Electricity Market Equilibrium under Information Asymmetry

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Abstract

We study a competitive electricity market equilibrium with two trading stages, day-ahead and real-time. The welfare of each market agent is exposed to uncertainty (e.g., induced by renewable production in our stylized setup), whose realization is only known at the real-time stage. Though renewable production is a common source of uncertainty, the information of market agents on the probability distribution of this uncertainty is not necessarily identical at the day-ahead stage. We prove the existence and uniqueness of such an equilibrium and show a high sensitivity of the equilibrium solution to the level of information asymmetry. In particular, we show that the overall social welfare reduces for any asymmetry in agent information, and there is economic, operational, and computational value for the system stemming from information sharing.

Keywords: Asymmetry of Information, Distributed Optimization, Electricity Market, Equilibrium, Existence and uniqueness, Uncertainty

1. Introduction

With increasing shares of renewable energy resources, electricity markets are exposed to uncertainty associated with intermittent power supply. To accommodate this uncertainty, electricity trading in short term has been arranged in several subsequent trading floors, such as day-ahead and real-time markets. At the first stage (day-ahead), market agents, such as power producers and consumers, compete to contract energy considering the forecast of renewable power production, while at the second stage (real-time) they settle power imbalances caused by forecast errors.

Such competition can be modeled as a stochastic *equilibrium* problem in the sense of [1], where each market agent is a self-optimizer and maximizes its welfare, e.g. expected profit for producers or expected utility for consumers. Each agent computes a day-ahead decision while anticipating the real-time outcomes using its *private* information (e.g., probabilistic forecast) about uncertainty. To find the equilibrium among such agents, distributed algorithms as in [2] have been proposed to let agents integrate their private forecasts. After a finite number of iterations, the agents find a set of equilibrium prices that reflect their private information and support the equilibrium. This distributed solution is promising for the operation of local markets [3] or parts of larger systems [4].

In contrast, large electricity markets, such as NordPool or CAISO, use a *centralized* optimization to compute the equilibrium, where the central entity called market operator collects bids from agents and clears the market on its own. To efficiently operate markets with renewables, it has been proposed to cast the centralized optimization as a two-stage stochastic model [5, 6]. This model considers that the operator generates a set of plausible renewable outcomes based on its *own* information about the probability distribution of renewable energy production, and clears the day-ahead market while accounting for the anticipated real-time imbalances.

The important property of the two problems is that they are equivalent in the case where all market agents in the equilibrium model optimize against the same probability distribution as that of the market operator in the centralized model. Under this scenario, the two problems yield the same market-clearing results and the centralized market is complete as it satisfies the preferences of the agents in the equilibrium problem. However, the equivalence between the centralized and equilibrium problems no longer holds when agents in the equilibrium problem optimize against different distributions. In this situation, the centralized market settlement is inefficient as it does not support the true preferences of agents. We refer to this situation as *information asymmetry* that typically holds for many reasons. Naturally, market agents use different data and forecast tools to build uncertainty distributions. Furthermore, for a given set of plausible outcomes, agents may explicitly assign different probabilities depending on whether they are rational or not, for example, in the sense of prospect theory [7].

In this line, this letter analyzes a competition among electricity market agents that have asymmetric information about a common source of uncertainty. We propose an equilibrium model in which agents may assign different probabilities over a common set of renewable power production outcomes. We show that the centralized model in

[5, 6] satisfies the preferences of all market agents only if they all agree on the probability distribution of renewable power production. We discuss the existence and uniqueness of the solution to the equilibrium problem, and refer the stability theory to discuss the challenges associates with its computation. With our analytic results, we point out a high sensitivity of equilibrium prices to the level of information asymmetry. We then propose a distributed algorithm to numerically assess the equilibrium outcomes. We eventually demonstrate that the system overall benefits from information sharing, as we show that for any asymmetry in agents information, there exist a loss of social welfare, an increase in real-time imbalances, and a decrease of the convergence rate of the distributed algorithm.

The letter is outlined as follows. In Section 2 we describe the setup and introduce the stochastic equilibrium model. We further proceed with an analytic solution to equilibrium prices as a function of the private information of agents in Section 3. In Section 4 we describe the distributed algorithm to compute equilibrium and provide extensive numerical experiments. All proofs are gathered in an Appendix.

