Chapter 7
Trading Stochastic Production in Electricity Pools

7.1 Introduction and Decision Framework

Recent years have witnessed an exceptional technological development that, along with increasing political pressure to cut CO_2 emissions and to create local jobs, spurred an unprecedented growth in installed production capacity from renewable sources. To sustain such growth, national governments have put in place special support schemes (tax credits, feed-in tariffs, etc.) for easing the market participation of renewable producers.

As the energy cost of renewables constantly decreases approaching grid parity, green power needs smaller and smaller incentives for being competitive. For this reason, renewable power producers are increasingly required to participate in electricity markets under the same rules as conventional power producers. In particular, as opposed to feed-in tariff schemes, they are more and more frequently subject to market prices and assigned *balance responsibility*. The former implies that renewable electricity producers are subject, like any other power producer, to price risk. Besides, the latter implies that they are financially accountable for the additional balancing costs incurred by the system operators, which in practice means that they need to correct their energy imbalances by trading in the balancing market.

Although renewable producers are asked to participate in the market in the same way as conventional producers, trading green energy presents substantial differences when compared to the case of conventional power sources. Firstly, the actual production is variable and uncertain at the time of offering. This uncertainty, coupled with the stochastic nature of power prices, results in uncertain returns depending on the realization of both power production and prices. Secondly, renewable producers are *forced* to participate in multiple markets, because markets with early gate closures—day-ahead- markets—have more stable prices and, in parallel, deviations of the actual production from the contractual positions at the day-ahead and adjustment markets must be settled at the balancing market.

As the market design often—although not always—penalizes real-time deviations from (and in general later corrections of) the day-ahead schedule, renewable power producers are in a disadvantaged position compared to conventional producers. As a partial solution to these disadvantages, renewable power producers can trade

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strategically, in the attempt to get the most out of the information they possess on the uncertain variables in play. This chapter focuses on the determination of optimal trading strategies for renewable power producers participating in electricity pools. Owing to the stochastic features and the multiple market layers described above, this is a multistage problem of decision-making under uncertainty.

Two basic assumptions are made in this chapter. The first one entails that incentives such as price premia added on top of market prices, feed-in tariffs, etc., are discarded. As a result, renewable energy is traded under exactly the same rules as the conventional one. This simplification is introduced for the sake of clarity and generality as different markets have different incentive schemes. Nevertheless, the tools developed here could be extended so as to account for these incentives with little effort. The second assumption is that renewable power producers participate in the electricity pool on their own. This means that the optimal trading strategies developed in this chapter do not account for possible associations with other market entities, e.g., owners of storage devices or flexible demand, which are treated in Chaps. 8 and 9, respectively.

This chapter is structured as follows. Section 7.2 presents the basic problem formulation and introduces some common concepts in decision-making under uncertainty. Optimal strategies are determined analytically in Sect. 7.3 for offering in the day-ahead and balancing markets considering deterministic electricity prices. Section 7.4 develops different strategies in a series of cases with stochastic market prices. Section 7.5 considers the case of a risk-averse stochastic producer. Thereafter, Sect. 7.6 models the problem in the framework of stochastic programming, which is more versatile as it can accommodate any type of probabilistic multivariate distribution of the uncertainty as well as different risk metrics. Finally, Sect. 7.7 concludes the chapter.

7.2 Revenue and Imbalance Cost: Concept and Definition

In this section, we introduce the basic formulation of the problem of optimal trading when the production volume is uncertain, which is the case for renewable energy sources such as wind and solar. In parallel, some concepts of decision-making under uncertainty are presented.

In order to set up the problem in a simple framework, we introduce the following assumptions. Later on in the development of this chapter, assumptions A1–A3 will be gradually removed.

- A1 The stochastic producer trades only at the day-ahead and at the balancing market, while adjustment markets are discarded from the analysis.
- A2 The only uncertainty is related to the production volume, while market prices are deterministic and known in advance.
- A3 The producer is risk-neutral, meaning that it aims at the maximization of the expected profits with disregard of possible losses.
- A4 The stochastic producer is a price-taker, i.e., market prices do not depend on the employed offering strategy.

Assumptions A1 and A4 are critical for obtaining an analytical solution to the trading problem. On the other hand, assumption A2, which is exploited in the derivations in Sect. 7.3, is mainly for presentation convenience. Indeed, Sect. 7.4.1 shows that, under the assumption that production and market prices are uncorrelated, prices can be substituted by their expected values. Analytical solutions are still available under less restrictive assumptions on the correlation between day-ahead and balancing market prices; they are presented in Sect. 7.4.1. Further results obtained considering the correlation between prices and production are presented in [6]. The risk-neutrality assumption A3 is justified by the relatively high frequency with which the producers offer in electricity markets. This implies that the possible losses incurred in a single trading period are small if aggregated over a reasonable time-span. Section 7.5 presents some analytical results available when considering a risk-averse power producer.

As we show in Sect. 7.6, the stochastic programming framework provides the necessary flexibility to treat the trading problem for stochastic power producers disregarding the assumptions A1–A3 above.

To get rid of the price-taker assumption A4, one can employ mathematical programs with equilibrium constraints (MPEC), see Appendix B and [7]. Given its complexity, this topic is not covered here. However, we refer the interested reader to [2] and [18], where stochastic MPECs are applied to trading problems of renewable suppliers.

In the remainder of this section, we consider the cases of one-price and two-price markets separately.

Before getting started, two important clarifications should be made. Firstly, we assume that the renewable electricity producer is located at a certain bus of the transmission network. For the sake of simplicity, we always use the term "market price." However, if considering a market with nodal pricing, "market price" has the meaning of "locational marginal price (LMP) at the bus of interest." Similarly in a market with zonal pricing, such term would refer to the relevant zonal price. Notice that the generalization of the models presented in this chapter to the case of a producer whose plants are located in different nodes, or zones, of the power network is straightforward.

Secondly, there are no intertemporal constraints in the problems we analyze. This means that we focus on single-period models, and thus time indexes are dropped from the mathematical formulations.

7.2.1 One-Price Market

In a one-price balancing market, deviations from day-ahead contracts are traded at a unique balancing price regardless of the sign of producer and system imbalances. Generally, the balancing market price is higher than the day-ahead price if the system is in up-regulation, i.e., when the system is in deficit of power production as a result of all the deviations from producers and consumers with respect to their day-ahead

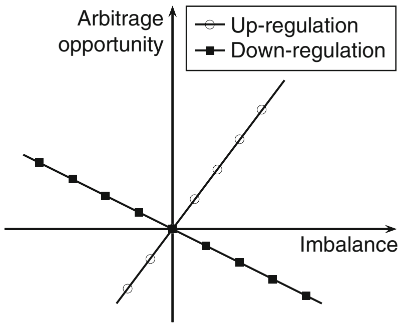


Fig. 7.1 Producer imbalance and arbitrage opportunity between balancing and day-ahead markets in the one-price system

positions. Conversely, in the down-regulation case (i.e., when the system has a surplus of generation) the balancing price is lower than the day-ahead price.

Due to the pricing rules described above, the participation at both the day-ahead and the balancing market opens arbitrage opportunities for power producers. When the producer's real-time imbalance with respect to the day-ahead contract is in the opposite direction compared to the overall system imbalance, power producers receive a more favorable price at the balancing market. Indeed, they can sell excess energy (positive imbalance) compared to their day-ahead position at a higher price than the day-ahead price when the system is in up-regulation, and repurchase their production deficit (negative imbalance) at a lower price in the down-regulation case. On the other hand, the balancing price is less favorable when the producer and the system deviations are in the same direction. Figure 7.1 summarizes these comments by showing the arbitrage opportunity as a function of the producer imbalance in the two regulation cases. Clearly, because the arbitrage opportunity is equal to the imbalance times the difference in price between the balancing and the day-ahead market, the relation is linear.

We begin our derivation by writing down the total profit during a single trading period for the stochastic power producer, which is equal to the product between the exchanged energy volume and the respective price, summed over all the market stages. Under assumption A1 in Sect. 7.1, only the day-ahead and the balancing markets are considered here.

Let us indicate prices with λ and traded production with E, with the superscripts D and B when these quantities refer to the day-ahead and to the balancing market, respectively. In a one-price system, a single balancing market price λ^{B} is applied for both sale and purchase of energy in real-time. Therefore, profits write down as

$$\widetilde{\rho} = \underbrace{\lambda^{\mathrm{D}} E^{\mathrm{D}}}_{\text{day-ahead market}} + \underbrace{\lambda^{\mathrm{B}} \widetilde{E}^{\mathrm{B}}}_{\text{balancing market}}. \tag{7.1}$$

The only decision variable in this formulation is E^{D}. This is because prices are exogenous variables under the price-taker assumption A4 made in the previous section. Furthermore, there are no degrees of freedom in the choice of the energy exchange $\widetilde{E}^{\mathrm{B}}$ at the balancing market. Indeed, this quantity is bound to match the difference between the day-ahead schedule and the actual production, i.e.,

$$\widetilde{E}^{\mathrm{B}} = \widetilde{E} - E^{\mathrm{D}}. \tag{7.2}$$

Being dependent on the uncertain production $\widetilde{E}$, the real-time exchange $\widetilde{E}^{\mathrm{B}}$ is stochastic, and so is the profit.

A relevant question when a decision-maker is exposed to stochastic profits is the definition of the objective of the optimal strategy. Owing to the risk-neutrality assumption A3, the producer is in this case interested in maximizing its profits *in expectation*, regardless of whether the shape of the profit distribution entails the possibility of incurring large losses. As we discuss in the following sections, decision-makers are not always risk-neutral, since in some circumstances possible losses are large enough to cause them financial problems.

In decision theory, the expected value of the profits for a certain decision goes under the name of *expected monetary value (EMV)*. Replacing (7.2) into (7.1), and taking the expectation yields the following expression for the EMV:

$$\mathbb{E}\{\widetilde{\rho}\} = \left(\lambda^{\mathrm{D}} - \lambda^{\mathrm{B}}\right) E^{\mathrm{D}} + \lambda^{\mathrm{B}} \widehat{E}, \tag{7.3}$$

where $\widehat{E}$ is the expected value of power production in the trading period considered.

7.2.2 *Two-Price Market*

In a two-price market, real-time deviations are priced differently depending on the imbalance sign. Deviations that are in the opposite direction to the overall system imbalance, which help the system restore the balance between production and consumption, are priced at the day-ahead market price. On the contrary, imbalances of the same sign as that of the system are settled at the clearing price of the balancing market. Let us denote the up-regulation and down-regulation prices with λ^{UP} and λ^{DW}, respectively, while the clearing price at the balancing market is still λ^{B}. The pricing rule in a two-price balancing market implies the following

$$\lambda^{\mathrm{UP}} = \begin{cases} \lambda^{\mathrm{B}} & \text{if} \quad \lambda^{\mathrm{B}} \geq \lambda^{\mathrm{D}}, \\ \lambda^{\mathrm{D}} & \text{if} \quad \lambda^{\mathrm{B}} < \lambda^{\mathrm{D}}, \end{cases} \tag{7.4}$$

$$\lambda^{\mathrm{DW}} = \begin{cases} \lambda^{\mathrm{D}} & \text{if} \quad \lambda^{\mathrm{B}} \geq \lambda^{\mathrm{D}}, \\ \lambda^{\mathrm{B}} & \text{if} \quad \lambda^{\mathrm{B}} < \lambda^{\mathrm{D}}. \end{cases} \tag{7.5}$$

As Fig. 7.2 shows, and contrarily to the one-price system, there is no arbitrage opportunity when the producer deviation is in the opposite direction to the overall system imbalance, since the price obtained at the balancing market is equal to the day-ahead market price. On the contrary, there is still an opportunity loss when producer and system deviations have the same sign.

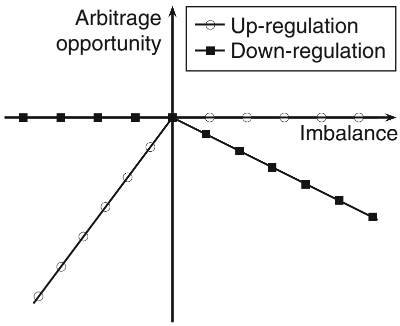


Fig. 7.2 Producer imbalance and arbitrage opportunity between balancing and day-ahead markets in the two-price system

As a result of this pricing rule, the term accounting for the balancing market profit in (7.1) splits in two when considering such a market settlement, resulting in the following formulation for the total profits

$$\widetilde{\rho} = \underbrace{\lambda^{\mathrm{D}} E^{\mathrm{D}}}_{\text{day-ahead market}} + \underbrace{\lambda^{\mathrm{UP}} \widetilde{E}^{\mathrm{UP}} + \lambda^{\mathrm{DW}} \widetilde{E}^{\mathrm{DW}}}_{\text{balancing market}}. \tag{7.6}$$

The symbols $\widetilde{E}^{\mathrm{UP}}$ and $\widetilde{E}^{\mathrm{DW}}$ refer to energy up-regulation and down-regulation for the producer at the balancing market, respectively. We recall that the power producer has to purchase upward regulation power at the balancing market when its actual production $\widetilde{E}$ is lower than the day-ahead position, E^{D}, while downward regulation is to be sold when $\widetilde{E}$ is larger than E^{D}. In mathematical terms, this writes

$$\widetilde{E}^{\mathrm{UP}} = \begin{cases} \widetilde{E} - E^{\mathrm{D}} & \text{if} \quad \widetilde{E} - E^{\mathrm{D}} \leq 0, \\ 0 & \text{if} \quad \widetilde{E} - E^{\mathrm{D}} > 0, \end{cases} \tag{7.7}$$

$$\widetilde{E}^{\mathrm{DW}} = \begin{cases} 0 & \text{if} \quad \widetilde{E} - E^{\mathrm{D}} \leq 0, \\ \widetilde{E} - E^{\mathrm{D}} & \text{if} \quad \widetilde{E} - E^{\mathrm{D}} > 0. \end{cases} \tag{7.8}$$

From the definition of up-regulation and down-regulation, it follows that

$$\widetilde{E} - E^{\mathrm{D}} = \widetilde{E}^{\mathrm{UP}} + \widetilde{E}^{\mathrm{DW}}. \tag{7.9}$$

Solving the previous equation for E^{D} and substituting the resulting expression in (7.6) yields

$$\widetilde{\rho} = \lambda^{\mathrm{D}} \widetilde{E} - \left[\left(\lambda^{\mathrm{D}} - \lambda^{\mathrm{UP}} \right) \widetilde{E}^{\mathrm{UP}} + \left(\lambda^{\mathrm{D}} - \lambda^{\mathrm{DW}} \right) \widetilde{E}^{\mathrm{DW}} \right]. \tag{7.10}$$

We notice that the first term of the sum in (7.10) is not under control of the power producer, in that neither λ^{D} nor $\widetilde{E}$ are dependent on its decisions. Furthermore, both terms inside brackets are nonnegative. This is because in a two-price system, pricing rules (7.4) and (7.5) entail that $\lambda^{\mathrm{UP}} \geq \lambda^{\mathrm{D}}$ and $\lambda^{\mathrm{DW}} \leq \lambda^{\mathrm{D}}$, and because (7.7) and (7.8) imply that $\widetilde{E}^{\mathrm{UP}} \leq 0$ and $\widetilde{E}^{\mathrm{DW}} \geq 0$.

