# Chapter 3
# Clearing the Day-Ahead Market with a High Penetration of Stochastic Production

## 3.1 Electricity Markets: Day-Ahead Market

Electricity markets are trading floors that allow electricity producers, on the one hand, and electricity consumers and retailers, on the other hand, to trade electricity.

Two trading floor categories are available depending on the immediacy of the trading: long-term trading (from 1 week to 1 year ahead of energy delivery), which takes place through *futures markets* and via private *bilateral contracts*; and short-term trading (from several minutes to 1 day ahead of energy delivery), which takes place through the *electricity pool*.

Futures markets allow the arrangement of long-term electricity trading through forward contracts and options.

A forward contract involves the trading of a prespecified amount of power during a future time period, e.g., 10 MW during the next week. A forward contract involves a seller that produces the energy sold and a buyer that consumes such energy.

An option associated with a forward contract provides the buyer of the option with the possibility of deciding at some future time whether to implement the forward contract. Buying an option involves a payment from the buyer of the option.

For instance, a producer may buy an option to sell 10 MW during 3 days in 2-week time to be decided in 1-week time, and pays for such an option a fee to the consumer willing to provide the buying flexibility required by the option.

Conversely, a consumer may acquire an option to buy 10 MW during 3 days in 2-week time to be decided in 1-week time, and pays for such an option a fee to the producer willing to provide the selling flexibility required by the option.

The electricity pool allows short-term trading and generally involves two trading arrangements, the day-ahead market and the balancing market.

The day-ahead market takes place the day prior to energy delivery, typically around noon. Producers submit to this market production offers (consisting of production quantities and minimum selling prices), while consumers and retailers submit consumption bids (consisting of consumption quantities and maximum buying prices). In turn, the market operator clears the market using a market-clearing tool that is generally an auction. This auction results in scheduled production and consumption levels and day-ahead market-clearing prices.

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The balancing market takes place several minutes before energy delivery and constitutes the last market mechanism to balance production and consumption. This market is particularly relevant for stochastic producers (e.g., wind and solar power producers) that cannot accurately predict their production levels prior to the closing of the day-ahead market. The balancing market is cleared by the market operator in a similar fashion as the day-ahead market through an auction. Its outcome involves production and consumption adjustments and balancing clearing prices.

Some electricity pools include intermediate market arrangements between the day-ahead and the balancing markets, intended to further hedge against uncertainty and to allow corrective actions in response to unexpected events and errors by market agents. These trading arrangements are generally called adjustment or intra-day markets.

This chapter focuses on the day-ahead market and provides clearing models particularly suited for markets including a significant number of stochastic producers. More specifically, the remainder of this chapter is structured as follows. Section 3.2 first introduces the concept of reserve capacity as a market commodity to cope with uncertainty in power systems, then briefly describes basic models for the dispatch of energy and reserve in electricity markets, and finally presents the market-clearing mechanism based on two-stage stochastic programming with recourse. Subsequently, Sect. 3.3 uses this two-stage stochastic programming model to derive consistent clearing prices for an energy-only market settlement. Alternatively to two-stage stochastic programming, Sect. 3.4 introduces a dispatch method for energy and reserve capacity that is built on adaptive robust optimization. Section 3.5 summarizes the chapter, and Sect. 3.6 provides a collection of selected readings on the topic. Finally, proofs related to some properties of the resulting stochastic and robust market-clearing procedures are provided in appendices at the end of the chapter.

It is worth mentioning that this chapter places emphasis on advanced methods for clearing electricity markets with a high penetration of stochastic generation. It is therefore highly recommended for the reader unfamiliar with the functioning of these markets to first learn the basics from more introductory manuals such as [6] and [16].

## 3.2 Clearing the Day-Ahead Market Under Uncertainty

Electricity markets for short-term energy transactions usually comprise, at least, two different trading stages in the form of a day-ahead energy exchange and a balancing market. The day-ahead energy exchange takes place 1 day in advance and settles contracts for the delivery of energy on an hourly basis. The balancing market serves to competitively settle the energy adjustments required to ensure the constant balance between electricity supply and demand.

The coexistence of both markets is well-justified. On the one hand, the day-ahead market is useful for those power plants that need advance planning in order to efficiently and reliably adjust their production levels. If major changes in the overall supply were left to be driven by the balancing market, some generating units would be limited or just unable to respond to market signals. On the other hand, if market participants were able to perfectly predict with enough lead time the amount of energy that they will produce or consume, there would be no need for taking balancing actions. However, there are always imbalances in practice, especially in power systems with a high penetration of stochastic production. The balancing market constitutes thus a competitive mechanism to efficiently cope with these energy imbalances by allowing flexible firms to adjust their day-ahead positions. No doubt that the balancing market is, therefore, of primary importance for stochastic producers given the limited predictability of their power production.

This chapter focuses on the day-ahead market, while Chap. 4 focuses on the balancing market.