2. Problem statement

2.1. Main notation and assumptions

We consider a finite set of uncertainty outcomes Ω indexed by $\omega = \{1, \dots, \Omega\}$. ξ_ω is the renewable power output that corresponds to outcome ω. The renewable producers are not modelled as market agents, but represented as an aggregated stochastic in-feed. The controllable generation (consumption) is represented by a single producer (consumer). The dispatch of power producer at the day-ahead stage is denoted by $p \in \mathcal{O}$, and it can be adjusted by $r_\omega \in \mathcal{O}$ in real-time if outcome ω realizes. The set $\mathcal{O}$ denotes the feasible operating region of the producer based on its technical constraints. The cost function of the producer is quadratic given by $c(x) = \frac{1}{2}\alpha x^2$, where α is a positive constant. The consumer procures energy at the day-ahead stage in amount of $d \in \mathcal{K}$ that is subject to adjustment $l_\omega \in \mathcal{K}$ in real-time if outcome ω realizes. The set $\mathcal{K}$ exhibits the feasible region of the decision-making problem of the consumer. The utility of the consumer is described by concave function $u(x) = \gamma x - \frac{1}{2}\beta x^2$, where γ and β are positive constants. We assume that both $\mathcal{O}$ and $\mathcal{K}$ are convex and compact sets. The electricity price in scenario ω is denoted by λ_ω in the optimization problem and by $\tilde{\lambda}_\omega$ in the equilibrium problem. The prices belong to a set of non-negative reals Λ_+.

2.2. Centralized model for market-clearing problem

Consider a centralized market organization, where the market operator collects bids of agents and finds socially optimal contracts $\{p, d\}$ at the day-ahead stage, followed by real-time recourse decisions $\{r_\omega, l_\omega\}_{\forall\omega}$. The market operator integrates its own information about underlying uncertainty that is described by a finite set of probabilities $\{\pi_\omega^{\text{mo}}\}_{\forall\omega}$ assigned to uncertain outcomes. This yields

$$\max_{p,r_\omega,d,l_\omega} \big[u(d) - c(p)\big] + \sum_{\omega\in\Omega} \pi_\omega^{\text{mo}}\big[u(l_\omega) - c(r_\omega)\big], \quad \text{(1a)}$$

$$\text{s.t.} \quad p + r_\omega + \xi_\omega - d - l_\omega \geqslant 0 : \lambda_\omega, \quad \forall\omega \in \Omega, \quad \text{(1b)}$$

$$(p, r_\omega) \in \mathcal{O}, \ (d, l_\omega) \in \mathcal{K}, \quad \forall\omega \in \Omega, \quad \text{(1c)}$$

where objective function (1a) represents the expected social welfare seen by the market operator, and constraint (1b) enforces the power balance for each outcome of renewable energy production. A set of dual prices $\{\lambda_\omega\}_{\forall\omega}$ shows the sensitivity of the expected social welfare to the stochastic in-feed and, therefore, is an implicit function of the information of market operator. Hence, the outcomes for market participants are subject to the information available to the market operator.

Remark 1. *Unlike settings in [5, 6], we do not explicitly model the day-ahead power balance constraint. Instead, we use the notion of price convergence between day-ahead and real-time stages [8] to obtain the day-ahead price as* $\lambda^{DA} = \Sigma_\omega \lambda_\omega$.

2.3. Equilibrium model for market-clearing problem

We now introduce an equilibrium model given by a set of individual optimization of three agents, i.e.,

$$\max_{\tilde{\lambda}_\omega\in\Lambda_+} J_\omega^{\text{ps}} := -\tilde{\lambda}_\omega\big[p + r_\omega + \xi_\omega - d - l_\omega\big], \ \forall\omega \in \Omega, \quad \text{(2a)}$$

$$\max_{(p,r_\omega)\in\mathcal{O}} J^{\text{p}} := \sum_{\omega\in\Omega} \pi_\omega^{\text{p}} \left[\frac{\tilde{\lambda}_\omega}{\pi_\omega^{\text{p}}}(p + r_\omega) - c(r_\omega)\right] - c(p), \quad \text{(2b)}$$

$$\max_{(d,l_\omega)\in\mathcal{K}} J^{\text{c}} := \sum_{\omega\in\Omega} \pi_\omega^{\text{c}} \left[u(l_\omega) - \frac{\tilde{\lambda}_\omega}{\pi_\omega^{\text{c}}}(d + l_\omega)\right] + u(d), \quad \text{(2c)}$$

The price-setting agent solves (2a) and optimizes a set of prices $\{\tilde{\lambda}_\omega\}_{\forall\omega}$ in response to the value of the system imbalance for each outcome of renewable production. For any surplus of generation, problem (2a) yields zero price, while it yields a strictly positive price in case of generation shortage. The power producer optimizes its first- and second-stage decisions p and $\{r_\omega\}_{\forall\omega}$ in (2b) to maximize the expected profit for a given set of prices $\{\tilde{\lambda}_\omega\}_{\forall\omega}$. In its optimization, the producer integrates its own information about the uncertain in-feed characterized by a finite set of probabilities $\{\pi_\omega^{\text{p}}\}_{\forall\omega}$. Finally, the consumer computes optimal first-stage and recourse decisions d and $\{l_\omega\}_{\forall\omega}$ in (2c) to maximize its expected utility using its own information set $\{\pi_\omega^{\text{c}}\}_{\forall\omega}$. Observe, that agents in (2b) and (2c) use the probability-removed prices obtained by dividing the equilibrium prices by the associated probabilities [5]. The probability-removed prices define the actual electricity price that each agent expects to receive once uncertainty is resolved.

