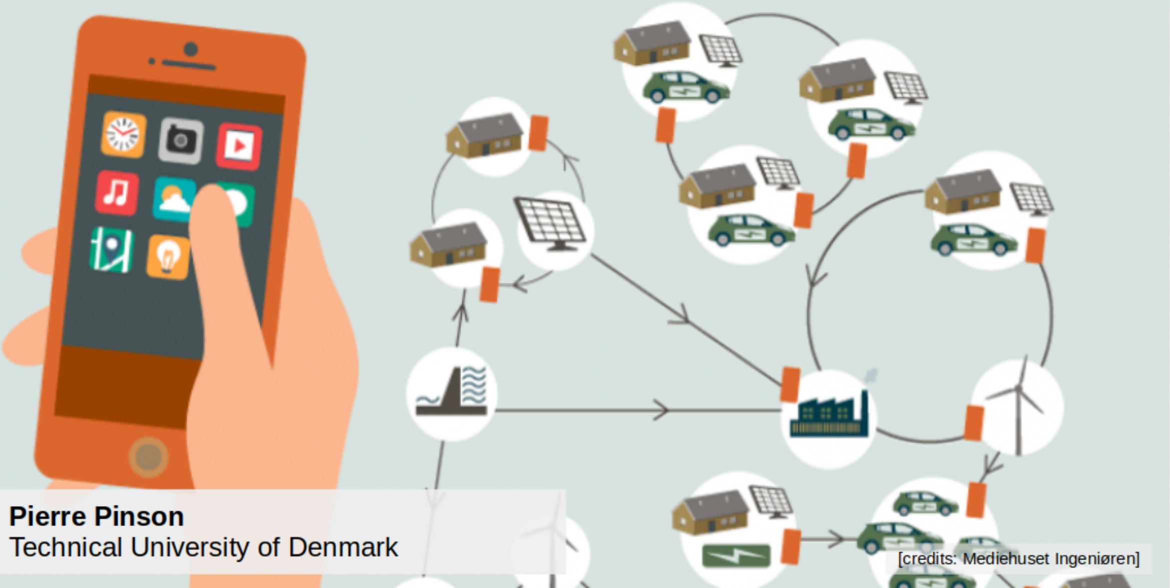
Module 6 – Participation of Renewables in Electricity Markets

6.2 Decision-making under uncertainty



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[credits: Mediehuset Ingeniøren]

The newsvendor problem

- The newsvendor problem is one of the most classical problems in **stochastic optimization** (or **statistical decision theory**)

- It can be traced back to:

FY Edgeworth (1888). The mathematical theory of banking. *Journal of the Royal Statistical Society* **51**(1): 113–127

even though in this paper the problem is about how much a bank should keep in its reserves to satisfy request for withdrawal (i.e., the *bank-cash-flow problem*)

- It applies to varied problems as long as:
  - one shot possibility to decide on the quantity of interest
  - outcome is uncertain
  - known marginal profit and loss
  - the aim is to maximize expected profit!

Revisited: The "Roskilde ticket pusher" problem©

- Everybody seem to want to go and see Eminem, right? (could also be Bruno Mars or Gorillaz, for those who don't like Eminem)
- Maybe some could have the idea of making a profit using this as an occasion...

[Note that this type of activity is not legal, as such purchased tickets cannot be re-sold at a price higher than the official retail price] - Do not get any idea here!

- 1-day tickets for the day Eminem is playing
- They are to be sold out fast, while you know that quite a lot of DTU students will not be able to buy the tickets on time...
- On 5 March 2018, you have an opportunity to make a good deal:
  - buy a batch tickets (up to 30) at an advantageous price!
  - sell them out to your fellow DTU students

