussed market designs that ignore cross-time elasticity continue to appear in the literature [21]. Reducing re-commitment frequency appears to be another option for reducing the Cobweb effect.

The question remains how well existing markets can control fast-moving, non-linear DR. Our market and demand models are unlikely to capture all sources of volatility, and our measure for social welfare does not count all the costs that stem from it. Voltage and frequency instability could result from a seemingly small amount of volatility, and future research should determine how much volatility is acceptable.

Future research should also investigate uplift payments [22] for markets considering cross-time elasticities, since no method has currently been shown to ensure prices that support all market outcomes.

Acknowledgment

The authors thank Henrik Bylling for scenario reduction advice. We are grateful for the support of all EcoGrid EU's partners. We also thank DMI and ECMWF for weather forecasts, and Nord Pool and ENFOR for market data and power system forecasts respectively.

References

[1] Ea Energianalyse, "The future requirements for flexibility in the energy system," 2012.

[2] "Energistatistik 2011," Danish Energy Agency, Tech. Rep., 2011.

[3] K. Heussen, S. You, B. Biegel, L. H. Hansen, and K. B. Andersen, "Indirect control for demand side management - A conceptual introduction," *IEEE PES Innovative Smart Grid Technologies Conference Europe*, pp. 1–8, 2012.

[4] M. Ezekiel, "The cobweb theorem," *The Quarterly Journal of Economics*, vol. 52, no. 2, pp. 255–280, 1938.

[5] J. L. Mathieu, D. S. Callaway, and S. Kiliccote, "Examining uncertainty in demand response baseline models and variability in automated responses to dynamic pricing," in *IEEE Conference on Decision and Control and European Control Conference*, dec 2011, pp. 4332–4339.

[6] Y. Ding, S. Pineda, P. Nyeng, J. Østergaard, E. M. Larsen, and Q. Wu, "Real-time market concept architecture for EcoGrid EU - A prototype for European smart grids," *IEEE Transactions on Smart Grid*, vol. 4, no. 4, pp. 2006–2016, 2013.

[7] C. H. Hommes, "Dynamics of the cobweb model with adaptive expectations and nonlinear supply and demand," *Journal of Economic Behavior & Organization*, vol. 24, no. 3, pp. 315–335, 1994.

[8] ——, "On the consistency of backward-looking expectations: The case of the cobweb," *Journal of Economic Behavior & Organization*, vol. 33, no. 3, pp. 333–362, 1998.

[9] D. Dufresne and F. Vázquez-Abad, "Cobweb theorems with production lags and price forecasting," *Economics Discussion Paper*, no. 2012-17, 2012.

[10] J. Sonnemans, C. Hommes, J. Tuinstra, and H. Velden, "The instability of a heterogeneous cobweb economy: A strategy experiment on expectation formation," *Journal of Economic Behavior & Organization*, vol. 54, no. 4, pp. 453–481, 2004.

[11] J. Nutaro and V. Protopopescu, "The impact of market clearing time and price signal delay on the stability of electric power markets," *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1337–1345, 2009.

[12] R. Masiello, J. Harrison, and R. Mukerji, "Market dynamics of integrating demand response into wholesale energy markets," *The Electricity Journal*, vol. 26, no. 6, pp. 8–19, 2013.

[13] M. Roozbehani, M. A. Dahleh, and S. K. Mitter, "Volatility of power grids under real-time pricing," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 1926–1940, 2012.

[14] C. D. Jonghe, B. F. Hobbs, and R. Belmans, "Optimal generation mix with short-term demand response and wind penetration," *IEEE Transactions on Power Systems*, vol. 27, no. 2, pp. 830–839, 2012.

[15] M. G. Lijesen, "The real-time price elasticity of electricity," *Energy Economics*, vol. 29, no. 2, pp. 249–258, mar 2007.

[16] E. M. Larsen and P. Pinson, "Demonstration of market-based real-time electricity pricing on a congested feeder," in *12th International Conference on the European Energy Market*, 2015.

[17] A. K. David and Y. Z. Li, "Effect of inter-temporal factors on the real time pricing of electricity," *IEEE Transactions on Power Systems*, vol. 8, no. 1, pp. 44–52, 1993.

[18] J. L. Mathieu, P. N. Price, S. Kiliccote, and M. A. Piette, "Quantifying changes in building electricity use, with application to demand response," *IEEE Transactions on Smart Grid*, vol. 2, no. 3, pp. 507–518, 2011.

[19] N. Growe-Kuska, H. Heitsch, and W. Romisch, "Scenario reduction and scenario tree construction for power management problems," *2003 IEEE Bologna Power Tech Conference Proceedings*, vol. 3, pp. 152–158, 2003.

[20] S. Downing and D. Socie, "Simple rainflow counting algorithms," *International Journal of Fatigue*, no. January, pp. 31–40, 1982.

[21] C. Ruiz, A. J. Conejo, and S. a. Gabriel, "Pricing non-convexities in an electricity pool," *IEEE Transactions on Power Systems*, vol. 27, no. 3, pp. 1334–1342, 2012.

[22] P. Andrianesis, G. Liberopoulos, G. Kozanidis, and A. D. Papalexopoulos, "Recovery mechanisms in day-ahead electricity markets with non-convexities - Part I: Design and Evaluation Methodology," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 960–968, 2013.