The first term in (7.10) represents the profit that could be obtained by the producer, in a two-price system, if it had *perfect information* on the future realization of the stochastic production $\widetilde{E}$. Clearly in this case, the producer would sell the (certain) production entirely at the day-ahead market, since as Fig. 7.2 shows there is never a strictly positive arbitrage opportunity between the balancing market and the day-ahead one. The term within brackets in (7.10) is the *imbalance costs* that the producer faces when settling its regulation volume at the balancing market. This sum represents an *opportunity loss*, in that it quantifies the missing profits stemming from not being able to sell the uncertain production entirely, or for selling more than the actual production, at the day-ahead market.

Taking the expectation on both sides of (7.10), we get the following

$$\underbrace{\mathbb{E}\{\widetilde{\rho}\}}_{\text{EMV}} = \lambda^{\text{D}}\widehat{E} - \underbrace{\mathbb{E}\left\{\left(\lambda^{\text{D}} - \lambda^{\text{UP}}\right)\widetilde{E}^{\text{UP}} + \left(\lambda^{\text{D}} - \lambda^{\text{DW}}\right)\widetilde{E}^{\text{DW}}\right\}}_{\text{EOL}}, \qquad (7.11)$$

where $\widehat{E}$ is the expected power production and therefore, $\lambda^{\text{D}}\widehat{E}$ is the expected profit given perfect information. We will refer to the expectation term on the right-hand side of (7.11) equivalently as the expected imbalance costs or as the *expected opportunity loss (EOL)*.

Equation (7.11) states a general fact of decision-making under uncertainty: for any strategy, the sum of the expected monetary value (EMV) and the EOL is constant and equal to the expected profit obtained with perfect information. Therefore, we can alternatively look at the optimal strategy of a risk-neutral decision-maker either as the strategy maximizing the EMV, or as the decision minimizing the EOL. In the next section, we will determine the optimal offer for a stochastic power producer in the two-price system by formulating the problem as the minimization of the EOL.

As a final remark of this section, we point out that the minimum value of the EOL among all the feasible decisions is of particular importance. Such quantity is referred to as the *expected value of perfect information (EVPI)*. The EVPI represents the profit improvement that the decision-maker would experience if it held perfect information on the realization of the uncertainty, compared to the best performance achievable with the employed characterization of the stochastic parameters. Hence, this quantity also indicates how much the decision-maker would pay *at most* for obtaining perfect information on the contingent process. As such, the EVPI represents an upper bound to the value of an improved forecasting model for the uncertainty.

7.3 Trading with Deterministic Prices: Bidding Quantities

This section presents some analytical results for the determination of optimal trading strategies for stochastic producers when market prices are deterministic, i.e., known in advance with certainty. Similarly to the previous section, results for the one-price and the two-price markets are presented separately.

7.3.1 One-Price Market

The final result in Sect. 7.2.1 was the formulation (7.3) for the expected profits of a stochastic power producer in a one-price market. Equation (7.3) reveals the triviality of the determination of the optimal offer in a one-price system when prices are deterministic. Indeed, the second term on the right-hand side of (7.3) does not depend on the producer's decision. Therefore, this constant term can be discarded from the determination of the optimum. As far as the optimization of the first term is concerned, the following cases can happen, all with a trivial solution.

1. If $\lambda^{\mathrm{D}} < \lambda^{\mathrm{B}}$, the power producer sells nothing at the day-ahead market, and waits to place all its production at the balancing market, where the price λ^{B} is higher.
2. If $\lambda^{\mathrm{D}} > \lambda^{\mathrm{B}}$, the power producer sells as much energy as possible at the day-ahead market, eventually buying back at the balancing market the energy needed to cover the difference between day-ahead trade and actual production (7.2). As $\lambda^{\mathrm{B}} < \lambda^{\mathrm{D}}$, the producer realizes a surplus on the energy that is sold at the day-ahead market but not delivered.
3. If $\lambda^{\mathrm{D}} = \lambda^{\mathrm{B}}$, the power producer is indifferent since any decision on E^{D} would yield the same profit.

Given that, in electricity markets, producers are usually imposed not to offer above their installed capacity $\overline{E}$, we have the following result.

> In a one-price system, under the assumption of deterministic market prices, the optimal offer for a risk-neutral stochastic power producer is price-inelastic and equal to zero volume if the balancing price is higher than the day-ahead price, while it is equal to the nominal capacity $\overline{E}$ if the balancing price is lower than the day-ahead price; if such prices are equal, any offer is optimal.

The expression "price-inelastic" in the previous sentence means that the optimal offer is either zero or the nominal capacity, regardless of the value of the day-ahead price. As we shall see in Sect. 7.4.2, this is particularly meaningful as market rules allow producers to specify their offer as a price-quantity curve.

The solution obtained in this simplified case is trivial and, besides, completely decoupled from the forecast of the stochastic power production, as it is only dependent on the forecast of the arbitrage opportunity between the day-ahead and the balancing markets.

Furthermore, it should be pointed out that the statement that the rightmost term in (7.3) is not under the control of the power producer is not completely true. In principle, power producers could spill their excess production if economically attractive. This has little impact on the optimal day-ahead strategy described above, but it would imply that, should the price λ^{B} be negative, the producer would rather spill its power production and gain additional profits when repurchasing power.

7.3.2 Two-Price Market

It is relevant to recall that the optimal strategy for a risk-neutral stochastic producer minimizes the expected imbalance costs, i.e., the EOL in (7.11). Before carrying out the necessary algebra, we define the following penalties, both nonnegative

$$\psi^{\text{UP}} = \lambda^{\text{UP}} - \lambda^{\text{D}}, \tag{7.12}$$

$$\psi^{\text{DW}} = \lambda^{\text{D}} - \lambda^{\text{DW}}. \tag{7.13}$$

It is important to notice that ψ^{UP} and ψ^{DW} represent the opportunity loss per energy unit, i.e., the profits lost by exchanging up-regulation and down-regulation energy at the balancing market instead of at the day-ahead stage. Substituting the above quantities in the EOL term in (7.11), we get the following expression

$$\text{EOL} = \mathbb{E}\left\{-\psi^{\text{UP}}\widetilde{E}^{\text{UP}} + \psi^{\text{DW}}\widetilde{E}^{\text{DW}}\right\}. \tag{7.14}$$

We determine the optimal day-ahead offer in the following way: first, we expand the expectation in (7.14) into an integral in the probability space of uncertain production; then, the first-order stationarity condition is enforced.

The terms inside the expectation operator in (7.14) can be expanded. Owing to the piecewise definitions of up-regulation and down-regulation in (7.7) and (7.8), respectively, each term in (7.14) expands into an integration in a half-space of the set of feasible day-ahead offers $\left[0, \overline{E}\right]$, split in two halves by the day-ahead offer E^{D}, i.e.,

$$\text{EOL} = -\int_0^{E^{\text{D}}} \psi^{\text{UP}}\left(E - E^{\text{D}}\right) p_{\widetilde{E}}(E)dE + \int_{E^{\text{D}}}^{\overline{E}} \psi^{\text{DW}}\left(E - E^{\text{D}}\right) p_{\widetilde{E}}(E)dE, \tag{7.15}$$

where $p_{\widetilde{E}}(\cdot)$ is the probability density function (pdf) of the stochastic power production $\widetilde{E}$. The reader is referred to Appendix A for an introduction to random variables and to the concept of probability density function.

To determine the optimum, we enforce the first-order stationarity condition by taking the derivative of expression (7.15) with respect to the day-ahead offer E^{D} and setting it equal to 0. Carrying out the differentiation under the integral sign, see [9], yields the following

$$\begin{aligned}\frac{d\text{EOL}}{dE^{\text{D}}} =& -\psi^{\text{UP}}\int_0^{E^{\text{D}}} -p_{\widetilde{E}}(E)dE + \psi^{\text{DW}}\int_{E^{\text{D}}}^{\overline{E}} -p_{\widetilde{E}}(E)dE \\ =& \psi^{\text{UP}}F_{\widetilde{E}}(E^{\text{D}}) + \psi^{\text{DW}}\left(F_{\widetilde{E}}(E^{\text{D}}) - 1\right) = 0,\end{aligned} \tag{7.16}$$

where $F_{\widetilde{E}}(\cdot)$ indicates the cumulative distribution function (cdf) of power production, see Appendix A. Solving (7.16) for the day-ahead offer E^{D} readily yields the following expression

$$E^{\text{D}*} = F_{\widetilde{E}}^{-1}\left(\frac{\psi^{\text{DW}}}{\psi^{\text{UP}} + \psi^{\text{DW}}}\right), \tag{7.17}$$

which involves the inverse $F_{\tilde{E}}^{-1}(\,\cdot\,)$ of the production cdf, i.e., the quantile function, which is defined in Appendix A. The optimality of E^{D*} in (7.17) can be easily checked by noticing that the second order derivative of EOL with respect to E^{D} is nonnegative everywhere

$$\frac{d^2\text{EOL}}{dE^{D2}}(E^{D}) = \psi^{UP} p_{\tilde{E}}(E^{D}) + \psi^{DW} p_{\tilde{E}}(E^{D}) \geq 0. \tag{7.18}$$

This is because probability density functions are nonnegative by definition, and so are the penalties according to (7.12) and (7.13). As a consequence of (7.18) and under mild continuity assumptions, the EOL is convex with respect to E^{D}, which implies that the first-order stationarity condition (7.16) is sufficient to ensure that E^{D*} is a minimum. The results obtained so far can be summarized in the following statement.

In a two-price system, under the assumption of deterministic market prices, the optimal day-ahead offer for a risk-neutral stochastic power producer is price-inelastic and equal to the quantile of the power production distribution corresponding to a probability equal to the down-regulation penalty divided by the sum of the up-regulation and down-regulation penalties.

Example 7.1 (Optimal bid in a two-price settlement with deterministic prices) Let us consider the following deterministic penalties due to the less favorable price at the balancing market

$$\psi^{UP} = \$9/\text{MWh},$$
$$\psi^{DW} = \$4/\text{MWh}.$$

An analyst provides us with a probabilistic forecast of the stochastic power production at trading period t of the following day. We refer the reader to Chap. 2 for an introduction to forecasting the production from renewable sources. According to the analyst, power production follows a uniform distribution, see Appendix A, between 100 MWh and 150 MWh. The cumulative distribution function is therefore

$$F_{\tilde{E}}(E) = \begin{cases} 0 & \text{if } E < 100, \\ \dfrac{E-100}{50} & \text{if } 100 \leq E < 150, \\ 1 & \text{if } E \geq 150. \end{cases} \tag{7.19}$$

According to (7.17), the optimal offer must satisfy

$$F_{\tilde{E}}(E^{D*}) = \frac{\psi^{DW}}{\psi^{UP} + \psi^{DW}} = \frac{4}{13}, \tag{7.20}$$

which yields, by inverting the function in the second case in (7.19)

$$E^{D*} = 100 + 50 \times \frac{4}{13}\,\text{MWh} = 115.38\,\text{MWh}. \tag{7.21}$$

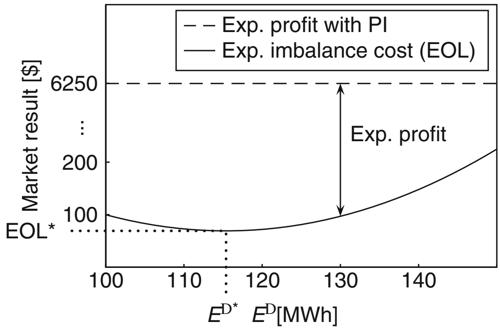


Fig. 7.3 Expected imbalance cost (EOL) as a function of the day-ahead offer. The expected profit is the difference between the expected profit with perfect information (PI) and the EOL

It is worth noticing that this result makes sense also from an intuitive point of view. Indeed, the optimal quantity offer is lower than 125 MWh, which is both the mean and the median of the power production distribution. This is in accordance with the fact that the penalty ψ^{UP} for underproducing with respect to the day-ahead market contract is higher than the penalty ψ^{DW} for overproducing, which makes a long position at the day-ahead market more attractive.

