Chapter 4
Balancing Markets

4.1 Why Are Balancing Markets Needed?

Balancing markets are generally used to balance as closely as possible production and consumption prior (e.g., half an hour in advance) to energy delivery. This is important as there is no economical way to store large quantities of electric energy.

Markets other than the balancing markets are cleared well in advance of energy delivery and thus the production and consumption levels scheduled in these markets can significantly differ from the actual production and consumption at balancing time. This is particularly so in markets with a significant amount of stochastic production units. Balancing markets bridge, or narrow, the balance gap between other forward markets and real-time energy delivery.

In the following we consider that the balancing market is an energy-only market, not including reserve or other ancillary services required for the proper functioning of the electric energy system. The analysis of reserves and ancillary services is outside the scope of this chapter.

4.1.1 Day-Ahead, Adjustment, and Balancing Markets

The day-ahead market allows electric energy trading one day ahead of energy delivery. It is typically cleared around noon the previous day to the day in which energy is to be delivered. Clearing at noon one day in advance implies 12-hour anticipation with respect to the first delivery hour and 35-hour anticipation with respect to the last delivery hour. Such anticipation is required by some production units (e.g., nuclear or coal plants) due to technical limitations on operation flexibility.

Adjustment or intraday markets allow electric energy trading after the day-ahead market clearing and typically up to one or few hours before energy delivery. For example in the Iberian Peninsula [3], four adjustment markets are arranged following the clearing of the day-ahead market for day d: the first adjustment market clears at 6 p.m. of day $d-1$ and expands the 24 h of day d, the second one clears at midnight of day $d-1$ and covers the last 23 h of day d, the third one clears at 6 a.m. of day

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d and spans the last 17 h of day d and finally, the fourth one clears at noon of day d and spans the last 11 h of day d.

Adjustment markets are particularly appropriate for nondispatchable, stochastic producers such as wind- or solar-based producers. Since such markets clear closer to actual energy delivery than the day-ahead market does, they allow stochastic producers to offer with increasing certainty on their production levels, which generally results in comparatively higher profits and reduced profit volatility. Intraday markets are implemented, although with different rules, in the majority of European markets. However, their liquidity is very limited [16], with the notable exception of the Iberian market.

The balancing market, also called real-time market, is the last market opportunity to balance production and consumption. The gate closure of this market is typically in the range between 30 min and 1 h before actual energy delivery.

4.1.2 Market Organization

Balancing markets are single-period markets as they take place just minutes before actual energy delivery. Since a multiperiod approach is not needed to clear the balancing market, time indexes are omitted in the formulations throughout this chapter.

Balancing markets should take into account network limitations as the network is a fundamental physical component of the electric energy system. However, in this chapter, for the sake of clarity and simplicity the network is not initially represented, i.e., we consider an unlimited transmission capacity. We revisit this assumption in Sect. 4.6.

In fact, balancing markets are partly needed due to transmission bottlenecks and transmission technical issues, which may straddle cheap production in some areas and force the use of expensive units elsewhere. These production allocation adjustments may actually take place in balancing markets.

In the following we consider that the market organization includes just two market floors:

1. The day-ahead market, which clears 12 h prior to the first hour of the delivery period.
2. A series of 24 balancing markets, each one cleared half an hour prior to energy delivery.

This market organization is sketched in Fig. 4.1. Notice that no adjustment market is considered for simplicity. If such a market exists, the day-ahead outcomes should be adjusted considering the adjustment market outcomes before analyzing the balancing markets.

Finally, it should be observed that hourly trading periods are considered in this chapter. For this reason, we will equivalently refer to 1 MWh of energy and 1 MW

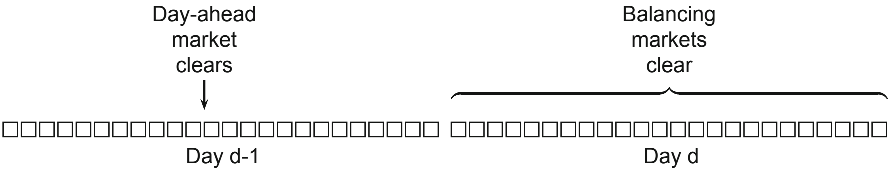


Fig. 4.1 Market decision timeline: one day-ahead market and 24 balancing markets

of power, since a constant power output of that magnitude generates 1 MWh in a 1-hour period.

4.2 Balancing Market Auction

In this section, the basic formulation of a balancing market auction is introduced. Such formulation refers to the so-called *one-price* or *single-price* imbalance settlement, which is adopted in many markets, especially in the USA.

4.2.1 Introduction

Once the day-ahead market is cleared, hourly production levels are assigned to generators and hourly consumption levels to demands. In addition, hourly clearing prices are derived. The reader is referred to Chap. 3 for a detailed description of the day-ahead market.

For the purpose of balancing markets, generators are divided into two sets:

1. Dispatchable generators that are denominated below *balancing generators*.
2. Stochastic generators, with no control over their production levels, e.g., wind- or solar-based generators.

The symbols used in the derivations below are defined in the following for clarity.

$P^{\mathrm{S}}_{\mathrm{B}i}$ is the scheduled power generation level for balancing generator i in the day-ahead market. Superscript S stands for *scheduled* and subscript B for *balancing*.

$P^{\mathrm{S}}_{\mathrm{B}}$ is the total scheduled power generation for all balancing generators in the day-ahead market.

$P^{\mathrm{S}}_{\mathrm{U}j}$ is the scheduled power generation level for stochastic generator j in the day-ahead market. Subscript U stands for *undispatchable*.

$P^{\mathrm{S}}_{\mathrm{U}}$ is the total scheduled power generation for all stochastic generators in the day-ahead market.

$P^{\mathrm{S}}_{\mathrm{D}k}$ is the scheduled power consumption level for demand k in the day-ahead market. Subscript D stands for *demand*.

$P^{\mathrm{S}}_{\mathrm{D}}$ is the total scheduled power consumption in the day-ahead market.

$P_{\mathrm{N}}^{\mathrm{S}}$ is the total scheduled net power consumption (demand minus stochastic production) in the day-ahead market. Subscript N stands for *net*.

λ^{S} is the clearing price in the day-ahead market.

Clearly,

$$\sum_{i \in \Xi_{\mathrm{B}}} P_{\mathrm{B}i}^{\mathrm{S}} = P_{\mathrm{B}}^{\mathrm{S}}, \tag{4.1a}$$

$$\sum_{j \in \Xi_{\mathrm{U}}} P_{\mathrm{U}j}^{\mathrm{S}} = P_{\mathrm{U}}^{\mathrm{S}}, \tag{4.1b}$$

$$\sum_{k \in \Xi_{\mathrm{D}}} P_{\mathrm{D}k}^{\mathrm{S}} = P_{\mathrm{D}}^{\mathrm{S}}, \tag{4.1c}$$

where Ξ_{B}, Ξ_{U}, and Ξ_{D} are the sets of indices of balancing generators, stochastic generators, and demands, respectively.

Since the day-ahead market is balanced, it holds that

$$P_{\mathrm{B}}^{\mathrm{S}} = P_{\mathrm{N}}^{\mathrm{S}} = P_{\mathrm{D}}^{\mathrm{S}} - P_{\mathrm{U}}^{\mathrm{S}}. \tag{4.2}$$

The balancing generators producing electric energy have control over their production levels and can either sell additional energy in the balancing market or buy back the energy already sold in the day-ahead market. Needless to say, they do so for a profit. Market rules generally bind balancing generators to provide both up-regulation (selling additional energy) and down-regulation (buying back energy already sold).

We assume that the bid of balancing generator i in the balancing market is characterized by the following parameters:

$\lambda_{\mathrm{B}i}^{\mathrm{U}}$ is the cost offer of balancing generator i for additional production at the balancing stage. The superscript U stands for *up-regulation*.

$P_{\mathrm{B}i}^{\mathrm{U,max}}$ is the capacity of production of balancing generator i at the balancing stage.

$\lambda_{\mathrm{B}i}^{\mathrm{D}}$ is the price offer of balancing generator i for repurchase at the balancing stage of own production scheduled at the day-ahead market. The superscript D stands for *down-regulation*.

$P_{\mathrm{B}i}^{\mathrm{D,max}}$ is the maximum quantity of scheduled own production that balancing generator i offers to repurchase at the balancing stage.

The activation of an up-regulation bid from a balancing generator implies that production, originally left out of the day-ahead market dispatch, be increased with a relatively short notice. Since such production is costlier than the generation dispatched day-ahead, it must hold that

$$\lambda_{\mathrm{B}i}^{\mathrm{U}} \geq \lambda^{\mathrm{S}}. \tag{4.3}$$

On the other hand, the submission of a down-regulation bid for repurchase of energy implies that the producer is willing to decrease (repurchase) part of its production dispatched at the day-ahead stage. Clearly, such a repurchase pays off only if it is

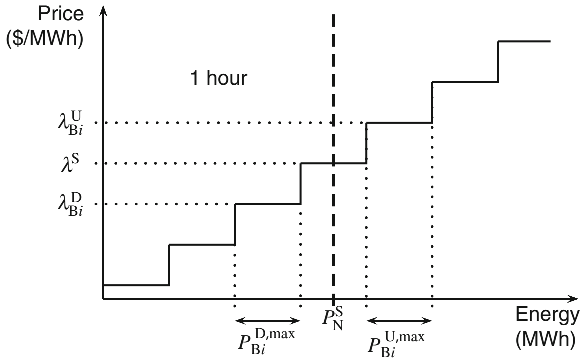


Fig. 4.2 Balancing supply function provided by all balancing generators

at a price not greater than the generation cost, which in turn must be lower than the day-ahead price λ^{S}. Therefore it holds that

$$\lambda^{\mathrm{D}}_{\mathrm{B}i} \leq \lambda^{\mathrm{S}}. \tag{4.4}$$

Considering jointly all balancing generators, the stepwise balancing supply function depicted in Fig. 4.2 is obtained. Observe that if the net demand level is $P^{\mathrm{S}}_{\mathrm{N}}$, the clearing price is λ^{S}.

Stochastic generators, e.g., wind- or solar-based generators, have no control over their production levels and thus they generally deviate from their scheduled productions in the day-ahead market. The production level at balancing time of stochastic generator j is denoted by $P^{\mathrm{B}}_{\mathrm{U}j}$, and generally,

$$P^{\mathrm{B}}_{\mathrm{U}j} \neq P^{\mathrm{S}}_{\mathrm{U}j}. \tag{4.5}$$

Superscript B in the earlier expression stands for *balancing market*.