The three problems are interconnected in the sense that the problem of the price-setter is parametrized by the decisions of the producer and consumer, while their problems are conditioned by the price provided by the price-setting agent. Therefore, a set of equilibrium prices $\{\tilde{\lambda}_\omega\}_{\forall\omega}$ is implicitly a function of the information that agents integrate into their optimization problems.

Proposition 1. *The solution to the equilibrium problem* (2) *exists and is unique for any agent information sets.*

Remark 2. *The proof of Proposition 1 relies on the strict monotonicity of agent preferences. In the case of linear preferences, other approaches would be required (see [9, Chapter 2]).*

2.4. Relation between centralized and equilibrium models

The equivalence between the centralized and equilibrium market-clearing models is established with the following proposition.

Proposition 2. *Let $\pi_\omega^{mo} = \pi_\omega^{p} = \pi_\omega^{c}, \forall\omega \in \Omega$. Then, there exists a set of prices $\{\tilde{\lambda}_\omega^\star\}_{\forall\omega}$ that yields the optimal solution $p^\star, d^\star, \{r_\omega^\star, l_\omega^\star\}_{\forall\omega}$ in the equilibrium model* (2) *that solves the centralized model* (1)*. Moreover, $\tilde{\lambda}_\omega^\star = \lambda_\omega^\star, \forall\omega$.*

However, this equivalence no longer holds when the information of market agents about the renewable in-feed in the equilibrium model is different from that of the market operator in the centralized model. In this scenario, the prices in (1) and (2) are not necessarily identical as they depend on different information sets, making the market based on (1) incomplete in terms of information. In the following we study model (2) that reveals the true equilibrium state among agents with private information on uncertainty. Eventually, we show that the system overall benefits when agents agree on a common information set that completes the market.

3. Analytic solution for equilibrium prices

Let us define the demand excess function for renewable power outcome ω as $z_\omega = d + l_\omega - p - r_\omega - \xi_\omega$. We derive the optimality conditions associated with (2b) and (2c) to define variables d, l_ω, p, and r_ω as a function of equilibrium prices $\tilde{\boldsymbol{\lambda}}$. Assuming the agent constraints are not binding, the demand excess function writes as:

$$z_\omega(\tilde{\boldsymbol{\lambda}}) = \frac{\gamma - \Sigma_\omega \tilde{\lambda}_\omega}{\beta} + \frac{\pi_\omega^{c}\gamma - \tilde{\lambda}_\omega}{\pi_\omega^{c}\beta} - \frac{\Sigma_\omega \tilde{\lambda}_\omega}{\alpha} - \frac{\tilde{\lambda}_\omega}{\pi_\omega^{p}\alpha} - \xi_\omega.$$

By solving $z_\omega(\tilde{\boldsymbol{\lambda}}) = 0, \forall\omega \in \Omega$, we obtain a closed-form characterization of equilibrium prices as a function of probabilities that agents assign to uncertain outcomes. In the interest of illustration, let us consider a set $\Omega \in \{h, \ell\}$ with only two outcomes with $\xi_\ell = 1$ and $\xi_h = 3$. For any agent it holds that $\pi_\ell + \pi_h = 1$. Let $\alpha = 1.5$, $\beta = 0.3$, and $\gamma = 5$. Figure 1 depicts the two equilibrium prices $\tilde{\lambda}_\ell$ and $\tilde{\lambda}_h$ as a function of π_ℓ and π_h. We find a clear relationship between the equilibrium prices and agent information. For instance in case (▲), when producer assigns the whole probability mass to outcome ℓ, it leads to a nearly zero price associated with outcome h. A similar situation holds in the opposite case (★). In a quite critical case (♦) with highly asymmetric assignment of probabilities, the equilibrium yields almost zero prices for both outcomes. Moreover, we find that the day-ahead price, i.e., $\tilde{\lambda}^{\text{DA}} = \tilde{\lambda}_\ell + \tilde{\lambda}_h$, attains its maximum value when both agents have symmetric information, i.e., $\pi_\ell^{\text{p}} = \pi_\ell^{\text{c}}$.

