Direct Current Flowgate Transmission Rights

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***Abstract*—This letter defines flowgate transmission rights for DC lines, based on a linearized power flow for the AC grid and network flow for the DC lines. The new rights are functionally identical to flowgate rights for AC lines, and therefore can be incorporated similarly into existing market frameworks. We also observe that, unlike in the AC case, the nodal price difference is always equal to the transmission capacity shadow price of a DC line. We show, using an illustrative example, how this financial mechanism can be used to hedge against nodal price variations.**

***Index Terms*—Financial transmission right, flowgate right, direct current transmission, electricity markets.**

I. INTRODUCTION

The favorable technical properties of direct current (DC) transmission lines can enhance the economic and secure operation of the power system [1]. DC links are well-suited for long-distance power delivery and interconnection of asynchronous power systems. The superior controllability of DC lines enables additional recourse actions during real-time operation and, for some topologies, alleviate problems due to congestion and loop flows. These features increase transmission capacity firmness and improve system reliability, which are essential coping with uncertainty from renewables.

To harness the advantages of DC transmission it is important to provide proper investment incentives and to ensure optimal operation of these assets according to system needs. Tradable transmission rights are a proven approach in AC networks, creating a revenue stream for the investors while decoupling physical dispatch from transmission ownership and congestion settlements. The two main forms of transmission rights are point-to-point (PTP) [2] and flowgate (FGR) rights [3].

The purpose of this letter is to extend FGRs to DC links. Existing literature [4] has only focused on PTP DC transmission rights. PTP rights are suitable for hedging bilateral contracts against congestion risk, since they are defined independently of network topology and implicitly account for loop flows in AC grids. On the contrary, FGRs are based on the physical capacity of a specific transmission asset and thus can be promptly used to finance grid investments. FGRs are especially appropriate for DC links, which are per se free of loop flows and often have prominent network locations with clear system effects, e.g., the interconnection of two AC systems.

II. DC FLOWGATE RIGHTS

Consider an electric power system comprising a set $\mathcal{N}$ of nodes with θ_i and p_i denoting the voltage angle and real power injection/withdrawal at node $i \in \mathcal{N}$, respectively. Let $\mathcal{L}^{\text{AC}}$ and $\mathcal{L}^{\text{DC}}$ be the sets of AC and DC lines and let $\mathcal{N}_i^{\text{AC}}$ and $\mathcal{N}_i^{\text{DC}}$ be the sets of nodes neighboring i through AC and DC lines, respectively. Using linearized and network flow approximations for AC and DC lines, respectively, the optimal power flow problem is written as

$$\min_{p,\theta,u} \qquad \mathcal{F}(p) \tag{1}$$

subject to

$$\lambda_i \qquad p_i = \sum_{j\in\mathcal{N}_i^{\text{AC}}} b_{ij}(\theta_i-\theta_j) + \sum_{j\in\mathcal{N}_i^{\text{DC}}} u_{ij}, i\in\mathcal{N}, \tag{2}$$

$$\xi_i^l, \xi_i^u \geq 0 \perp \qquad \underline{p}_i \leq p_i \leq \overline{p}_i, \quad i\in\mathcal{N}, \tag{3}$$

$$\mu_{ij}^{\text{AC}} \geq 0 \perp \qquad b_{ij}(\theta_i-\theta_j) \leq \overline{s}_{ij}, \quad (i,j)\in\mathcal{L}^{AC}, \tag{4}$$

$$\mu_{ij}^{\text{DC}} \geq 0 \perp \qquad u_{ij} \leq \overline{s}_{ij}, \quad (i,j)\in\mathcal{L}^{DC}, \tag{5}$$