Demands are considered to have no control over their consumption levels. However, we revisit this assumption in Sect. 4.4. Demand k at balancing time has a demand level denoted by $P^{\mathrm{B}}_{\mathrm{D}k}$, and generally,

$$P^{\mathrm{B}}_{\mathrm{D}k} \neq P^{\mathrm{S}}_{\mathrm{D}k}. \tag{4.6}$$

The total up- and down-regulation provided by the balancing generators, $P^{\mathrm{U}}_{\mathrm{B}}$ and $P^{\mathrm{D}}_{\mathrm{B}}$, respectively, are

$$P^{\mathrm{U}}_{\mathrm{B}} = \sum_{i \in \Xi_{\mathrm{B}}} P^{\mathrm{U}}_{\mathrm{B}i}, \tag{4.7a}$$

$$P^{\mathrm{D}}_{\mathrm{B}} = \sum_{i \in \Xi_{\mathrm{B}}} P^{\mathrm{D}}_{\mathrm{B}i}. \tag{4.7b}$$

The total generation from balancing generators, $P^{\mathrm{B}}_{\mathrm{B}}$, from stochastic generators, $P^{\mathrm{B}}_{\mathrm{U}}$, the total demand, $P^{\mathrm{B}}_{\mathrm{D}}$, and the net demand, $P^{\mathrm{B}}_{\mathrm{N}}$, at balancing time are, respectively,

$$P_{\mathrm{B}}^{\mathrm{B}} = \sum_{i \in \Xi_{\mathrm{B}}} \left(P_{\mathrm{B}i}^{\mathrm{S}} + P_{\mathrm{B}i}^{\mathrm{U}} - P_{\mathrm{B}i}^{\mathrm{D}} \right) = P_{\mathrm{B}}^{\mathrm{S}} + P_{\mathrm{B}}^{\mathrm{U}} - P_{\mathrm{B}}^{\mathrm{D}}, \tag{4.8a}$$

$$P_{\mathrm{U}}^{\mathrm{B}} = \sum_{j \in \Xi_{\mathrm{U}}} P_{\mathrm{U}j}^{\mathrm{B}}, \tag{4.8b}$$

$$P_{\mathrm{D}}^{\mathrm{B}} = \sum_{k \in \Xi_{\mathrm{D}}} P_{\mathrm{D}k}^{\mathrm{B}}, \tag{4.8c}$$

$$P_{\mathrm{N}}^{\mathrm{B}} = P_{\mathrm{D}}^{\mathrm{B}} - P_{\mathrm{U}}^{\mathrm{B}}. \tag{4.8d}$$

The different form of (4.8a) with respect to (4.8b) and (4.8c) depends on the fact that $P_{\mathrm{B}i}^{\mathrm{U}}$ and $P_{\mathrm{B}i}^{\mathrm{D}}$ represent *adjustments* at the balancing stage with respect to the day-ahead schedule. On the contrary, $P_{\mathrm{U}j}^{\mathrm{B}}$ and $P_{\mathrm{D}k}^{\mathrm{B}}$ denote the actual total production and consumption in the balancing market, respectively.

4.2.2 Auction Formulation

The simplest form of a balancing auction to match production and consumption and to minimize the balancing costs for the system is

$$\min_{P_{\mathrm{B}i}^{\mathrm{U}}, P_{\mathrm{B}i}^{\mathrm{D}}} \quad \sum_{i \in \Xi_{\mathrm{B}}} \lambda_{\mathrm{B}i}^{\mathrm{U}} P_{\mathrm{B}i}^{\mathrm{U}} - \lambda_{\mathrm{B}i}^{\mathrm{D}} P_{\mathrm{B}i}^{\mathrm{D}} \tag{4.9a}$$

$$\text{s.t.} \quad P_{\mathrm{B}}^{\mathrm{S}} + \sum_{i \in \Xi_{\mathrm{B}}} P_{\mathrm{B}i}^{\mathrm{U}} - P_{\mathrm{B}i}^{\mathrm{D}} = P_{\mathrm{N}}^{\mathrm{B}} : \quad \lambda^{\mathrm{B}}, \tag{4.9b}$$

$$0 \le P_{\mathrm{B}i}^{\mathrm{U}} \le P_{\mathrm{B}i}^{\mathrm{U,max}}, \quad \forall i, \tag{4.9c}$$

$$0 \le P_{\mathrm{B}i}^{\mathrm{D}} \le P_{\mathrm{B}i}^{\mathrm{D,max}}, \quad \forall i, \tag{4.9d}$$

where λ^{B} is the dual variable associated with the balance constraint (4.9b), i.e., the marginal price, see Sect. B.2.

It is important to note that auction (4.9) clears the market for the total net demand, i.e., demand minus stochastic production, at the balancing stage. In a market without stochastic production, balancing markets would only serve to balance the deviations stemming from the demand and equipment (generating unit or transmission line) failures.

The solution of auction (4.9) provides each balancing generator i with the cleared production increase, $P_{\mathrm{B}i}^{\mathrm{U}}$, or decrease, $P_{\mathrm{B}i}^{\mathrm{D}}$, at the balancing stage, as well as the price λ^{B}. In a balancing market employing the one-price system as in this case, all market agents sell or buy energy at this marginal price.

Furthermore, from the balancing auction formulation (4.9), and because of the relationships (4.3) and (4.4), note that if $P_{\mathrm{N}}^{\mathrm{B}} > P_{\mathrm{N}}^{\mathrm{S}}$ (up-regulation), then $\lambda^{\mathrm{B}} \ge \lambda^{\mathrm{S}}$, else if $P_{\mathrm{N}}^{\mathrm{B}} < P_{\mathrm{N}}^{\mathrm{S}}$ (down-regulation), then $\lambda^{\mathrm{B}} \le \lambda^{\mathrm{S}}$, and if $P_{\mathrm{N}}^{\mathrm{B}} = P_{\mathrm{N}}^{\mathrm{S}}$ (no imbalance), then $\lambda^{\mathrm{B}} = \lambda^{\mathrm{S}}$. Finally, note that auction (4.9) ensures power balance at balancing

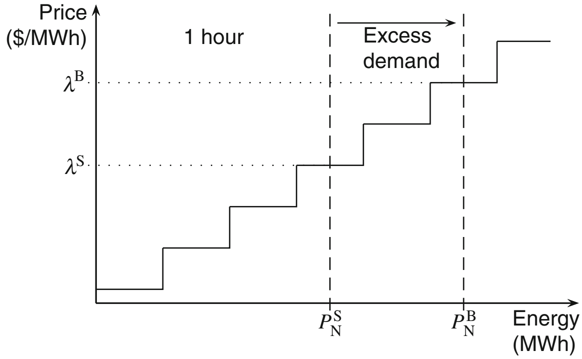


Fig. 4.3 Excess demand at balancing time: the net demand at balancing time (P_N^B) exceeds the scheduled net demand at the day-ahead market (P_N^S)

time, i.e.,

$$P_B^B = P_N^B = P_D^B - P_U^B. \tag{4.10}$$

The different imbalance alternatives that can occur are analyzed in the two following subsections.

4.2.3 *Excess Consumption*

If at balancing time total net demand exceeds total scheduled generation, we have an *excess demand* situation. This situation is depicted in Fig. 4.3 and analyzed later.

In an excess demand situation, $P_N^B > P_N^S$ and $\lambda^B \geq \lambda^S$. The extra production needed, $\left(P_N^B - P_N^S\right)$, is produced by the balancing generators, whose total balancing revenue is

$$\left(P_N^B - P_N^S\right)\lambda^B > 0. \tag{4.11}$$

Regarding demands and stochastic generators, the four alternatives discussed later are possible.

1. If stochastic generator j deviates producing below its scheduled production in the day-ahead market, it has to buy energy. The amount

$$\left(P_{Uj}^B - P_{Uj}^S\right)\lambda^B < 0 \tag{4.12}$$

is the negative revenue (payment) incurred by stochastic generator j for buying energy in the balancing market. Note that the additional energy required is bought at a higher price than the price in the day-ahead market, which entails an extra loss of revenue for the generator.

2. On the other hand, if stochastic generator j deviates producing above its scheduled production in the day-ahead market, it has to sell this excess energy. The amount

$$\left(P_{Uj}^B - P_{Uj}^S\right)\lambda^B > 0 \tag{4.13}$$

is the revenue achieved by stochastic generator j for selling its excess energy in the balancing market. Note that this additional energy is sold at a higher price than the price in the day-ahead market, which entails an additional revenue for the generator. This is so because this stochastic generator (although involuntarily) helps restoring the system balance.

3. If demand k deviates increasing its consumption, the amount

$$\left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right) \lambda^{\mathrm{B}} > 0 \tag{4.14}$$

represents a payment by this demand to buy the additional energy required. Note that this additional energy is bought at a higher price than the price in the day-ahead market, which entails an extra cost for the deviating demand.

4. On the other hand, if demand k deviates decreasing its consumption, the amount

$$\left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right) \lambda^{\mathrm{B}} < 0 \tag{4.15}$$

represents a negative payment (revenue) to demand k for selling back the excess energy. Note that this excess energy is sold back at a higher price than the price paid to buy it in the day-ahead market, which constitutes an extra revenue. This is so because this demand (although involuntarily) helps restoring the system balance.

To sum up, the required increase in energy production is

$$\begin{aligned}\sum_{k \in \Xi_{\mathrm{D}}} &\left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right) - \sum_{j \in \Xi_{\mathrm{U}}} \left(P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right) \\ &= \left(P_{\mathrm{D}}^{\mathrm{B}} - P_{\mathrm{D}}^{\mathrm{S}}\right) - \left(P_{\mathrm{U}}^{\mathrm{B}} - P_{\mathrm{U}}^{\mathrm{S}}\right) \\ &= \left(P_{\mathrm{D}}^{\mathrm{B}} - P_{\mathrm{U}}^{\mathrm{B}}\right) - \left(P_{\mathrm{D}}^{\mathrm{S}} - P_{\mathrm{U}}^{\mathrm{S}}\right) \\ &= P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}}.\end{aligned} \tag{4.16}$$

Therefore, the total payment due to deviations by demands and stochastic generators is

$$\left(P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}}\right) \lambda^{\mathrm{B}}, \tag{4.17}$$

which coincides with the total payment for deviations to balancing generators given by (4.11).

4.2.4 Excess Production

If at balancing time total production exceeds total net demand, we have an *excess production* situation. This situation is depicted in Fig. 4.4. In an excess production situation, $P_{\mathrm{N}}^{\mathrm{B}} < P_{\mathrm{N}}^{\mathrm{S}}$ and $\lambda^{\mathrm{B}} \leq \lambda^{\mathrm{S}}$.

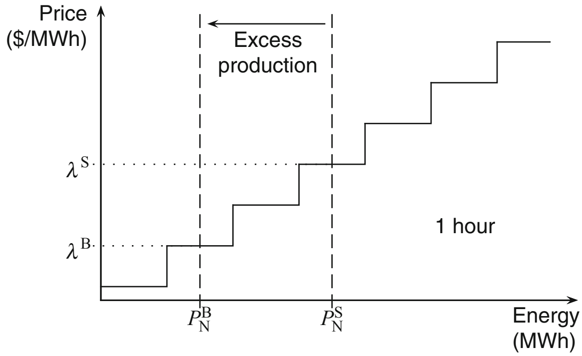


Fig. 4.4 Excess production at balancing time: the scheduled net demand (P_N^S) in the day-ahead market exceeds the net demand at balancing time (P_N^B)

As generally required by market rules, balancing generators need to buy back the excess production $P_N^S - P_N^B$ and pay for it the amount

$$\left(P_N^S - P_N^B\right)\lambda^B > 0. \tag{4.18}$$

Regarding demands and stochastic generators, the four alternatives discussed below are possible.

1. If stochastic generator j deviates producing below its scheduled production in the day-ahead market, it has to buy energy. The quantity

$$\left(P_{Uj}^B - P_{Uj}^S\right)\lambda^B < 0 \tag{4.19}$$

is the negative revenue (payment) incurred by the stochastic generator for buying energy. Note that the additional energy required is bought at a lower price than the price in the day-ahead market, which constitutes an extra revenue. This is so because this stochastic generator (although involuntarily) helps restoring the system balance.
2. On the other hand, if stochastic generator j deviates producing above its scheduled production in the day-ahead market, it has to sell this excess energy. The amount

$$\left(P_{Uj}^B - P_{Uj}^S\right)\lambda^B > 0 \tag{4.20}$$

is the revenue achieved by the stochastic producer for selling its excess energy. Note that the additional energy is sold at a lower price than the price in the day-ahead market, which constitutes an opportunity loss.
3. If demand k deviates increasing its consumption, the value

$$\left(P_{Dk}^B - P_{Dk}^S\right)\lambda^B > 0 \tag{4.21}$$

represents a payment by the demand to buy the additional energy required. Note that this additional energy is bought at a lower price than the price in the day-ahead market, which ensures lower costs for the demand than if all the consumption was bought at the day-ahead market. This is so because this demand (although involuntarily) helps restoring the system balance.
4. On the other hand, if demand k deviates decreasing its consumption, the quantity

$$\left(P_{Dk}^B - P_{Dk}^S\right)\lambda^B < 0 \tag{4.22}$$

represents a negative payment (revenue) to the demand for selling back the excess energy. Note that this excess demand is sold back at a lower price than the price paid to buy it in the day-ahead market, which entails an opportunity loss.