### *3.2.1 Cooptimizing Energy and Reserve Capacity*

In order to ensure that enough balancing resources are available during the real-time operation of the power system, the system operator allocates *reserve capacity* in advance. In practice, the procurement and *scheduling* of reserve capacity implies operating the system at less than its full capacity, while its use or *deployment* usually translates into the redispatch of units previously committed in the day-ahead market, the voluntary curtailment of loads, and/or the quick start-up of extra power plants to cover unexpected shortages of energy supply in real time.

There exist two schools of thought on how reserve should be traded in electricity markets. On the one hand, reserve capacity may be *sequentially* procured in a series of auctions run once the day-ahead energy dispatch has been determined. These auctions are organized to procure reserves with different activation times. The rationale behind this approach is that the free capacity that has not been successfully placed in one market can then be offered in the following auctions where the required activation time for the traded reserve is not as demanding. Consequently, reserve capacity offers that are successful in one market are not considered in the subsequent ones.

On the other hand, energy and reserve may be *simultaneously* procured in the same auction using a co-optimization algorithm that captures the strong coupling between the supply of energy and the provision of reserve capacity. The following illustrative example serves to get a more intuitive understanding of this coupling.

*Example 3.1 (Cooptimization of Energy and Reserve)* Consider an electricity market that solely includes two power producers, A and B. Each of these producers runs a power plant with a capacity of 100 MW. Producer A offers to sell energy at \$10/MWh, while producer B does it at \$30/MWh. A demand of 130 MWh is to be supplied.

Additionally, with the aim of dealing with unforseen events, the system operator estimates that 20 MW of reserve capacity are required. Producer A is willing to provide reserve at no cost, whereas producer B offers reserve capacity at \$25/MW.

To start with, let us suppose that energy and reserve capacity are *sequentially* settled in this order. Thus, the energy-only dispatch is first determined as follows:

$$\text{Min. } 10P_\text{A} + 30P_\text{B} \tag{3.1a}$$

$$\text{s.t. } P_\text{A} + P_\text{B} = 130, \tag{3.1b}$$

$$0 \le P_\text{A} \le 100, \tag{3.1c}$$

$$0 \le P_\text{B} \le 100, \tag{3.1d}$$

where $P_\text{A}$ and $P_\text{B}$ are the amounts of energy sold by producers A and B, respectively. Optimization problem (3.1) is trivial, and its solution is given by $P_\text{A}^* = 100$ MWh and $P_\text{B}^* = 30$ MWh. The clearing (marginal) price for energy, which is defined as the dual variable of constraint (3.1b), results in \$30/MWh.

Once the energy dispatch is determined, the reserve capacity market is cleared as follows:

$$\text{Min. } 0R_\text{A} + 25R_\text{B} \tag{3.2a}$$

$$\text{s.t. } R_\text{A} + R_\text{B} = 20, \tag{3.2b}$$

$$0 \le R_\text{A} \le 100 - P_\text{A}^*, \tag{3.2c}$$

$$0 \le R_\text{B} \le 100 - P_\text{B}^*, \tag{3.2d}$$

where $R_\text{A}$ and $R_\text{B}$ are the amounts of reserve capacity sold by producers A and B, respectively. Note that the reserve scheduling takes the energy dispatch $\left\{P_\text{A}^*, P_\text{B}^*\right\}$ as input. The solution to problem (3.2) is also trivial and is given by $R_\text{A}^* = 0$ and $R_\text{B}^* = 20$MW. That is, since producer A has been dispatched at full capacity in the energy market, reserve needs are entirely covered by producer B. Thus, the total system operation costs $\text{TC}^\text{seq}$, including both the procurement costs of energy and reserve capacity, are computed as

$$\begin{aligned} \text{TC}^\text{seq} &= 10P_\text{A}^* + 30P_\text{B}^* + 0R_\text{A}^* + 25R_\text{B}^* \\ &= 10 \times 100 + 30 \times 30 + 0 + 25 \times 20 = \$2400. \end{aligned} \tag{3.3}$$

The clearing (marginal) price for reserve capacity is \$25/MW, which is the value taken by the dual variable associated with the reserve requirement constraint (3.2b). Therefore, the profits made by producers A and B, respectively, under the sequential market organization are calculated as follows:

$$\text{Profit}_\text{A}^\text{seq} = (30 - 10)P_\text{A}^* + (25 - 0)R_\text{A}^* = 20 \times 100 + 25 \times 0 = \$2000, \tag{3.4a}$$

$$\text{Profit}_\text{B}^\text{seq} = (30 - 30)P_\text{B}^* + (25 - 25)R_\text{B}^* = 0. \tag{3.4b}$$

Let us now consider that energy and reserve capacity are *simultaneously* traded in the same auction. To this end, both commodities are jointly dispatched using optimization problem (3.5) below, which minimizes the total system operation costs.