$$\nu_{ij} \qquad u_{ij}+u_{ji}=0, \quad (i,j)\in\mathcal{L}^{DC}. \tag{6}$$

The objective function (1) to be minimized is the real power cost over all nodes with $\mathcal{F}(p)$ being convex and continuously differentiable. The equality constraint (2) is the nodal power balance equation and the inequalities (3) impose the upper and lower real power limits denoted as $\underline{p}_i$ and $\overline{p}_i$, respectively. Constraints (4) and (5) enforce the transmission capacity limits $\overline{s}_{ij}$ for AC and DC lines, respectively, where b_{ij} is the susceptance of AC line (i,j). These constraints are enforced in both directions since both orientations are in the line sets, i.e., if $(i,j)\in\mathcal{L}^{\text{AC}}$ then $(j,i)\in\mathcal{L}^{\text{AC}}$ and likewise for DC lines. This implies that $u_{ij}=-u_{ji}$ for each DC line (i,j) as stated in constraint (6). Each constraint's dual multipliers are listed to their left, with complementarity relationships indicated by the $\perp$ symbol. In particular, λ_i and μ_{ij}^{AC} (μ_{ij}^{DC}) are the conventional nodal prices and shadow prices associated with AC (DC) lines.

We remark that this model assumes the existence of converters at both ends of DC lines. It does not apply to systems in which DC lines are directly bus connected. As an approximation, it does not capture converter limits and inefficiencies, and therefore is not well-suited for systems in which the subnetwork of DC lines is highly meshed. In such cases FGRs could be defined using convex relaxations of power flow in mixed AC and DC networks [5].

Differentiating the Lagrangian by the primal variables yields the following equations, which are part of the KKT conditions:

$$\frac{d\mathcal{F}(p)}{dp_i} - \lambda_i + \xi_i^u - \xi_i^l = 0, \quad i\in\mathcal{N}, \tag{7}$$

$$\sum_{j\in\mathcal{N}_i^{\text{AC}}} b_{ij}(\lambda_i - \lambda_j + \mu_{ij}^{\text{AC}} - \mu_{ji}^{\text{AC}}) = 0, \quad i\in\mathcal{N}, \tag{8}$$

$$\lambda_i + \mu_{ij}^{\text{DC}} + \nu_{ij} + \nu_{ji} = 0, \quad (i,j)\in\mathcal{L}^{DC}. \tag{9}$$

Multiplying (2) by λ_i, summing over i, and then making substitutions based on the above KKT conditions and complementary slackness, we obtain the budget balance equation

$$\sum_{i\in\mathcal{N}} \lambda_i p_i + \sum_{ij\in\mathcal{L}^{\text{AC}}} \mu_{ij}^{\text{AC}}\overline{s}_{ij} + \sum_{ij\in\mathcal{L}^{\text{DC}}} \mu_{ij}^{\text{DC}}\overline{s}_{ij} = 0. \tag{10}$$

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The first term in (10) is the budget surplus of the system operator resulting from the nodal payments. The second and third terms define the FGRs for AC and DC lines, respectively. In the event of network congestion, either in the AC or DC part of the grid, the associated dual multipliers of constraints (4) and (5) are positive, which in turn yields a nonzero budget surplus that can be redistributed through FGRs. In particular, the owner of a DC (AC) FGR is entitled to a payment of μ_{ij}^{DC} (μ_{ij}^{AC}) times the contracted power flow quantity. The derivation of (10) can be found in [6].

Observe from (9) that

$$\lambda_i - \lambda_j = \mu_{ji}^{\text{DC}} - \mu_{ij}^{\text{DC}}, \quad (i,j) \in \mathcal{L}^{\text{DC}}. \tag{11}$$

By complementary slackness and the fact that power must flow in one direction, at least one of μ_{ji}^{DC} and μ_{ij}^{DC} must be zero at an optimal solution. This means that the nodal price difference across a DC line is equal to its shadow price regardless of network topology. This differs from AC lines, in which the nodal price difference is only guaranteed to be equal to the shadow price in radial networks.

III. Illustrative example

We illustrate the main properties of DC FGRs using the four-node system depicted in Fig. 1. The susceptance of all AC lines is equal to 5 p.u. and their transmission capacity limits are shown in Fig. 1. We consider two network setups where branch (i_4, i_3) is either an AC or DC line.