Figure 7.3 shows the expected imbalance cost (expected opportunity loss) as a function of the day-ahead offer. According to (7.11), the expected profit obtained with a given day-ahead offer E^{D} is the difference between the expected profit with perfect information (PI) and the EOL. The optimal quantity offer $E^{\mathrm{D}*}$ is a minimum point for the expected imbalance cost and, as a consequence, a maximum point for the expected profit. Observe that the expected profit with perfect information is calculated with a day-ahead price $\lambda^{\mathrm{D}} = \$50/\mathrm{MWh}$.

It is relevant to note that the closed formula (7.17) for the optimal day-ahead offer rests on the assumption that the cumulative distribution function $F_{\widetilde{E}}(\,\cdot\,)$ be invertible. In fact, we already know that $F_{\widetilde{E}}(\cdot)$ is, by definition of cdf, monotonically increasing, though the monotonicity may be nonstrict. In the latter case, the closed formula (7.17) is still valid using the generalized inverse function

$$F_{\widetilde{E}}^{-1}(\alpha) = \inf\left\{x \in \left[0,\ \overline{E}\right] : F_{\widetilde{E}}(x) \geq \alpha\right\}. \tag{7.22}$$

In fact, any amount E^{D} such that $F_{\widetilde{E}}(E^{\mathrm{D}}) = \psi^{\mathrm{DW}}/(\psi^{\mathrm{DW}} + \psi^{\mathrm{UP}})$ is optimal in this case.

Finally, we underline that expression (7.17) always makes sense, in that the ratio $\psi^{\mathrm{DW}}/(\psi^{\mathrm{UP}} + \psi^{\mathrm{DW}})$ is always included in the interval $[0,\ 1]$. This follows trivially from the nonnegativity of ψ^{UP} and ψ^{DW}.

7.4 Trading with Stochastic Prices

In the previous section, we derived some results on the optimal trading of stochastic power producers under simplified assumptions, among which was the restrictive hypothesis of deterministic prices. In what follows, we relax this simplification by assuming that prices are also uncertain.

The trading problem presents an increased level of difficulty if prices are stochastic. This additional difficulty stems from the fact that taking expectations of the power producer's profit in (7.1) and (7.10) entails the integration in two variables, namely the power production and a market price (or penalty), which requires knowledge of their joint probability density function. Furthermore, because market rules allow power producers to differentiate the offered quantity depending on the day-ahead clearing price, these expectations are to be conditioned on the latter quantity.

As usual, we increase the difficulty of the problem gradually. Section 7.4.1 treats the special case of trading with stochastic prices, where the difference between the balancing price and the day-ahead price (i.e., the penalty) is uncorrelated both with the day-ahead price itself and with the stochastic power production. The results obtained in the previous section still hold, though with some formal modifications. In Sect. 7.4.2, the case where the penalty is uncorrelated with the power production, but not with the day-ahead price, is considered.

7.4.1 Stochastic Generalizations of Quantity Bidding

We now turn our focus to the analysis of a special case, where the results obtained in the previous section still hold under the assumption of uncertain prices—provided that deterministic prices are replaced by their expected values. The critical assumption here is that the difference between the balancing market price and the day-ahead price is uncorrelated both with the day-ahead price itself and with the stochastic power production. As customary, the cases of the one-price and the two-price markets are considered separately.

7.4.1.1 One-Price Market

Resuming the analysis in Sect.7.2.1, the equivalent of the expected profit in (7.3) if prices are stochastic is given by

$$\mathbb{E}\{\widetilde{\rho}\} = \mathbb{E}\left\{\left(\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}\right) E^{\mathrm{D}}\right\} + \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{B}} \widetilde{E}\right\}. \tag{7.23}$$

The last term in (7.23) is not under control of the stochastic power producer. Therefore, the optimal bid must maximize the first term on the right-hand side of (7.23). Assuming that the difference $\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}$ is uncorrelated with the day-ahead price, there is no additional benefit in differentiating the offered quantity with respect to the day-ahead price through a bidding curve. In other words, the optimal offer still consists of a single quantity. Taking the expectation of (7.23), we obtain

$$\mathbb{E}\{\widetilde{\rho}\} = \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}\right\} E^{\mathrm{D}} + \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{B}} \widetilde{E}\right\}. \tag{7.24}$$

The conclusion on the optimal day-ahead offer is similar to the one in Sect. 7.3.1. The following cases can happen.

1. If $\mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}\right\} < 0$, the optimal bid is 0.
2. If $\mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}\right\} > 0$, the optimal bid is the nominal capacity.
3. If $\mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}\right\} = 0$, the power producer is indifferent since any decision on E^{D} would yield the same profit in expectation.

In a one-price system with stochastic prices, under the assumption that the difference between the balancing and the day-ahead prices is uncorrelated with both the day-ahead price and the power production, the optimal offer for a stochastic power producer is price-inelastic and equal to zero volume if the expectation of the balancing price is higher than the expected day-ahead price, while it is equal to the nominal capacity $\overline{E}$ in the opposite case; if the expected prices are equal, any offer is optimal.

We remark that the assumption of the difference between the day-ahead and the balancing market prices being independent from the day-ahead price itself is rather restrictive. If this assumption is relaxed, as we show in the following part of this section, the optimal offer is no longer a single quantity.

Example 7.2 (Optimal bid in a one-price settlement with uncorrelated price difference and day-ahead price) The price at the balancing market is equal to

$$\widetilde{\lambda}^{\mathrm{B}} = \widetilde{\lambda}^{\mathrm{D}} + \widetilde{u}, \tag{7.25}$$

where $\widetilde{u}$ is uniformly distributed in the interval between −\$20/MWh and \$30 /MWh. As a result, the difference between the day-ahead and the balancing market prices is independent of and therefore uncorrelated with the day-ahead price itself. The expected value of the price difference is

$$\mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{B}} - \widetilde{\lambda}^{\mathrm{D}}\right\} = \mathbb{E}\left\{\widetilde{u}\right\} = \int_{-20}^{30} u\frac{1}{50}du = \$5/\mathrm{MWh}. \tag{7.26}$$

As the price difference is positive in expectation, the profit is maximized by bidding 0 at the day-ahead market and placing all the production at the balancing market.

7.4.1.2 Two-Price Market

In a two-price market, under the assumption that the market penalties (7.12) and (7.13) are stochastic and uncorrelated with the day-ahead price and the power production, the expectation of the imbalance costs in (7.15) writes as

$$\begin{aligned}\mathrm{EOL} = &-\int_0^{E^{\mathrm{D}}}\int_0^{\infty} \psi^{\mathrm{UP}}\left(E - E^{\mathrm{D}}\right) p_{\widetilde{\psi}^{\mathrm{UP}},\widetilde{E}}(\psi^{\mathrm{UP}}, E)d\psi^{\mathrm{UP}}dE \\ &+\int_{E^{\mathrm{D}}}^{\overline{E}}\int_0^{\infty} \psi^{\mathrm{DW}}\left(E - E^{\mathrm{D}}\right) p_{\widetilde{\psi}^{\mathrm{DW}},\widetilde{E}}(\psi^{\mathrm{DW}}, E)d\psi^{\mathrm{DW}}dE.\end{aligned} \tag{7.27}$$

Exploiting the definition of conditional probability (see Appendix A), (7.27) can be rewritten as follows

$$\begin{aligned}\mathrm{EOL} = &- \int_0^{E^{\mathrm{D}}} \left(\int_0^{\infty} \psi^{\mathrm{UP}} p_{\widetilde{\psi}^{\mathrm{UP}}|\widetilde{E}}(\psi^{\mathrm{UP}}|E) d\psi^{\mathrm{UP}} \right) \left(E - E^{\mathrm{D}}\right) p_{\widetilde{E}}(E)dE \\ &+ \int_{E^{\mathrm{D}}}^{\overline{E}} \left(\int_0^{\infty} \psi^{\mathrm{DW}} p_{\widetilde{\psi}^{\mathrm{DW}}|\widetilde{E}}(\psi^{\mathrm{DW}}|E) d\psi^{\mathrm{DW}} \right) \left(E - E^{\mathrm{D}}\right) p_{\widetilde{E}}(E)dE.\end{aligned} \tag{7.28}$$

We notice that the integrals inside the parentheses in the above equations are the expectation of the market penalties conditional on the realization of the stochastic production. Since market penalties and power production are uncorrelated, the terms inside the brackets are equal to their expected values $\widehat{\psi}^{\mathrm{UP}}$ and $\widehat{\psi}^{\mathrm{DW}}$. At this point, the derivation follows precisely the same steps as in Sect. 7.3.2, yielding the following optimal quantity offer

$$E^{\mathrm{D}*} = F_{\widetilde{E}}^{-1} \left(\frac{\widehat{\psi}^{\mathrm{DW}}}{\widehat{\psi}^{\mathrm{UP}} + \widehat{\psi}^{\mathrm{DW}}} \right). \tag{7.29}$$

This result is similar to the one obtained in Sect. 7.3.2 and can be summarized in the following statement.

In a two-price market with stochastic prices, the optimal offer for a stochastic power producer under the assumption that imbalance penalties are uncorrelated with its power production and the day-ahead price, is price-inelastic and equal to the quantile of the power distribution corresponding to a probability equal to the expected value of the down-regulation penalty divided by its sum with the expected value of the up-regulation penalty.

Example 7.3 (Optimal bid in a two-price settlement with uncorrelated price difference and day-ahead price) We assume that the clearing price at the balancing market is distributed as the balancing price $\widetilde{\lambda}^{\mathrm{B}}$ in Example 7.2, and that the stochastic power production is distributed as in Example 7.1.

Figure 7.4 shows the probability density function of the balancing market price, and the resulting distributions for the up-regulation and down-regulation prices $\widetilde{\lambda}^{\mathrm{UP}}$, $\widetilde{\lambda}^{\mathrm{DW}}$ in the two-price system. According to the pricing rules (7.4) and (7.5), the up-regulation price $\widetilde{\lambda}^{\mathrm{UP}}$ follows the same distribution as $\widetilde{\lambda}^{\mathrm{B}}$ for values greater than the day-ahead price λ^{D}, while the part of the density function on the left of the day-ahead price λ^{D} is compressed and placed at λ^{D}, which thus has a positive probability of occurrence $\mathrm{P}\{\widetilde{\lambda}^{\mathrm{UP}} = \lambda^{\mathrm{D}}\} = \mathrm{P}\{\widetilde{\lambda}^{\mathrm{B}} \leq \lambda^{\mathrm{D}}\} = 0.4$. In a similar fashion, $\mathrm{P}\{\widetilde{\lambda}^{\mathrm{DW}} = \lambda^{\mathrm{D}}\} = \mathrm{P}\{\widetilde{\lambda}^{\mathrm{B}} \geq \lambda^{\mathrm{D}}\} = 0.6$. The expected values of the up-regulation and down-regulation penalties are given by

$$\widehat{\psi}^{\mathrm{UP}} = 0.4 \times 0 + \int_0^{30} \psi^{\mathrm{UP}} \frac{1}{50} d\psi^{\mathrm{UP}} = \$9/\mathrm{MWh}, \tag{7.30}$$

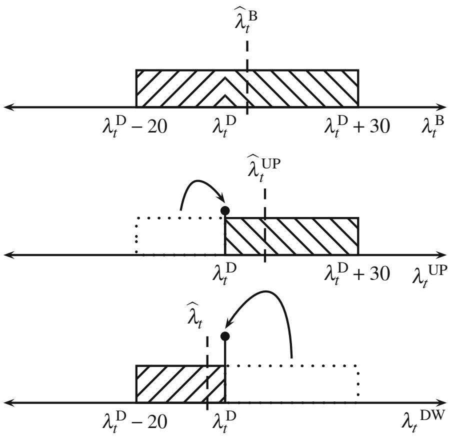


Fig. 7.4 Probability density function of a uniformly distributed clearing price $\widetilde{\lambda}^{\mathrm{B}}$ at the balancing market (*top axes*), and the resulting distributions of the up-regulation and down-regulation prices $\widetilde{\lambda}^{\mathrm{UP}}$, $\widetilde{\lambda}^{\mathrm{DW}}$ in a two-price settlement (*middle* and *bottom* axes, respectively)

$$\widehat{\psi}^{\mathrm{DW}} = \int_0^{20} \psi^{\mathrm{DW}} \frac{1}{50} d\psi^{\mathrm{DW}} + 0.6 \times 0 = \$4/\mathrm{MWh}. \tag{7.31}$$

Noticing that the expected values of the imbalance penalties are equal to their deterministic values in Example 7.1, and that the uncertain power production follows the same distribution, the optimal day-ahead offer is again $E^{\mathrm{D}*} = 115.38\,\mathrm{MWh}$.

7.4.2 Correlated Penalties and Day-Ahead Price: Bidding Curves

Electricity market rules allow power producers to submit supply curves rather than single quantities at day-ahead markets. Indeed, they can specify a certain number of production-price pairs, where they declare how much power they are willing to deliver at every price level indicated in the offer. By doing so, generators can offer power produced by units employing different technologies, while being confident that cost recovery is guaranteed for any realization of the stochastic price.

As such offer curves were conceived as an instrument for conventional power producers, and due to technical reasons related to market pricing, supply-curves must be nondecreasing, i.e., production-price pairs are to be ordered increasingly in price. Since the ordering of production-price pairs determines the scheduling preference, i.e., which supply blocks are chosen first when determining the power dispatch, a nondecreasing supply curve entails that blocks with a lower offered price are scheduled first. This is intuitively consistent with the preference of conventional power producers. Indeed, under the assumption that bids reflect the true marginal cost of generation, and disregarding non-convexities such as startup costs, conventional power producers would obviously prefer the scheduling of units with the lowest marginal cost to the more expensive ones, as this guarantees higher profits. Figure 7.5 shows an example of a supply curve for a producer employing different conventional generation technologies. Only the first two (cheapest) blocks on the left, indicated

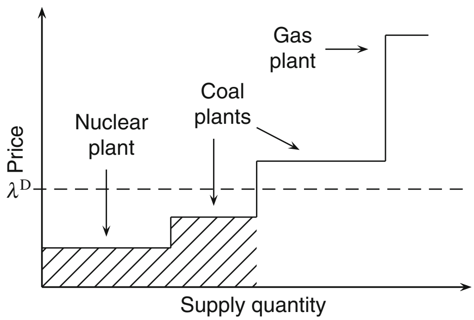


Fig. 7.5 Supply curve offered by a conventional power producer in the day-ahead market

with dashed fill and whose price offer is not greater than the cleared day-ahead price λ^{D}, are dispatched.