$$
\begin{aligned}
\text{Min.} \quad & 10P_{\mathrm{A}} + 30P_{\mathrm{B}} + 0R_{\mathrm{A}} + 25R_{\mathrm{B}} && \text{(3.5a)}\\
\text{s.t.} \quad & P_{\mathrm{A}} + P_{\mathrm{B}} = 130, && \text{(3.5b)}\\
& R_{\mathrm{A}} + R_{\mathrm{B}} = 20, && \text{(3.5c)}\\
& P_{\mathrm{A}} + R_{\mathrm{A}} \leq 100, \quad P_{\mathrm{A}} \geq 0, \quad R_{\mathrm{A}} \geq 0, && \text{(3.5d)}\\
& P_{\mathrm{B}} + R_{\mathrm{B}} \leq 100, \quad P_{\mathrm{B}} \geq 0, \quad R_{\mathrm{B}} \geq 0. && \text{(3.5e)}
\end{aligned}
$$

The solution to this problem is $P_{\mathrm{A}}^{*} = 80$ MWh, $R_{\mathrm{A}}^{*} = 20$ MW, $P_{\mathrm{B}}^{*} = 50$ MWh, and $R_{\mathrm{B}}^{*} = 0$ MW. The total system operation costs in this case ($\mathrm{TC}^{\mathrm{sim}}$) are calculated as

$$
\begin{aligned}
\mathrm{TC}^{\mathrm{sim}} &= 10P_{\mathrm{A}}^{*} + 30P_{\mathrm{B}}^{*} + 0R_{\mathrm{A}}^{*} + 25R_{\mathrm{B}}^{*} \\
&= 10 \times 80 + 30 \times 50 + 0 \times 20 + 25 \times 0 = \$2300.
\end{aligned} \tag{3.6}
$$

Prices for energy and reserve capacity, defined as the dual variables of constraints (3.5b) and (3.5c), respectively, are \$30/MWh and \$20/MW in that order. Therefore, the profits made by producers A and B under the simultaneous market clearing of energy and reserve are given by

$$\mathrm{profit}_{\mathrm{A}}^{\mathrm{sim}} = (30 - 10)P_{\mathrm{A}}^{*} + (20 - 0)R_{\mathrm{A}}^{*} = 20 \times 80 + 20 \times 20 = \$2000, \tag{3.7a}$$

$$\mathrm{profit}_{\mathrm{B}}^{\mathrm{sim}} = (30 - 30)P_{\mathrm{B}}^{*} + (20 - 25)R_{\mathrm{B}}^{*} = 0 \times 50 - 5 \times 0 = 0, \tag{3.7b}$$

which turn out to be the same as the profits made by both producers in the sequential setup. However, the simultaneous dispatch of energy and reserve captures the coupling existing between these two commodities, thus reducing the total costs by \$100. Actually, in this illustrative example, the interaction between energy and reserve is inferred from the following results:

1. Producer A cannot sell as much energy as it might do otherwise. Indeed, this producer is committed to producing 80 MWh of energy, so that it can provide its spare capacity (20 MW) as reserve. Reserve requirements are thus satisfied.
2. On the contrary, producer B, which runs a more expensive power plant, has to produce more energy in order to meet the electricity demand.
3. The price for reserve capacity in the simultaneous arrangement (\$20/MW) does not correspond to any of the reserve offer costs submitted by the producers. It is, in fact, given by the difference between the marginal energy costs of producer B (\$30/MWh) and A (\$10/MWh). This is so because a 1-MW increase of the reserve needs in constraint (3.5c) is covered by producer A. To this end, this producer must decrease its energy production by 1 MWh, while producer B must increase it by the same amount. This action does not involve any additional reserve cost, but increases the cost of the energy dispatch by \$20.

### *3.2.2 Reserve Requirements*

In developed countries, electricity has become a necessity of everyday life, an asset essential for the functioning of society and economy. Being deprived of electricity may be thus extremely costly and troublesome for many consumers. Therefore, determining the amount of reserve capacity necessary to ensure a secure and efficient balancing operation in real time is paramount. Furthermore, the reserve determination must comply with market principles, i.e., the procurement cost of reserve should match the value it provides to power system users.

When estimating reserve capacity needs, two different approaches, namely *deterministic* or *probabilistic*, can be adopted. The deterministic approach often relies on rule-of-thumb criteria such as procuring enough reserve capacity to cover the loss of the largest generating unit (the so-called $\mathrm{N}-1$ security criterion), or to supply a percentage of the hourly demand, or even a combination of these two. These standards, nevertheless, ignore the stochastic nature of the factors that call for balancing energy, and consequently, reserve requirements are estimated independent of the magnitude of the uncertainties affecting the power system and their impact on system operation costs. On the other hand, in the probabilistic approach, reserve needs are determined based on a probabilistic description of these uncertainties. This approach, therefore, exploits concepts and methods from stochastic process theory, such as those presented in Appendix A of this book, to quantify the optimal amount of reserve capacity to be procured from a market perspective.

