A dynamic programming approach to the ramp-constrained intra-hour stochastic single-unit commitment problem

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Abstract

We consider the problem of single-unit commitment (1UC) in a power system with renewable energy. To account for variability and uncertainty in electricity prices magnified by the intermittency of renewable generation we use a Markov chain to describe prices, and formulate the 1UC as a multi-stage stochastic problem with a high resolution of the time horizon and including ramping constraints. In particular, we assume intra-hour electricity prices are available on an hourly basis and allow for hourly binary unit commitment (UC) decisions and intra-hour continuous economic dispatch (ED) decisions. As multi-stage stochastic models challenge computational tractability, we solve the hourly UC problem by dynamic programming and the intra-hour ED problem as a convex quadratic program. Depending on how UC decisions are adapted to the evolution of prices, the dynamic programming problem is either approximated or an exact solution can be found. In a case study, we illustrate the significant profit potential from considering

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the high time resolution and taking into account the ability to ramp.
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1. Introduction

In a regulated market, the unit commitment problem (UC) is solved by a central planner operating all units in the system to minimize total production cost and ensure that total supply matches total demand. An early review on the UC problem is found in Sheble and Fahd (1994) and for a state-of-the-art model, see Gollmer et al. (2000). Previously, there has been an interest in stochastic extensions considering uncertain demand, see e.g. Bunn and Paschentis (1986). As the quality of demand forecasts has increased, however, demand is often assumed to be known and more effort has been put into different solution methods. An example is the dynamic programming (DP) solution by Rong et al. (2008), in which they reduce the dimension of the DP problem by relaxing the integrality conditions of the on/off state variables and committing subsets of units sequentially. An alternative is Lagrangian relaxation of unit coupling balance constraints, see Zhuang and Galiana (1988), Muckstadt and Koenig (1977) and Frangioni et al. (2008). This decouples the units, and the UC can be solved for one unit at a time, making DP directly applicable.

During the 1990s, the electricity market went through a deregulation. As a result, planning is no longer made by a central planner, but rather by independent power producers with the objective of profit maximization, since the market now ensures that total supply matches total demand. This problem has likewise been studied in the literature, often as a subproblem of the Lagrangian relaxation of the system wide UC problem, but also independently, see e.g. Arroyo and Conejo (2000) and Frangioni and Gentile (2006).

In recent years, the increasing deployment of intermittent renewable generation in power systems challenges the balancing of supply and demand. This also makes electricity prices more varying and less predictable. To account for the resulting uncertainty in supply, the literature has proposed stochastic programming formulations of the UC problem, see for example Papavasiliou and Oren (2013), Bouffard and Galiana (2008), Morales et al. (2009) and Pritchard et al. (2010). Typically, uncertainty is represented by a so-called scenario tree that branches in each stage. This implies a copying of decision variables for each scenario and, thus, the number of variables grows exponentially with the number of stages. Especially the copying of binary variables is likely to make multi-stage problems computationally intractable. Consequently, most stochastic programming formulations are two-stage (as all of the above), and even when multi-stage models are considered, as in Nowak and Römisch (2000), the number of stages is rather small. Hence, these models do not capture the frequent update of production forecasts etc. Furthermore, many problem formulations suggested in the literature (again, as all of the above) consider hourly time intervals. Renewable generation and wind power in particular, however, shows significant variations within an hour that are not visible at an hourly level. For this reason, time resolutions higher than the usual hourly resolution of the scheduling horizon are necessary, as demonstrated by the simulation models of sequential electricity markets in Jaehnert and Doorman (2012) and Ela and O'Malley (2012). This further increases the complexity of the problem and thereby also the task of solving it. In the application of Lagrangian relaxation to the stochastic central planner UC problem, variability and uncertainty of renewable generation is reflected in the shadow price of electricity (i.e. the Lagrangian multiplier). In the UC problems of independent power producers, renewable generation likewise produces variability and uncertainty in the market price of electricity Tseng and Barz (2002). With renewables in the system, the variability of prices is clearly visible on an intra-hour level, as shown in Figure 1.

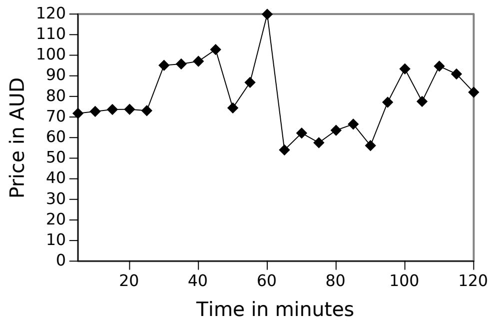

Figure 1: The electricity price on February 15th 2013 from 6 to 8 AM in New South Wales, Australia, which at this time had a wind capacity penetration of 27%.

We refer to the 1UC problem with uncertainty in the objective function as the stochastic 1UC. The large number of binary variables in this problem has also occasionally been handled by DP, see e.g. Tseng and Barz (2002) for a backward dynamic programming approach. The drawback of the dynamic programming approach is that the straightforward application cannot handle constraints on the continuous variables across time intervals, such as ramping constraints without approximation by discretization. An example is provided by Tseng and Barz (2002), who propose to either discretize the production variables or handle the constraints heuristically. Another approach is presented in Frangioni and Gentile (2006), in which the authors propose a forward moving dynamic programming algorithm in which the online or offline status of the unit determines the stages. In particular, the dynamic programming subproblem in online stages consists of production scheduling between the start-up of the unit in the beginning of the stage and the shut-down in the end. This way, ramping of generating units applies only to time intervals within a stage and not across stages.

Different market designs have been implemented throughout various power markets. Today, many markets such as Nord Pool[2] for the Nordic countries, clears the day-ahead market with an hourly time resolution. The intra-day market subsequently handles unplanned changes in production and consumption by a settling of the balancing market an hour ahead of operation. Other markets such as the Australian National Electricity Market (NEM) are operated as real-time spot markets that settle every half hour, calculating electricity prices every five minutes[3]. Such markets settled several times a day may easier accommodate the variations in renewable energy production, making the modelling of this market design increasingly relevant.