Owing to the fact that the marginal cost of power generation from stochastic sources such as wind and solar is null (or close to zero), it may appear that supply curves are not relevant for producers solely employing such technologies. Indeed, the optimal supply curve for producers of firm (deterministic) power is, under the price-taker assumption, the marginal cost of generation. From a simplistic analysis, one may expect that the same holds for stochastic power producers, who would thus be willing to sell the optimal quantity determined in the previous section at any (positive) price. This holds true in the special case presented in Sect. 7.4.1, but not in the general case.

The possibility of submitting a supply curve allows stochastic power producers to define in advance the quantity to be delivered to the market as a function $E^{D}(\lambda^{D})$ of the realization of the stochastic day-ahead price. Clearly, the determination of several quantity-price pairs is a far less constrained decision problem than the determination of a single quantity to be offered for any realization of the day-ahead price. This implies that the expected profit obtained with the optimal supply curve is at least not lower than the one resulting from bidding a fixed quantity. As we shall see in the following, the extent of this improvement depends on the level of correlation between imbalance penalties, power production, and the day-ahead price. In the case of uncorrelated variables, the optimal supply curve boils down to the fixed optimal quantity already determined in Sect. 7.4.1. In a more general case, the expected balancing prices, and therefore the optimal quantile of the power distribution, are correlated with the day-ahead price.

A last remark before going deeper into the problem regards the shape of the optimal curve. From the discussion above, it is clear that the requirement that the supply curve be nondecreasing is not restrictive for a conventional power producer. Indeed, it follows naturally from economic considerations that cheaper production blocks are offered first, as the dispatch preference of the producer is completely aligned with the increasing marginal costs of its production blocks. On the contrary, an optimal bidding curve for a stochastic power producer does not obviously follow this requirement in the general case. Determining the optimal nondecreasing supply curve analytically is not a trivial problem. As we shall see in Chap. 8, this problem can be solved by employing stochastic programming.

In the remainder of the section, we deal with the closed-form determination of the optimal bidding curve, individually for the one-price and the two-price case.

7.4.2.1 One-Price Market

In the general case where the quantities involved in the determination of the expected profit (7.23) are correlated with the day-ahead price $\widetilde{\lambda}^{\mathrm{D}}$, the stochastic power producer can benefit from specifying the quantity offered at the day-ahead market as a function of the cleared price, i.e., $E^{\mathrm{D}}(\lambda^{\mathrm{D}})$. The expected profit for the producer conditioned on the realization of the day-ahead price $\widetilde{\lambda}^{\mathrm{D}} = \lambda^{\mathrm{D}}$ writes as

$$\mathbb{E}\left\{\widetilde{\rho}|\lambda^{\mathrm{D}}\right\} = \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}|\lambda^{\mathrm{D}}\right\} E^{\mathrm{D}}(\lambda^{\mathrm{D}}) + \mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{B}}\widetilde{E}|\lambda^{\mathrm{D}}\right\}. \tag{7.32}$$

As in the previous sections, the second term on the right-hand side of (7.32) is not dependent on the choice of the offered quantity $E^{\mathrm{D}}(\lambda^{\mathrm{D}})$. In the absence of constraints on the offer, the optimal quantity to be offered at the day-ahead price λ^{D} would be either 0 or nominal capacity, depending on whether the conditional expectation of $\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}$ is negative or positive, respectively. The determination of the optimal bidding curve can then be carried out in a pointwise fashion for any value λ^{D}.

Example 7.4 (Optimal bidding curve in a one-price settlement) The expectation of the price difference between the day-ahead and the balancing market, conditional on the day-ahead market price, is

$$\mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}|\lambda^{\mathrm{D}}\right\} = -17.5 + \frac{1}{4}\lambda^{\mathrm{D}}. \tag{7.33}$$

We also expect the day-ahead price to be between \$20/MWh and \$80/MWh. Therefore, the day-ahead offer curve should specify a production value for each of these prices.

We notice that the value in (7.33) is zero for $\lambda^{\mathrm{D}} =$ \$70/MWh, and strictly negative (positive) for day-ahead prices lower (greater) than this value. Therefore, the optimal offering curve prescribes to offer a zero quantity for \$20 /MWh $\leq \lambda^{\mathrm{D}} <$ \$70/MWh and the nominal capacity for \$70 /MWh $< \lambda^{\mathrm{D}} \leq$ \$80/MWh. For $\lambda^{\mathrm{D}} =$ \$70/MWh, any offer maximizes the expected revenues, as the expected value of the balancing market price is equal to the price at the day-ahead stage.

Figure 7.6 illustrates the conditional expectation of the price difference and the optimal offer curve for a unit with generation capacity equal to 200 MW.

Notice that if the expected day-ahead price is \$50 /MWh, the expected value of the balancing market price exceeds the former quantity by \$5 /MWh, just like in Example 7.2. However, bidding a zero quantity at any price in this case is suboptimal.

In the general case, the optimal bidding curve resulting from the pointwise calculation (7.32) may not fulfill the requirement that the supply curve be nondecreasing in its domain. Indeed, it is easy to realize that the optimal bidding curve determined in a pointwise fashion is not a supply curve whenever $\mathbb{E}\left\{\widetilde{\lambda}^{\mathrm{D}} - \widetilde{\lambda}^{\mathrm{B}}|\lambda^{\mathrm{D}}\right\}$ switches in

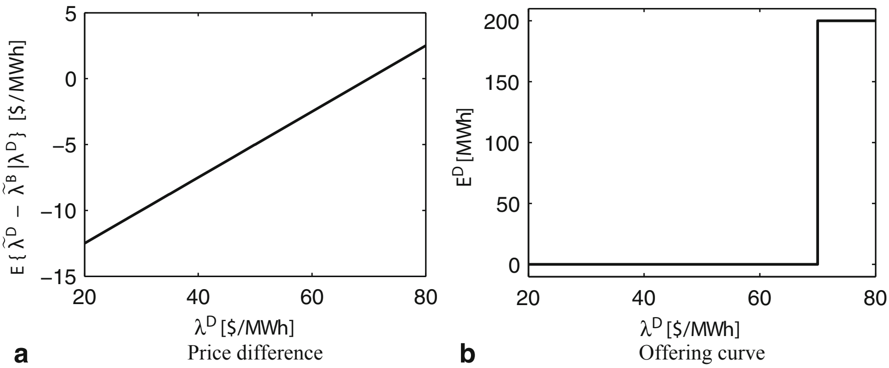


Fig. 7.6 Expected price difference between balancing and day-ahead markets, conditional on the day-ahead price (**a**), and resulting optimal offering curve (**b**)

sign from strictly positive to strictly negative for some values of λ^{D}. The stochastic programming framework can be used to determine the optimal offering curve in that case. We refer the reader to Chap. 8, where a similar offering problem including bidding curves is presented for a virtual power plant using stochastic programming.

7.4.2.2 Two-Price Market

The expected opportunity loss under the day-ahead price λ^{D} writes, by replacing the probability density functions of the penalties in (7.27) by probability distributions conditional on λ^{D}, as

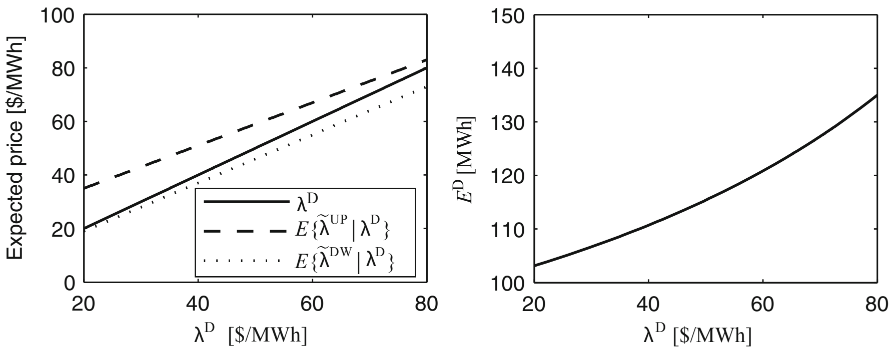
$$\begin{aligned}\mathrm{EOL}(\lambda^{\mathrm{D}}) = &- \int_0^{E^{\mathrm{D}}(\lambda^{\mathrm{D}})} \mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}|E,\lambda^{\mathrm{D}}\right\}\left[E - E^{\mathrm{D}}(\lambda^{\mathrm{D}})\right] p_{\widetilde{E}}(E)dE \\ &+ \int_{E^{\mathrm{D}}(\lambda^{\mathrm{D}})}^{\overline{E}} \mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|E,\lambda^{\mathrm{D}}\right\}\left[E - E^{\mathrm{D}}(\lambda^{\mathrm{D}})\right] p_{\widetilde{E}}(E)dE. \qquad (7.34)\end{aligned}$$

By exploiting the fact that the imbalance penalties are uncorrelated with the stochastic power production, we can bring the conditional expectations of the penalties out of the integral operator

$$\begin{aligned}\mathrm{EOL}(\lambda^{\mathrm{D}}) = &- \mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}|\lambda^{\mathrm{D}}\right\} \int_0^{E^{\mathrm{D}}(\lambda^{\mathrm{D}})} \left[E - E^{\mathrm{D}}(\lambda^{\mathrm{D}})\right] p_{\widetilde{E}}(E)dE \\ &+ \mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|\lambda^{\mathrm{D}}\right\} \int_{E^{\mathrm{D}}(\lambda^{\mathrm{D}})}^{\overline{E}} \left[E - E^{\mathrm{D}}(\lambda^{\mathrm{D}})\right] p_{\widetilde{E}}(E)dE. \qquad (7.35)\end{aligned}$$

Requiring that the first order derivative of the imbalance cost in (7.35) be equal to 0 yields the following expression for the optimal bidding curve

$$E^{\mathrm{D}*}(\lambda^{\mathrm{D}}) = F_{\widetilde{E}}^{-1}\left(\frac{\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|\lambda^{\mathrm{D}}\right\}}{\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}|\lambda^{\mathrm{D}}\right\} + \mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|\lambda^{\mathrm{D}}\right\}}\right). \qquad (7.36)$$



a Price expectation **b** Offer curve

Fig. 7.7 Expected balancing prices, conditional on the day-ahead price (**a**), and resulting optimal offering curve (**b**)

As in the one-price market case, we remark that the optimal curve resulting from the pointwise calculation of the optimal quantities in (7.36) does not necessarily yield a valid nondecreasing supply curve in the general case.

Example 7.5 (Optimal bidding curve in a two-price settlement) Let us once again consider that the stochastic power production is uniformly distributed between 100 MWh and 150 MWh. The expectations of the up-regulation and down-regulation penalties, conditional on the realization of the day-ahead price, are affine functions of the latter quantity, defined as follows

$$\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}|\lambda^{\mathrm{D}}\right\} = 19 - \frac{1}{5}\lambda^{\mathrm{D}}, \tag{7.37}$$

$$\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|\lambda^{\mathrm{D}}\right\} = -1 + \frac{1}{10}\lambda^{\mathrm{D}}. \tag{7.38}$$

The expected up-regulation and down-regulation prices at the balancing market are shown in Fig. 7.7(a) as functions of the day-ahead price.

It is worth to notice that $\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}|\lambda^{\mathrm{D}}\right\}$ and $\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|\lambda^{\mathrm{D}}\right\}$ are a decreasing and an increasing function of λ^{D}, respectively. According to (7.36), and due to the zero correlation between power production and day-ahead price, the optimal bidding curve for the producer is given by

$$E^{\mathrm{D}*}(\lambda^{\mathrm{D}}) = F_{\widetilde{E}}^{-1}\left(\frac{-1+\frac{1}{10}\lambda^{\mathrm{D}}}{19-\frac{1}{5}\lambda^{\mathrm{D}}-1+\frac{1}{10}\lambda^{\mathrm{D}}}\right) = F_{\widetilde{E}}^{-1}\left(\frac{-1+\frac{1}{10}\lambda^{\mathrm{D}}}{18-\frac{1}{10}\lambda^{\mathrm{D}}}\right). \tag{7.39}$$

The resulting optimal bidding curve is shown in Fig. 7.7(b). It is assumed that the support of the distribution of the day-ahead price $\widetilde{\lambda}^{\mathrm{D}}$ is included in the interval between \$20 /MWh and \$80/MWh.

We point out that, since the argument of the quantile function in (7.39) is an increasing function of the day-ahead price, it results that the optimal bidding curve is a valid offer (i.e., nondecreasing) at the day-ahead market.

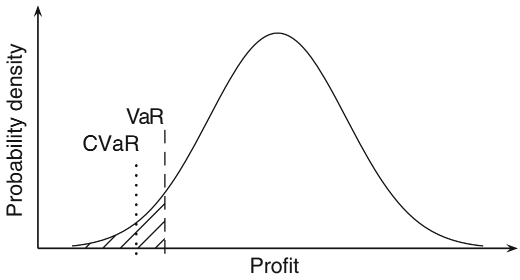


Fig. 7.8 Example of profit distribution and its relative VaR and CVaR

To conclude the example, we notice that, if the distribution of the day-ahead price $\widetilde{\lambda}^{\mathrm{D}}$ has mean \$50/MWh, the expected values $\widehat{\psi}^{\mathrm{UP}}$, $\widehat{\psi}^{\mathrm{DW}}$ of the up-regulation and down-regulation penalties are \$9/MWh and \$4/MWh, respectively, as in Example 7.3. The fixed quantity offer, though, is suboptimal in this case.