A natural way to compute reserve needs using a probabilistic approach is by means of the *expected load not served* (ELNS). The ELNS is a stochastic security metric that represents the average amount of energy not supplied as a result of load-shedding actions. It is cast as a weighted average energy value that accounts for the probability of uncertain factors and the damage that these factors cause to the system in the form of involuntarily curtailed load. Moreover, the ELNS can be expressed linearly, and hence, easily included within a market-clearing problem. Indeed, as illustrated in the example below, the ELNS allows determining reserve requirements *endogenously*, i.e., as a byproduct of the dispatch problem itself.

*Example 3.2 (Estimating Reserve Requirements)* Consider again the electricity market described in Example 3.1. Recall that this market is a duopoly made up of producers A and B, in which reserve requirements are estimated by the system operator at 20 MW. The reason for this estimate is that the electricity demand may increase from 130 MWh to 150 MWh without prior notice, and the system operator decides to protect the electrical infrastructure against this unexpected growth of consumption by scheduling 20 MW of reserve capacity in advance. The probability of this happening is, though, relatively small, specifically 0.05.

Let us now rethink this problem using a probabilistic approach. For this purpose, note that, in response to a sudden increase of load, three different balancing actions may be taken, namely:

1. Producer A may increase its production from $P_{\mathrm{A}}$ to $P_{\mathrm{A}}+r_{\mathrm{A}}$. The energy increase $r_{\mathrm{A}}$ is obtained from the reserve capacity $R_{\mathrm{A}}$ scheduled beforehand for this producer.
2. Similarly, producer B may increase its production from $P_{\mathrm{B}}$ to $P_{\mathrm{B}}+r_{\mathrm{B}}$. The energy increase $r_{\mathrm{B}}$ results from deploying the reserve capacity $R_{\mathrm{B}}$ dispatched beforehand for this producer.
3. A part of the load increase, $L^{\text{shed}}$, may be simply not supplied. This action, however, entails huge economic losses, which are estimated at \$1000/MWh.

Based on these three possible balancing measures, the energy-reserve dispatch problem can be reformulated as follows:

$$\text{Min.} \quad 10P_{\mathrm{A}} + 30P_{\mathrm{B}} + 0R_{\mathrm{A}} + 25R_{\mathrm{B}} + 0.05\left(10r_{\mathrm{A}} + 30r_{\mathrm{B}} + 1000\,L^{\text{shed}}\right) \tag{3.8a}$$

$$\text{s.t.} \quad r_{\mathrm{A}} + r_{\mathrm{B}} + L^{\text{shed}} = 20, \tag{3.8b}$$

$$r_{\mathrm{A}} \leq R_{\mathrm{A}}, \tag{3.8c}$$

$$r_{\mathrm{B}} \leq R_{\mathrm{B}}, \tag{3.8d}$$

$$P_{\mathrm{A}} + R_{\mathrm{A}} \leq 100, \tag{3.8e}$$

$$P_{\mathrm{B}} + R_{\mathrm{B}} \leq 100, \tag{3.8f}$$

$$L^{\text{shed}} \leq 20, \tag{3.8g}$$

$$P_{\mathrm{A}},\ P_{\mathrm{B}},\ R_{\mathrm{A}},\ R_{\mathrm{B}},\ r_{\mathrm{A}},\ r_{\mathrm{B}},\ L^{\text{shed}} \geq 0. \tag{3.8h}$$

The solution to this problem is $P_{\mathrm{A}}^{*} = 80$ MWh, $R_{\mathrm{A}}^{*} = 20$ MW, $P_{\mathrm{B}}^{*} = 50$ MWh, $R_{\mathrm{B}}^{*} = 0$ MW, $r_{\mathrm{A}}^{*} = 20$ MWh, $r_{\mathrm{B}}^{*} = 0$, and $L^{\text{shed}*} = 0$. Therefore, the energy and reserve capacity dispatches, i.e., $\left\{P_{\mathrm{A}}^{*}, P_{\mathrm{B}}^{*}\right\}$ and $\left\{R_{\mathrm{A}}^{*}, R_{\mathrm{B}}^{*}\right\}$, respectively, obtained from problem (3.8) are the same as those resulting from problem (3.5) in Example 3.1. This is just pure coincidence. Actually, dispatch models (3.5) and (3.8) are essentially different inasmuch as the following:

1. Market-clearing problem (3.8) takes into account explicitly both the probability of occurrence of the 20-MWh demand increase and its potential impact on system operation costs through the utilization of balancing resources. Indeed, the expression $0.05\left(10r_{\mathrm{A}} + 30r_{\mathrm{B}} + 1000\,L^{\text{shed}}\right)$ in (3.8a) represents the expected cost incurred at the balancing stage. This cost component is, in contrast, ignored in dispatch model (3.5).
2. The reserve dispatch yielded by market-clearing model (3.8) is directly determined based on how valuable this reserve is to consumers by including the cost of the expected load not served in objective function (3.8a), where this cost appears as $0.05 \times 1000 \times L^{\text{shed}}$. For the particular instance solved above, this cost is equal to zero, meaning that consumers are willing to pay for 20 MW of reserve capacity that can be deployed to satisfy a potential consumption increase, if needed. In contrast, if the probability of occurrence of the 20-MWh demand growth is small enough, say 0.005, or the value of lost load is sufficiently low,

e.g., \$100/MWh, no reserve capacity is dispatched, i.e., $\left\{R_{\mathrm{A}}^{*}, R_{\mathrm{B}}^{*}\right\} = \{0, 0\}$, and the whole demand increase is shed instead ($L^{\mathrm{shed}*} = 20$ MWh), if it comes to it.