In this paper, we formulate the stochastic 1UC problem in a real-time market similar to the NEM with a 24 hour time horizon and updated information of five minute electricity prices every hour. This framework captures daily variations in the electricity price and allows generating units to start up and ramp up if it is profitable in the longer run. The resulting model is a multi-stage stochastic problem with hourly stages and high time resolution. To the best of our knowledge such a model has not previously been consid-

[2]See www.nordpoolspot.com.

[3]See www.aemo.com.au.

ered. To model the uncertainty of prices, we assume a time-inhomogeneous first-order finite-state discrete-time Markov chain (MC). The Markov property allows for the inclusion of temporal correlation in prices while facilitating the application of the dynamic programming algorithm, since the current state only depends on the immediately preceding stage.

The paper is structured as follows. First, the decomposition of the UC to the 1UC is explained in Section 2, which also contains the DP approaches. The DP application to the stochastic 1UC model with ramping is presented in Section 3, while details on the Markov chain for electricity prices are found in Section 4. Section 5 contains the numerical results from the model. Finally, the contributions of the paper are summarized and discussed in Section 6.

2. Methodology

The unit commitment problem determines when to start up and shut down a power producing unit. We consider a given time horizon, $\mathcal{T} = \{1, \ldots, T\}$, and a set of power production units, $\mathcal{I} = \{1, \ldots, I\}$. For every $i \in \mathcal{I}$ and $h \in \mathcal{T}$, the online status is represented by a binary variable u_{ih}, which is 1 when the unit is online, and 0 otherwise. The production level is a continuous variable q_{ih}. Furthermore, v_{ih} is a binary variable, which is 1 if unit i is started up at time h, and 0 otherwise. We let c_i^o denote the cost of being online, c_i^s represent the start-up cost and $c_i(q) = aq^2 + bq + c$ the convex quadratic production cost for unit i. Now, we obtain the objective

$$\min_{q_{ih}, u_{ih}, v_{ih}} \sum_{i=1}^{I} \sum_{h=1}^{T} \left(c(q_{ih}) - c_i^o u_{ih} - c_i^s v_{ih} \right). \tag{1}$$

The problem is subject to a set of balance constraints matching supply with demand, d_h, in every time interval, h,

$$\sum_{i=1}^{I} q_{ih} = d_h, \;\; h \in \mathcal{T}. \tag{2}$$

Further constraints include technical restrictions on every unit, usually including minimum and maximum production limits, q_i^{min} and q_i^{max},

$$q_i^{min} u_{ih} \leq q_{ih} \leq q_i^{max} u_{ih} \;\; h \in \mathcal{T}, i \in \mathcal{I}, \tag{3}$$

ramping restrictions allowing the production level in two consecutive time intervals only to decrease by r_i^{down} or increase by r_i^{up},

$$-r_i^{down} \leq q_{ih+1} - q_{ih} \leq r_i^{up} \;\; h \in \mathcal{T}, \tag{4}$$

and minimum up- and down-time constraints ensuring that the unit is online in at least T_i^{up} consecutive time intervals and offline in at least T_i^{down} consecutive time intervals when the online/offline status changes

$$u_{ih} - u_{ih-1} \leq u_{ik}, \;\; k = h+1, \ldots, h + T^{up} - 1, h \in \mathcal{T}, i \in \mathcal{I}, \tag{5}$$

$$u_{ih-1} - u_{ih} \leq 1 - u_{ik}, \;\; k = h+1, \ldots, h + T^{down} - 1, h \in \mathcal{T}, i \in \mathcal{I}. \tag{6}$$

Finally, logical constraints relate the binary start-up variable to the binary unit commitment variable

$$u_{ih} - u_{ih-1} \leq v_{ih}, \;\; h \in \mathcal{T}, i \in \mathcal{I}. \tag{7}$$

This is a deterministic mixed integer programming formulation of the problem, which, as mentioned in Section 1, can be solved e.g. by Lagrangian relaxation. If we denote the Lagrangian multipliers, also known as the shadow prices, by $\lambda_h, h \in \mathcal{T}$, the Lagrangian single unit problem is

$$\mathcal{L}_i(\lambda) = \max_{q_h, u_h, v_h} \sum_{h \in \mathcal{T}} \left(\lambda_h q_h - c_i(q_h) - c_i^o u_h - c_i^s v_h \right), \tag{8}$$

subject to (3)-(7) with the unit index of the variables omitted. To accelerate the solution process these subproblems could be solved in parallel as the subproblems of the Lagrangian relaxation in Papavasiliou et al. (2015). The Lagrangian dual is

$$\min_{\lambda} \sum_{i \in \mathcal{I}} \mathcal{L}_i(\lambda) - \sum_{h \in \mathcal{T}} d_h,$$

which provides an upper bound to the UC problem.

Solving the Lagrangian 1UC subproblem corresponds to solving the profit maximising 1UC problem in a deregulated market. Denoting by p_h the market price for every $h \in \mathcal{T}$, the problem is

$$\max_{q_h, u_h, v_h} \sum_{h \in \mathcal{T}} \left(p_h q_h - c_i(q_h) - c_i^o u_h - c_i^s v_h \right), \tag{9}$$

under the constraints (3)-(7), again with the unit index of the variables omitted.

We propose two formulations of the stochastic 1UC problem, which differ in how decisions are adapted to the evolution of prices. In the first formulation, we assume that the online/offline status of the unit is adapted to the price on an hourly basis (as is the production schedule). This corresponds to extending the stochastic approach of Tseng and Barz (2002) to a backwards dynamic programming algorithm with high time resolution and a discretization of the production variables at an hourly level to include ramping constraints. In the second formulation, we further assume the number of hours online/offline is adapted to the prices (as is the production schedule for the entire online/offline period). This results in an extension of Frangioni and Gentile (2006) likewise to a stochastic backwards dynamic programming algorithm with high time resolution, but with stages consisting of online and offline periods. The latter approach allows us to consider ramping restrictions without discretizing the continuous production variables. It therefore has the advantage of an exact solution, whereas the solution to the first has to be approximated. Both models enable a higher time resolution than hourly without compromising computational tractability even for a stochastic model with many binary variables. We show that the second formulation provides a lower bound to the first, as this can be seen as a stronger non-anticipativity condition in stochastic programming. We use a case study to illustrate and quantify and importance of a fine time resolution, ramping constraints, and the inclusion of uncertainty in a power system with a significant share of renewable energy.