To sum up, the required decrease in energy production is

$$\begin{aligned}\sum_{j \in \Xi_{\mathrm{U}}} \left(P^{\mathrm{B}}_{\mathrm{U}j} - P^{\mathrm{S}}_{\mathrm{U}j}\right) - \sum_{k \in \Xi_{\mathrm{D}}} \left(P^{\mathrm{B}}_{\mathrm{D}k} - P^{\mathrm{S}}_{\mathrm{D}k}\right) \\ = \left(P^{\mathrm{B}}_{\mathrm{U}} - P^{\mathrm{S}}_{\mathrm{U}}\right) - \left(P^{\mathrm{B}}_{\mathrm{D}} - P^{\mathrm{S}}_{\mathrm{D}}\right) \\ = \left(P^{\mathrm{S}}_{\mathrm{D}} - P^{\mathrm{S}}_{\mathrm{U}}\right) - \left(P^{\mathrm{B}}_{\mathrm{D}} - P^{\mathrm{B}}_{\mathrm{U}}\right) \\ = P^{\mathrm{S}}_{\mathrm{N}} - P^{\mathrm{B}}_{\mathrm{N}} > 0. \end{aligned} \tag{4.23}$$

Thus, the total revenue by demands and stochastic generators for selling back energy due to deviations is

$$\left(P^{\mathrm{S}}_{\mathrm{N}} - P^{\mathrm{B}}_{\mathrm{N}}\right) \lambda^{\mathrm{B}}, \tag{4.24}$$

which coincides with the total payment due to deviations by balancing generators given in (4.18).

4.2.5 *Payments and Revenues*

Payment and revenues are calculated as follows.

1. The revenue for balancing generator i is given by

$$P^{\mathrm{S}}_{\mathrm{B}i} \lambda^{\mathrm{S}} + \left(P^{U}_{\mathrm{B}i} - P^{\mathrm{D}}_{\mathrm{B}i}\right) \lambda^{\mathrm{B}}. \tag{4.25}$$

2. The revenue for stochastic generator j is given by

$$P^{\mathrm{S}}_{\mathrm{U}j} \lambda^{\mathrm{S}} + \left(P^{\mathrm{B}}_{\mathrm{U}j} - P^{\mathrm{S}}_{\mathrm{U}j}\right) \lambda^{\mathrm{B}}. \tag{4.26}$$

3. The payment by demand k is given by

$$P^{\mathrm{S}}_{\mathrm{D}k} \lambda^{\mathrm{S}} + \left(P^{\mathrm{B}}_{\mathrm{D}k} - P^{\mathrm{S}}_{\mathrm{D}k}\right) \lambda^{\mathrm{B}}. \tag{4.27}$$

The total payment by demands is

$$\begin{aligned}\lambda^{\mathrm{S}} \sum_{k \in \Xi_{\mathrm{D}}} P^{\mathrm{S}}_{\mathrm{D}k} + \lambda^{\mathrm{B}} \sum_{k \in \Xi_{\mathrm{D}}} \left(P^{\mathrm{B}}_{\mathrm{D}k} - P^{\mathrm{S}}_{\mathrm{D}k}\right) \\ = \lambda^{\mathrm{S}} P^{\mathrm{S}}_{\mathrm{D}} + \lambda^{\mathrm{B}} \left(P^{\mathrm{B}}_{\mathrm{D}} - P^{\mathrm{S}}_{\mathrm{D}}\right). \end{aligned} \tag{4.28}$$

The total revenue for generators is

$$\begin{aligned}
&\lambda^{\mathrm{S}} \sum_{i \in \Xi_{\mathrm{B}}} P_{\mathrm{B}i}^{\mathrm{S}} + \lambda^{\mathrm{B}} \sum_{i \in \Xi_{\mathrm{B}}} \left(P_{\mathrm{B}i}^{\mathrm{U}} - P_{\mathrm{B}i}^{\mathrm{D}}\right) \\
&\quad + \lambda^{\mathrm{S}} \sum_{j \in \Xi_{\mathrm{U}}} P_{\mathrm{U}j}^{\mathrm{S}} + \lambda^{\mathrm{B}} \sum_{j \in \Xi_{\mathrm{U}}} \left(P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right) \\
&\quad = \lambda^{\mathrm{S}} \left(P_{\mathrm{B}}^{\mathrm{S}} + P_{\mathrm{U}}^{\mathrm{S}}\right) + \lambda^{\mathrm{B}} \left[\left(P_{\mathrm{B}}^{\mathrm{U}} - P_{\mathrm{B}}^{\mathrm{D}}\right) + \left(P_{\mathrm{U}}^{\mathrm{B}} - P_{\mathrm{U}}^{\mathrm{S}}\right)\right] \\
&\quad = \lambda^{\mathrm{S}} P_{\mathrm{D}}^{\mathrm{S}} + \lambda^{\mathrm{B}} \left(P_{\mathrm{D}}^{\mathrm{B}} - P_{\mathrm{D}}^{\mathrm{S}}\right), \qquad (4.29)
\end{aligned}$$

where the last equality is a result of the balance Eqs. (4.2) and (4.9b). Since the right hand sides of (4.29) and of (4.28) coincide, the balancing market is revenue adequate, i.e., it does not involve losses for the market operator.

In some American balancing markets the whole energy interchange takes place at balancing prices, i.e.,

1. The revenue for balancing generator i is given by

$$P_{\mathrm{B}i}^{\mathrm{B}} \lambda^{\mathrm{B}}. \qquad (4.30)$$

2. The revenue for stochastic generator j is given by

$$P_{\mathrm{U}j}^{\mathrm{B}} \lambda^{\mathrm{B}}. \qquad (4.31)$$

3. The payment by demand k is given by

$$P_{\mathrm{D}k}^{\mathrm{B}} \lambda^{\mathrm{B}}, \qquad (4.32)$$

where

$P_{\mathrm{B}i}^{\mathrm{B}}$ is the actual power generation level for balancing generator i.
$P_{\mathrm{U}j}^{\mathrm{B}}$ is the actual power generation level for stochastic generator j.
$P_{\mathrm{D}k}^{\mathrm{B}}$ is the actual power consumption level for demand k.

The earlier pricing scheme also results in revenue adequacy.

Two examples that illustrate the functioning of a balancing market auction are provided below. The examples are based on a simplified power system, and consider the cases of excess consumption and production, respectively.

Example 4.1 (Balancing auction with excess consumption) To illustrate the functioning of a balancing auction, we consider a simple system with three dispatchable generators (units B1, B2, and B3), two stochastic generators (U1 and U2) and two loads (D1 and D2). The outcome of the day-ahead market clearing is shown in Table 4.1.

Half of the total load, $P_{\mathrm{D}}^{\mathrm{S}} = 140$ MWh, is covered by dispatchable producers, which are scheduled $P_{\mathrm{B}}^{\mathrm{S}} = 70$ MWh in total. Likewise, stochastic producers are scheduled $P_{\mathrm{U}}^{\mathrm{S}} = 70$ MWh. The scheduled net demand is therefore $P_{\mathrm{N}}^{\mathrm{S}} = P_{\mathrm{D}}^{\mathrm{S}} - P_{\mathrm{U}}^{\mathrm{S}} =$ 70 MWh. Besides, it is useful to remark that the dispatchable producers' installed capacity is 50, 50, and 70 MW for units B1, B2, and B3, respectively. Hence, unit B1 is fully scheduled at the day-ahead market.

Table 4.1 Results of the day-ahead market clearing

Variable		Value
P^S (MWh)	Unit B1	50
	Unit B2	20
	Unit B3	0
	Unit U1	40
	Unit U2	30
	Load D1	40
	Load D2	100
λ^S (\$/MWh)		20

Table 4.2 Realization of stochastic production and consumption at the balancing stage, and deviation from the day-ahead dispatch

Participant	P^B (MWh)	$P^B - P^S$ (MWh)
Unit U1	50	10
Unit U2	20	− 10
Load D1	35	− 5
Load D2	120	20

Table 4.3 Up-regulation offers of the dispatchable generators in the balancing market

Unit	λ^U (\$/MWh)	$P^{U,\max}$ (MWh)
B1	–	–
B2	30	10
B3	50	50

At the balancing stage, stochastic plants and loads produce and consume as shown in Table 4.2.

As the reader can notice, unit U1 is producing 10 MWh in excess of its day-ahead schedule. On the contrary, generator U2 has an underproduction of the same amount. As far as the loads are concerned, D1 is consuming 5 MWh less than scheduled day-ahead, while D2 exceeds the day-ahead demand schedule by 20 MWh. The total net demand at the balancing market is

$$P_N^B = \left(P_{D1}^B + P_{D2}^B\right) - \left(P_{U1}^B + P_{U2}^B\right) = 85\,\text{MWh}\,. \tag{4.33}$$

Since $P_N^B > P_N^S$, the system is in a situation of excess consumption.

The offers made by dispatchable producers in the balancing market are included in Table 4.3.

Since the system needs to increase the total production to restore balance, it suffices to consider only the sale offers in this example. Notice that unit B1 is not offering to sell any additional production, since it was fully dispatched (i.e., dispatched at its production capacity) at the day-ahead market.

Under these assumptions, the auction in (4.9) translates to

$$\min_{P_{B2}^U, P_{B3}^U} \quad 30P_{B2}^U + 50P_{B3}^U \tag{4.34a}$$

$$\text{s.t.} \quad 70 + P_{B2}^U + P_{B3}^U = 85 \quad : \lambda^B, \tag{4.34b}$$

$$0 \le P_{B2}^U \le 10, \tag{4.34c}$$

$$0 \le P_{B3}^U \le 50. \tag{4.34d}$$

Table 4.4 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	0	1000
Unit B2	400	500	900
Unit B3	0	250	250
Unit U1	800	500	1300
Unit U2	600	– 500	100
Load D1	– 800	250	– 550
Load D2	– 2000	– 1000	– 3000

Clearly, the optimal solution consists in fully dispatching generator B2, which has the cheaper offer; the remainder of the consumption is to be covered by the costlier unit B3:

$$P^{\mathrm{U}}_{\mathrm{B2}} = 10\,\mathrm{MWh}, \tag{4.35}$$

$$P^{\mathrm{U}}_{\mathrm{B3}} = 5\,\mathrm{MWh}. \tag{4.36}$$

The dual price of (4.34b), i.e., the balancing market price, is equal to the per unit cost of the marginal generator, i.e.,

$$\lambda^{\mathrm{B}} = \lambda^{\mathrm{U}}_{\mathrm{B3}} = \$50\,/\mathrm{MWh}, \tag{4.37}$$

which is higher than the day-ahead market price λ^{S}.

Revenues and payments for all the producers and loads are summarized in Table 4.4.

The first column provides the financial results for the participants in the day-ahead market, which are obtained by multiplying the day-ahead schedule P^{S} with the market clearing price λ^{S} in that market. The results for the balancing stage, included in the second column, are calculated as the multiplication of the schedule deviation $P^{\mathrm{B}} - P^{\mathrm{S}}$ and the balancing market price λ^{B}. Since $\lambda^{\mathrm{B}} > \lambda^{\mathrm{S}}$, the balancing market rewards the producers that, voluntarily or not, are increasing their production at the balancing stage (B2, B3, and U1). Indeed, these producers are paid at the higher price $\lambda^{\mathrm{B}} = \$50\,/\mathrm{MWh}$ as compared to the day-ahead price $\lambda^{\mathrm{S}} = \$20\,/\mathrm{MWh}$. Similarly, load D1, which is decreasing its consumption, sells back 5 MWh of its scheduled consumption at the price λ^{B}, which is higher than the price of initial purchase λ^{S}. On the contrary, the unit U2 and the load D2, which are underproducing and overconsuming with respect to their day-ahead schedule, are penalized as they must purchase power at a higher cost than at the day-ahead market. In particular, unit U2 realizes a total revenue of only $100, despite producing 20 MWh. In comparison, the balancing unit B3, which produces only 5 MWh, receives a payment of $250 from the market operator. As a final remark, we underline that the total payments collected and made by the market operator are zero at both market stages. Indeed, the sums of the quantities in the first and the second columns of Table 4.4 are both zero.