3. While the 20-MW reserve requirement enters dispatch model (3.5) as an *input* in constraint (3.5c), reserve needs are an *outcome* of market-clearing model (3.8). In fact, there is no reserve requirement constraint in this problem. Instead, we enforce constraint (3.8b), in which all the variables involved, namely $r_{\mathrm{A}}$, $r_{\mathrm{B}}$, and $L^{\mathrm{shed}}$, represent balancing *energy* quantities. But if there is no such reserve requirement constraint, how do we determine the reserve capacity price? We will get to the answer of this question in due time.

### 3.2.3 A Two-Stage Stochastic Programming Approach

One of the main functions of the system operator is to ensure that enough reserve capacity is scheduled in advance so that a sufficient level of balancing resources are available in real time to cope with system uncertainties. In Example 3.1, we came to the conclusion that system operation costs are minimized if energy and reserve capacity are simultaneously dispatched in the day-ahead market, as the supply of energy and the provision of reserve capacity are complementary services. Subsequently, in Example 3.2, we showed that, by including the expectation of the balancing costs in the objective function of the dispatch problem, reserve needs are determined as a byproduct of the clearing process itself. This way, the amount of reserve capacity that is scheduled matches the value it provides to system users. Besides, the direct connection between reserve and system uncertainties bestows a *probabilistic* sense on this value.

The market-clearing model (3.8) in Example 3.2 is, in fact, a two-stage stochastic programming model, in which the *here-and-now* decisions make up the energy-reserve dispatch and the *wait-and-see* decisions correspond to the real-time operation. The reader is referred to Appendix C for a brief introduction to stochastic programming.

The objective function in (3.8) aims at minimizing the so-called *expected system operation costs*, which include both the cost related to the day-ahead energy-reserve dispatch and the expected cost of the *anticipated* balancing actions to be taken during the real-time operation of the power system. These costs are computed based on the energy and reserve capacity offers submitted by market participants to the day-ahead market.

This objective function is subject to three different sets of constraints, namely, the constraints involving energy and reserve capacity variables in the day-ahead dispatch; the equations constraining the utilization of balancing resources, some of which may involve day-ahead decision variables; and the constraints declaring the non-negative nature of energy- and reserve-related variables.

A generalization of such a market-clearing model is outlined below.

*Minimize* Day-ahead dispatch cost + Expected balancing cost

subject to
- Day-ahead market constraints:
  - Power balance equations at the day-ahead stage
  - Reserve capacity determination constraints
  - Bounds of reserve and energy offers
- Operation constraints:
  - Power balance equations at the balancing stage
  - Network constraints
  - Deployed reserve determination constraints
- Declarations of non-negative variables

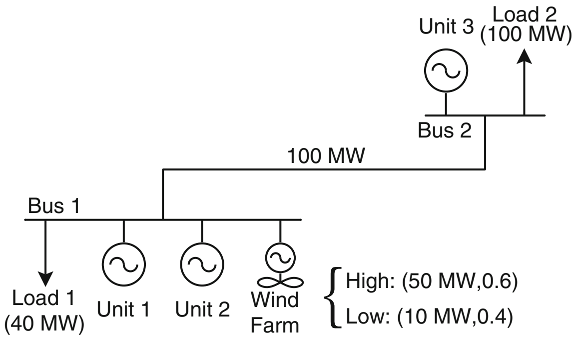
This approach, based on a two-stage stochastic programming model, naturally describes the interaction between the day-ahead and the real-time operation. In particular, the economic performance of the day-ahead energy-reserve dispatch is improved by implicitly accounting for its projected impact on the subsequent balancing costs. This way, enough flexible capacity is made available for balancing to efficiently cope with uncertain factors. The following illustrative example highlights the main features of this approach.

*Example 3.3 (A Two-Stage Stochastic Programming Approach)* Consider the two-node system in Fig. 3.1.