3. Stochastic single-unit commitment and intra-hour dispatch

3.1. A dynamic programming formulation for the 1UC

In the dynamic programming formulation of the 1UC problem we let a state be defined by the status of the unit in the preceding hour, u_{h-1}, i.e. whether the unit was online or offline, how long the unit has been online or offline, τ, and a price vector, $\mathbf{p}_h$, of the intra-hour electricity prices in hour h. We denote the stages by $h = 1, \ldots, T$, which in our case represents the hours of a $T = 24$ hour time horizon. We solve the intra-hour subproblem for each stage deciding the production level in each intra-hour time interval. These subproblems are ED problems since the binary online/offline decision variables are decided on the hourly basis in the DP formulation.

As above we let start-up (/shut-down) cost be denoted by $c^s_h(u_{h-1}, u_h)$, and online (/offline) cost, $c^o_h(u_h)$. Offline and shut-down costs may be zero as in Section 2, but it is not necessary for the following to work.

We denote the value function of the intra-hour ED problem by $f_h(\mathbf{p}_h)$.

The dynamic programming recursion is as follows. If the unit was offline in stage $h-1$ we denote the expected future profit from stage h and onwards by F_h^0. If, furthermore, the unit has been offline for at least T^{down} hours it can either start up or remain offline, and hence,

$$\begin{aligned} F_h^0(T^{down}, \mathbf{p}_h) = \max\Big\{ & c_h^s(0, u_h) + c_h^o(u_h) \\ & + \big(f_h(\mathbf{p}_h) + \mathbb{E}[F_{h+1}^1(0, \mathbf{p}_{h+1})|\mathbf{p}_h]\big)u_h \\ & + \mathbb{E}[F_{h+1}^0(T^{down}, \mathbf{p}_{h+1})|\mathbf{p}_h](1-u_h) : u_h \in \{0,1\}\Big\}. \end{aligned}$$

For $\tau \in \{0, \ldots, T^{down}-1\}$ the unit has to remain offline due to the minimum down-time restrictions, and thus,

$$F_h^0(\tau, \mathbf{p}_h) = c_h^o(0) + \mathbb{E}[F_{h+1}^0(\tau+1, \mathbf{p}_{h+1})|\mathbf{p}_h].$$

If the unit was online in stage $h-1$ we denote the expected future profit for from stage h and onwards by F_h^1. If the unit has been online for at least T^{up} hours and the production level in the previous hour is at minimum capacity it can remain online or shut down,

$$\begin{aligned} F_h^1(T^{up}, \mathbf{p}_h) = \max\Big\{ & c_h^s(1, u_h) + c_h^o(u_h) \\ & + \big(f_h(\mathbf{p}_h) + \mathbb{E}[F_{h+1}^1(T^{up}, \mathbf{p}_{h+1})|\mathbf{p}_h]\big)u_h \\ & + \mathbb{E}[F_{h+1}^0(0, \mathbf{p}_{h+1})|\mathbf{p}_h](1-u_h) : u_h \in \{0,1\}\Big\}. \end{aligned}$$

For $\tau \in \{0, \ldots, T^{up}-1\}$, the unit has to remain online due to the minimum up-time restrictions,

$$F_h^1(\tau, \mathbf{p}_h) = c_h^o(1) + f_h(\mathbf{p}_h) + \mathbb{E}[F_{h+1}^1(\tau+1, \mathbf{p}_{h+1})|\mathbf{p}_h].$$

3.2. Economic dispatch with high time resolution

We solve the economic dispatch (ED) problem as a convex quadratic programming problem assuming the variable costs, $c(q_t)$, are quadratic and convex. In accordance with operational practice, production levels may be changed every few minutes, whereas start up or shut down of units takes longer and is often made on an hourly basis. We therefore allow for a higher time resolution in the ED problem by assuming that each time interval of the UC problem is divided into a number of intra-hour time intervals $t = 1, \ldots, T_h$. The production level in each time interval is subject to capacity constraints and the objective is to maximize profit. Letting $\mathbf{p}_h = (p_t)_{t=1}^{T_h}$, the ED problem is

$$\begin{aligned} f_h(\mathbf{p}_h) = \max \ & \sum_{t=1}^{T_h} (p_t q_t - c(q_t)) \\ \text{st} \ & q^{min} \leq q_t \leq q^{max}, \ \ t = 1, \ldots, T_h. \end{aligned}$$

3.3. Non-anticipativity

In the above, we assume that the online/offline status of the unit is adapted to the price on an hourly basis. More specifically, we assume that the electricity prices of the current hour are known in the beginning of the hour enabling the producer to decide whether to remain online/offline or shut down/start up. In stochastic programming terms, this corresponds to the following so-called non-anticipativity constraints:

Assumption 1(i) $\mathbb{E}[u_h | \mathbf{p}_h] = u_h$.

We likewise assume that the ED decisions are adapted to the price on an hourly basis, i.e.

Assumption 1(ii) Let $\mathbf{q}_h = (q_t)_{t=1}^{T_h}$. Then $\mathbb{E}[\mathbf{q}_h | \mathbf{p}_h] = \mathbf{q}_h$.

We refer to this model as single-hour planning as opposed to the multi-hour planning presented in the following.