7.5 Modeling Risk-Aversion

When the power producer is not risk-neutral, a different objective than the maximization of the expected profit is sought. Generally speaking, a suitable objective function for a risk-averse power producer penalizes the lowest profit, i.e., the tail on the left-hand side of the profit distribution.

Two metrics widely used to quantify risk are the *Value at Risk (VaR)* and the *Conditional Value at Risk (CVaR)*. For a confidence level $0 \leq \alpha < 1$, $\mathrm{VaR}_{1-\alpha}$ is the $(1-\alpha)$-quantile of the profit. Denoting the (uncertain) profit with $\widetilde{\rho}$ and the support of its distribution with R, $\mathrm{VaR}_{1-\alpha}(\widetilde{\rho})$ is defined as

$$\mathrm{VaR}_{1-\alpha}(\widetilde{\rho}) = \max\left\{\rho \in \mathrm{R} : \mathrm{P}\left(\widetilde{\rho} < \rho\right) \leq 1-\alpha\right\}. \tag{7.40}$$

The definition of $\mathrm{CVaR}_{1-\alpha}$ is related to the previous definition [16]. For continuously distributed profits, it is the expected value of the profits that are lower than or equal to $\mathrm{VaR}_{1-\alpha}$:

$$\mathrm{CVaR}_{1-\alpha}(\widetilde{\rho}) = \mathbb{E}\left\{\widetilde{\rho} | \widetilde{\rho} \leq \mathrm{VaR}_{1-\alpha}(\widetilde{\rho})\right\} = \frac{1}{1-\alpha}\int_0^{\mathrm{VaR}_{1-\alpha}(\widetilde{\rho})} \rho p_{\widetilde{\rho}}(\rho) d\rho, \tag{7.41}$$

where $p_{\widetilde{\rho}}(\cdot)$ is the probability density function of the profit. According to this definition, CVaR often goes under the name of *expected shortfall*. Appendix C includes further details on these two risk measures and valid definitions in the case of discretely distributed profits.

In the recent years, CVaR has gained increasing attention, partly due to the fact that, differently from VaR, CVaR satisfies some properties that make it a *coherent* risk measure [1]. Figure 7.8 shows VaR and CVaR for an example of profit distribution. The dashed area in the illustration measures $1-\alpha$. VaR is the $(1-\alpha)$-quantile of the profit, while CVaR is the expected value of the profits falling below VaR.

An intuitive approach when defining a risk-averse strategy consists in seeking a compromise between the maximization of the expected profit and a term accounting for the chosen risk metric. Employing CVaR at the α confidence level, a suitable objective function z is

$$z = (1-k)\mathbb{E}\{\widetilde{\rho}\} + k \times \mathrm{CVaR}_{1-\alpha}(\widetilde{\rho}). \tag{7.42}$$

For $k = 0$, the objective function consists in the maximization of the expected profit, which corresponds to the risk-neutral case. For increasing values of k, the second term in the objective function weighs more and more, implying that the maximization of the worst outcomes has more and more importance. When $k = 1$, the decision is completely risk-averse. It is worth noticing that the objective defined in (7.42) depends on two parameters arbitrarily set by the decision-maker: α and k.

An alternative approach for a risk-averse decision-maker is the direct maximization of the CVaR, i.e.,

$$z = \mathrm{CVaR}_{1-\alpha}(\widetilde{\rho}). \tag{7.43}$$

Setting $\alpha = 0$ yields the risk-neutral case; increasing values of α represent situations with higher aversion to risk.

In the next section, we consider the short-term trading problem of a risk-averse stochastic power producer employing the objective function (7.43).

7.5.1 Risk-Averse Strategy in a Two-Price Market

A relevant application of the risk criteria described above in the trading problem for stochastic power producers is the case of a two-price settlement for the balancing market with deterministic penalties. In this case, the only risk is introduced by the uncertainty in power production.

We make the further assumption that market prices are always nonnegative. In this situation, it holds that the higher the realized power production, the higher the profit. If objective (7.43) is employed, the α fraction of highest returns corresponds to the α fraction of highest production, which is therefore discarded from the objective function. Taking into account the expression of the producer's returns (7.10), the expectation of the $1-\alpha$ lowest profits is given by

$$\begin{aligned} z = &\int_0^{E^{1-\alpha}} \lambda^{\mathrm{D}} E \frac{p_{\widetilde{E}}(E)}{1-\alpha} dE + \int_0^{E^{\mathrm{D}}} \psi^{\mathrm{UP}} \left(E - E^{\mathrm{D}}\right) \frac{p_{\widetilde{E}}(E)}{1-\alpha} dE \\ &- \int_{E^{\mathrm{D}}}^{E^{1-\alpha}} \psi^{\mathrm{DW}} \left(E - E^{\mathrm{D}}\right) \frac{p_{\widetilde{E}}(E)}{1-\alpha} dE, \end{aligned} \tag{7.44}$$

where $E^{1-\alpha}$ is the $(1-\alpha)$-quantile of the power production distribution, i.e., it satisfies $\mathrm{P}\{\widetilde{E} \leq E^{1-\alpha}\} = 1-\alpha$. Observe that in Eq. (7.44), it is assumed that the

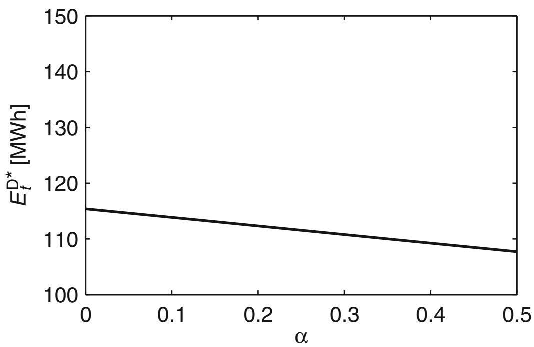


Fig. 7.9 Optimal day-ahead bid for a risk-averse power producer as a function of the risk-aversion parameter α

optimal value of the day-ahead offer E^{D} is not greater than the quantile $E^{1-\alpha}$, which makes sense for small values of α.

Proceeding in a similar way as in Sect. 7.3.2, the stationary point must satisfy

$$\frac{1}{1-\alpha}\psi^{\mathrm{UP}} F_{\widetilde{E}}(E^{\mathrm{D}}) + \frac{1}{1-\alpha}\psi^{\mathrm{DW}}\left(F_{\widetilde{E}}(E^{\mathrm{D}}) - (1-\alpha)\right) = 0, \tag{7.45}$$

which readily yields the optimal risk-averse bid

$$E^{\mathrm{D}*} = F_{\widetilde{E}}^{-1}\left\{(1-\alpha)\frac{\psi^{\mathrm{DW}}}{\psi^{\mathrm{UP}} + \psi^{\mathrm{DW}}}\right\}. \tag{7.46}$$

The result (7.46) is rather intuitive. Indeed, since the rightmost part of the power production distribution is discarded from the objective function (7.44), the risk-averse power producer is more concerned about negative (deficit) deviations from the day-ahead position. The coefficient $1-\alpha$ in the argument of the quantile function in (7.46) scales the optimal quantile, reducing the quantity placed at the day-ahead market and therefore decreasing the possibility and the size of power deficits at the balancing market.

Example 7.6 (Risk-averse offering strategy in a two-price market) We consider the same market penalties and the same distribution of power production as in Example 7.1. From (7.46) and the cdf (7.19) it follows that the optimal day-ahead bid for the risk-averse power producer is

$$E^{\mathrm{D}*} = 100 + 50 \times \frac{4}{13} \times (1-\alpha)\,\mathrm{MWh}. \tag{7.47}$$

Figure 7.9 shows the optimal quantity bid as a function of the risk-aversion parameter α. For $\alpha = 0$, the bid is the same as in the risk-neutral case, while it decreases for higher values of the parameter.

The expected value and the $\mathrm{CVaR}_{5\%}$ of the profit (i.e., the expectation of the 5% lowest profits), obtained with the day-ahead price $\lambda^{\mathrm{D}} = \$20$/MWh, are shown in Fig. 7.10. The expected profit in Fig. 7.10(a) decreases as α grows, signaling that risk-averse strategies achieve worse financial results in expectation than the

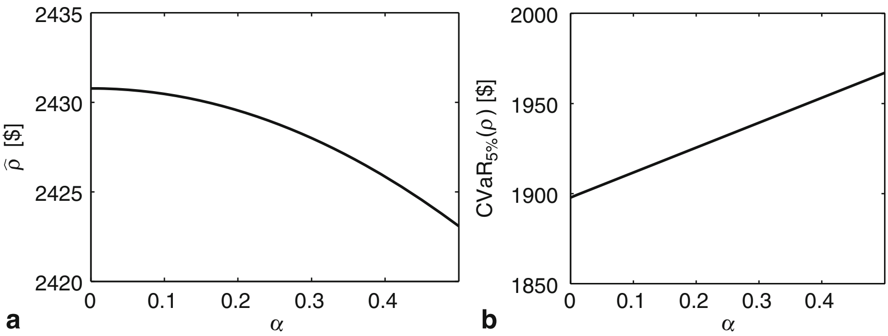


Fig. 7.10 Expected profit (**a**) and $\mathrm{CVaR}_{5\%}$ of the profit (**b**) as functions of the risk-aversion parameter α

risk-neutral strategy. In turn, the $\mathrm{CVaR}_{5\%}$ of the profit, shown in Fig. 7.10(b), increases, which implies that high values of α better hedge the power producer.

Notice that such a direct application of risk management to the case of a one-price market with deterministic prices is not particularly interesting. Indeed, a price-inelastic zero offer maximizes $\mathrm{CVaR}_{1-\alpha}(\widetilde{\rho})$ for any α when $\lambda^{\mathrm{B}} > \lambda^{\mathrm{D}}$. Similarly, nominal capacity is still the optimal risk-averse offer if $\lambda^{\mathrm{B}} < \lambda^{\mathrm{D}}$ for any level of risk aversion.

The risk-averse strategy presented in this section, due to the use of deterministic prices, is only aimed at hedging the power generator from the uncertainty in power production. In practice, when imbalance penalties are stochastic, the risk stemming from both prices and production should be considered. The determination of the optimal risk-averse strategy would then involve double integrals with possibly joint probability distributions. Obviously, such calculations can be rather cumbersome. As we shall see in Sect. 7.6, stochastic programming can be used to account for risk in a simple and intuitive manner.

Before turning to the stochastic programming approach, we summarize the analytical results obtained so far in Table 7.1.

7.6 Bidding Strategies: Stochastic Programming Approach

The existence of an analytical solution to the short-term trading problem of a stochastic producer is limited to a number of simplified cases, all relying on at least one of the assumptions stated in Sect. 7.2 of this chapter. Such a solution is thus no longer available as soon as the dependence structure exhibited by market prices and production volume becomes more intricate and/or the trading problem is enriched with new elements and features.

In this section, we present an alternative approach to solving the trading problem of a stochastic producer. This approach is based on stochastic programming, which provides us with a powerful and flexible modeling framework to easily account for all the relevant factors in this problem. The stochastic programming approach starts from

Table 7.1 Summary of the analytical results

Case	Market	Optimal offer	Offering rule
Deterministic prices	1-price	$\begin{cases} 0 & \text{if} \quad \lambda^{\text{B}} > \lambda^{\text{D}} \\ \overline{E} & \text{if} \quad \lambda^{\text{B}} < \lambda^{\text{D}} \end{cases}$	fixed quantity to be offered at any day-ahead price λ^{D}
	2-price	$F_{\widetilde{E}}^{-1}\left(\frac{\psi^{\text{DW}}}{\psi^{\text{UP}}+\psi^{\text{DW}}}\right)$	
Stochastic prices, no penalties/day-ahead price correlation	1-price	$\begin{cases} 0 & \text{if} \quad \mathbb{E}\left\{\widetilde{\lambda}^{\text{D}}-\widetilde{\lambda}^{\text{B}}\right\}<0 \\ \overline{E} & \text{if} \quad \mathbb{E}\left\{\widetilde{\lambda}^{\text{D}}-\widetilde{\lambda}^{\text{B}}\right\}>0 \end{cases}$	fixed quantity to be offered at any day-ahead price λ^{D}
	2-price	$F_{\widetilde{E}}^{-1}\left(\frac{\mathbb{E}\{\widetilde{\psi}^{\text{DW}}\}}{\mathbb{E}\{\widetilde{\psi}^{\text{UP}}\}+\mathbb{E}\{\widetilde{\psi}^{\text{DW}}\}}\right)$	
Stochastic prices, nonzero penalties/day-ahead price correlation	1-price	$\begin{cases} 0 & \text{if} \quad \mathbb{E}\left\{\widetilde{\lambda}^{\text{D}}-\widetilde{\lambda}^{\text{B}}\mid\lambda^{\text{D}}\right\}<0 \\ \overline{E} & \text{if} \quad \mathbb{E}\left\{\widetilde{\lambda}^{\text{D}}-\widetilde{\lambda}^{\text{B}}\mid\lambda^{\text{D}}\right\}>0 \end{cases}$	price-quantity curve; valid only if nondecreasing
	2-price	$F_{\widetilde{E}}^{-1}\left(\frac{\mathbb{E}\left\{\widetilde{\psi}^{\text{DW}}\mid\lambda^{\text{D}}\right\}}{\mathbb{E}\left\{\widetilde{\psi}^{\text{UP}}\mid\lambda^{\text{D}}\right\}+\mathbb{E}\left\{\widetilde{\psi}^{\text{DW}}\mid\lambda^{\text{D}}\right\}}\right)$	
Deterministic prices, risk-averse	1-price	$\begin{cases} 0 & \text{if} \quad \lambda^{\text{B}} > \lambda^{\text{D}} \\ \overline{E} & \text{if} \quad \lambda^{\text{B}} < \lambda^{\text{D}} \end{cases}$	fixed quantity to be offered at any day-ahead price λ^{D}
	2-price	$F_{\widetilde{E}}^{-1}\left((1-\alpha)\frac{\psi^{\text{DW}}}{\psi^{\text{UP}}+\psi^{\text{DW}}}\right)$	

the premise that the uncertain parameters influencing the decision-making process faced by the stochastic producer can be efficiently approximated by a finite set Ω of plausible outcomes or *scenarios*.