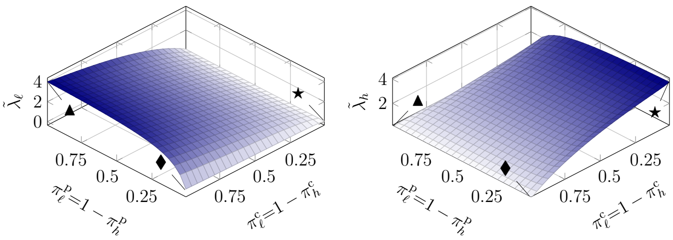
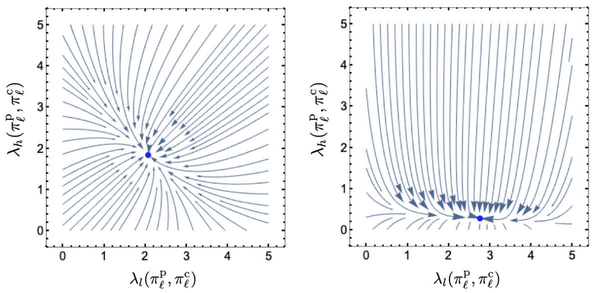


Figure 1: Equilibrium prices $\tilde{\lambda}_\ell$ and $\tilde{\lambda}_h$ as a function of probabilities that agents assign to the two uncertainty outcomes. The black markers indicate the three boundary equilibrium cases.



(a) $\pi_\ell^{\text{p}} = 0.5, \pi_\ell^{\text{c}} = 0.5$ (b) $\pi_\ell^{\text{p}} = 0.99, \pi_\ell^{\text{c}} = 0.5$

Figure 2: Equilibrium point and vector field around equilibrium point in case of (a) symmetric and (b) asymmetric information.

As shown in [2], the unstable equilibrium may not be computable by standard distributed algorithms. To verify the stability of the equilibrium solution under different assignments of probabilities, we consider a dynamic price adjustment process as the following first order differential equation [10]:

$$\frac{\mathrm{d}\tilde{\boldsymbol{\lambda}}(t)}{\mathrm{d}t} = \tau \boldsymbol{z}(\tilde{\boldsymbol{\lambda}}(t)), \quad \tilde{\boldsymbol{\lambda}}(0) = \tilde{\boldsymbol{\lambda}}_0, \tag{3}$$

where τ is some positive constant, and $\tilde{\boldsymbol{\lambda}}_0$ is a vector of initial prices. We discuss the stability of the equilibrium solution using the following proposition.

Proposition 3 (Adapted from [11])**.** *If $\tilde{\boldsymbol{\lambda}}$ is a solution of* (3) *and all the eigenvalues of the Jacobian matrix of $\boldsymbol{z}$ have strictly negative real parts, then $\tilde{\boldsymbol{\lambda}}$ is locally stable. If at least one eigenvalue has strictly positive real part, then $\tilde{\boldsymbol{\lambda}}$ is unstable.*

By verifying the eigenvalues of the Jacobian of $\boldsymbol{z}$, we find that for any assignment of probabilities, the equilibrium solution is locally stable and, thus, supposedly computable. However, we observe that for asymmetric cases the ratio of the two eigenvalues significantly increases. This ratio heavily affects the convergence rate for gradient search algorithms [12], as illustrated by vector fields in Figure 2 for some choice of $\tilde{\boldsymbol{\lambda}}_0$. In particular, in Figure 2(a), a gradient search is almost uniform in both directions $\tilde{\lambda}_\ell$ and $\tilde{\lambda}_h$, while in Figure 2(b) the gradient in direction $\tilde{\lambda}_\ell$ is notably smaller than that in direction $\tilde{\lambda}_h$. In the following section, we will demonstrate that the high eigenvalue ratios significantly affect the convergence of the distributed market-clearing algorithms.

4. Equilibrium computation

In this section, we first introduce a distributed algorithm to compute the solution to the equilibrium problem. We then describe the setup and provide numerical results.

4.1. Algorithm

To compute the equilibrium solution, we use a distributed algorithm that naturally embodies the Walrasian tatonnement [13]. The price-setter problem (2a) updates the prices based on the optimal response of the producer and consumer optimization problems (2b) and (2c), respectively.

We first show that the price-setter optimization (2a) reduces to a single analytic expression.