Table 4.5 Realization of stochastic production and consumption at the balancing stage, and deviation from the day-ahead dispatch

Participant	P^{B} (MWh)	$P^{\mathrm{B}} - P^{\mathrm{S}}$ (MWh)
Unit U1	65	25
Unit U2	25	− 5
Load D1	22	− 18
Load D2	120	20

Table 4.6 Down-regulation bids of the dispatchable generators in the balancing market

Unit	λ^{D} (\$/MWh)	$P^{\mathrm{D,max}}$ (MWh)
B1	8	40
B2	15	10
B3	–	–

Example 4.2 (Balancing auction with excess production) Let us consider the simplified power system already introduced in Example 4.1, with the day-ahead market clearing summarized in Table 4.1.

In this case, at the balancing stage, the actual productions from stochastic generators and the demand realize as indicated in Table 4.5.

As in Example 4.1, there are an overproducing unit (U1) and an underproducing one (U2), a load that consumes below schedule (D1) and another load that consumes above schedule (D2). Differently from the previous example, though, the total net demand at the balancing market

$$P_{\mathrm{N}}^{\mathrm{B}} = \left(P_{\mathrm{D1}}^{\mathrm{B}} + P_{\mathrm{D2}}^{\mathrm{B}}\right) - \left(P_{\mathrm{U1}}^{\mathrm{B}} + P_{\mathrm{U2}}^{\mathrm{B}}\right) = 52\,\mathrm{MWh} \tag{4.38}$$

is lower than its corresponding quantity in the day-ahead market: $P_{\mathrm{N}}^{\mathrm{B}} < P_{\mathrm{N}}^{\mathrm{S}}$. Therefore, the system is in a situation of excess production.

Table 4.6 shows the bids submitted by balancing generators in the balancing market. In this case only the bids for down-regulation need to be shown.

Notice that unit B3 is not able to bid for repurchasing scheduled production, since it was not dispatched in the day-ahead market (Table 4.1).

In this case, the balancing market auction (4.9) writes as follows

$$\underset{P_{\mathrm{B1}}^{\mathrm{D}}, P_{\mathrm{B2}}^{\mathrm{D}}}{\mathrm{Min.}} \quad -8P_{\mathrm{B1}}^{\mathrm{D}} - 15P_{\mathrm{B2}}^{\mathrm{D}} \tag{4.39a}$$

$$\text{s.t.} \quad 70 - P_{\mathrm{B1}}^{\mathrm{D}} - P_{\mathrm{B2}}^{\mathrm{D}} = 52 \quad : \lambda^{\mathrm{B}}, \tag{4.39b}$$

$$0 \le P_{\mathrm{B1}}^{\mathrm{D}} \le 40, \tag{4.39c}$$

$$0 \le P_{\mathrm{B2}}^{\mathrm{D}} \le 10. \tag{4.39d}$$

Since unit B2 has a higher benefit in repurchasing energy, the optimal clearing of the balancing market implies that this unit repurchases as much as possible (10 MWh), while the remainder of excess production is assigned to producer B1:

$$P_{\mathrm{B1}}^{\mathrm{D}} = 8\,\mathrm{MWh}, \tag{4.40}$$

$$P_{\mathrm{B2}}^{\mathrm{D}} = 10\,\mathrm{MWh}. \tag{4.41}$$

Table 4.7 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	– 64	936
Unit B2	400	– 80	320
Unit B3	0	0	0
Unit U1	800	200	1000
Unit U2	600	– 40	560
Load D1	– 800	144	– 656
Load D2	– 2000	– 160	– 2160

The balancing market price, obtained as the dual variable of (4.39b), is given by the benefit per unit of the marginal generator

$$\lambda^{\mathrm{B}} = \$8\ /\mathrm{MWh}, \tag{4.42}$$

which is lower than the day-ahead market price $\lambda^{\mathrm{S}} = \$20/\mathrm{MWh}$.

Table 4.7 summarizes revenues and payments for producers and loads, respectively.

The balancing units B1 and B2 have to pay the market operator for repurchasing energy at the balancing stage. Since this power was initially sold at the day-ahead market at the price $\lambda^{\mathrm{S}} = \$20\ /\mathrm{MWh}$, these producers realize a net profit on the energy they are asked not to produce, as the repurchasing price $\lambda^{\mathrm{B}} = \$8\ /\mathrm{MWh}$ is lower than the initial selling price. Regarding the stochastic generators, unit U1 is penalized in the balancing market, since it sells its production surplus at a lower price than it could have received at the day-ahead market. On the contrary, unit U2 is rewarded, since it repurchases energy at a lower price than it was sold. The situation for the loads is similar, with D2 being rewarded (its additional consumption is paid a lower rate) and D1 being penalized (it sells energy back at a lower price than the purchase price). As in the previous example, we point out that payments and revenues cancel out both at the day-ahead and at the balancing markets.

4.3 Two-Price Imbalance Settlement

The auction and pricing mechanism presented in the previous section, where all the power exchanged in the balancing market is priced at the marginal cost of the power balance constraint (4.9b), describe the functioning of markets based on the so-called one-price imbalance settlement.

The rationale behind one-price balancing markets implies that deviations from the day-ahead schedule are settled at a price that is more favorable than the day-ahead price if the sign of the participant's imbalance is opposite to the sign of the overall system deviation. Indeed, generators producing more power than contracted in the day-ahead market, and demands consuming less power, receive a higher price for their sale in the balancing market if the system is in excess of demand. On the contrary, generators that produce less power than contracted, and loads consuming more, when the system is in excess of production have to pay a price that is lower than the day-ahead market price, thus achieving a profit. The one-price imbalance settlement, therefore, rewards participants that help restore system balance with their deviations, regardless of whether such deviations are wanted or not.

Other balancing markets are designed according to the *two-price* (or *dual-price*) imbalance settlement principle. In this type of market design, only wanted deviations (i.e., from dispatchable producers) opposite in sign from the system imbalance are rewarded financially with a balancing market price that is more favorable than the day-ahead price. On the contrary, deviations from stochastic producers are either settled at the day-ahead price (if opposite to the system imbalance) or, just like in a one-price market, at the less favorable balancing market price (if in the same direction as the system imbalance). Such a design is common in European electricity markets.

The determination of the optimal dispatch is performed in the same fashion both for the one-price and the two-price balancing markets by solving an optimization problem, whose basic formulation is given in (4.9). While there is no difference in the determination of the clearing price either, which is the dual variable of the power balance constraint (4.9b), the two market designs differ as to how prices apply to conventional and stochastic producers.

In the remainder of this section, we analyze how producers are priced in a balancing market following the two-price design. Since the auction model for the balancing market is equal to that in the one-price case, the notation is the same as in the previous section.

4.3.1 Excess Consumption

As pointed out in Sect. 4.2.3, in the excess demand situation $P_{\mathrm{N}}^{\mathrm{B}} > P_{\mathrm{N}}^{\mathrm{S}}$ and $\lambda^{\mathrm{B}} \geq \lambda^{\mathrm{S}}$. The production needed to cover the difference $\left(P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}}\right)$ is generated by the balancing generators. The pricing for these generators is exactly the same as in the one-price settlement, and their total balancing revenue is still given by

$$\left(P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}}\right) \lambda^{\mathrm{B}} > 0. \tag{4.43}$$

Contrary to the case of balancing generators, the pricing of demands and stochastic generators in the two-price model differs from the one-price market. This is discussed in the remainder of this section.

1. Stochastic generator j that deviates producing less than its day-ahead schedule has to buy energy. Since such a generator is increasing the total system imbalance, this exchange is settled at the less favorable price λ^{B} for the producer, implying the following negative revenue (payment)

$$\left(P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right) \lambda^{\mathrm{B}} < 0 \tag{4.44}$$

   for repurchasing energy in the balancing market. As in the one-price model, energy is purchased at a higher price than the day-ahead market price, which entails a loss of revenue for the generator.

2. On the other hand, stochastic generator j producing above its scheduled production in the day-ahead market is involuntarily helping restoring the system balance. In the two-price model, the energy in excess is sold at the day-ahead market price, generating the following revenue

$$\left(P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right) \lambda^{\mathrm{S}} > 0. \tag{4.45}$$

Differently from the one-price model, there is no price premium for selling this additional energy at the balancing market compared to the day-ahead market.

3. If demand k deviates increasing its consumption, thus increasing the system imbalance, it must pay the following amount

$$\left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right) \lambda^{\mathrm{B}} > 0 \tag{4.46}$$

in order to buy the energy needed to balance its position. Once more, note that this additional energy is bought at a higher price, and thus less favorable, than the day-ahead market price.

4. If demand k decreases its consumption, thus reducing (although involuntarily) the overall system imbalance, its negative payment (revenue) is

$$\left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right) \lambda^{\mathrm{S}} < 0 \tag{4.47}$$

for selling back the energy in excess. Contrary to the one-price system, the two-price settlement entails no extra revenue for the demand involuntarily reducing the system imbalance compared to the day-ahead market.

Differently from the one-price model, the total payment due to deviations by demands and stochastic generators is not a linear function of the required increase $P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}}$ in energy production, but it depends on the sign and magnitude of the individual deviations. Denoting positive and negative part functions by, respectively,

$$[\,\cdot\,]^{+} = \max\,(\cdot, 0)\,, \tag{4.48}$$

$$[\,\cdot\,]^{-} = -\min\,(\cdot, 0)\,, \tag{4.49}$$

the net of the payments that the market operator receives from demands and stochastic generators is

$$\begin{aligned}
&\Bigg(\sum_{k\in\Xi_{\mathrm{D}}} \left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{+} + \sum_{j\in\Xi_{\mathrm{U}}} \left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{-}\Bigg)\lambda^{\mathrm{B}} \\
&\qquad - \Bigg(\sum_{k\in\Xi_{\mathrm{D}}} \left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{-} + \sum_{j\in\Xi_{\mathrm{U}}} \left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{+}\Bigg)\lambda^{\mathrm{S}} \\
&\qquad \geq \Bigg(\sum_{k\in\Xi_{\mathrm{D}}} \left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{+} + \sum_{j\in\Xi_{\mathrm{U}}} \left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{-}\Bigg)\lambda^{\mathrm{B}} \\
&\qquad - \Bigg(\sum_{k\in\Xi_{\mathrm{D}}} \left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{-} + \sum_{j\in\Xi_{\mathrm{U}}} \left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{+}\Bigg)\lambda^{\mathrm{B}} = (P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}})\lambda^{\mathrm{B}},
\end{aligned} \tag{4.50}$$

where the inequality is due to the fact that $\lambda^{B} \geq \lambda^{S}$ in situations of excess of demand. Since the market operator may receive higher net payments than the total payment for deviations to balancing generators given by (4.43), the two-price model may entail a surplus at the balancing stage for the market operator in the case of excess consumption. More specifically, this surplus is generated whenever at least one stochastic generator produces more or one demand consumes less than scheduled at the day-ahead market.

4.3.1.1 Payments and Revenues

Payments and revenues in the case of excess demand in a two-price model are calculated as follows.

1. The revenue for balancing generator i is given by

$$P_{Bi}^{S}\lambda^{S} + \left(P_{Bi}^{B} - P_{Bi}^{S}\right)\lambda^{B}. \tag{4.51}$$

2. The revenue for stochastic generator j is given by

$$P_{Uj}^{S}\lambda^{S} + \left[P_{Uj}^{B} - P_{Uj}^{S}\right]^{+}\lambda^{S} - \left[P_{Uj}^{B} - P_{Uj}^{S}\right]^{-}\lambda^{B}. \tag{4.52}$$

3. The payment by demand k is given by

$$P_{Dk}^{S}\lambda^{S} + \left[P_{Dk}^{B} - P_{Dk}^{S}\right]^{+}\lambda^{B} - \left[P_{Dk}^{B} - P_{Dk}^{S}\right]^{-}\lambda^{S}. \tag{4.53}$$

Because all the transactions at the day-ahead market are settled at price λ^{S}, and since the day-ahead market is balanced, see (4.2), the total day-ahead payment by demands always equals the total day-ahead revenue for generators

$$P_{D}^{S}\lambda^{S} = (P_{B}^{S} + P_{U}^{S})\lambda^{S}. \tag{4.54}$$

Furthermore, the previous subsection showed that there is at least revenue adequacy (total revenue is higher than or equal to total cost) in the balancing market (see (4.50)), with the possibility for the market operator to realize a surplus. Therefore, we conclude that there is revenue adequacy for the aggregation of these two market floors in the case of excess demand.