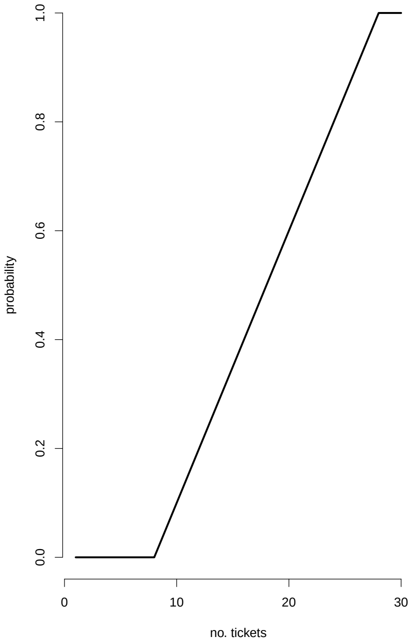
"Roskilde ticket pusher" problem©: detailed setup

- Sets of prices:
  - 1-day tickets for Eminem: **1050 dkk**
  - retail price to DTU students: **1100 dkk**
  - unsold tickets can be given to your RUC pusher friend at **930 dkk**

- Why is it a newsvendor problem?
  - this is a one-shot opportunity - batch buy on **5 March 2018** (here and now!)
  - actual DTU demand is uncertain
  - the marginal profit and loss are known - a profit of 50 dkk per ticket sold, and a loss of 120 dkk per ticket unsold
  - the aim definitely is to maximize expected profit!!

- If you were that "Roskilde ticket pusher", *how many tickets would you buy?*

You need to know your probabilities



- Based on an expert assessment, here is the *cumulative distribution function* F for the number of tickets (X) we may be able to sell to our DTU fellow students
- It shows $\mathrm{P}[X \leq n]$ as a function of n
- Examples:
  - $\mathrm{P}[X \leq 8] = 0$: we are 100% sure to sell at least 8 tickets
  - $\mathrm{P}[X \leq 10] = 0.1$: we are 90% sure to sell more than 10 tickets
  - $\mathrm{P}[X \leq 20] = 0.6$: we are 40% sure to sell more than 20 tickets
  - $\mathrm{P}[X \leq 28] = 1$: there is no way we sell more than 28 tickets

In terms of marginal profit and loss

n	$\mathrm{P}[X=n]$	P_{sell}	$\mathrm{P}_{\text{no-sell}}$	$\mathbb{E}[\text{profit}]$	$\mathbb{E}[\text{loss}]$	$\mathbb{E}[\text{net}]$
≤ 8	0					
9	0.05	1	0	50	0	**50**
10	0.05	0.95	0.05	47.5	6	**41.5**
11	0.05	0.9	0.1	45	12	**33**
...						

where

- $\mathrm{P}[X=n]$: probability that demand is **EXACTLY** n tickets
- P_{sell}: probability of selling the n^{th} ticket
- $\mathrm{P}_{\text{no-sell}}$: probability of **NOT** selling the n^{th} ticket
- $\mathbb{E}[\text{profit}]$: expected profit from selling the n^{th} ticket
- $\mathbb{E}[\text{loss}]$: expected loss from **NOT** selling the n^{th} ticket
- $\mathbb{E}[\text{net}]$: expected net profit related to the n^{th} ticket

With the full table

n	$P[X=n]$	P_{sell}	$P_{\text{no-sell}}$	$\mathbb{E}$[profit]	$\mathbb{E}$[loss]	$\mathbb{E}$[net]
≤ 8	0					
9	0.05	1	0	50	0	**50**
10	0.05	0.95	0.05	47.5	6	**41.5**
11	0.05	0.9	0.1	45	12	**33**
12	0.05	0.85	0.15	42.5	18	**24.5**
13	0.05	0.8	0.2	40	24	**16**
14	0.05	0.75	0.25	37.5	30	**7.5**
15	0.05	0.7	0.3	35	36	**-1**
16	0.05	0.75	0.35	32.5	42	**-9.5**
...						
28	0.05	0	1	0	120	**-120**
>28	0					

- So, *how many tickets should our "Roskilde ticket pusher" buy?*

Mathematical formulation

- If we have
  - λ^P: purchase cost for a ticket (1050 dkk)
  - λ^R: re-sell price of a ticket (1100 dkk)
  - λ^T: transfer price for unsold tickets (930 dkk)