This small system includes three thermal units, two loads, and a 50-MW wind farm placed at bus 1. The single transmission line in the system has a per-unit reactance of 0.13. For a given time period in the future, the system operator must determine *here-and-now* both the energy dispatch and the reserve capacity needs. Naturally, reserve capacity is required to cope with the uncertain wind power production, which is represented via two scenarios, namely *high* (50 MW) and *low* (10 MW), with probability 0.6 and 0.4, respectively. Specifically, the sequence of decisions that the system operator has to face is as follows:

1. Determine the production levels of thermal units and the quantity and allocation of reserves to deal with the uncertain wind power production.
2. Deploy reserves in the form of balancing energy during the real-time operation of the power system to accommodate the actual realization of wind power production. Four different types of balancing actions can be undertaken for this purpose, namely:
   a) The power output of thermal unit $i$ can be *increased* from $P_i$ to $P_i + r_i^{\mathrm{U}}$, where $r_i^{\mathrm{U}}$ is the balancing energy obtained from the *upward* reserve capacity of unit $i$, denoted as $R_i^{\mathrm{U}}$. This action entails a cost given by $C_i r_i^{\mathrm{U}}$, where $C_i$ is the marginal production cost declared by unit $i$.
   b) Conversely, the power output of unit $i$ can be *decreased* from $P_i$ to $P_i - r_i^{\mathrm{D}}$, where $r_i^{\mathrm{D}}$ is the balancing energy resulting from the deployment of the

**Fig. 3.1** Two-bus system




*downward* reserve capacity of unit $i$, represented by $R_i^{\mathrm{D}}$. This action implies cost savings of $C_i r_i^{\mathrm{D}}$.

c) A part of the wind power production, $W^{\mathrm{spill}}$, can be curtailed (spilled). This action is cost free, as long as the marginal cost of wind energy production is considered to be zero.

d) A part of the load $j$, $L_j^{\mathrm{shed}}$, can be also curtailed. This action involves, though, the so-called value of lost load, $V_j^{\mathrm{LOL}}$, which is estimated for this small example at \$200/MWh.

These balancing actions may be taken in either of the two wind power scenarios considered. Subscripts $h$ and $l$ are used to indicate to which scenario, *high* or *low*, respectively, each balancing action refers to. For example, $W_h^{\mathrm{spill}}$ is the amount of wind power production that is curtailed in scenario *high*.

The market-clearing process is driven by the minimization of the expected system operation cost, which is made up of the energy-reserve dispatch costs plus the expected cost involved in the balancing actions. These costs are computed from the energy and reserve offers submitted by market agents to the electricity market. In this illustrative example, we assume that each thermal unit offers a single block of energy and up- and down-reserve capacity at prices $C$, $C^{\mathrm{RU}}$, and $C^{\mathrm{RD}}$, respectively. The value of these offer prices are shown in Table 3.1 together with the maximum power output, $P^{\max}$, of every unit $i$. These units are assumed to be fully dispatchable between 0 and $P^{\max}$.

*Objective Function:* The expected system operation cost (EC) is calculated as

$$
\begin{aligned}
\mathrm{EC} = {} & \underbrace{\underbrace{10P_1 + 30P_2 + 35P_3}_{\text{Day-ahead energy costs}} + \underbrace{16R_1^{\mathrm{U}} + 15R_1^{\mathrm{D}} + 13R_2^{\mathrm{U}} + 12R_2^{\mathrm{D}} + 10R^{\mathrm{U}}{}_3 + 9R^{\mathrm{D}}{}_3}_{\text{Reserve capacity costs}}}_{\text{Day-ahead dispatch costs}} \\
& + 0.6\underbrace{\left[10\left(r_{1h}^{\mathrm{U}} - r_{1h}^{\mathrm{D}}\right) + 30\left(r_{2h}^{\mathrm{U}} - r_{2h}^{\mathrm{D}}\right) + 35\left(r_{3h}^{\mathrm{U}} - r_{3h}^{\mathrm{D}}\right) + 200\left(L_{1h}^{\mathrm{shed}} + L_{2h}^{\mathrm{shed}}\right)\right]}_{\text{Balancing costs in scenario } high} \\
& + 0.4\underbrace{\left[10\left(r_{1l}^{\mathrm{U}} - r_{1l}^{\mathrm{D}}\right) + 30\left(r_{2l}^{\mathrm{U}} - r_{2l}^{\mathrm{D}}\right) + 35\left(r_{3l}^{\mathrm{U}} - r_{3l}^{\mathrm{D}}\right) + 200\left(L_{1l}^{\mathrm{shed}} + L_{2l}^{\mathrm{shed}}\right)\right]}_{\text{Balancing costs in scenario } low}.
\end{aligned}
$$

**Table 3.1** Unit data

| Unit $i$ | 1 | 2 | 3 |
|---|---|---|---|
| $P^{\max}$ (MW) | 50 | 110 | 100 |
| $C$ ($/MWh) | 10 | 30 | 35 |
| $C^{\mathrm{RU}}$ ($/MW) | 16 | 13 | 10 |
| $C^{\mathrm{RD}}$ ($/MW) | 15 | 12 | 9 |