4.3.2 Excess Production

As in Sect. 4.2.4, in an excess production situation $P_{N}^{B} < P_{N}^{S}$ and $\lambda^{B} \leq \lambda^{S}$. The price for balancing generators scheduled to buy the excess production is, just like in the single-price settlement, the marginal price λ^{B} of (4.9b). Therefore, these producers pay at the balancing stage the following amount

$$\left(P_{N}^{S} - P_{N}^{B}\right)\lambda^{B} > 0. \tag{4.55}$$

The pricing of demands and stochastic producers follows the rules below.

1. If stochastic producer j produces below its day-ahead market dispatch, it has to repurchase energy at the balancing stage. Since such a deviation, opposite in sign to the overall system imbalance, is involuntary, the less favorable day-ahead market price λ^{S} is charged, resulting in the negative revenue (payment)

$$\left(P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right)\lambda^{\mathrm{S}} < 0 \tag{4.56}$$

   for the repurchase. Compared to the case of the one-price settlement, there is no extra revenue involved in this transaction.
2. Else, if stochastic generator j overproduces with respect to the day-ahead market schedule, it has to sell this energy surplus. Since the deviation has the same sign as the overall system deviation, the price λ^{B} applies, resulting in the following revenue

$$\left(P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right)\lambda^{\mathrm{B}} > 0 \tag{4.57}$$

   for selling the excess energy. Like in the one-price market settlement, the additional energy is sold at a lower price than the day-ahead market price, thus resulting in an opportunity loss.
3. If the consumption from demand k is higher than scheduled, the day-ahead price applies, resulting in the payment

$$\left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right)\lambda^{\mathrm{S}} > 0 \tag{4.58}$$

   for purchasing the extra energy. Differently from the one-price settlement described in Sect. 4.2.4, there is no price premium for the demand. This is because the deviation, although contributing to restore the system balance, is involuntary.
4. Finally, if load k instead deviates decreasing its demand, it must sell back its unrealized consumption. Since the market is already in an excess of production, the price λ^{B}, less favorable than the day-ahead price λ^{S}, applies. The negative payment (revenue) for selling back the unrealized consumption is therefore

$$\left(P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right)\lambda^{\mathrm{B}} < 0. \tag{4.59}$$

Similar to the case of excess demand, the amount the market operator has to pay to demands and stochastic producers is not a linear function of the required decrease $P_{\mathrm{N}}^{\mathrm{B}} - P_{\mathrm{N}}^{\mathrm{S}}$ in energy production. This payment is given by

$$\begin{aligned}
&\Big(\sum_{j\in\Xi_{\mathrm{U}}}\left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{+} + \sum_{k\in\Xi_{\mathrm{D}}}\left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{-}\Big)\lambda^{\mathrm{B}} \\
&\qquad - \Big(\sum_{j\in\Xi_{\mathrm{U}}}\left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{-} + \sum_{k\in\Xi_{\mathrm{D}}}\left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{+}\Big)\lambda^{\mathrm{S}} \\
&\qquad \le \Big(\sum_{j\in\Xi_{\mathrm{U}}}\left[P_{\mathrm{U}j}^{\mathrm{B}} - P_{\mathrm{U}j}^{\mathrm{S}}\right]^{+} + \sum_{k\in\Xi_{\mathrm{D}}}\left[P_{\mathrm{D}k}^{\mathrm{B}} - P_{\mathrm{D}k}^{\mathrm{S}}\right]^{-}\Big)\lambda^{\mathrm{B}}
\end{aligned}$$

$$- \Big(\sum_{j \in \Xi_{\mathrm{U}}} \big[P^{\mathrm{B}}_{\mathrm{U}j} - P^{\mathrm{S}}_{\mathrm{U}j}\big]^{-} + \sum_{k \in \Xi_{\mathrm{D}}} \big[P^{\mathrm{B}}_{\mathrm{D}k} - P^{\mathrm{S}}_{\mathrm{D}k}\big]^{+} \Big) \lambda^{\mathrm{B}} = \big(P^{\mathrm{S}}_{\mathrm{N}} - P^{\mathrm{B}}_{\mathrm{N}}\big) \lambda^{\mathrm{B}}, \tag{4.60}$$

where the inequality is due to the fact that $\lambda^{\mathrm{B}} \leq \lambda^{\mathrm{S}}$ when the system is in surplus of production. Since the term on the right hand side of (4.60) is equal to the payment received from the balancing generators (4.55), it is clear that the market operator realizes a surplus whenever either one stochastic generator produces less than its day-ahead dispatch, or one load consumes more than planned.

4.3.2.1 Payments and Revenues

Payments and revenues in the case of excess production can be formulated as follows.

1. Balancing generator i receives

$$P^{\mathrm{S}}_{\mathrm{B}i} \lambda^{\mathrm{S}} + \big(P^{\mathrm{B}}_{\mathrm{B}i} - P^{\mathrm{S}}_{\mathrm{B}i}\big) \lambda^{\mathrm{B}}. \tag{4.61}$$

2. For stochastic generator j, the revenues is

$$P^{\mathrm{S}}_{\mathrm{U}j} \lambda^{\mathrm{S}} + \big[P^{\mathrm{B}}_{\mathrm{U}j} - P^{\mathrm{S}}_{\mathrm{U}j}\big]^{+} \lambda^{\mathrm{B}} - \big[P^{\mathrm{B}}_{\mathrm{U}j} - P^{\mathrm{S}}_{\mathrm{U}j}\big]^{-} \lambda^{\mathrm{S}}. \tag{4.62}$$

3. Demand k pays the following amount

$$P^{\mathrm{S}}_{\mathrm{D}k} \lambda^{\mathrm{S}} + \big[P^{\mathrm{B}}_{\mathrm{D}k} - P^{\mathrm{S}}_{\mathrm{D}k}\big]^{+} \lambda^{\mathrm{S}} - \big[P^{\mathrm{B}}_{\mathrm{D}k} - P^{\mathrm{S}}_{\mathrm{D}k}\big]^{-} \lambda^{\mathrm{B}}. \tag{4.63}$$

Following the same reasoning as in Sect. 4.3.1.1, it follows that there is always perfect balance between payments and revenues for the market operator at the day-ahead market stage. Furthermore, as shown in the previous section (see (4.60)), the balancing market, and therefore the aggregation of the two market floors, is always revenue adequate in the case of excess production, with the possibility that the market operator realizes a surplus.

In conclusion, a balancing market with two-price settlement is revenue adequate.

Example 4.1 is revisited below employing a two-price system for settling imbalances in the balancing market.

Example 4.3 (Balancing auction with excess consumption in a two-price market) Let us consider the day-ahead dispatch, the realizations of production and consumption and the regulation bids of Example 4.1, described in Tables 4.1, 4.2 and 4.3, respectively. Since the balancing market auction (4.9) is still valid in a two-price system, the clearing of the balancing market of Example 4.1, summarized in (4.35), (4.36) and (4.37), is still optimal.

Table 4.8 summarizes revenues and payments for all the market participants.

Clearly, the column regarding the day-ahead market is unchanged as compared to the one in Table 4.4. As far as payments and revenues in the balancing market are concerned, we notice that the situation changes with respect to Example 4.1

Table 4.8 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	0	1000
Unit B2	400	500	900
Unit B3	0	250	250
Unit U1	800	200	1000
Unit U2	600	– 500	100
Load D1	– 800	100	– 700
Load D2	– 2000	– 1000	– 3000

Table 4.9 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	– 64	936
Unit B2	400	– 80	320
Unit B3	0	0	0
Unit U1	800	200	1000
Unit U2	600	– 100	500
Load D1	– 800	144	– 656
Load D2	– 2000	– 400	– 2400

only for the participants that are involuntarily restoring the system balance with their deviation. Indeed, producer U1, which is producing 10 MWh above its schedule, and load D1, which is consuming 5 MWh less than scheduled, are now only paid λ^{S} = \$20 /MWh for their deviation. This way, their revenues at the balancing market reduce to \$200 and \$100 from \$500 and \$250, respectively. Contrarily, revenues for the balancing generators (B1, B2, and B3) and payments for the participants that are increasing the total system deviations (U2 and D2) are the same as in Table 4.4 for the one-price market. Finally, we underline that the net balance of payments for the market operator is not 0 at the balancing stage. Indeed, while \$1500 are collected from U2 and D2, only \$1050 are paid to B2, B3, U1, and D1, resulting in a net profit of \$450.

Next, we revisit Example 4.2 employing the two-price system for settling imbalances in the balancing market.

Example 4.4 (Balancing auction with excess production in a two-price market) The day-ahead dispatch, the realization of production and consumption and the down-regulation bids of Example 4.2, included in Tables 4.1, 4.5, and 4.6, are considered again. Once more, we only need to consider revenues and payments, since the optimal market clearing is the same as in the one-price market case of Example 4.2, i.e., the quantities and prices in (4.40), (4.41), and (4.42).

Revenues and payments are summarized in Table 4.9.

Once again, the only difference with respect to Table 4.7 regards the balancing market results for the participants that are involuntarily decreasing the overall system imbalance. Indeed, generator U2, which is producing less than scheduled in the day-ahead market, and load D2, which is consuming more than planned, buy energy from the balancing market at the price λ^{S} = \$20 /MWh instead of at λ^{B} = \$8 /MWh. For this reason, their payment at the balancing market increase to \$100 and \$400 from \$40 and \$160, respectively. Again, the reader can notice that there is a net profit for the market operator at the balancing stage. Indeed, it receives in total \$644 from B1, B2, U2, and D2, but pays only \$344 to U1 and D1 and, therefore, realizes a net profit of \$300.

4.4 Balancing Auction with Proactive Demand

In this section, the balancing auction mechanism for a single-price market, introduced in Sect. 4.2, is extended to consider flexibility on the demand side.

Proactive demands counteract the net deviation and achieve a profit in doing so. In case of excess demand, a proactive demand sells back energy for a profit, and in doing so contributes to balancing the market and reduces the balancing price moving it toward the day-ahead clearing price, which benefits some balancing market participants.

On the contrary, in case of excess production a proactive demand buys energy at a lower price than the day-ahead price. This way, it contributes to balancing the market and as a result, to increasing the balancing price, which moves toward the day-ahead price. Again, this benefits some balancing market participants.