For example, consider the random variable $\widetilde{E}$ describing the uncertain production in a future time period and let E_ω denote the realization of this random variable under scenario ω. The set $\{(E_\omega, \pi_\omega), \omega \in \Omega\}$, such that $\sum_{\omega\in\Omega} \pi_\omega = 1$ and $\pi_\omega \geq 0$ for all ω, is a discrete approximation of the probability distribution of $\widetilde{E}$, with π_ω being the probability of occurrence assigned to realization E_ω. Similarly, one might construct a scenario set $\{(E_\omega, \psi_\omega^{\text{UP}}, \psi_\omega^{\text{DW}}, \pi_\omega), \omega \in \Omega\}$ to model, if needed, the interdependence structure between the uncertain production volume $\widetilde{E}$ and the imbalance penalties $\widetilde{\psi}^{\text{UP}}$ and $\widetilde{\psi}^{\text{DW}}$. Be that as it may, the stochastic programming approach to the trading problem assumes that this scenario set is available. Chapter 2 provides the concepts and tools required to properly construct scenarios for the stochastic processes of interest within the scope of this book.

Next, we illustrate the stochastic programming approach to the offering problem of a stochastic producer using a small example. For a brief introduction to stochastic programming, we refer the interested reader to Appendix C.

Example 7.7 (Stochastic programming approach) Let us try to solve Example 7.1 using the concept of *scenario*. Recall that the imbalance penalties, ψ^{UP} and ψ^{DW}

are considered here deterministic and equal to \$9/MWh and \$4/MWh, respectively. Besides, the stochastic power production at trading period t, i.e., $\widetilde{E}$, is given by a uniform distribution between 100 MWh and 150 MWh.

The stochastic programming solution approach requires that this uniform distribution be approximated by N_Ω scenarios, i.e., $\widetilde{E} \approx \{(E_1, \pi_1), \ldots, (E_\omega, \pi_\omega), \ldots, (E_{N_\Omega}, \pi_{N_\Omega})\}$, with $\sum_{\omega=1}^{N_\Omega} \pi_\omega = 1$ and $\pi_\omega \geq 0$. By means of this discretization, the offering problem of the stochastic producer can be cast as the linear programming problem (7.48), which can be readily processed by optimization solvers.

$$\text{Min.} \sum_{\omega=1}^{N_\Omega} \pi_\omega \left(-\psi^{\text{UP}} E_\omega^{\text{UP}} + \psi^{\text{DW}} E_\omega^{\text{DW}}\right) \tag{7.48a}$$

$$\text{s.t. } 0 \leq E^{\text{D}} \leq \overline{E}, \tag{7.48b}$$

$$E_\omega - E^{\text{D}} = E_\omega^{\text{UP}} + E_\omega^{\text{DW}}, \forall \omega, \tag{7.48c}$$

$$E_\omega^{\text{UP}} \leq 0, \quad E_\omega^{\text{DW}} \geq 0, \quad \forall \omega. \tag{7.48d}$$

Variables E^{D}, E_ω^{UP}, and E_ω^{DW}, for all ω, are the decisions to be optimized. In stochastic programming terminology, the amount of energy sold by the stochastic producer in the day-ahead market, E^{D}, is referred to as a *here-and-now decision variable*, because it must be decided before knowing the eventual realization of the stochastic production $\widetilde{E}$. Consequently, E^{D} is independent of the scenario index ω. On the other hand, the amounts of up-regulation and down-regulation energy acquired by the stochastic producer in the balancing market, i.e., E_ω^{UP} and E_ω^{DW} are called *wait-and-see decision variables*, because they are decided after knowing the specific realization E_ω of the stochastic production $\widetilde{E}$. Therefore, E_ω^{UP} and E_ω^{DW} do depend on the scenario index ω.

It should be noticed that the objective function (7.48a) is the equivalent scenario-based formulation of the expected opportunity loss as expressed in (7.14). Analogously, Eq. (7.48c) is the scenario-based definition of the imbalance of the stochastic producer, also stated in (7.9) in a more general form.

Needless to say, the solution to the stochastic programming problem (7.48) is strongly dependent on the scenario set that we use to approximate the uniformly distributed power output $\widetilde{E}$. In a first attempt, we can just consider a set of one single scenario consisting in the expected power production, i.e., $\widehat{E} = \mathbb{E}\{\widetilde{E}\} = (100 + 150)/2 = 125$ MWh, with a probability of 1. In such a case, problem (7.48) becomes

$$\text{Min.} \; -9E_1^{\text{UP}} + 4E_1^{\text{DW}} \tag{7.49a}$$

$$\text{s.t. } 0 \leq E^{\text{D}} \leq 150, \tag{7.49b}$$

$$125 - E^{\text{D}} = E_1^{\text{UP}} + E_1^{\text{DW}}, \tag{7.49c}$$

$$E_1^{\text{UP}} \leq 0, \quad E_1^{\text{DW}} \geq 0, \tag{7.49d}$$

whose solution is trivial. Indeed, the minimum of (7.49) is attained at $E^{\text{D}*} = 125$ MWh, which zeroes the objective function (7.49a). However, we know from

Example 7.1 that the actual optimal strategy is to offer 115.38 MWh in the day-ahead market. Obviously, the difference is caused by the scenario-based representation of the stochastic power production.

In order to enhance the accuracy of the stochastic programming solution approach, we can clearly build a better scenario set. For instance, we can approximate the uncertain power output $\widetilde{E}$ this time using a two-scenario model that contains the two extremes of the associated uniform distribution with the same probability, i.e., $\widetilde{E} \approx \{(E_1, \pi_1), (E_2, \pi_2)\} = \{(100\text{ MWh}, 0.5), (150\text{ MWh}, 0.5)\}$. Thus the stochastic programming problem (7.48) becomes

$$\text{Min. } 0.5\left(-9E_1^{\text{UP}} + 4E_1^{\text{DW}}\right) + 0.5\left(-9E_2^{\text{UP}} + 4E_2^{\text{DW}}\right) \tag{7.50a}$$

$$\text{s.t. } 0 \leq E^{\text{D}} \leq 150, \tag{7.50b}$$

$$100 - E^{\text{D}} = E_1^{\text{UP}} + E_1^{\text{DW}}, \tag{7.50c}$$

$$150 - E^{\text{D}} = E_2^{\text{UP}} + E_2^{\text{DW}}, \tag{7.50d}$$

$$E_1^{\text{UP}}, E_2^{\text{UP}} \leq 0, \quad E_1^{\text{DW}}, E_2^{\text{DW}} \geq 0, \tag{7.50e}$$

which results in $E^{\text{D}*} = 100$ MWh, still far from the actual optimal bid of 115.38 MWh. We can further increase the size of the scenario set by adding the expected power production $\widehat{E}$ to the previous two-scenario model. That is, we approximate $\widetilde{E}$ by $\{(100\text{ MWh}, 1/3), (125\text{ MWh}, 1/3), (150\text{ MWh}, 1/3)\}$, which leads to the following stochastic programming problem:

$$\text{Min. } \frac{1}{3}\left(-9E_1^{\text{UP}} + 4E_1^{\text{DW}}\right) + \frac{1}{3}\left(-9E_2^{\text{UP}} + 4E_2^{\text{DW}}\right) + \frac{1}{3}\left(-9E_3^{\text{UP}} + 4E_3^{\text{DW}}\right) \tag{7.51a}$$

$$\text{s.t. } 0 \leq E^{\text{D}} \leq 150, \tag{7.51b}$$

$$100 - E^{\text{D}} = E_1^{\text{UP}} + E_1^{\text{DW}}, \tag{7.51c}$$

$$125 - E^{\text{D}} = E_2^{\text{UP}} + E_2^{\text{DW}}, \tag{7.51d}$$

$$150 - E^{\text{D}} = E_3^{\text{UP}} + E_3^{\text{DW}}, \tag{7.51e}$$

$$E_1^{\text{UP}}, E_2^{\text{UP}}, E_3^{\text{UP}} \leq 0, \quad E_1^{\text{DW}}, E_2^{\text{DW}}, E_3^{\text{DW}} \geq 0. \tag{7.51f}$$

However, problem (7.51) also yields $E^{\text{D}*} = 100$ MWh.

Finally, we construct a set of $N+1$ equiprobable scenarios uniformly spaced over the interval [100, 150] MWh, i.e., we model the stochastic production $\widetilde{E}$ by $\{(E_1, \pi_1), \ldots, (E_\omega, \pi_\omega), \ldots, (E_{N+1}, \pi_{N+1})\}$, where $E_\omega = 100 + (\omega - 1)(150 - 100)/N$ and $\pi_\omega = 1/(N+1)$, for all $\omega = 1, \ldots, N+1$. Figure 7.11 illustrates the optimal offer $E^{\text{D}*}$ given by the stochastic programming problem (7.48) as a function of N. Observe that as the size of the scenario set increases, the stochastic programming solution converges to the optimal bid that was obtained analytically in Example 7.1, namely 115.38 MWh.

In short, the reliance of the stochastic programming solution approach on the scenario set used to model the uncertain parameters is both its greatest virtue and its

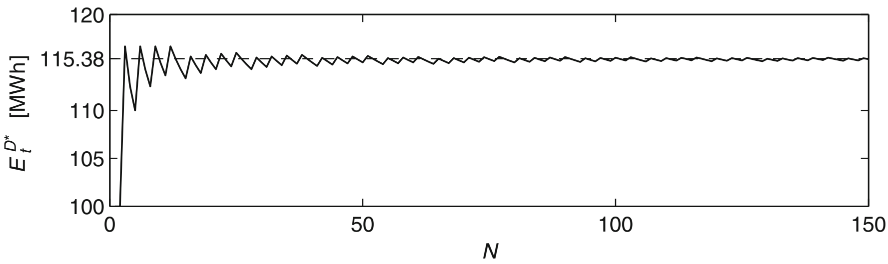


Fig. 7.11 Energy offer E^{D*} (in megawatt-hour) obtained from the stochastic linear programming problem (7.48) as a function of the size of the scenario set that approximates the uniformly distributed power production $\widetilde{E}$. Note that this offer approaches the analytical solution (115.38 MWh) as N increases

Achilles' heel. On the one hand, the scenario-based formulation of the trading problem allows us to determine the optimal offering strategy of the stochastic producer by means of an equivalent (deterministic) optimization problem that can be directly tackled by conventional optimization solvers. On the other, the actual value of the solution provided by this optimization problem becomes strongly contingent on the quality of the scenario set. This may render the stochastic solution approach computationally prohibitive if, for example, the quality of the scenario set is conditional on its size.

What is certain, though, is that the stochastic programming solution approach allows us to easily consider other facets of the trading problem. For instance, it is straightforward to determine risk-averse offering strategies using the conditional value at risk within a stochastic programming framework. In the following, we build on Example 7.7 to illustrate how to manage risk in the trading problem of a stochastic producer using stochastic programming.

Example 7.8 (Risk management via stochastic programming) Problem (7.48) in Example 7.7 determines the optimal offering strategy of a risk-neutral stochastic producer. We can now extend this problem to account for risk-averse behavior as follows:

$$\text{Max.}\ (1-k)\sum_{\omega\in\Omega}\pi_\omega\rho_\omega + k\left(\zeta - \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi_\omega\eta_\omega\right) \tag{7.52a}$$

$$\text{s.t.}\ 0 \le E^{\text{D}} \le \overline{E}, \tag{7.52b}$$

$$\rho_\omega = \lambda^{\text{D}}E_\omega - \left(-\psi^{\text{UP}}E_\omega^{\text{UP}} + \psi^{\text{DW}}E_\omega^{\text{DW}}\right)\forall\omega, \tag{7.52c}$$

$$E_\omega - E^{\text{D}} = E_\omega^{\text{UP}} + E_\omega^{\text{DW}}, \quad \forall\omega, \tag{7.52d}$$

$$\zeta - \rho_\omega \le \eta_\omega, \quad \forall\omega, \tag{7.52e}$$

$$E_\omega^{\text{UP}} \le 0,\ \ E_\omega^{\text{DW}} \ge 0,\ \ \eta_\omega \ge 0 \quad \forall\omega, \tag{7.52f}$$

where, for simplicity, we have omitted the time subscript t. Note that, as opposed to (7.48a), the new objective function (7.52a) is formulated in terms of profits, not opportunity costs. Accordingly, the newly defined variable ρ_ω represents the profit made by the stochastic producer in scenario ω, as stated by (7.52c), where variable ρ_ω is computed scenario-wise as the profit with perfect information minus the opportunity cost. At the optimum, the term $\left(\zeta^* - \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi_\omega\eta^*_\omega\right)$ in the objective function coincides with the conditional value at risk at confidence level α ($\text{CVaR}_{1-\alpha}$), which here represents the average value of the $1-\alpha$ cases with lowest profits. Variables ζ and η_ω are auxiliary. For further details on how to model the conditional value at risk within a stochastic programming optimization problem, we refer the reader to Appendix C.

Note that the objective function (7.52a) establishes the tradeoff between the expected value and the conditional value at risk of the profit distribution. This tradeoff is resolved by means of the user-defined constant $k \in [0, 1]$, which is usually referred to as *risk-aversion parameter*. The higher the value of k, the more risk averse the stochastic producer is, as it becomes more concerned with the maximization of the lowest profits. As seen later, the multi-objective form of (7.52a) permits us to introduce the concept of *efficient frontier* in a very intuitive manner.