*Day-Ahead Market Constraints:* The day-ahead energy dispatch must satisfy the power balance equations, i.e.,

$$P_1 + P_2 + W^{\mathrm{S}} - 40 = \frac{(\delta_1^0 - \delta_2^0)}{0.13} \qquad \text{(bus 1)},$$

$$P_3 - 100 = \frac{(\delta_2^0 - \delta_1^0)}{0.13} \qquad \text{(bus 2)},$$

where $W^{\mathrm{S}}$ is the amount of wind power production scheduled in the day-ahead market. We define bus 1 as the reference node by setting $\delta_1^0$ to 0. The power flow resulting from the day-ahead energy dispatch must satisfy the transmission capacity limits, i.e.,

$$\frac{(\delta_1^0 - \delta_2^0)}{0.13} \leq 100, \tag{3.9a}$$

$$\frac{(\delta_2^0 - \delta_1^0)}{0.13} \leq 100. \tag{3.9b}$$

Furthermore, energy and reserve capacity are mutually exclusive. Therefore, it holds

$$P_1 + R_1^{\mathrm{U}} \leq 50, \tag{3.10a}$$

$$P_1 - R_1^{\mathrm{D}} \geq 0, \tag{3.10b}$$

$$P_2 + R_2^{\mathrm{U}} \leq 110, \tag{3.10c}$$

$$P_2 - R_2^{\mathrm{D}} \geq 0, \tag{3.10d}$$

$$P_3 + R_3^{\mathrm{U}} \leq 100, \tag{3.10e}$$

$$P_3 - R_3^{\mathrm{D}} \geq 0. \tag{3.10f}$$

*Operation Constraints:* Now we focus on the balancing market stage. Needless to say, balancing actions must ensure the real-time balance between supply and demand under each possible scenario, i.e.,

$$r_{1h}^{\mathrm{U}} + r_{2h}^{\mathrm{U}} - r_{1h}^{\mathrm{D}} - r_{2h}^{\mathrm{D}} + L_{1h}^{\mathrm{shed}} + 50 - W^{\mathrm{S}} - W_h^{\mathrm{spill}} = \frac{(\delta_{1h} - \delta_1^0 + \delta_2^0 - \delta_{2h})}{0.13} \quad \text{(bus 1)},$$

$$r_{1l}^{\mathrm{U}}+r_{2l}^{\mathrm{U}}-r_{1l}^{\mathrm{D}}-r_{2l}^{\mathrm{D}}+L_{1l}^{\mathrm{shed}}+10-W^{\mathrm{S}}-W_{l}^{\mathrm{spill}}=\frac{(\delta_{1l}-\delta_{1}^{0}+\delta_{2}^{0}-\delta_{2l})}{0.13} \quad \text{(bus 1)},$$

$$r_{3h}^{\mathrm{U}}-r_{3h}^{\mathrm{D}}+L_{2h}^{\mathrm{shed}}=\frac{(\delta_{2h}-\delta_{2}^{0}+\delta_{1}^{0}-\delta_{1h})}{0.13} \quad \text{(bus 2)},$$

$$r_{3l}^{\mathrm{U}}-r_{3l}^{\mathrm{D}}+L_{2l}^{\mathrm{shed}}=\frac{(\delta_{2l}-\delta_{2}^{0}+\delta_{1}^{0}-\delta_{1l})}{0.13} \quad \text{(bus 2)}.$$

We also consider bus 1 as the reference node in the balancing stage by setting $\delta_{1h}=\delta_{1l}=0$. Due to the implementation of balancing actions, the power flowing between buses 1 and 2 is altered. The new power flow must also satisfy the transmission capacity limits. This is stated as follows:

$$\frac{(\delta_{1h}-\delta_{2h})}{0.13}\leq 100, \qquad \frac{(\delta_{2h}-\delta_{1h})}{0.13}\leq 100 \qquad \text{(scenario } high\text{)},$$

$$\frac{(\delta_{1l}-\delta_{2l})}{0.13}\leq 100, \qquad \frac{(\delta_{2l}-\delta_{1l})}{0.13}\leq 100 \qquad \text{(scenario } low\text{)}.$$

Clearly, the amount of wind power production that is curtailed under each scenario must be lower than or equal to the actual wind power output, i.e.,

$$W_{h}^{\mathrm{spill}}\leq 50 \quad \text{(scenario } high\text{)},$$

$$W_{l}^{\mathrm{spill}}\leq 10 \quad \text{(scenario } low\text{)}.$$

Similarly, the amount of load that is shed in each scenario has to be lower than or equal to the actual consumption value,

$$L_{1h}^{\mathrm{shed}}\leq 40, \qquad L_{1l}^{\mathrm{shed}}\leq 40 \qquad \text{(load 1)},$$

$$L_{2h}^{\mathrm{shed}}\leq 100, \qquad L_{2l}^{\mathrm{shed}}\leq 100 \qquad \text{(load 2)}.$$