A balancing auction with proactive demand has the form

$$\underset{P^{\mathrm{U}}_{\mathrm{B}i}, P^{\mathrm{D}}_{\mathrm{B}i}, P^{\mathrm{U}}_{\mathrm{D}k}, P^{\mathrm{D}}_{\mathrm{D}k}}{\text{Min.}} \quad \sum_{i \in \Xi_{\mathrm{B}}} \lambda^{\mathrm{U}}_{\mathrm{B}i} P^{\mathrm{U}}_{\mathrm{B}i} - \lambda^{\mathrm{D}}_{\mathrm{B}i} P^{\mathrm{D}}_{\mathrm{B}i} + \sum_{k \in \Xi_{\mathrm{C}}} \lambda^{\mathrm{U}}_{\mathrm{D}k} P^{\mathrm{U}}_{\mathrm{D}k} - \lambda^{\mathrm{D}}_{\mathrm{D}k} P^{\mathrm{D}}_{\mathrm{D}k} \tag{4.64a}$$

$$\text{s.t.} \quad P^{\mathrm{S}}_{\mathrm{B}} + \sum_{i \in \Xi_{\mathrm{B}}} P^{\mathrm{U}}_{\mathrm{B}i} - P^{\mathrm{D}}_{\mathrm{B}i} = P^{\mathrm{B}}_{\mathrm{N}} + \sum_{k \in \Xi_{\mathrm{C}}} \left(P^{\mathrm{D}}_{\mathrm{D}k} - P^{\mathrm{U}}_{\mathrm{D}k} \right) : \quad \lambda^{\mathrm{B}}, \tag{4.64b}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{B}i} \le P^{\mathrm{U,max}}_{\mathrm{B}i}, \quad \forall i, \tag{4.64c}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B}i} \le P^{\mathrm{D,max}}_{\mathrm{B}i}, \quad \forall i, \tag{4.64d}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{D}k} \le P^{\mathrm{U,max}}_{\mathrm{D}k}, \quad k \in \Xi_{\mathrm{C}}, \tag{4.64e}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{D}k} \le P^{\mathrm{D,max}}_{\mathrm{D}k}, \quad k \in \Xi_{\mathrm{C}}, \tag{4.64f}$$

where

- $P^{\mathrm{U}}_{\mathrm{D}k}$ represents how much demand k is asked to decrease its consumption at the balancing stage. The superscript U stands for *up-regulation*, since demand k is effectively selling power when accepting to reduce its consumption.
- $P^{\mathrm{D}}_{\mathrm{D}k}$ represents how much demand k is asked to increase its consumption at the balancing stage. Because demand k is buying additional power, the superscript D stands for *down-regulation*.
- $\lambda^{\mathrm{U}}_{\mathrm{D}k}$ is the per unit price offered by demand k to reduce its consumption.
- $\lambda^{\mathrm{D}}_{\mathrm{D}k}$ is the per unit price offered by demand k to increase its consumption.
- $P^{\mathrm{U,max}}_{\mathrm{D}k}$ is the maximum consumption decrease from demand k.

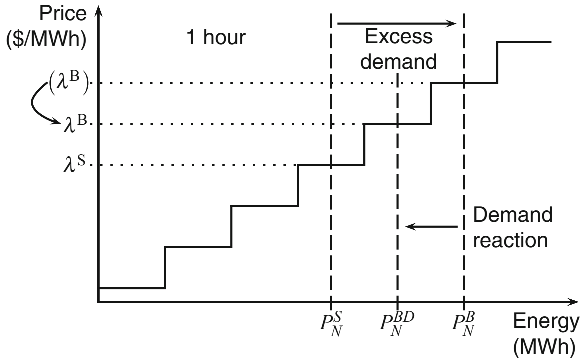


Fig. 4.5 Role of proactive demands in the case of excess consumption

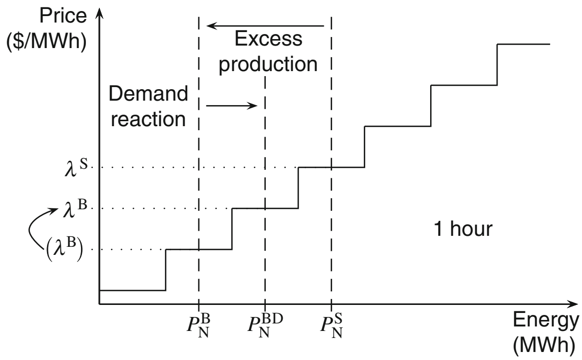


Fig. 4.6 Role of proactive demands in the case of excess production

$P_{Dk}^{D,max}$ is the maximum consumption increase from demand k.

Ξ_C is the set of indices of proactive demands.

The effect of proactive demands is illustrated in Figs. 4.5 and 4.6. Figure 4.5 depicts the action of proactive demands in a case of excess demand. Proactive demands reduce their respective consumptions. As a result, the net consumption including demand response (P_N^{BD}) and the balancing price drop.

Figure 4.6 refers to the excess production case. In that situation, proactive demands increase their respective consumptions. As a result, the net consumption including demand response (P_N^{BD}) and the balancing price rise.

The examples below revisit Examples 4.1 and 4.2, considering a simplified power system where all demands are proactive, and illustrate the functioning and characteristics of a balancing auction with proactive demands.

Example 4.5 (Balancing auction with excess consumption and proactive demands) We consider the same day-ahead dispatch as well as the realization of production and consumption of Example 4.1, included in Tables 4.1 and 4.2.

In addition to the bids submitted by the balancing generators shown in Table 4.3, bids are also submitted to the market by proactive demands. Table 4.10 shows the characteristics of these bids.

Table 4.10 Up-regulation bids of the proactive demands in the balancing market

Demand	λ^{U} (\$/MWh)	$P^{\mathrm{U,max}}$ (MWh)
D1	22	5
D2	24	5

Both demands are willing to deviate marginally from their scheduled consumption for a small profit, given by the price difference between their offer and the day-ahead market price $\lambda^{\mathrm{S}} = \$20\,/\mathrm{MWh}$.

Under these assumptions, the balancing auction (4.64) is the following

$$\min_{P^{\mathrm{U}}_{\mathrm{B2}},P^{\mathrm{U}}_{\mathrm{B3}},P^{\mathrm{U}}_{\mathrm{D1}},P^{\mathrm{U}}_{\mathrm{D2}}} \quad 30P^{\mathrm{U}}_{\mathrm{B2}} + 50P^{\mathrm{U}}_{\mathrm{B3}} + 22P^{\mathrm{U}}_{\mathrm{D1}} + 24P^{\mathrm{U}}_{\mathrm{D2}} \tag{4.65a}$$

$$\text{s.t.} \quad 70 + P^{\mathrm{U}}_{\mathrm{B2}} + P^{\mathrm{U}}_{\mathrm{B3}} = 85 - P^{\mathrm{U}}_{\mathrm{D1}} - P^{\mathrm{U}}_{\mathrm{D2}} \quad : \lambda^{\mathrm{B}}, \tag{4.65b}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{B2}} \le 10, \tag{4.65c}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{B3}} \le 50, \tag{4.65d}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{D1}} \le 5, \tag{4.65e}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{D2}} \le 5. \tag{4.65f}$$

The optimal solution is again evident. Indeed, reducing the consumption from the proactive demands is significantly cheaper than activating up-regulation bids from balancing generators. Thus, the optimal dispatch at the balancing market is the following

$$P^{\mathrm{U}}_{\mathrm{B2}} = 5\,\mathrm{MWh}, \tag{4.66}$$

$$P^{\mathrm{U}}_{\mathrm{D1}} = 5\,\mathrm{MWh}, \tag{4.67}$$

$$P^{\mathrm{U}}_{\mathrm{D2}} = 5\,\mathrm{MWh}. \tag{4.68}$$

The dual variable associated with (4.65b) is equal to the per unit cost of the marginal activated bid, i.e., the one from unit B2, and sets the balancing price

$$\lambda^{\mathrm{B}} = \$30\,/\mathrm{MWh}. \tag{4.69}$$

We underline that the price in (4.69) is significantly lower than the balancing price obtained in Example 4.1.

Clearly, since the balancing market clearing changes when demand response is introduced, revenues and payments are also different from the ones reported in Table 4.16. As Table 4.11 shows, the balancing generators obtain lower revenues in the balancing market than in Example 4.1. This is an effect both of the reduced balancing price and of the lower dispatch of the balancing units. Because the balancing price is lower, unit U1, overproducing with respect to its day-ahead schedule, receives a lower payment than in Example 4.1. On the contrary, underproducing unit U2 pays a lower amount to the market operator. Finally, as a result of their flexibility, loads D1 and D2 are rewarded with a higher revenue and a lower cost in the balancing market, respectively.

Table 4.11 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	0	1000
Unit B2	400	150	550
Unit B3	0	0	0
Unit U1	800	300	1100
Unit U2	600	– 300	300
Load D1	– 800	300	– 500
Load D2	– 2000	– 450	– 2450

Table 4.12 Down-regulation bids of the proactive demands in the balancing market

Demand	λ^{D} ($/MWh)	$P^{\mathrm{D,max}}$ (MWh)
D1	16	5
D2	19	10

Example 4.6 (Balancing auction with excess production and proactive demands) Let us again consider the day-ahead dispatch, and the realization of production and consumption of Example 4.2, summarized in Tables 4.1 and 4.5.

The loads submit the bids for down-regulation illustrated in Table 4.12.

The offers submitted by the dispatchable generators in the balancing market are the ones shown in Table 4.6.

Auction (4.64) writes as follows:

$$\min_{P^{\mathrm{D}}_{\mathrm{B1}},P^{\mathrm{D}}_{\mathrm{B2}},P^{\mathrm{D}}_{\mathrm{D1}},P^{\mathrm{D}}_{\mathrm{D2}}} \quad -8P^{\mathrm{D}}_{\mathrm{B1}} - 15P^{\mathrm{D}}_{\mathrm{B2}} - 16P^{\mathrm{D}}_{\mathrm{D1}} - 19P^{\mathrm{D}}_{\mathrm{D2}} \tag{4.70a}$$

$$\text{s.t.} \quad 70 - P^{\mathrm{D}}_{\mathrm{B1}} - P^{\mathrm{D}}_{\mathrm{B2}} = 52 + P^{\mathrm{D}}_{\mathrm{D1}} + P^{\mathrm{D}}_{\mathrm{D2}} \quad : \lambda^{\mathrm{B}}, \tag{4.70b}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B1}} \le 40, \tag{4.70c}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B2}} \le 10, \tag{4.70d}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{D1}} \le 5, \tag{4.70e}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{D2}} \le 10. \tag{4.70f}$$

Since the loads have the highest benefit in purchasing the additional power available at the balancing stage, their bids are fully accepted. The extra supply left is bought back by the dispatchable generator B2. The resulting dispatch is

$$P^{\mathrm{D}}_{\mathrm{B2}} = 3\,\mathrm{MWh}, \tag{4.71}$$

$$P^{\mathrm{D}}_{\mathrm{D1}} = 5\,\mathrm{MWh}, \tag{4.72}$$

$$P^{\mathrm{D}}_{\mathrm{D2}} = 10\,\mathrm{MWh}. \tag{4.73}$$

The balancing market price, i.e., the dual variable of (4.70b), is determined on the basis of the benefit of the marginal generator, i.e., unit B2, and equal to

$$\lambda^{\mathrm{B}} = \$15\,/\mathrm{MWh}. \tag{4.74}$$

Table 4.13 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	0	1000
Unit B2	400	– 45	355
Unit B3	0	0	0
Unit U1	800	375	1175
Unit U2	600	– 75	525
Load D1	– 800	195	– 605
Load D2	– 2000	– 450	– 2450

The reader should notice that this price is higher than the one in (4.42). This results from deploying demand response in a situation of excess production.

Revenues and payments are summarized in Table 4.13.

As a result of the increase in price for balancing power as compared to Example 4.2, sellers of energy, i.e., unit U1 and load D1, are less penalized when demand response is introduced.

4.5 Balancing Auction with Stepwise Offers

In the previous sections, for the sake of simplicity, producers were bound to submit a single price offer in the balancing market. This assumption is revisited in this section, where the auction model is extended so as to consider multiblock, stepwise offers from the producers.

Balancing generators offer generally a number of power blocks at increasing marginal offer costs, which represent the actual cost of energy production. This is recognized in the formulation of the balancing auction

$$\underset{P^{\mathrm{U}}_{\mathrm{B}i,s},P^{\mathrm{D}}_{\mathrm{B}i,s}}{\mathrm{Min.}} \quad \sum_{i\in\Xi_{\mathrm{B}}}\sum_{s\in\Xi_{\mathrm{B}i}} \lambda^{\mathrm{U}}_{\mathrm{B}i,s}P^{\mathrm{U}}_{\mathrm{B}i,s} - \lambda^{\mathrm{D}}_{\mathrm{B}i,s}P^{\mathrm{D}}_{\mathrm{B}i,s} \tag{4.75a}$$

$$\text{s.t.} \quad P^{\mathrm{S}}_{\mathrm{B}} + \sum_{i\in\Xi_{\mathrm{B}}}\sum_{s\in\Xi_{\mathrm{B}i}} P^{\mathrm{U}}_{\mathrm{B}i,s} - P^{\mathrm{U}}_{\mathrm{B}i,s} = P^{\mathrm{B}}_{\mathrm{N}} : \quad \lambda^{\mathrm{B}}, \tag{4.75b}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{B}i,s} \le P^{\mathrm{U,max}}_{\mathrm{B}i,s}, \quad \forall i,s, \tag{4.75c}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B}i,s} \le P^{\mathrm{D,max}}_{\mathrm{B}i,s}, \quad \forall i,s, \tag{4.75d}$$

where

$P^{\mathrm{U}}_{\mathrm{B}i,s}$ is the power increase from the level scheduled within block s by balancing generator i.