3.4. A compact formulation

The UC problem can be formulated in a more compact fashion, which we will exploit in the following sections. Let $\mathbf{p}_{hk} = (\mathbf{p}_h, \ldots, \mathbf{p}_k)$, where $\mathbf{p}_h = (p_t)_{t=1}^{T_h}$, and let the multi-hour ED problem be

$$f_{hk}(\mathbf{p}_{hk}) = \max \sum_{i=h}^{k} \sum_{t=1}^{T_i} (p_t q_t - c(q_t))$$
$$\text{st } q^{min} \leq q_t \leq q^{max}, \;\; t = 1, \ldots, T_i, i = h, \ldots, k.$$

Moreover, let $c^o_{hk}(u_h) = \sum_{i=h}^{k} c^o_i(u_h)$. By iterating until the unit is no longer forced online or offline (using the law of iterated expectations), the problem is equivalent to

$$\begin{aligned} F^0_h(\mathbf{p}_h) = \max \Big\{ & c^s_h(0, u_h) + \big(c^o_{hh+T^{up}-1}(u_h) \\ & + \mathbb{E}[f_{hh+T^{up}-1}(\mathbf{p}_{hh+T^{up}-1}) + F^1_{h+T^{up}}(\mathbf{p}_{h+T^{up}})|\mathbf{p}_h]\big)u_h \\ & + \big((c^o_{hh}(u_h) + \mathbb{E}[F^0_{h+1}(\mathbf{p}_{h+1})|\mathbf{p}_h]\big)(1 - u_h) : u_h \in \{0, 1\}\Big\}, \end{aligned}$$

and

$$\begin{aligned} F^1_h(\mathbf{p}_h) = \max \Big\{ & c^s_h(1, u_h) + \big(c^o_{hh}(u_h) + f_{hh}(\mathbf{p}_{hh}) + \mathbb{E}[F^1_{h+1}(\mathbf{p}_{h+1}))|\mathbf{p}_h]\big)u_h \\ & + \big(c^o_{hh+T^{down}-1}(u_h) \\ & + \mathbb{E}[F^0_{h+T^{down}}(\mathbf{p}_{h+T^{down}})|\mathbf{p}_h]\big)(1 - u_h) : u_h \in \{0, 1\}\Big\}. \end{aligned}$$

Note that in this formulation, the multi-hour ED subproblem of F^0 is a multi-stage problem (with hourly stages) due to the non-anticipativity constraints.

3.5. Strong non-anticipativity

As an alternative to Assumption 1, we may assume that the decision to start up/shut down and *the number of hours* to remain offline/online depend on the prices in a given hour. In stochastic programming terms, this corresponds to a stronger non-anticipativity condition and can be formulated as

Assumption 2(i) Let $k_h = \min\{k \geq h : u_h \neq u_{k+1}\}$, i.e. the first time the unit changes status. Then $\mathbb{E}[k_h|\mathbf{p}_h] = k_h$.

We likewise assume strong non-anticipativity in the ED problem. Formally, we impose

Assumption 2(ii) Let $\mathbf{q}_h = (q_t)_{t=1}^{T_h}$ and $k_h = \min\{k \geq h : u_h \neq u_{k+1}\}$. Then $\mathbb{E}[\mathbf{q}_k|\mathbf{p}_h] = \mathbf{q}_k,\ \ k = h, \ldots, k_h$.

3.6. Multi-hour planning

We denote the UC and ED value functions under Assumption 2 by $\tilde{F}_h^0, \tilde{F}_h^1$ and $\tilde{f}_{hk}$. Note that with a stronger non-anticipativity condition, the multi-hour ED problem becomes the deterministic problem

$$\begin{aligned}\tilde{f}_{hk_h}(\mathbf{p}_{hk_h}) = \max\ \ & \sum_{k=h}^{k_h} \sum_{t=1}^{T_k} (\mathbb{E}[p_t|\mathbf{p}_h] q_t - c(q_t)) \\ \text{st}\ \ & q^{min} \leq q_t \leq q^{max},\ \ t = 1, \ldots, T_k, k = h, \ldots, k_h.\end{aligned}$$

Moreover, the DP problem becomes a shortest path problem. To see this, we may further iterate until the unit changes status. As a result, the DP formulation is equivalent to

$$\begin{aligned}\tilde{F}_h^0(\mathbf{p}_h) = \max \Big\{ & c_h^s(0,1) + c_{hk_h}^o(1) + \tilde{f}_{hk_h}(\mathbf{p}_{hk_h}) \\ & + \mathbb{E}[\tilde{F}_{k_h+1}^1(\mathbf{p}_{k_h+1})|\mathbf{p}_h] : k_h \in \{h + T^{up} - 1, \ldots, T\} \Big\},\end{aligned}$$

and

$$\tilde{F}_h^1(\mathbf{p}_h) = \max\Big\{c_h^s(1,0) + c_{hk_h}^o(0) \\ + \mathbb{E}[\tilde{F}_{k_h+1}^0(\mathbf{p}_{k_h+1})|\mathbf{p}_h] : k_h \in \{h + T^{down} - 1, \dots, T\}\Big\}.$$

Now consider a directed graph with nodes corresponding to feasible online periods (i.e. respecting minimum up-time restrictions) and arcs representing feasible offline periods going from one node corresponding to an online stage to another, while respecting minimum down-time restrictions. Furthermore, let arc costs represent start-up costs of the unit and let node costs represent the sum of online costs and the negative profits associated with each stage. Then the solution to the problem can be found as the shortest path between a source node with arcs to all nodes and a sink node with arcs coming in from all nodes (see Frangioni and Gentile (2006)). Note that in this formulation, the binary decisions to start-up/shut-down are replaced by the integral number of hours to remain online/offline.

The value function of the multi-hour planning problem provides a lower bound on the single-hour problem, i.e.

Proposition 1

$$\tilde{F}_h^j(\mathbf{p}_h) \leq F_h^j(\mathbf{p}_h), j = 0, 1.$$

For a formal proof, see Appendix A.

Without the stronger non-anticipativity, intra-hour dispatch *with ramping* requires discretization of the production level on an hourly basis. This is unnecessary in the case of the stronger non-anticipativity condition, which has an exact solution. We consider the inclusion of ramping constraints in the following section.

3.7. Ramp-constrained single-hour planning

To accommodate the inclusion of ramping constraints, we introduce an additional state variable, q_{h-1}, in the DP formulation that accounts for the production level in the last intra-hour time interval of the preceding hour.

The ramp-constrained intra-hour ED problem is then

$$
\begin{aligned}
f_h(\mathbf{p}_h, q_{h-1}, q_h) = \max \ & \sum_{t=1}^{T_h} (p_t q_t - c(q_t)) \\
\text{st} \ & q^{min} \leq q_t \leq q^{max}, \ \ t = 1, \ldots, T_h \\
& -r^{down} \leq q_{t+1} - q_t \leq r^{up} \ \ t = 1, \ldots, T_h \\
& q_0 = \begin{cases} q_{h-1}, & \text{if} \ \ q_{h-1} > 0 \\ q^{min} - r^{up}, & \text{if} \ \ q_{h-1} = 0, \end{cases} \\
& q_{T_h} = q_h,
\end{aligned}
$$

where the condition on q_0 ensures that the unit always starts production at q^{min} in the first time interval of the hour when it has been offline the previous hour.