Now we assume the same imbalance penalties and the same distribution of power production as in Example 7.7. Likewise, we approximate such a distribution using a set of 1000 scenarios, built as explained in that example. We consider the conditional value at risk at a confidence level α of 95%, i.e., $\text{CVaR}_{5\%}$, which accounts for the 5% of scenarios with lowest profits. The day-ahead market price λ^{D} is assumed to be equal to $20/MWh.

Figure 7.12(a) shows the optimal energy offer $E^{\text{D}*}$ in the day-ahead market as a function of the risk-aversion parameter k. Observe that higher values of k lead to lower values of E^{D}. Under risk aversion, poorer outcomes are weighted more than good outcomes. In the trading problem of a stochastic producer with positive prices, the cases with the lowest profits correspond to the scenarios in which the stochastic producer is short and therefore, must purchase its generation deficit in the balancing market. Figure 7.12(b) represents the so-called *efficient frontier*, which is made up of the optimal pairs $(\text{CVaR}_{5\%}, \widehat{\rho})$ for different degrees of risk aversion. True to form, the stochastic producer can reduce its risk exposure—by increasing k in problem (7.52)—at the expense of decreasing the expected monetary value of its optimal sale offer in the day-ahead market.

The stochastic programming approach to the trading problem of a stochastic producer becomes particularly attractive in those instances for which an analytical solution is not available. This occurs, for example, in the case where the stochastic producer has the opportunity to participate in one or several adjustment markets. Generally speaking, these markets allow consumers and producers to adapt their forward consumption or production schedule to unplanned eventualities such as equipment failures, technical constraints, or sudden changes in load. For this reason, adjustment markets are placed in between the clearing of the day-ahead and balancing markets. Since the magnitude of the forecast error of stochastic production is usually strongly dependent on the lead time—the forecasts of wind/solar power issued

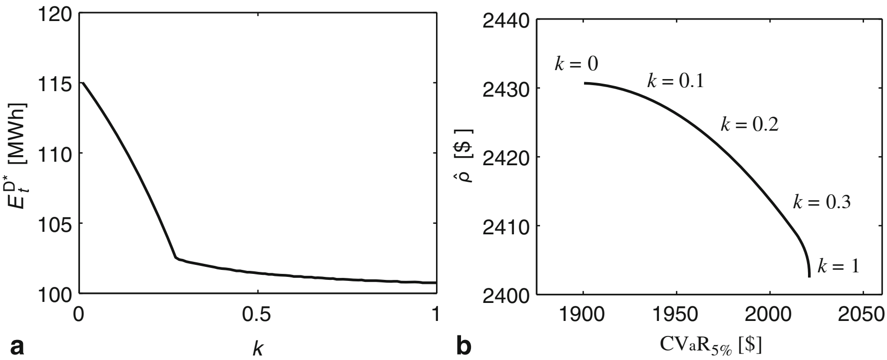


Fig. 7.12 The optimal amount of energy to be sold in the day-ahead market by the stochastic producer decreases as the degree of risk aversion increases (**a**). Offering strategies aimed at increasing the profit associated with the least favorable production outcomes are possible at the expense of decreasing the expected profit (**b**)

one hour ahead tend to be much more accurate than the forecast issued, e.g., 40 h ahead—stochastic producers may largely benefit from adjustment markets as they can trade in these markets with a lower degree of uncertainty on their eventual power production. In practice, this means that the future stochastic production is known better in the adjustments markets than in the day-ahead market. In the following example, we illustrate how to deal with an adjustment market in the trading problem of a stochastic producer by means of stochastic programming.

Example 7.9 (Adjustment market) Let us consider a stochastic producer that participates in an electricity market with the structure and time framework represented in Fig. 7.13. The day-ahead market is cleared at time t^{D}. After the closure of the day-ahead market, bidding in the adjustment market is allowed until Δt time units prior to the energy delivery period t. In the balancing market, the energy deviations incurred by the stochastic producer during the delivery period t are determined with respect to the dispatch program agreed in the day-ahead and adjustment markets, and priced accordingly.

Suppose that the day-ahead and adjustment market prices, i.e., λ^{D} and λ^{A}, are equal to \$20/MWh and \$19/MWh, respectively. Besides, the imbalance penalties, ψ^{UP} and ψ^{DW}, are \$9/MWh and \$4/MWh in that order, which means that the balancing market prices for upward and downward regulation, i.e., λ^{UP} and λ^{DW}, are given by $\lambda^{\mathrm{UP}} = \lambda^{\mathrm{D}} + \psi^{\mathrm{UP}} = \$29/\mathrm{MWh}$ and $\lambda^{\mathrm{DW}} = \lambda^{\mathrm{D}} - \psi^{\mathrm{DW}} = \$16/\mathrm{MWh}$.

The stochastic producer owns a 100-MW wind farm whose power output in time period t can be described as follows:

- The amount of energy $E_{t-\Delta t}$ produced by the wind farm in time period $t - \Delta t$ may be *high* (60 MWh) with a probability of 0.4 or low (30 MWh) with a probability of 0.6.

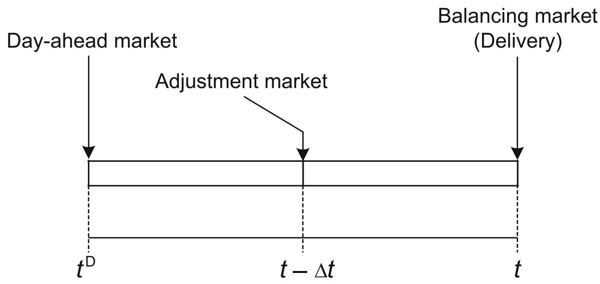


Fig. 7.13 Market organization and time framework including day-ahead, adjustment and balancing markets

- If the wind power production $E_{t-\Delta t}$ is *high* (60 MWh), the amount of energy E produced during the delivery period t may be *extremely high* (100 MWh) with a probability of 0.75 or *relatively high* (50 MWh) with a probability of 0.25.
- If the wind power production $E_{t-\Delta t}$ is *low* (30 MWh), the amount of energy E produced during the delivery period t may be *extremely low* (0 MWh) or *relatively low* (40 MWh), both with a probability of 0.5.

The sequence of stages and decisions that the stochastic producer has to face is described below.

1. Decide the amount of energy E^{D} to be sold in the day-ahead market with inaccurate information on the eventual power output of its wind farm in the delivery period t.
2. Decide the amount of energy E^{A} to be traded in the adjustment market. At this stage, the energy produced by its wind farm in time period $t - \Delta t$ is known and this information improves the stochastic producer's knowledge of its power production in the delivery period t.
3. Lastly, once the wind power production in the delivery period t becomes known, the stochastic producer must cover the amount of energy deviating from that scheduled in the day-ahead and adjustment markets by selling or purchasing its generation surplus or shortage, respectively, in the balancing market.

The decision-making process faced by the stochastic producer can be represented in the form of a tree, as depicted in Fig. 7.14. Each scenario ω in the tree is characterized by a certain wind power outcome in time periods $t - \Delta t$ and t, i.e., $E_{t-\Delta t\omega}$ and E_ω, with π_ω being its probability of occurrence. The scenario tree is made up of three sets of nodes corresponding to the three stages of the stochastic producer decision-making process. Each stage represents the trading in a different market.

For comparison purposes, let us first suppose that the stochastic producer ignores the adjustment market. In this case, its optimal energy offer in the day-ahead market is obtained as the solution to the opportunity loss minimization problem (7.48) stated in Example 7.7, that is

$$\text{Min. } 0.3\left(-9E_1^{\text{UP}} + 4E_1^{\text{DW}}\right) + 0.1\left(-9E_2^{\text{UP}} + 4E_2^{\text{DW}}\right) + \tag{7.53a}$$

$$0.3\left(-9E_3^{\text{UP}} + 4E_3^{\text{DW}}\right) + 0.3\left(-9E_4^{\text{UP}} + 4E_4^{\text{DW}}\right) \tag{7.53b}$$

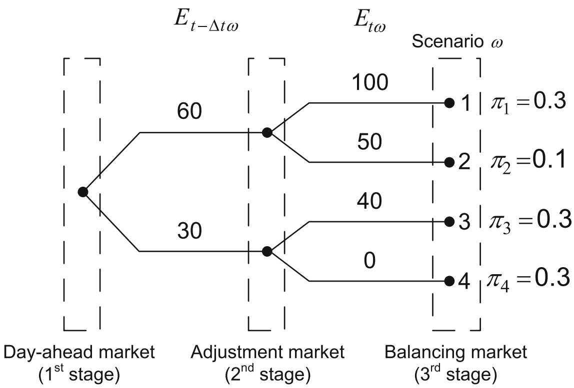


Fig. 7.14 Scenario tree describing the three-stage decision-making process faced by the stochastic producer. The nodes represent points in time where trading decisions are to be made. The branches represent the realization of wind power

$$\text{s.t. } 0 \leq E^{\text{D}} \leq 150, \tag{7.53c}$$
$$100 - E^{\text{D}} = E_1^{\text{UP}} + E_1^{\text{DW}}, \tag{7.53d}$$
$$50 - E^{\text{D}} = E_2^{\text{UP}} + E_2^{\text{DW}}, \tag{7.53e}$$
$$40 - E^{\text{D}} = E_3^{\text{UP}} + E_3^{\text{DW}}, \tag{7.53f}$$
$$0 - E^{\text{D}} = E_4^{\text{UP}} + E_4^{\text{DW}}, \tag{7.53g}$$
$$E_1^{\text{UP}}, E_2^{\text{UP}}, E_3^{\text{UP}}, E_4^{\text{UP}} \leq 0, \quad E_1^{\text{DW}}, E_2^{\text{DW}}, E_3^{\text{DW}}, E_4^{\text{DW}} \geq 0, \tag{7.53h}$$

which yields $E^{\text{D}*} = 40$ MWh. The expected opportunity loss (EOL) associated with this bid is \$184. Therefore, the expected profit $\widehat{\rho}$ made by the stochastic producer can be calculated as $\widehat{\rho} = \lambda^{\text{D}} \sum_{\omega=1}^{4} \pi_\omega E_\omega - \text{EOL} = \$20/\text{MWh} \times 47\text{MWh} - \$184 = \$756$.

Let us now consider that the stochastic producer trades in the adjustment market as well. In this case, the best strategy the stochastic producer can adopt is given by the following expected profit maximization problem,

$$\text{Max. } \lambda^{\text{D}} E^{\text{D}} + \sum_{\omega=1}^{N_\Omega} \pi_\omega \left(\lambda^{\text{A}} E_\omega^{\text{A}} + \lambda^{\text{UP}} E_\omega^{\text{UP}} + \lambda^{\text{DW}} E_\omega^{\text{DW}} \right) \tag{7.54a}$$
$$\text{s.t. } 0 \leq E^{\text{D}} \leq \overline{E}, \tag{7.54b}$$
$$0 \leq E^{\text{D}} + E_\omega^{\text{A}} \leq \overline{E}, \quad \forall \omega, \tag{7.54c}$$
$$E_\omega - E^{\text{D}} - E_\omega^{\text{A}} = E_\omega^{\text{UP}} + E_\omega^{\text{DW}}, \forall \omega, \tag{7.54d}$$
$$E_1^{\text{A}} = E_2^{\text{A}}, \quad E_3^{\text{A}} = E_4^{\text{A}}, \tag{7.54e}$$
$$E_\omega^{\text{UP}} \leq 0, \quad E_\omega^{\text{DW}} \geq 0, \forall \omega, \tag{7.54f}$$

where E^{A}_{ω} is the amount of energy sold, if positive, or purchased, if negative, in the adjustment market by the stochastic producer. The objective function (7.54a) to be maximized is the expected profit, which includes three terms: (i) the profit made in the day-ahead market, $\lambda^{\mathrm{D}} E^{\mathrm{D}}$; (ii) the expected profit obtained in the adjustment market, $\lambda^{\mathrm{A}} \sum_{\omega=1}^{N_{\Omega}} \pi_{\omega} E^{\mathrm{A}}{}_{\omega}$; and the expected profit made in the balancing market, $\sum_{\omega=1}^{N_{\Omega}} \pi_{\omega} \left(\lambda^{\mathrm{UP}} E^{\mathrm{UP}}_{\omega} + \lambda^{\mathrm{DW}} E^{\mathrm{DW}}_{\omega}\right)$. Note that the last two terms referring to the adjustment and balancing markets may actually represent a cost in those scenarios where the stochastic producer purchases electricity from these markets. Logically, the energy deviations incurred by the stochastic producer are to be computed here with respect to the forward production program resulting from the day-ahead and adjustment markets, as stated by the set of constraints (7.54d). Equations (7.54e) enforce the nonanticipatory character of the information, which requires the amount of energy traded in the adjustment market be unique irrespective of the wind production outcome in the future delivery period t. However, the energy offer in the adjustment market may be indeed conditional on the wind power produced in time period $t - \Delta t$. In stochastic programming, constraints of the nature of (7.54e) are usually called *nonanticipativity constraints*.