$P^{\mathrm{D}}_{\mathrm{B}i,s}$ is the power decrease from the level scheduled within block s by balancing generator i.

$\lambda^{\mathrm{U}}_{\mathrm{B}i,s}$ is the production cost offer for block s of balancing generator i in the balancing market.

Table 4.14 Up-regulation bids of the dispatchable generators in the balancing market

Unit	Block	λ^{U} (\$/MWh)	$P^{U,\max}$ (MWh)
B1	–	–	–
B2	1	25	5
	2	45	5
B3	1	40	15
	2	60	35

$\lambda^{D}_{Bi,s}$ is the price offer for block s of balancing generator i for repurchase of energy in the balancing market.

Ξ_{Bi} is the set of indices of the power blocks of balancing generator i.

The objective function (4.75a) includes two summations, one pertaining to the balancing generators and the other one to the blocks of each of them. Similarly, a double summation appears in the balancing Eq. (4.75b). Constraints (4.75c) and (4.75d) impose bounds on the production blocks of all balancing generators.

Example 4.1 is modified next in order to allow the submission of stepwise offers in the balancing market.

Example 4.7 (Balancing Auction with Excess Consumption and Stepwise Offers) Once again, the day-ahead dispatch and the realization of production and consumption are the same as in Example 4.1, illustrated in Tables 4.1 and 4.2.

The dispatchable generators submit the stepwise up-regulation offers described in Table 4.14 in the balancing market.

The auction resulting from model (4.75) is

$$\min_{P^{U}_{B2,1},P^{U}_{B2,2},P^{U}_{B3,1},P^{U}_{B3,2}} \quad 25P^{U}_{B2,1} + 45P^{U}_{B2,2} + 40P^{U}_{B3,1} + 60P^{U}_{B3,2} \tag{4.76a}$$

$$\text{s.t.} \quad 70 + P^{U}_{B2,1} + P^{U}_{B2,2} + P^{U}_{B3,1} + P^{U}_{B3,2} = 85 : \quad \lambda^{B}, \tag{4.76b}$$

$$0 \le P^{U}_{B2,1} \le 5, \tag{4.76c}$$

$$0 \le P^{U}_{B2,2} \le 5, \tag{4.76d}$$

$$0 \le P^{U}_{B3,1} \le 15, \tag{4.76e}$$

$$0 \le P^{U}_{B3,2} \le 35. \tag{4.76f}$$

We notice that the balancing market clearing differs from the one in the basic auction of Example 4.1. This is because the first block of unit B3 is cheaper than the second block of unit B2, which results in the following optimal dispatch:

$$P^{U}_{B2} = P^{U}_{B2,1} = 5\,\text{MWh}, \tag{4.77}$$

$$P^{U}_{B3} = P^{U}_{B3,1} = 10\,\text{MWh}. \tag{4.78}$$

Furthermore, the balancing market price is now

$$\lambda^{B} = \$40\,/\text{MWh}. \tag{4.79}$$

Table 4.15 Down-regulation bids of the dispatchable generators in the balancing market

Unit	Block	λ^{D} (\$/MWh)	$P^{\mathrm{D,max}}$ (MWh)
B1	1	12	10
	2	5	30
B2	1	18	5
	2	10	5
B3	–	–	–

Example 4.2 is revisited next, with the inclusion of stepwise offers from the producers participating in the balancing market.

Example 4.8 (Balancing Auction with Excess Production and Stepwise Offers) In this example, we reconsider the day-ahead dispatch and the realization of production and consumption of Example 4.2, shown in Tables 4.1 and 4.5, respectively.

The stepwise down-regulation bids received from the balancing generators are illustrated in Table 4.15.

Under these assumptions, auction (4.75) translates to

$$\underset{P^{\mathrm{D}}_{\mathrm{B1,1}},P^{\mathrm{D}}_{\mathrm{B1,2}},P^{\mathrm{D}}_{\mathrm{B2,1}},P^{\mathrm{D}}_{\mathrm{B2,2}}}{\text{Min.}} \quad -12P^{\mathrm{D}}_{\mathrm{B1,1}} - 5P^{\mathrm{D}}_{\mathrm{B1,2}} - 18P^{\mathrm{D}}_{\mathrm{B2,1}} - 10P^{\mathrm{D}}_{\mathrm{B2,2}} \tag{4.80a}$$

$$\text{s.t.} \quad 70 - P^{\mathrm{D}}_{\mathrm{B1,1}} - P^{\mathrm{D}}_{\mathrm{B1,2}} - P^{\mathrm{D}}_{\mathrm{B2,1}} - P^{\mathrm{D}}_{\mathrm{B2,2}} = 52 : \quad \lambda^{\mathrm{B}}, \tag{4.80b}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B1,1}} \le 10, \tag{4.80c}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B1,2}} \le 30, \tag{4.80d}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B2,1}} \le 5, \tag{4.80e}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B2,2}} \le 5. \tag{4.80f}$$

Since the first block of unit B1 has a higher marginal benefit than the second block of unit B2, the former has preference over the latter in the optimal balancing market dispatch, which is therefore the following:

$$P^{\mathrm{D}}_{\mathrm{B1}} = P^{\mathrm{D}}_{\mathrm{B1,1}} = 10\,\mathrm{MWh}, \tag{4.81}$$

$$P^{\mathrm{D}}_{\mathrm{B2}} = P^{\mathrm{D}}_{\mathrm{B2,1}} + P^{\mathrm{D}}_{\mathrm{B2,2}} = (5+3)\,\mathrm{MWh} = 8\,\mathrm{MWh}. \tag{4.82}$$

The balancing market price is determined from the price offer of the marginal block called in, i.e., the second block of unit B2

$$\lambda^{\mathrm{B}} = \$10\,/\mathrm{MWh}. \tag{4.83}$$

4.6 Network-Constrained Balancing Auction

For the sake of simplicity, the auction models presented in the previous sections assume that there is infinite transmission capacity, so that the location of generators and demands in the power grid has no influence on the optimal balancing market dispatch and on the resulting prices.

In this section, network constraints are considered in the balancing auction through a DC load flow representation. The output of this optimization model is the optimal dispatch of balancing power, which results in flows that satisfy the network capacity limits. Besides, it yields *Locational Marginal Prices* (LMPs), i.e., the marginal cost of electricity at each node. These are used to price electricity in balancing markets designed according to the *nodal pricing* rationale, e.g., the majority of power exchanges in the USA.

This section describes a network-constrained balancing auction and removes the unlimited transmission capacity assumption previously used.

For the sake of simplicity, we assume that at most one stochastic generator, one balancing generator, and one demand is located at each node of the transmission network. These generators and demands are identified by the node at which they are located.

A network-constrained balancing auction can be formulated as

$$\underset{P^{\mathrm{U}}_{\mathrm{B}n},P^{\mathrm{D}}_{\mathrm{B}n}}{\text{Min.}} \quad \sum_{n \in \Xi_{\mathrm{N}}} \lambda^{\mathrm{U}}_{\mathrm{B}n} P^{\mathrm{U}}_{\mathrm{B}n} - \lambda^{\mathrm{D}}_{\mathrm{B}n} P^{\mathrm{D}}_{\mathrm{B}n} \tag{4.84a}$$

$$\text{s.t.} \quad P^{\mathrm{S}}_{\mathrm{B}n} + P^{\mathrm{U}}_{\mathrm{B}n} - P^{\mathrm{D}}_{\mathrm{B}n} - P^{\mathrm{B}}_{\mathrm{N}n} = \sum_{m \in \Xi_{\mathrm{N}(n)}} B_{nm} (\delta_n - \delta_m) : \quad \lambda^{\mathrm{B}}_n, \quad \forall n, \tag{4.84b}$$

$$0 \le P^{\mathrm{U}}_{\mathrm{B}n} \le P^{\mathrm{U,max}}_{\mathrm{B}n}, \quad \forall n, \tag{4.84c}$$

$$0 \le P^{\mathrm{D}}_{\mathrm{B}n} \le P^{\mathrm{D,max}}_{\mathrm{B}n}, \quad \forall n, \tag{4.84d}$$

$$B_{nm} (\delta_n - \delta_m) \le C^{\max}_{nm}, \quad \forall n, m \in \Xi_{\mathrm{N}(n)}, \tag{4.84e}$$

$$- B_{nm} (\delta_n - \delta_m) \le C^{\max}_{nm}, \quad \forall n, m \in \Xi_{\mathrm{N}(n)}, \tag{4.84f}$$

$$\delta_1 = 0. \tag{4.84g}$$

where

- $P^{\mathrm{S}}_{\mathrm{B}n}$ is the production level of the balancing generator at node n as scheduled in the day-ahead market.
- $P^{\mathrm{U}}_{\mathrm{B}n}$ is the increase in production of the balancing generator at node n in the balancing market.
- $P^{\mathrm{D}}_{\mathrm{B}n}$ is the decrease in production of the balancing generator at node n in the balancing market.
- B_{nm} is the absolute value of susceptance (physical constant) of the interconnection between nodes n and m.
- δ_n is the voltage angle (state variable) at node n.
- $C^{\max}_{nm}$ is the capacity of the interconnection between nodes n and m.
- $P^{\mathrm{B}}_{\mathrm{N}n}$ is the net demand (demand minus stochastic production) at balancing time at bus n.
- Ξ_{N} is the set of nodes.
- $\Xi_{\mathrm{N}(n)}$ is the set of nodes adjacent to node n.

Objective function (4.84a) is the balancing cost. Equality constraints (4.84b) enforce energy balance per node of the transmission network and have associated dual variables $\lambda_n^{\mathrm{B}}, \forall n$, which are the LMPs. Constraints (4.84c) and (4.84d) enforce bounds on increments and decrements, respectively, of the production levels of the balancing generators. Inequalities (4.84e) and (4.84f) ensure that the power transfer between nodes n and m is below the capacity of the corresponding line. Finally, (4.84g) sets the first node as the reference bus with angle zero.

It is relevant to note that in case of network congestion, the payments made by demands exceed the revenues obtained by balancing generators, resulting in a merchandizing surplus to be administered by the market operator, e.g., for network improvement.

Example 4.1 is revised next considering a simple two-node network, where the power transfer capacity between the two buses is limited. The system is illustrated in Fig. 4.7. Balancing generators B1 and B2, stochastic unit U1 and demand D1 are located at node 1, while B3, U2, and D2 are located at node 2. The capacity of the interconnection between the two nodes is limited to 90 MW.

Example 4.9 (Balancing Auction with Excess Demand and Network Congestion) Let us consider again the day-ahead dispatch, the realization of production and consumption, and the up-regulation bids of Example 4.1, included in Tables 4.1, 4.2 and 4.3.