Proposition 4. *Consider the response of producer* $p^\nu, \{r^\nu_\omega\}_{\forall\omega}$ *and the response of consumer* $d^\nu, \{l^\nu_\omega\}_{\forall\omega}$ *to a set of prices* $\{\tilde{\lambda}^{\nu-1}_\omega\}_{\forall\omega}$ *at some iteration* ν. *Then, the solution of* (2a) *converges to optimum over iterations using*

$$\tilde{\lambda}^\nu_\omega = \max\Big\{0, \tilde{\lambda}^{\nu-1}_\omega - \rho\big[p^\nu + r^\nu_\omega + \xi_\omega - d^\nu - l^\nu_\omega\big]\Big\},\ \forall\omega \in \Omega,$$

for some positive constant ρ.

Using analytic expression for the price-update, we can compute the solution of the equilibrium problem (2a)-(2c) using Algorithm 1. The algorithm is implemented using JuMP [14] and MOSEK solver [15], and is benchmarked against conventional PATH Solver [16].

4.2. Setup

We choose $\alpha = 1.5$, $\gamma = 5$, $\beta = 0.3$, $\{\tilde{\lambda}^0_\omega\}_{\forall\omega} = 0$, $\rho = \epsilon = 10^{-5}$. The outcomes of uncertain renewable production are described by 100 samples drawn from a normal distribution $\mathcal{N}(\mu, \sigma^2)$ with $\mu = 1.5$ and $\sigma^2 = 0.25$. We consider the reference distribution Ⓡ that assigns equally likely probabilities over all outcomes. We then generate a series of distributions that tweak either mean or variance of the reference distribution Ⓡ using the probability weighting function of the following form [17, Eq.(3)]:

$$\Phi(\boldsymbol{\xi}) = \frac{\delta\big[\Phi^{Ⓡ}(\boldsymbol{\xi})\big]^\gamma}{\delta\big[\Phi^{Ⓡ}(\boldsymbol{\xi})\big]^\gamma + \big[1 - \Phi^{Ⓡ}(\boldsymbol{\xi})\big]^\gamma}, \tag{4}$$

Data: ν_{MAX}, ρ, $\tilde{\lambda}^0_\omega\ \forall\omega$, ϵ
for ν *from 1 to* ν_{MAX} **do**

① For $\{\tilde{\lambda}^{\nu-1}_\omega\}_{\forall\omega}$, update producer response

$$p^\nu, \{r^\nu_\omega\}_{\forall\omega} \leftarrow \underset{(p,r_\omega)\in\mathcal{O}}{\text{argmax}} \quad J^{\text{p}}(p, r_\omega)$$

② For $\{\tilde{\lambda}^{\nu-1}_\omega\}_{\forall\omega}$, update consumer response

$$d^\nu, \{l^\nu_\omega\}_{\forall\omega} \leftarrow \underset{(d,l_\omega)\in\mathcal{K}}{\text{argmax}} \quad J^{\text{c}}(d, l_\omega)$$

③ For $p^\nu, \{r^\nu_\omega\}_{\forall\omega}$ and $d^\nu, \{l^\nu_\omega\}_{\forall\omega}$, update price for each outcome ω:

$$\tilde{\lambda}^\nu_\omega = \max\Big\{0, \tilde{\lambda}^{\nu-1}_\omega - \rho\big[p^\nu + r^\nu_\omega + \xi_\omega - d^\nu - l^\nu_\omega\big]\Big\}$$

④ Return ϵ-equilibrium prices and dispatch if:

$$\big\|p^\nu + r^\nu_\omega + \xi_\omega - d^\nu - l^\nu_\omega\big\|^2 \leqslant \epsilon,\ \forall\omega \in \Omega,$$

otherwise go to ①.

end

Algorithm 1: Solution algorithm

Table 1: Descriptive statistics of distributions: Ⓜ-labeled distributions primarily tweak the mean of the reference distribution Ⓡ, while Ⓢ-labeled distributions primarily tweak the variance of Ⓡ.

Label	$\mu^\uparrow_3$	$\mu^\uparrow_2$	$\mu^\uparrow_1$	Ⓡ	$\mu^\downarrow_1$	$\mu^\downarrow_2$	$\mu^\downarrow_3$
μ	2.02	1.79	1.65	1.56	1.34	1.22	1.07
σ^2	0.59	0.59	0.58	0.58	0.56	0.55	0.52
Label	$\sigma^\uparrow_3$	$\sigma^\uparrow_2$	$\sigma^\uparrow_1$	Ⓡ	$\sigma^\downarrow_1$	$\sigma^\downarrow_2$	$\sigma^\downarrow_3$
μ	1.63	1.60	1.57	1.56	1.55	1.55	1.56
σ^2	1.27	0.96	0.73	0.58	0.31	0.21	0.13

where Φ represents the cumulative distribution function of stochastic renewable production, $\delta \in \mathbb{R}_+$ primarily affects the mean of the reference distribution Ⓡ, and $\gamma \in \mathbb{R}_+$ primarily impacts the variance. By applying (4) to the reference distribution for different δ and γ, we obtain a collection of probability assignments to the same set of outcomes. Table 1 summarizes the distributions that we use in the following analysis.