We let $Q = [q^{min}, q^{max}]$ and

$$
\tilde{Q}(q) = \begin{cases} Q \cap [q - T_h r^{down}, q + T_h r^{up}], & \text{if} \ \ q > 0, \\ Q \cap [q^{min}, q^{min} - r^{up} + T_h r^{up}], & \text{if} \ \ q = 0, \end{cases}
$$

such that the production level complies with the minimum and maximum production and can be reached within an hour (T_h intra-hour intervals) and thereby also comply with the ramping restrictions. The DP recursion is as follows. If the unit has been offline for at least T^{down} hours it can either remain offline or start up,

$$F_h^0(T^{down}, \mathbf{p}_h) = \max\Big\{c_h^s(0, u_h) + c_h^o(u_h) + \max\{f_h(\mathbf{p}_h, 0, q_h) \\ + \mathbb{E}[F_{h+1}^1(0, \mathbf{p}_h, q_h)|\mathbf{p}_h] : q_h \in \tilde{Q}(0)\}u_h \\ + \mathbb{E}[F_{h+1}^0(T^{down}, \mathbf{p}_{h+1})|\mathbf{p}_h](1-u_h) : u_h \in \{0,1\}\Big\}.$$

For $\tau \in \{0, \ldots, T^{down}-1\}$ it must stay offline,

$$F_h^0(\tau, \mathbf{p}_h) = c_h^o(0) + \mathbb{E}[F_{h+1}^0(\tau+1, \mathbf{p}_{h+1})|\mathbf{p}_h].$$

If the unit has been online for at least T^{up} hours it can shut down if the production level in the previous hour ends at q^{min}, otherwise it has to remain online. Hence,

$$F_h^1(T^{up}, \mathbf{p}_h, q^{min}) = \max\Big\{c_h^s(1, u_h) + c_h^o(u_h) + \max\{f_h(\mathbf{p}_h, q^{min}, q_h) \\ + \mathbb{E}[F_{h+1}^1(T^{up}, \mathbf{p}_{h+1}, q_h)|\mathbf{p}_h] : q_h \in \tilde{Q}(q^{min})\}u_h \\ + \mathbb{E}[F_{h+1}^0(0, \mathbf{p}_{h+1})|\mathbf{p}_h](1-u_h) : u_h \in \{0,1\}\Big\}.$$

and for $q_{h-1} > q^{min}$

$$F_h^1(T^{up}, \mathbf{p}_h, q_{h-1}) = c_h^o(1) + \max\Big\{f_h(\mathbf{p}_h, q_{h-1}, q_h) \\ + \mathbb{E}[F_{h+1}^1(T^{up}, \mathbf{p}_{h+1}, q_h)|\mathbf{p}_h] : q_h \in \tilde{Q}(q_{h-1})\Big\}.$$

For $\tau \in \{0, \ldots, T^{up}-1\}$, the unit likewise has to remain online,

$$F_h^1(\tau, \mathbf{p}_h, q_{h-1}) = c_h^o(1) + \max\Big\{f_h(\mathbf{p}_h, q_{h-1}, q_h) \\ + \mathbb{E}[F_{h+1}^1(\tau+1, \mathbf{p}_{h+1}, q_h)|\mathbf{p}_h] : q_h \in \tilde{Q}(q_{h-1})\Big\}$$

In this problem, we have imposed Assumption 1. To solve it, we discretize the production level. Note that in spite of solving a problem with high-resolution time horizon, it is sufficient to discretize the production level in the beginning of an hour and let the ED problem handle the ramping for the remaining intra-hour intervals as continuous variables. Thus, intra-hour ramping can be handled in exact manner, whereas we approximate hour by hour ramping.

By alternatively imposing Assumption 2 we avoid the discretization of the production level, and hence, an exact solution can be found, in spite of including ramping restrictions. Indeed, the DP recursion is the same with or without ramping restrictions since the ED covers the entire online or offline period and thus handles all the ramping.

4. Electricity price modelling

The modelling of electricity prices has been extensively studied, especially with the deregulation of electricity markets in the 1990s and the increasing deployment of renewable power sources in the following decade, see Weron (2014) for a comprehensive study of the literature. Here, we model the hourly sets of five minute electricity prices by means of an inhomogeneous, first-order, finite-state, discrete-time Markov chain. The Markov Property allows for the inclusion of temporal correlation in prices while facilitating the application of the dynamic programming algorithm. Although current prices may not only depend on the immediately preceding prices but also on prices further back in time, Markov chains have been argued to be a reasonable choice for modelling of electricity prices, see for instance González et al. (2005). Finally, it is well-known that electricity prices exhibit daily patterns, Weron (2014), which is accommodated by considering a time-inhomogeneous Markov chain, see e.g. Iversen et al. (2014).

Recall that T_h is the number of intra-hour intervals in hour h. For every $h = 1, \ldots, H$, we assume that the intra-hour prices of a given hour, h, are represented by a discrete stochastic vector, $\mathbf{P}_h = (P_{ht_1}, \ldots, P_{ht_{T_h}})$, that takes values within a finite set of states $\{\mathbf{p}_h^1, \ldots, \mathbf{p}_h^B\}$ with $\mathbf{p}_h^b = (p_{ht_1}^b, \ldots, p_{ht_{T_h}}^b)$. Each of these vectors represents a high-resolution intra-hour price path corresponding to the price bin. The Markov Property ensures that

$$\mathbb{P}(\mathbf{P}_{h+1} = \mathbf{p}_{h+1}^{b_{h+1}} | \mathbf{P}_1 = \mathbf{p}_1^{b_1}, \ldots, \mathbf{P}_h = \mathbf{p}_h^{b_h}) = \mathbb{P}(\mathbf{P}_{h+1} = \mathbf{p}_{h+1}^{b_{h+1}} | \mathbf{P}_h = \mathbf{p}_h^{b_h}),$$

where $b_1, \ldots, b_{h+1} \in \{1, \ldots, B\}$. Note that we may have that $\mathbf{p}_{h_1}^b \neq \mathbf{p}_{h_2}^b$, since the Markov chain is time-inhomogeneous.