Denoting with $B = B_{12} = B_{21}$ the susceptance of the interconnection linking nodes 1 and 2, balancing auction (4.84) writes as

$$\min_{P_{\mathrm{B2}}^{\mathrm{U}}, P_{\mathrm{B3}}^{\mathrm{U}}} \quad 30 P_{\mathrm{B2}}^{\mathrm{U}} + 50 P_{\mathrm{B3}}^{\mathrm{U}} \tag{4.85a}$$

$$\text{s.t.} \quad 70 + P_{\mathrm{B2}}^{\mathrm{U}} + 15 = -B\delta_2 : \quad \lambda_1^{\mathrm{B}}, \tag{4.85b}$$

$$0 + P_{\mathrm{B3}}^{\mathrm{U}} - 100 = B\delta_2 : \quad \lambda_2^{\mathrm{B}}, \tag{4.85c}$$

$$0 \le P_{\mathrm{B2}}^{\mathrm{U}} \le 10, \tag{4.85d}$$

$$0 \le P_{\mathrm{B3}}^{\mathrm{U}} \le 50, \tag{4.85e}$$

$$-B\delta_2 \le 90, \tag{4.85f}$$

$$B\delta_2 \le 90. \tag{4.85g}$$

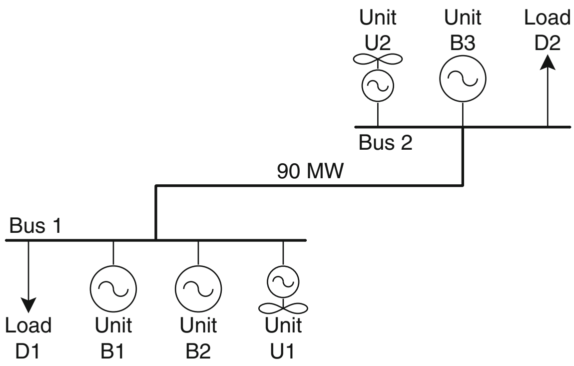
Since the capacity of the line is limited to 90 MW, generator B2 can only produce additionally 5 MWh at the balancing stage. The remaining excess of consumption, caused by the increase in load by D2 and the production decrease of U2, must be covered by the only dispatchable unit at node 2, i.e., B3. The optimal dispatch is then

$$P_{\mathrm{B2}}^{\mathrm{U}} = 5\,\mathrm{MWh}, \tag{4.86}$$

$$P_{\mathrm{B3}}^{\mathrm{U}} = 10\,\mathrm{MWh}. \tag{4.87}$$

Since the network is congested, the LMPs are different at the two nodes of the system. Indeed, the reader can notice that an increase in the right-hand side of (4.85b), i.e.,

Fig. 4.7 Two-bus system



an increase in the net demand at the balancing stage at node 1, could be balanced by an increase in production by dispatchable generator B2. The marginal increase of the system cost would then be the price bid by this generator, i.e., \$30 /MWh. Note that this is not possible when the net demand at node 2 is increased. Indeed, the upper bound to the power transfer from bus 1 to bus 2 ($-B\delta_2 \leq 90$) implies that any additional increase in demand at node 2 is to be balanced by the costlier generator B3, whose price bid is \$50 /MWh. The balancing prices at each node are therefore

$$\lambda_1^{\mathrm{B}} = \$30\,/\mathrm{MWh}, \tag{4.88}$$

$$\lambda_2^{\mathrm{B}} = \$50\,/\mathrm{MWh}. \tag{4.89}$$

Revenues and payments are summarized in Table 4.16.

Notice that λ_1^{B} applies to units B1, B2, U1, and load D1, while B3, U2, and D2 trade at price λ_2^{B}. The reader should also note that the market operator realizes a profit at the balancing stage. Indeed, it collects \$1500 from U2 and D2, but only pays \$1100 to B2, B3, U1, and D1.

The following example revisits Example 4.2 introducing network congestion.

Example 4.10 (Balancing Auction with Excess Production and Network Congestion) Let us consider one more time the day-ahead schedule and the realization of production and consumption of Example 4.2, summarized in Tables 4.1 and 4.5. As in the previous example, we consider a simplified power system with a two-bus network, depicted in Fig. 4.7.

In this example, we have to consider both the offers for up-regulation included in Table 4.3 and the ones for down-regulation summarized in Table 4.6.

In this case, auction (4.84) writes as

$$\underset{P_{\mathrm{B2}}^{\mathrm{U}},P_{\mathrm{B3}}^{\mathrm{U}},P_{\mathrm{B1}}^{\mathrm{D}},P_{\mathrm{B2}}^{\mathrm{D}}}{\text{Min.}} \quad 30P_{\mathrm{B2}}^{\mathrm{U}} + 50P_{\mathrm{B3}}^{\mathrm{U}} - 8P_{\mathrm{B1}}^{\mathrm{D}} - 15P_{\mathrm{B2}}^{\mathrm{D}} \tag{4.90a}$$

$$\text{s.t.} \quad 70 + P_{\mathrm{B2}}^{\mathrm{U}} - P_{\mathrm{B1}}^{\mathrm{D}} - P_{\mathrm{B2}}^{\mathrm{D}} + 43 = -B\delta_2 : \quad \lambda_1^{\mathrm{B}}, \tag{4.90b}$$

$$0 + P_{\mathrm{B3}}^{\mathrm{U}} - 95 = B\delta_2 : \quad \lambda_2^{\mathrm{B}}, \tag{4.90c}$$

Table 4.16 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	0	1000
Unit B2	400	150	550
Unit B3	0	500	500
Unit U1	800	300	1100
Unit U2	600	– 500	100
Load D1	– 800	150	– 650
Load D2	– 2000	– 1000	– 3000

$$0 \leq P^{\mathrm{U}}_{\mathrm{B2}} \leq 10, \tag{4.90d}$$

$$0 \leq P^{\mathrm{U}}_{\mathrm{B3}} \leq 50, \tag{4.90e}$$

$$0 \leq P^{\mathrm{D}}_{\mathrm{B1}} \leq 40, \tag{4.90f}$$

$$0 \leq P^{\mathrm{D}}_{\mathrm{B2}} \leq 10, \tag{4.90g}$$

$$-B\delta_2 \leq 90, \tag{4.90h}$$

$$B\delta_2 \leq 90. \tag{4.90i}$$

Despite the system as a whole has a production surplus of 18 MWh, this does not hold true at node 2, where the net demand at the balancing stage (95 MWh) is greater than the one in the day-ahead market (70 MWh). Furthermore, the interconnection capacity is not sufficiently high to allow the large power production at node 1 to cover the net demand at node 2 completely. Therefore, the market operator has to activate down-regulation at node 1

$$P^{\mathrm{D}}_{\mathrm{B1}} = 13\,\mathrm{MWh}, \tag{4.91}$$

$$P^{\mathrm{D}}_{\mathrm{B2}} = 10\,\mathrm{MWh}, \tag{4.92}$$

and up-regulation at node 2

$$P^{\mathrm{U}}_{\mathrm{B3}} = 5\,\mathrm{MWh}. \tag{4.93}$$

The LMPs at the balancing market spread significantly in this case. Indeed, it is easy to verify that

$$\lambda^{\mathrm{B}}_1 = \$8\,/\mathrm{MWh}, \tag{4.94}$$

$$\lambda^{\mathrm{B}}_2 = \$50\,/\mathrm{MWh}. \tag{4.95}$$

Payments and revenues are summarized in Table 4.17.

We emphasize again that the market operator realizes a profit in the balancing market. Indeed, it collects \$1434 from participants B1, B2, U2, and D2. On the other hand, the total payment to units B3, U1, and load D1 is only \$594.

Table 4.17 Revenues and payments for producers and loads at each market stage and in total

Participant	Payment from the market operator ($)		
	Day-ahead market	Balancing market	Total
Unit B1	1000	– 104	896
Unit B2	400	– 80	320
Unit B3	0	250	250
Unit U1	800	200	1000
Unit U2	600	– 250	350
Load D1	– 800	144	– 656
Load D2	– 2000	– 1000	– 3000

4.7 Relevant Worldwide Experiences

Some North American and European balancing markets are briefly described in this section. The two-price settlement rule is most widespread in Europe while American systems generally use a single-price settlement scheme.

4.7.1 The Americas

Balancing markets in North America are generally denominated *real-time* markets and are organized as follows:

1. Some time before energy delivery, a security-constrained dispatch is carried out with the most updated information available. This dispatch is a sophisticated version of auction (4.84). This is an ex ante dispatch as it takes place prior to energy delivery.
2. A state estimation algorithm [6] is used after each dispatch period (e.g., every 5 min) to identify the actual production level of each unit and the actual consumption level of each demand. This is an ex post calculation.
3. A pricing algorithm is used in turn to determine the price to be paid by each demand and to be received by each generator for each dispatch period. This price is similar to the marginal price derived from auction (4.84) but ex post not ex ante.

Details of specific implementations of this procedure in the cases of ISO New England, PJM, and Midwest ISO are available in [9], [11], and [7], respectively.

4.7.2 Europe

Balancing markets in Europe are organized in a variety of ways. They are loosely characterized by the following features:

1. The *zonal pricing* criterion is generally preferred to the nodal one. As opposed to the latter one, which potentially can assign a different price, i.e., the locational marginal price, at each node in the network (see Sect. 4.6), markets with zonal pricing are divided in a number of areas, across which the price is uniform.
2. An ex-post calculation is carried out to determine payments and revenues for the market participants based on their metered power injection to or withdrawal from the grid.
3. The prevailing imbalance settlement scheme is the two-price system. However, the German EEX and the Dutch APX electricity markets are two remarkable exceptions where the one-price system is adopted, see [5] and [14], respectively.
4. An important issue in European markets is how imbalances are determined. Indeed, differently from the American electricity markets where day-ahead schedules are determined unit-wise, payments and revenues in the balancing markets are calculated on the basis of the total net imbalance of several units associated with a single market participant. For example, mixed load and generation portfolios are allowed in the Iberian MIBEL market [12]. Contrarily, the imbalances for load and generation are accounted for separately in the Danish balancing market [4].

4.8 Balancing Prices

The statistical characterization of clearing prices is important in all markets and thus in balancing markets too.

This statistical characterization and the actual forecasting of balancing prices have an important complication emanating from the fact that deviations are either excess demand, leading to an increase in price with respect to the day-ahead price, or excess production, leading to a decrease in price. This above/below phenomenon that has an impact on clearing prices is complex to model if using time series models [2]. A relevant reference describing a methodology for balancing price forecasting is [10]. Finally, note that further statistical characterization of balancing prices is beyond the scope of this chapter.

4.9 Summary and Conclusions

The remarks pertaining to balancing markets are worth mentioning:

1. Balancing markets are needed to balance as close as possible production and consumption just before energy delivery. This is so because electric energy cannot be stored in large quantities.
2. Balancing markets play an important role in case of equipment failures (transmission lines and/or production facilities) and in case of high penetration of stochastic production facilities.

3. Balancing auctions are conceptually and mathematically simple, but very important in practice.
4. Balancing auctions are easily formulated as simple optimization problems (e.g., linear programming problems) that can be solved in a robust and efficient manner.
5. Characterizing the behavior of balancing prices through time series models is a challenging research task.

4.10 Further Reading

Some relevant references on balancing markets are [17], [10], [1], [13], and [15]. Reference [17] describes the functioning of the balancing market of ISO New England. References [1] and [15] describe balancing market implementation and price derivation algorithms. References [13] and [8] consider the impact of balancing market on the strategies of wind producers. Balancing markets become more and more important as renewable stochastic producers increasingly penetrate the electric energy systems. Finally, reference [10] pertains to price forecasting in balancing markets.

Exercises

4.1. Extend the formulation of auction (4.9) to include a minimum power output level per balancing generator. Note that the minimum power output pertains to the total production (day-ahead and balancing) of any given generating unit.

4.2. Illustrate using an example the model in the previous exercise.

4.3. Extend the formulation of auction (4.75) to include proactive demands and a minimum power output levels for balancing generators.

4.4. Illustrate using an example the model in the previous exercise.

4.5. Within a four-bus network, use an instance of auction (4.84) to illustrate how clearing prices can vary throughout the network.

4.6. Extend the formulation of auction (4.84) to include multiblock production offers, a minimum power output level per balancing generator and proactive demands.

4.7. Illustrate using an example the model in the previous exercise.

4.8. Show revenue adequacy (the market administrator collects a higher amount of money that the amount it has to distribute) for auction (4.75).

4.9. Show revenue adequacy for auction (4.84).

4.10. Are balancing prices more difficult to characterize statistically than day-ahead prices? If so, why?

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