- It then defines
  - π^+: unit cost of buying less than needed

    $$\pi^+ = \lambda^R - \lambda^P \text{ (50 dkk)}$$

  - π^-: unit cost of buying more than needed

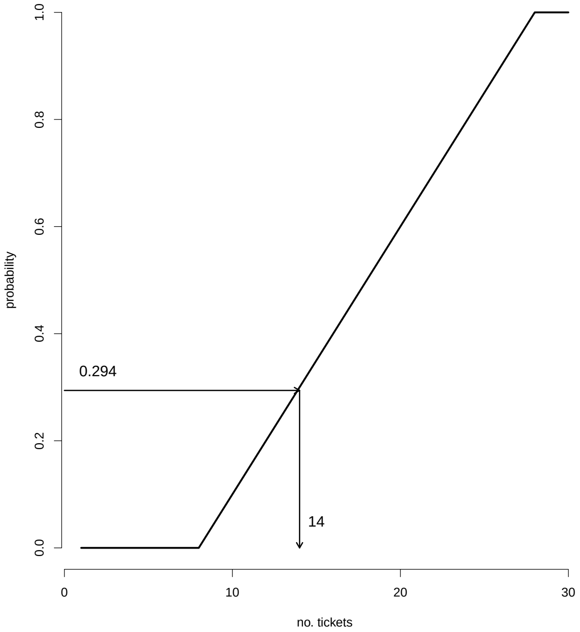
    $$\pi^- = \lambda^P - \lambda^T \text{ (120 dkk)}$$

- Then the optimal number n^* of tickets to purchase is such that:

  $$\mathrm{P}[X \leq n^*] = \frac{\pi^+}{\pi^+ + \pi^-} \text{ (here, 0.294)}$$

  This defines the nominal level α^* of our original *cumulative distribution function* F

The optimal quantile

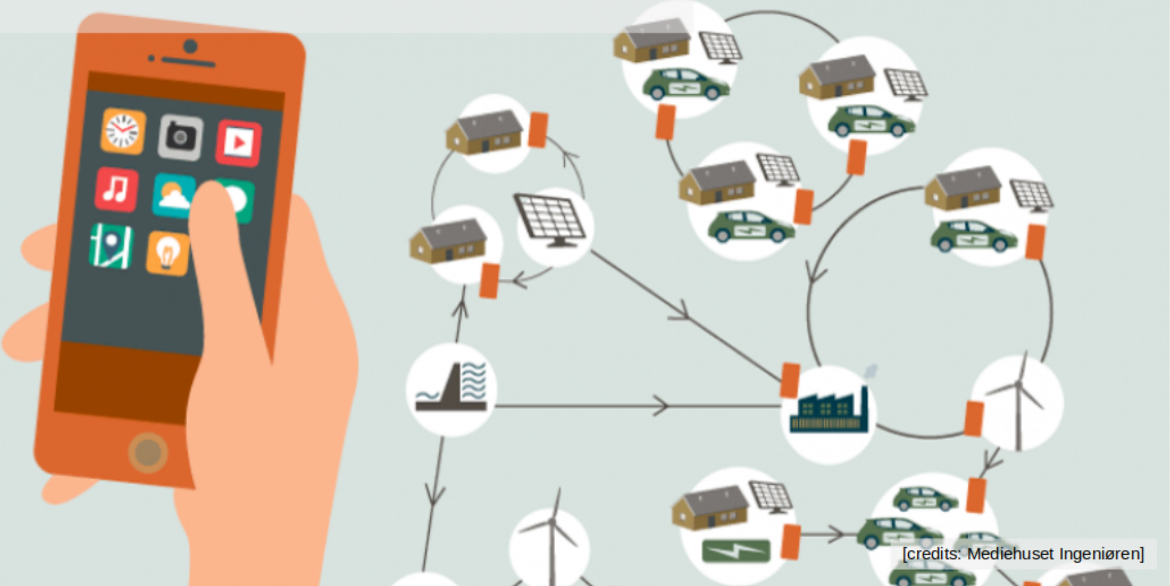


- The optimal decision of the "Roskilde ticket pusher" is to pick the quantile with nominal level α^* of his predictive *cumulative distribution function F*

- Graphically:

$$\boxed{n^* = F^{-1}(\alpha^*) = 14}$$

Use the self-assessment quizz to check your understanding!



[credits: Mediehuset Ingeniøren]