We construct the price paths by clustering of historical data. For every $h = 1, \ldots, H$, we define B price bins $]\underline{p}_{hb}, \bar{p}_{hb}], b = 1, \ldots, B$ such that each price in the first time interval of hour h is in exactly one of these bins. For consecutive stages h and $h+1$, we consider the observed price paths beginning in bin $]\underline{p}_{hb}, \bar{p}_{hb}]$ in the first time period of the current stage, h, and ending in bin $]\underline{p}_{h+1b'}, \bar{p}_{h+1b'}]$, in the first time period of the next stage, $h+1$, for $b, b' \in \{1, \ldots, B\}$. As a representative price vector $\mathbf{p}_h^b = (p_{ht_1}^b, \ldots, p_{ht_{T_h}}^b)$ we choose the price path with smallest Euclidean distance to the rest of the paths (the centre). By choosing the centre of the cluster, we represent price variations throughout the hour, which enables us able to assess the effect of ramping. Another possibility would be the average of the cluster, but as this would diminish variations in prices, we would underestimate the costs of ramping restrictions.

The probability, $\pi_{b_h, b_{h+1}}$, that the Markov chain is in state b_{h+1} in stage $h+1$ given that it is in state b_h in stage h is called the one-step transition probability and can be estimated as the number of times the price started in bin b_h in the beginning of hour h and ended in bin b_{h+1} in the beginning of hour $h+1$ divided the number of times the price started in bin b_h,

$$\hat{\pi}_{b_h,b_{h+1}} = \frac{n_{b_h,b_{h+1}}}{\sum_{b=1}^{B} n_{b_h,b}},$$

where $n_{b_h,b_{h+1}}$ is the number of observed price paths beginning in state b_h and ending in state b_{h+1}.

An example of the price paths and the chosen representative can be seen in Figure 2.

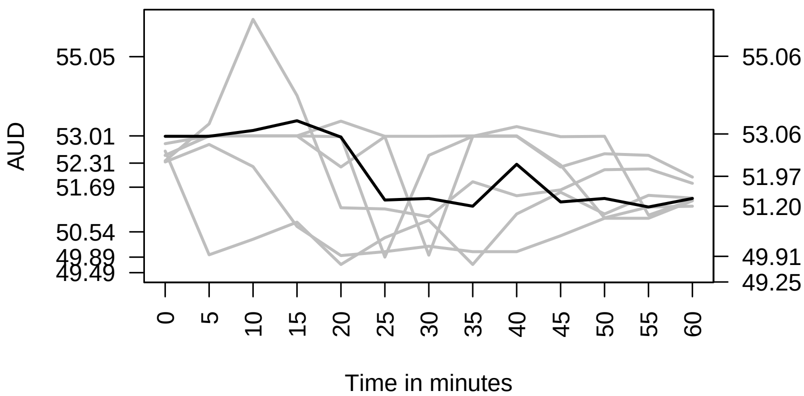

Figure 2: Choosing a price path between two bins. The intra-hour price path has 5-minute time resolution. The grey lines are the observed price paths and the black is the chosen representative. The left and right y-axis represents the price bin values for the current and coming hour, respectively.

5. Computational results

5.1. Price data

Price data with 5 minute resolution from July 2012 to December 2013 from New South Wales has kindly been provided by the Australian system operator. The data is separated into a set of week days and a set of special days (including weekends, school holidays and public holidays). Outliers are removed based on the histogram of the log transformed data, see Appendix B in the electronic supplemental material. This gives us a total price range of 0–150 AUD.

As mentioned in Section 4 the price range is divided into a number of bins that represent the states of the Markov chain. The upper and lower bounds of the bins are chosen such that the fraction of prices in each bin is $1/B$, where B is the number of bins. We consider 8 bins for our case studies to represent the price data properly. Increasing the number of bins would require additional data to estimate the parameters of the Markov chain appropriately. Excluding the first and last bin, this results in an average price range of 1.74 AUD per bin for the week day data set and 1.78 AUD for the special day data set. The price range of the first and last bin is naturally somewhat larger as they contain the rare very low and very high prices, respectively.

In order to provide meaningful comparative models with 15 and 30 minute time resolution, we aggregate the price trajectories from the Markov chain model by taking the mean over 15 and 30 minutes, respectively.

5.2. Unit data

We study and compare the ramp constrained intra-hour stochastic 1UC problem for power producing units with different production characteristics.

Table 1 displays their technical data regarding capacity, ramp rates, etc. For the 1a unit, the data is obtained from Conejo et al. (2010). The other units are variants of the first unit, constructed to compare the influence of ramping constraints and higher costs on the solution to the stochastic 1UC problem. Table 2 holds the start-up, online and production costs. These numbers are likewise based on Conejo et al. (2010), but adjusted to comply with the quadratic production cost function. Unit 1a resembles a base load unit and is as such assumed to be online at midnight which is the beginning of the time horizon. Furthermore, we assume that it produces at 103MW in the last time period of the hour previous to the beginning of the time horizon. To enable comparison we make the same initial assumptions for the rest of the units.

Table 1: Technical data of generating units. Capacity bounds, ramp rates and minimum up and down times for each unit. Unit 1a represents a base load unit. The other units are variants of it increasingly resembling peak load units.

Unit	q^{max} MW	q^{min} MW	r^{up} MW/min	r^{down} MW/min	T^{up} h	T^{down} h
1a & 1c	152	30.4	2.53	2.53	8	4
1b & 1d	152	30.4	6	6	8	4
1e	152	30.4	6	6	4	2

Table 2: Unit costs. Start-up cost and coefficients of the quadratic production cost function for the units.

Unit	c^{su} \$	c^{on} \$	b \$/MWh	a \$/(MWh)^2
1a & 1b	1430.4	250	13	0.002
1c & 1d & 1e	1430.4	300	52.9	0.002

5.3. Implementation

The dynamic programming algorithm for solving the UC problem is implemented in Java. The convex quadratic programming formulation ED problem is likewise solved in Java with the Cplex callable library using Cplex 12.6. Both are run with a 2.7GHz processor and 4GB RAM.