In order to demonstrate the use of DC FGRs as a hedging instrument against transmission congestion, we assume that a consumer at node i_3 has signed a contract of difference (CfD) with the generator at node i_4 for the supply of 10MW at 8$/MWh during a specific hour. The reference price for the contract settlement is the nodal price at node i_4. In addition, the consumer at node i_3 has to decide on the amount of FGRs that needs to buy in order to protect himself against nodal price variations.

Table I provides the dispatch results and the associated nodal electricity prices for the DC and AC setups. In both setups, line (i_4, i_3) is congested with the corresponding dual multipliers being μ_{43}^{DC} = 2.5$/MWh and μ_{43}^{AC} = 16$/MWh. To settle the CfD in the DC setup, the consumer has to pay $10 \times 10 = \$100$ to the market operator for withdrawing 10 MW at node i_3 and $10 \times (8 - 7.5) = \$5$ to the generator at node i_4 to settle the CfD. The generator is also paid $10 \times 7.5 = \$75$ from the market operator for injecting 10 MW at node i_4 and thus receives in total $80, which is equivalent to a price of 8$/MWh, i.e., the strike price of the CfD. Hence, the consumer has paid so far $105 or equivalently 10.5$/MWh due to different prices in nodes i_3 and i_4. To hedge this nodal price differential, the consumer can buy 10MW of DC FRGs on line (i_4, i_3) that will entitle a payment of $10 \times \mu_{43}^{\text{DC}} = \$25$. In turn, the total consumer payment is $105 - 25 = 80\$$ or 8$/MWh. Given property (11) of DC FGRs to perfectly hedge nodal price differences irrespective of network topology, the procurement of 10MW of FGRs on the DC line (i_4, i_3) is straightforward.

This problem becomes more complex in the AC setup. Following the same approach as in the DC setup for deriving payments and revenues, the consumer has to pay $160 to the market operator and 20$ to the generator at node i_4 or equivalently 18$/MWh. In this case, the decision on the amount of AC FGRs is not that obvious, since in a full AC network 1MW of power injected at node i_3 and withdrawn at node i_4 does not flow directly on line (i_4, i_3). In fact, the consumer at node i_3 must know the factors relating the nodal power injections and the line flows, i.e., power transfer distribution factors (PTDFs), in order to determine the amount of AC FGRs that will insure its CfD against spatial price difference. For this example, where line (i_4, i_3) is the only congested branch in the AC setup, the consumer has to buy 6.25 MW of AC FGRs to be entitled for a payment of $6.25 \times \mu_{43}^{\text{AC}} = \$100$ from the market operator, resulting to an equivalent 8$/MWh price for the 10 MW of the CfD.

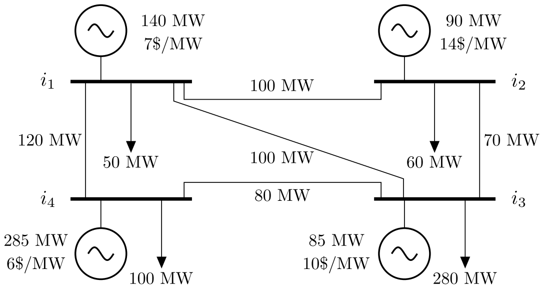

Fig. 1. Four-node power system.

TABLE I
Dispatch results and electricity prices

	AC setup		DC setup	
Node	p_i [MW]	λ_i [$/MWh]	p_i [MW]	λ_i [$/MWh]
i_1	140	12	125	7.5
i_2	76.25	14	0	8.75
i_3	85	16	80	10
i_4	188.75	6	285	7.5

IV. Conclusion

In this letter, we define FGRs for DC lines based on the same principles as the AC FGRs. We show that value of DC FGR is always equal to the shadow price of the respective DC line in contrast to the value of their AC counterparts that depends on network PTDFs. We demonstrate the properties of this financial mechanism using a small illustrative example in which DC FGRs are used to hedge nodal price volatility.

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