In our setup, consumer always optimizes against the reference distribution Ⓡ, while producer optimizes against one of the distributions in Table 1. When producer uses Ⓡ in its local optimization, the equilibrium solution corresponds to the symmetric case, and any deviation from Ⓡ corresponds to the asymmetric equilibrium.

4.3. Numerical results

We first consider the impact of information asymmetry on the price at the day-ahead stage depicted in Figure 3. We see that the day-ahead price is maximized when the two agents use the same uncertainty distribution Ⓡ. Any deviation from Ⓡ in producer optimization decreases

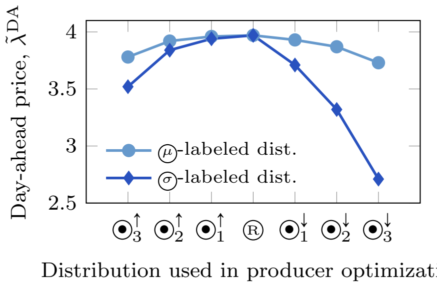


Figure 3: Impacts of information asymmetry on the day-ahead price.

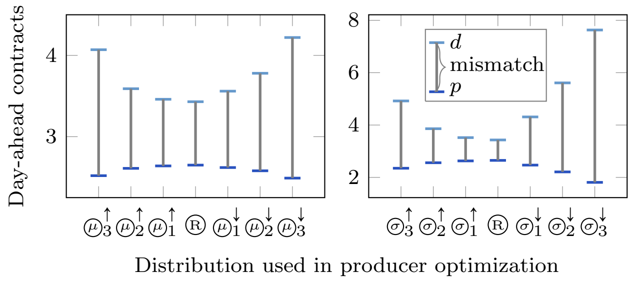


Figure 4: Impacts of information asymmetry on the contracted quantities of consumer (d) and producer (p) at the day-ahead stage.

the equilibrium price. The resulting price-supported day-ahead contracts illustrated in Figure 4 show that such deviations in terms of either mean or variance lead to increasing power mismatch between controllable generation and consumption.

Next, we compute the realization of the social welfare for each uncertainty outcome for the fixed day-ahead decisions of the producer and consumer. They are computed considering symmetric information and some asymmetric information cases. The results are summarized in Figure 5. We observe that the social welfare improves in larger realizations of renewable output, and records the maximum when producer deploys Ⓡ. For any deviation of the producer from the reference distribution, we find a social loss, that is smaller for deviations in terms of the mean rather than variance for given distributions. Moreover, we see the welfare reduces more significantly if the producer assigns smaller variance relatively to that of the consumer.

Finally, we show how the computational performance of the algorithm is affected by the asymmetry of information. Table 2 collects the number of iterations required by the algorithm to converge along with the ratio between the largest and the smallest eigenvalues of Jacobian of the demand excess function. We see that apart from the case of $\mu_1^{\uparrow}$ distribution, the asymmetry of agent information yields larger ratio of eigenvalues, and thus requires more iterations to converge. Moreover, for a highly asymmetric case of a low-variance distribution $\sigma_3^{\downarrow}$, this ratio boosts so that the algorithm does not converge for any iteration limit.

Table 2: Number of iterations and the ratio between the largest and smallest eigenvalues for various distributions in producer optimization.

Label	$\mu_3^{\uparrow}$	$\mu_2^{\uparrow}$	$\mu_1^{\uparrow}$	Ⓡ	$\mu_1^{\downarrow}$	$\mu_2^{\downarrow}$	$\mu_3^{\downarrow}$
# iters	369	347	312	321	362	372	381
ratio	2.4	2.2	2.0	2.0	2.2	2.3	2.5
Label	$\sigma_3^{\uparrow}$	$\sigma_2^{\uparrow}$	$\sigma_1^{\uparrow}$	Ⓡ	$\sigma_1^{\downarrow}$	$\sigma_2^{\downarrow}$	$\sigma_3^{\downarrow}$
# iters	379	372	353	321	359	499	∞
ratio	2.7	2.4	2.2	2.0	18.8	1.8e3	1.8e7

Appendix A. Proof of Proposition 1

Appendix A.1. Preliminaries

We connect the solution of the equilibrium problem to the solution of variational inequalities.