5.4. Hourly benchmark

In order to evaluate the models and the high time resolution we implement a simplified model to obtain an hourly benchmark. We assume that the market continue to have 5 minute prices, but as for the hourly commitment decisions, the dispatch decisions are hourly. To represent ramping we let production increase or decrease linearly throughout the hour between the production level of the previous hour and the production level at the end of the current hour.

5.5. Profit opportunity from intra-hour dispatch

The results from the single-hour planning model can be found in Table 3. Obviously, the expensive units have lower profits than less expensive units. As should always be the case, the fast ramping units, 1b and 1d, have at least as high profits as their slower counterparts, 1a and 1c. For the 5 minute stochastic model the profits are also higher. In fact, the just more than double ramp rate results in a profit increase of only 0.12%, comparing 1b with 1a in the 5 minute stochastic model, but 2.12% when comparing 1d with 1c. The corresponding 60 minute resolution profits shows no difference between the fast and slow ramping units. Thus the high time resolutions reveals profit opportunity mainly for the expensive fast ramping units. In contrast, when comparing the profit of 1e to that of unit 1d, the 60 minute stochastic model shows a difference of 1.59% whereas the 5 minute stochastic model shows a 1.64% difference. Hence, there is only little profit opportunity in the reduction of up- and down-times to be revealed with the high time resolution in this setting.

Table 3: Unit profits for the single-hour model and the week day data set. The profits for 5, 15 and 30 minute time resolutions are listed along with their corresponding 60 minute benchmarks. Finally, the difference between high time resolution profit and 60 minute benchmark profit relative to the high time resolution profit is shown. Except for the relative difference all values are in AUD.

		Deterministic			Stochastic		
	Unit	Solution	60 min bench.	Diff.	Solution	60 min bench.	Diff.
5 min res.	1a	145691.71	145056.64	0.44%	147328.63	146668.68	0.45%
	1b	145858.90	145056.64	0.55%	147500.55	146668.68	0.56%
	1c	2085.52	1973.53	5.37%	6921.47	6874.90	0.67%
	1d	2291.53	1973.53	13.88%	7059.58	6874.90	2.62%
	1e	2291.53	1973.53	13.88%	7131.34	6930.12	2.82%
15 min res.	1a	145627.87	145029.56	0.41%	147268.14	146647.98	0.42%
	1b	145734.04	145029.56	0.48%	147377.25	146647.98	0.49%
	1c	1905.82	1825.22	4.23%	6819.12	6782.84	0.53%
	1d	2120.89	1858.22	12.38%	6958.07	6830.04	1.84%
	1e	2120.89	1858.22	12.38%	7026.95	6885.11	2.02%
30 min res.	1a	145456.79	144988.59	0.32%	147100.01	146615.55	0.33%
	1b	145456.79	144988.59	0.32%	147100.01	146615.55	0.33%
	1c	1608.58	1514.63	5.84%	6634.71	6548.29	1.30%
	1d	1815.48	1675.39	7.72%	6784.21	6754.35	0.44%
	1e	1815.48	1675.39	7.72%	6846.11	6808.17	0.55%

Once more for obvious reasons there is a positive difference between the high resolution profits and the 60 minute benchmark. Especially the units 1d and 1e experience a positive profit difference of 2.8% and 2.84%, respectively, for the stochastic 5 minute model, compared to the 60 minute benchmark. This is higher than for the 1a, 1b and 1c units due to 1d and 1e having high costs and at the same time having high ramping ability, thus exploiting the possibility of adjusting production to the price.

By running a deterministic model using expected prices we notice the same pattern with even greater differences than in the stochastic model. This indicates that the lower the price volatility is, the more important are the higher time resolutions.

Finally, the table shows that the profit difference when compared with the 60 minute benchmark for unit 1e in the stochastic model decreases from 2.84% with 5 minute resolution to 2.05% and 0.55% as the time resolution decreases to 15 and 30 minutes, respectively. Hence, a time resolution of 5 minutes reveals more profit potential than the 15 and 30 minute resolution in our case.

Table 4 in general shows the same patterns for the multi-hour model. The intra-hour planning allows fast ramping units to exploit their ramping abilities resulting in higher profits compared to the slow ramping units both in absolute terms and the relative difference from the hourly benchmarks. Finally, we see again that relative profit difference between the high time resolution cases and the 60 minute benchmarks is decreased as the time resolution decreases except for some cases in which profits are very low. What is also evident, however, is that for the stochastic cases and the fast ramping units the multi-hour model has in general a much lower profit than its single-hour counterpart, indicating that the lower bound is not tight. For the slow and inexpensive units 1a and 1b we see that the profits are almost the same for the multi-hour model and the single-hour model, even a little higher for the multi-hour model. In this case the lower bound is in fact so tight that the discretization of the production levels for the single-hour

Table 4: Unit profits for the multi-hour model and week day data set. The profits for 5, 15 and 30 minute time resolutions are listed along with their corresponding 60 minute benchmarks. Finally, the difference between high time resolution profit and 60 minute benchmark profit relative to the high time resolution profit is shown. Except for the relative difference all values are in AUD.

		Deterministic			Stochastic		
	Unit	Solution	60 min bench.	Diff.	Solution	60 min bench.	Diff.
5 min res.	1a	145691.71	145056.64	0.44%	147330.80	146670.85	0.45%
	1b	145858.90	145056.64	0.55%	147502.72	146670.85	0.56%
	1c	2049.61	1962.49	4.25%	3130.16	3064.79	2.09%
	1d	2190.05	1962.49	10.39%	3267.21	3064.79	6.20%
	1e	2190.05	1962.49	10.39%	3687.65	3441.69	6.67%
15 min res.	1a	145627.87	145029.56	0.41%	147270.31	146650.14	0.42%
	1b	145734.04	145029.56	0.48%	147379.42	146650.14	0.49%
	1c	1882.13	1824.03	3.09%	2969.09	2925.72	1.46%
	1d	2027.94	1852.45	8.65%	3107.60	2956.01	4.88%
	1e	2027.94	1852.45	8.65%	3530.10	3354.58	4.97%
30 min res.	1a	145456.79	144988.59	0.32%	147102.17	146617.72	0.33%
	1b	145456.79	144988.59	0.32%	147379.42	146650.14	0.49%
	1c	1619.65	1525.56	5.81%	2724.16	2636.27	3.23%
	1d	1773.80	1675.39	5.55%	2866.46	2788.90	2.71%
	1e	1773.80	1675.39	5.55%	3274.62	3219.78	1.67%

model yields a slightly worse result. Thus a production planner could benefit from planning only one hour ahead, when planning for fast ramping units with high variable costs, but we may use the exact and computationally less expensive multi-hour planning for inexpensive slow ramping units.