By replacing parameters in optimization problem (7.54) with their actual values, we end up with the following optimization problem,

$$\begin{aligned}
\text{Max. } & 20E^{\mathrm{D}} + 0.3\left(19E^{\mathrm{A}}_1 + 29E^{\mathrm{UP}}_1 + 16E^{\mathrm{DW}}_1\right) + \\
& 0.1\left(19E^{\mathrm{A}}_2 + 29E^{\mathrm{UP}}_2 + 16E^{\mathrm{DW}}_2\right) + 0.3\left(19E^{\mathrm{A}}_3 + 29E^{\mathrm{UP}}_3 + 16E^{\mathrm{DW}}_3\right) \quad (7.55a)\\
& 0.3\left(19E^{\mathrm{A}}_4 + 29E^{\mathrm{UP}}_4 + 16E^{\mathrm{DW}}_4\right)\\
\text{s.t. } & 0 \le E^{\mathrm{D}} \le 100, \quad (7.55b)\\
& 0 \le E^{\mathrm{D}} + E^{\mathrm{A}}_1 \le 100, \quad (7.55c)\\
& 0 \le E^{\mathrm{D}} + E^{\mathrm{A}}_2 \le 100, \quad (7.55d)\\
& 0 \le E^{\mathrm{D}} + E^{\mathrm{A}}_3 \le 100, \quad (7.55e)\\
& 0 \le E^{\mathrm{D}} + E^{\mathrm{A}}_4 \le 100, \quad (7.55f)\\
& 100 - E^{\mathrm{D}} - E^{\mathrm{A}}_1 = E^{\mathrm{UP}}_1 + E^{\mathrm{DW}}_1, \quad (7.55g)\\
& 50 - E^{\mathrm{D}} - E^{\mathrm{A}}_2 = E^{\mathrm{UP}}_2 + E^{\mathrm{DW}}_2, \quad (7.55h)\\
& 40 - E^{\mathrm{D}} - E^{\mathrm{A}}_3 = E^{\mathrm{UP}}_3 + E^{\mathrm{DW}}_3, \quad (7.55i)\\
& 0 - E^{\mathrm{D}} - E^{\mathrm{A}}_4 = E^{\mathrm{UP}}_4 + E^{\mathrm{DW}}_4, \quad (7.55j)\\
& E^{\mathrm{A}}_1 = E^{\mathrm{A}}_2, \quad E^{\mathrm{A}}_3 = E^{\mathrm{A}}_4, \quad (7.55k)\\
& E^{\mathrm{UP}}_1, E^{\mathrm{UP}}_2, E^{\mathrm{UP}}_3, E^{\mathrm{UP}}_4 \le 0, \quad E^{\mathrm{DW}}_1, E^{\mathrm{DW}}_2, E^{\mathrm{DW}}_3, E^{\mathrm{DW}}_4 \ge 0, \quad (7.55l)
\end{aligned}$$

which results in $E^{\mathrm{D}*} = 100$ MWh, $E^{\mathrm{A}*}_1 = E^{\mathrm{A}*}_2 = -50$ MWh, $E^{\mathrm{A}*}_3 = E^{\mathrm{A}*}_4 = -100$ MWh, and $\widehat{\rho} = \$912$. This trading strategy, therefore, entails an increase in the expected profit equal to \$912 − \$756 = \$156—a 20.64% increment in relative terms—with respect to the expected profit associated with the offering strategy derived from problem (7.53), which ignores the adjustment market. This increase is partly due to the fact that the future wind power production in the delivery period t is known with higher accuracy in the adjustment market. Indeed, from the day-ahead market, the stochastic producer foresees a future energy generation in period t ranging from 0 to 100 MWh. In contrast, from the adjustment market, the stochastic producer forecasts an energy production varying either from 50 to 100 MWh or from 0 to 40 MWh depending on whether the wind energy produced in time period $t - \Delta t$, i.e., $E_{t-\Delta t}$, is *high* (60 MWh) or *low* (30 MWh). This way, if $E_{t-\Delta t}$ is *high*, the stochastic producer purchases 50 MWh in the adjustment market, while if it is *low*, its energy purchase in this market is increased up to 100 MWh.

In short, the fact that the clearing of the adjustment market is closer in time to the energy delivery period enhances the profitability of the stochastic producer. Indeed, if the pair of constraints (7.54e) is replaced with the single equation $E_1^{\mathrm{A}} = E_2^{\mathrm{A}} = E_3^{\mathrm{A}} = E_4^{\mathrm{A}}$, in order to disregard this effect, the expected profit made by the stochastic producer drops to \$852, which represents a 6.6% decrease with regard to the expected profit resulting from problem (7.54).

In reality, the electricity prices in the day-ahead and adjustment markets are also uncertain. Furthermore, since the trading volume in adjustments markets is generally lower than in the day-ahead market, the electricity price in adjustment markets is often more volatile. As a result, the stochastic producer must face a tradeoff between selling in the day-ahead market at less volatile prices and trading in the adjustment market with a reduced level of uncertainty about its future energy production. This tradeoff can be resolved by means of the stochastic programming solution approach presented in this section.

7.7 Summary and Conclusions

Renewable energy producers are increasingly required to participate in electricity markets under the same rules as conventional power producers. However, the competitive sale of renewable energy by stochastic producers is cursed with the weather dependency of the underlying energy source, e.g., sunlight or wind. Stochastic producers are thus *forced* to channel part of their business into the balancing market, where they can take part with perfect knowledge of their production.

Compared to the electricity prices in markets with early gate closures, such as day-ahead and adjustment markets, balancing market prices are generally less predictable and/or less competitive. Consequently, as the stochastic producer becomes more and more dependent on the balancing market, its profitability may rapidly deteriorate. Therefore, the stochastic producer must smartly decide its involvement in day-ahead, adjustment, and balancing markets according to its best guess on market prices and future power production.

The trading problem of a stochastic producer is described in this chapter as a multi-stage decision-making problem under uncertainty. We identify the assumptions that render this problem analytically solvable and provide the corresponding exact solutions. For those cases in which these assumptions prove to be unrealistic, simplistic or too constraining, a more general modeling approach based on stochastic programming and the concept of scenarios is introduced to determine the best trading strategy for the stochastic producer.

7.8 Further Reading

For a basic introduction to statistical decision theory, the reader is referred to [14]. From the same author, [15] offers a more complete treatment of the subject, including the *newsvendor problem*, which is the general formulation of the power trading problem.

Further reading on analytical results on the trading problem for stochastic power producers consists mainly of research articles focusing on the case of wind power producers. The optimal strategy based on the offer of a quantile of the wind power distribution is proposed and tested in [4; 12; 17]. Further analytical results including and extending part of the results presented in this chapter can be found in [3; 6]. Finally, [8] provides a general discussion, including an application to electricity markets, on the relationship between loss functions and the optimal forecasts.

On the other hand, the short-term trading problem of a wind power producer is tackled in [10; 11; 13] using stochastic programming. Reference [5] also describes a number of trading models for retailers, consumers and producers (including nondispatchable agents) that are built upon stochastic programming, and provides insight into risk management and the scenario-based modeling of the uncertainties affecting trading decisions.

The trading problem for price-maker producers employing renewable sources with variable and stochastic nature is considered in [2; 18], making use of stochastic Mathematical Programs with Equilibrium Constraints.

Exercises

7.1 A forecaster predicts that the energy production $\widetilde{E}$ of a wind farm during a given trading period t is characterized by the following probability density function $p_{\widetilde{E}}(\cdot)$:

$$p_{\widetilde{E}}(E) = \begin{cases} \dfrac{E-100}{625}, & 100 \leq E < 125, \\ \\ \dfrac{150-E}{625}, & 125 \leq E < 150, \\ 0, & \text{elsewhere.} \end{cases} \tag{7.56}$$

1. Determine the cumulative distribution function $F_{\widetilde{E}}(\cdot)$ for wind power production.
2. Assume that the imbalance penalties in a two-price market are $\psi^{\mathrm{UP}} = \$9$ /MWh and $\psi^{\mathrm{DW}} = \$4$/MWh. Determine the optimal offer and the resulting expectation of the imbalance cost.
3. Now determine the optimal offer and the resulting expectation of the imbalance cost assuming that the imbalance penalties are $\psi^{\mathrm{UP}} = \$4$/MWh and $\psi^{\mathrm{DW}} = \$9$/MWh.

7.2 Consider the definition of $\widetilde{u}$ in Example 7.2. Let us assume that the probability density function for $\widetilde{u}$ is

$$p_{\widetilde{u}}(u) = \begin{cases} \dfrac{u+20}{625}, & -20 \leq u < 5, \\ \dfrac{30-u}{625}, & 5 \leq u < 30, \\ 0, & \text{elsewhere.} \end{cases} \tag{7.57}$$

1. Determine the expected value of the balancing price in a one-price market. If the forecast for wind power production is the same as in Exercise 7.1, what is the optimal offer?
2. Determine the expected values of the imbalance penalties $\widehat{\psi}^{\mathrm{UP}}$ and $\widehat{\psi}^{\mathrm{DW}}$ in a two-price market. If the forecast for wind power production is the same as in Exercise 7.1, what is the optimal offer?

7.3 In a two-price market, the expectation of the imbalance penalties, conditional on the day-ahead price, is given by the following.

$$\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{UP}}|\lambda^{\mathrm{D}}\right\} = 10 - \frac{1}{10}\lambda^{\mathrm{D}}, \tag{7.58}$$

$$\mathbb{E}\left\{\widetilde{\psi}^{\mathrm{DW}}|\lambda^{\mathrm{D}}\right\} = \frac{1}{8}\lambda^{\mathrm{D}}. \tag{7.59}$$

Let us consider a uniform distribution for the stochastic production with lower and upper bounds equal to 100 MWh and 150 MWh, respectively. Determine the optimal offering curve, assuming that the day-ahead price is nonnegative and can take values up to \$100/MWh.

7.4 Consider the distribution of wind power production in Exercise 7.1, and the deterministic penalties $\psi^{\mathrm{UP}} = \$9$/MWh and $\psi^{\mathrm{DW}} = \$4$/MWh. Determine the risk-averse bid with parameter $\alpha = 0.2$. Assuming a day-ahead price $\lambda^{\mathrm{D}} = \$50$/MWh, determine the expected value of the profit, the *value at risk* ($\mathrm{VaR}_{5\%}$) and the *conditional value at risk* ($\mathrm{CVaR}_{5\%}$) of the profit.

7.5 Suppose that the energy production of a certain wind farm in a given trading period t can be modeled by a uniform distribution between 0 MWh and 50 MWh.

1. Construct a set of four equiprobable and uniformly spaced scenarios that approximates this uniform distribution.
2. Based on this scenario set and knowing that the imbalance penalties, ψ^{UP} and ψ^{DW}, are deterministic and equal to \$3/MWh and \$6/MWh, respectively, for-

mulate and solve a stochastic programming model to calculate the energy offer in the day-ahead market that minimizes the expected imbalance cost of the wind farm.

7.6 Reformulate the stochastic programming model of the previous exercise to account for the risk aversion of the wind power producer using the Conditional Value-at-Risk of its profit distribution at a confidence level of 99%. Then, obtain the optimal bid of the wind power producer in the day-ahead market as a function of a risk-aversion parameter and draw the resulting efficient frontier.

7.7 Consider a solar power producer that participates in an electricity market with the structure and time framework depicted in Fig. 7.13 of Example 7.9. The solar producer owns a 50-MW photovoltaic power plant whose energy output in time period t is stochastic and can be described as follows:

- The amount of energy $E_{t-\Delta t}$ produced by the solar power plant in time period $t - \Delta t$ may be *high* (45 MWh) with a probability of 0.7 or low (5 MWh) with a probability of 0.3.
- If the solar power production $E_{t-\Delta t}$ is *high* (45 MWh), the amount of energy E produced during the delivery period t may be *extremely high* (50 MWh) with a probability of 0.35 or *relatively high* (41 MWh) with a probability of 0.65.
- If the solar power production $E_{t-\Delta t}$ is *low* (5 MWh), the amount of energy E produced during the delivery period t may be *extremely low* (0 MWh) with a probability of 0.70 or *relatively low* (10 MWh) with a probability of 0.30.

Furthermore, the day-ahead and adjustment market prices, i.e., λ^{D} and λ^{A}, are known to be equal to \$45/MWh and \$46/MWh, respectively, and the imbalance penalties, ψ^{UP} and ψ^{DW}, equal to \$10/MWh and \$8/MWh, in that order.

1. Construct a scenario tree similar to that in Fig. 7.14 of Example 7.9 to describe the decision-making process faced by the solar power producer.
2. Compute the optimal bid of the solar power producer in the day-ahead market if the adjustment market is disregarded.
3. Compute the optimal bid of the solar power producer in the day-ahead market if the adjustment market is taken into account and calculate how much the expected profit of the solar power producer increases with respect to the previous case. Determine also how much of this increase is due to the fact that the future energy production of the photovoltaic power plant in period t is known with higher accuracy in the adjustment market than in the day-ahead market.

7.8 Consider a 60-MW solar power plant whose power output in a given trading period t can be either 35 and 60 MW, with probabilities 0.6 and 0.4, respectively. It is known that:

- The price in the day-ahead market for period t can be either \$25/MWh or \$50/MWh, with probabilities 0.3 and 0.7, in that order.
- If the day-ahead market price is equal to \$25/MWh, the imbalance penalties for upward and downward balancing energy are \$10 /MWh and \$2/MWh, respectively.

- In contrast, if the day-ahead market price is equal to \$50/MWh, these imbalance penalties take on the values \$5/MWh and \$4/MWh instead.

Use the information about offering curves provided in Sect. 8.4 of Chap. 8 (see, in particular, Example 8.9 in this chapter) to formulate and solve a stochastic programming model that computes, as an increasing function of the day-ahead electricity price, the optimal energy offer in the day-ahead market that maximizes the expected profit of the solar power producer.

Based on the comments and explanations in Sect. 7.4.2, justify the obtained solution.

7.9 Repeat Exercise 7.8 for the case that:

- If the day-ahead market price is \$25/MWh, the imbalance penalties for upward and downward balancing energy are \$5 /MWh and \$4/MWh, respectively.
- If the day-ahead market price is equal to \$50/MWh, the imbalance penalties are \$10/MWh and \$2/MWh instead.

7.10 Try to solve Exercises 7.8 and 7.9 analytically. Can both problems be solved this way? If not, motivate why.

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