Definition 1. *Consider a mapping $F : \mathbb{R}^n \to \mathbb{R}^n$ and a set $K \subseteq \mathbb{R}$. A solution set* SOL*(K,F) to the variational inequality problem* VI*(K,F) is a vector $x^\star \in K$ such that $\langle F(x^\star), x - x^\star\rangle \geqslant 0,\ \forall x \in K$.*

In the following, we use the results from [9] to establish the existence and uniqueness of the solution to equilibrium.

Theorem 1 (Corollary 2.2.5 [9])**.** *Suppose that K is a compact and convex set, and that the mapping F is continuous. Then, the set* SOL*(K,F) is nonempty and compact.*

Theorem 2 (Theorem 1.3.1 [9])**.** *Let $F : U \to \mathbb{R}$ be continuously differentiable on the open convex set $U \subseteq \mathbb{R}$. The following three statements are equivalent: (a) there exists a real-valued function θ such that $F(x) = \nabla\theta(x)\ \forall x \in U$; (b) the Jacobian matrix of $F(x)$ is symmetric $\forall x \in U$; (c) F is integrable on U.*

In terms of equilibrium problem (2a)-(2c), $K = \mathcal{O} \times \mathcal{K} \times \Lambda_+$, vector x consists of decision variables of all market agents, and

$$F^\top = \big[\, \nabla_p J^{\mathrm{p}}(p,\boldsymbol{r})\ \nabla_{\boldsymbol{r}} J^{\mathrm{p}}(p,\boldsymbol{r})\ \nabla_d J^{\mathrm{c}}(d,\boldsymbol{l})\ \nabla_{\boldsymbol{l}} J^{\mathrm{c}}(d,\boldsymbol{l})\ \nabla_{\tilde{\boldsymbol{\lambda}}} \boldsymbol{J}^{\mathrm{ps}}(\tilde{\boldsymbol{\lambda}}) \,\big],$$

where symbols in bold are properly dimensioned vectors.

Appendix A.2. Proof

① Existence. Recall that by definition $\mathcal{O}$, $\mathcal{K}$ and Λ_+ are convex and compact. The map F is continuous as agents' objective functions are differentiable. Thus, the solution to equilibrium exists by Theorem 1.

② Uniqueness. We rely on the symmetry principle that states that if Jacobian of F is symmetric, there exists an equivalent optimization problem that solves VI(K,F). Consider the map F that consists of separable functions, and each component function depends only on the single variable. Thus, the Jacobian of F is a diagonal matrix. Therefore, the Jacobian is symmetric and conditions (b,c) of Theorem 2 hold, such that there exists a function $\theta(x)$ given by

$$\theta(x) = \int_0^1 F(x^0 + t(x - x^0))^\top (x - x^0) dt$$

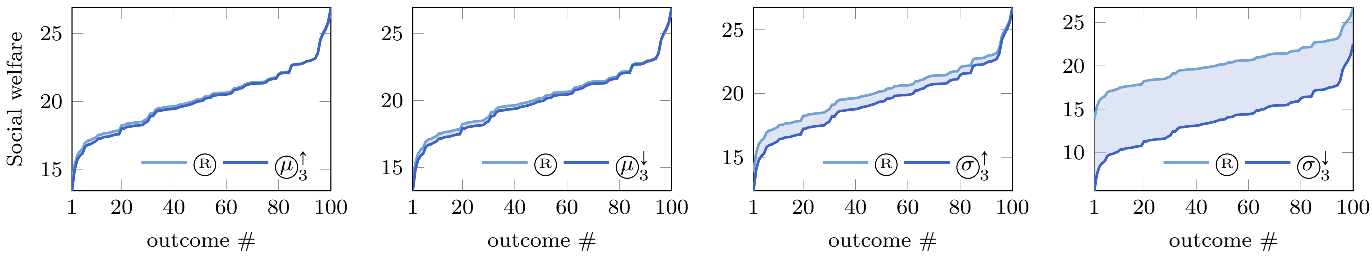


Figure 5: Social welfare for each outcome of renewable production. The 100 outcomes are ordered from the smallest to largest. The colored area between the curves shows the welfare loss caused by asymmetry of information.