Tendencies are the same for the week day and special days data sets, although the lower prices in the special days data result in reduced profits overall. The results can be found in the online supplementary material.

5.6. Precision

Since we have discretized the hourly production levels to exploit the DP solution of the stochastic 1UC problem with ramping, we briefly consider the effect of the number of production levels. We find that increasing the number of production levels from 16 to 31 generates no profit increase for unit 1a and only 0.00021% profit increase for unit 1e which would be most vulnerable to price volatility and thus have the greatest risk of having suboptimal hourly production levels. Finally, we consider an extra unit, 1f, which corresponds to 1e, but with lower minimum up and down time and lower start-up costs to see if a unit more susceptible to shutting down and starting up would change this picture. Here the profit increase remains insignificant at 0.00025%. We conclude that for these units the 16 production values are sufficient to appropriately represent the hourly production levels.

5.7. Running times

The running times of the model heavily depends on the number of states in the Markov chain and for the single-hour model the number of production levels. In general the multi-hour model solves quickly with running times around 350 seconds for the stochastic model and less than 10 seconds for the deterministic model. Due to the increased number of states for the single-hour model the running times are higher, around 1000 seconds for the stochastic 5 minute model and 30 seconds for the deterministic model. The running times for the special days data set are of the same order of magnitude. For both data sets we consider 8 price bins and 16 production levels. Finally, since the running times are reasonable it would be possible to use the single-hour or multi-hour 1UC as the Lagrangian relaxation subproblem of a large system model, as discussed in Section 1, especially if the Lagrangian subproblems were run in parallel.

6. Conclusions and discussion

We consider the stochastic multi-stage 1UC problem with hourly updating of intra-hour electricity price information. We present two DP formulations that both make hourly binary UC decisions whereas continuous ED decisions are made in the DP subproblem, which is a convex quadratic program with higher time resolution, but differ in their assumptions regarding non-anticipativity. In our single-hour model the hourly plans are adapted to current prices. In our multi-hour model, however, plans are made for an entire online period on the basis of current prices. We show that the multi-hour model can handle ramping in an exact manner and with little computational effort, whereas the single-hour model requires discretization of the hourly production levels. However, multi-hour planning results in a substantial reduction in profits compared to single-hour planning. Moreover, the single-hour model can be solved with a sufficiently fine discretization to obtain a satisfactory computational precision within reasonable running times.

Our results show that there is a significant difference in profits between low and high time resolution cases, especially when prices are close to marginal costs and the ramping ability is high. Furthermore, the ability of intra-hour ramping is revealed with the high time resolution with profit differences up to 2.00% which was not detectable in the low time resolution model. However, these effects are mitigated when the intra-hour time resolution is decreased to 15 and 30 minutes, confirming the importance of the high time resolution.

Future work includes the implementation of Lagrangian relaxation of the system-wide stochastic UC problem to further investigate computational tractability of our approach.

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Appendix A. Proof of Proposition 1

Corrollary 1 Denote the value functions under Assumption 1 and 2 by f_{hk_h} and $\tilde{f}_{hk_h}$, respectively. Then,

$$\tilde{f}_{hk_h}(\mathbf{p}_{hk_h}) \leq f_{hk_h}(\mathbf{p}_{hk_h}).$$

Proof: Let $\mathbf{q}_h = (q_t)_{t=1}^{T_h}$, and let

$$\tilde{f}_{hk_h}(\mathbf{p}_{hk_h}) = \max \ \mathbb{E}\Big[\sum_{k=h}^{k_h}\sum_{t=1}^{T_k}(p_t q_t - c(q_t))\Big|\mathbf{p}_h\Big]$$

$$\text{st } q^{min} \leq q_t \leq q^{max}, \ \ t \in T_k, k = h, \ldots, k_h$$

$$\mathbb{E}[\mathbf{q}_k|\mathbf{p}_h] = \mathbf{q}_k, k = h, \ldots, k_h.$$

Due to the stronger non-anticipativity constraints, the multi-hour ED problem provides a lower bound to the multi-hour ED problem. Under Assumption 2, this becomes

$$\tilde{f}_{hk_h}(\mathbf{p}_{hk_h}) = \max \ \sum_{k=h}^{k_h}\sum_{t=1}^{T_k}(\mathbb{E}[p_t|\mathbf{p}_h]q_t - c(q_t))$$

$$\text{st } q^{min} \leq q_t \leq q^{max}, \ \ t \in T_k, k = h, \ldots, k_h$$

$$\mathbb{E}[\mathbf{q}_k|\mathbf{p}_h] = \mathbf{q}_k, \ \ k = h, \ldots, k_h,$$

which is a deterministic problem.

Proposition 1 Denote the value functions under Assumption 1 and 2 by F_h^j and $\tilde{F}_h^j$, respectively. Then,

$$\tilde{F}_h^j(\mathbf{p}_h) \leq F_h^j(\mathbf{p}_h), j = 0, 1.$$

Proof: For $k = h, \ldots, k_h$, the constraint $\mathbb{E}[u_k|\mathbf{p}_k] = u_k$ in the single-hour UC problem is replaced by $\mathbb{E}[u_k|\mathbf{p}_h] = u_k$ in the multi-hour UC problem, which is a restriction of the problem. By definition, $u_h = u_{h+1} = \cdots = u_k \neq u_{k+1}$ if and only if $k_h = k$. Combining this with Corollary 1, we obtain the desired result.

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