$$
\begin{aligned}
=&\int_0^1 \begin{pmatrix} t\alpha p - t\Sigma_\omega \tilde{\lambda}_\omega \\ t\alpha \boldsymbol{\pi}^{\mathrm{p}\top} \boldsymbol{r} - t\tilde{\boldsymbol{\lambda}} \\ -\gamma + t\beta d + t\Sigma_\omega \tilde{\lambda}_\omega \\ -\gamma \boldsymbol{\pi}^{\mathrm{c}} + t\beta \boldsymbol{\pi}^{\mathrm{c}} \boldsymbol{l} + t\tilde{\boldsymbol{\lambda}} \\ td - t\boldsymbol{l} - tp - t\boldsymbol{r} - \boldsymbol{\xi} \end{pmatrix}^{\top} \begin{pmatrix} p \\ \boldsymbol{r} \\ d \\ \boldsymbol{l} \\ \tilde{\boldsymbol{\lambda}} \end{pmatrix} dt \\
=&[\alpha p^2 - 2p\Sigma_\omega \tilde{\lambda}_\omega + \Sigma_\omega \pi_\omega^{\mathrm{p}} \alpha r_\omega^2 - 2\Sigma_\omega \tilde{\lambda}_\omega r_\omega + \beta d^2 \\
&+ 2d\Sigma_\omega \tilde{\lambda}_\omega + \Sigma_\omega \pi_\omega^{\mathrm{c}} \beta l_\omega^2 + 2\Sigma_\omega \tilde{\lambda}_\omega l_\omega] \int_0^1 t dt \\
&+ [-\gamma d - \Sigma_\omega \pi_\omega^{\mathrm{c}} \gamma l_\omega - \Sigma_\omega \tilde{\lambda}_\omega \xi_\omega] \\
=&\frac{1}{2}\alpha p^2 - [\gamma d - \frac{1}{2}\beta d^2] + \Sigma_\omega \pi_\omega^{\mathrm{p}} \frac{1}{2}\alpha r_\omega^2 \\
&- \Sigma_\omega \pi_\omega^{\mathrm{c}} [\gamma l_\omega - \frac{1}{2}\beta l_\omega^2] + \tilde{\lambda}_\omega [p + r_\omega + \xi_\omega - d - l_\omega],
\end{aligned}
$$

which gives rise to the following optimization:

$$\max_{p, r_\omega, d, l_\omega} \big[u(d) - c(p)\big] + \sum_{\omega \in \Omega} \pi_\omega^{\mathrm{c}} u(l_\omega) - \sum_{\omega \in \Omega} \pi_\omega^{\mathrm{p}} c(r_\omega), \tag{A.1a}$$

$$\text{s.t.} \quad p + r_\omega + \xi_\omega - d - l_\omega \geqslant 0 : \tilde{\lambda}_\omega, \quad \forall \omega \in \Omega, \tag{A.1b}$$

$$(p, r_\omega) \in \mathcal{O}, \ (d, l_\omega) \in \mathcal{K}, \quad \forall \omega \in \Omega, \tag{A.1c}$$

whose stationarity conditions correspond to the map F. We know that optimization (A.1) yields a unique solution due to strict concavity of objective function and convex and compact constraint set. Since the solution of (A.1) constitutes set SOL(K,F), the solution of original equilibrium problem (2a)-(2c) is also unique, as desired.

Appendix B. Proof of Proposition 2.

Since producer and consumer optimize over independent variables, we can optimize problems (2b) and (2c) jointly. If we constrain the joint problem by the optimality conditions of price-setter problem (2a), we obtained the following optimization:

$$\max_{p, r_\omega, d, l_\omega} J^{\mathrm{p}}(p, r_\omega) + J^{\mathrm{c}}(d, l_\omega), \tag{B.1a}$$

$$\text{s.t.} \quad (p, r_\omega) \in \mathcal{O}, \ (d, l_\omega) \in \mathcal{K}, \ \forall \omega \in \Omega, \tag{B.1b}$$

$$0 \leqslant p + r_\omega + \xi_\omega - d - l_\omega \perp \tilde{\lambda}_\omega \geqslant 0, \ \forall \omega \in \Omega. \tag{B.1c}$$

This is equivalent to the optimality condition of the centralized problem (1) given that expectations over uncertain renewable production are the same.

Appendix C. Proof of Proposition 4

The descent direction of the price-setter problem writes as

$$-\nabla_{\tilde{\lambda}_\omega} J_\omega^{\mathrm{ps}}(\tilde{\lambda}_\omega) = p^\nu + r_\omega^\nu + \xi_\omega - d^\nu - l_\omega^\nu.$$

Then, the solution of the price-setter problem evolves along the decent direction with a suitable step size ρ as follows:

$$\tilde{\lambda}_\omega^\nu = \tilde{\lambda}_\omega^{\nu-1} - \rho \nabla_{\tilde{\lambda}_\omega} J_\omega^{\mathrm{ps}}(\tilde{\lambda}_\omega),$$

that is bounded from below by zero due to $\tilde{\lambda}_\omega \in \Lambda_+$.

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