The balancing energy comes from the reserve capacity that has been previously scheduled in the day-ahead market. Consequently, we have

$$r_{1h}^{\mathrm{U}}\leq R_{1}^{\mathrm{U}}, \quad r_{1h}^{\mathrm{D}}\leq R_{1}^{\mathrm{D}}, \quad r_{2h}^{\mathrm{U}}\leq R_{2}^{\mathrm{U}}, \quad r_{2h}^{\mathrm{D}}\leq R_{2}^{\mathrm{D}}, \quad r_{3h}^{\mathrm{U}}\leq R_{3}^{\mathrm{U}}, \quad r_{3h}^{\mathrm{D}}\leq R_{3}^{\mathrm{D}},$$

$$r_{1l}^{\mathrm{U}}\leq R_{1}^{\mathrm{U}}, \quad r_{1l}^{\mathrm{D}}\leq R_{1}^{\mathrm{D}}, \quad r_{2l}^{\mathrm{U}}\leq R_{2}^{\mathrm{U}}, \quad r_{2l}^{\mathrm{D}}\leq R_{2}^{\mathrm{D}}, \quad r_{3l}^{\mathrm{U}}\leq R_{3}^{\mathrm{U}}, \quad r_{3l}^{\mathrm{D}}\leq R_{3}^{\mathrm{D}}.$$

*Declarations of Non-Negative Variables:* Lastly, reserve, production, and consumption quantities in both the day-ahead and balancing stages must be non-negative,

$$R_{1}^{\mathrm{U}}, R_{2}^{\mathrm{U}}, R_{3}^{\mathrm{U}}, R_{1}^{\mathrm{D}}, R_{2}^{\mathrm{D}}, R_{3}^{\mathrm{D}}, P_{1}, P_{2}, P_{3}, W^{\mathrm{S}}\geq 0 \qquad \text{(day-ahead stage)},$$

$$r_{1h}^{\mathrm{U}}, r_{2h}^{\mathrm{U}}, r_{3h}^{\mathrm{U}}, r_{1h}^{\mathrm{D}}, r_{2h}^{\mathrm{D}}, r_{3h}^{\mathrm{D}}, L_{1h}^{\mathrm{shed}}, L_{2h}^{\mathrm{shed}}, W_{h}^{\mathrm{spill}}\geq 0 \quad \text{(balancing stage, scenario } high\text{)},$$

$$r_{1l}^{\mathrm{U}}, r_{2l}^{\mathrm{U}}, r_{3l}^{\mathrm{U}}, r_{1l}^{\mathrm{D}}, r_{2l}^{\mathrm{D}}, r_{3l}^{\mathrm{D}}, L_{1l}^{\mathrm{shed}}, L_{2l}^{\mathrm{shed}}, W_{l}^{\mathrm{spill}}\geq 0 \qquad \text{(balancing stage, scenario } low\text{)}.$$

*Complete Model Formulation:* We compile below all these constraints to formulate the two-stage stochastic programming problem to be solved.

$$\begin{aligned}\text{Min. } & 10\mathrm{P}_1 + 30\mathrm{P}_2 + 35\mathrm{P}_3 + 16\mathrm{R}_1^{\mathrm{U}} + 15\mathrm{R}_1^{\mathrm{D}} + 13\mathrm{R}_2^{\mathrm{U}} + 12\mathrm{R}_2^{\mathrm{D}} + 10\mathrm{R}_3^{\mathrm{U}} + 9\mathrm{R}_3^{\mathrm{D}} \\ & + 0.6\big[10\left(r_{1h}^{\mathrm{U}} - r_{1h}^{\mathrm{D}}\right) + 30\left(r_{2h}^{\mathrm{U}} - r_{2h}^{\mathrm{D}}\right) + 35\left(r_{3h}^{\mathrm{U}} - r_{3h}^{\mathrm{D}}\right) + 200\left(L_{1h}^{\mathrm{shed}} + L_{2h}^{\mathrm{shed}}\right)\big] \\ & + 0.4\big[10\left(r_{1l}^{\mathrm{U}} - r_{1l}^{\mathrm{D}}\right) + 30\left(r_{2l}^{\mathrm{U}} - r_{2l}^{\mathrm{D}}\right) + 35\left(r_{3l}^{\mathrm{U}} - r_{3l}^{\mathrm{D}}\right) + 200\left(L_{1l}^{\mathrm{shed}} + L_{2l}^{\mathrm{shed}}\right)\big]\end{aligned} \tag{3.11a}$$

$$\text{s.t. } \mathrm{P}_1 + \mathrm{P}_2 + \mathrm{W}^{\mathrm{S}} - 40 = \frac{(\delta_1^0 - \delta_2^0)}{0.13}, \tag{3.11b}$$

$$P_3 - 100 = \frac{(\delta_2^0 - \delta_1^0)}{0.13}, \tag{3.11c}$$

$$\frac{(\delta_1^0 - \delta_2^0)}{0.13} \leq 100, \tag{3.11d}$$

$$\frac{(\delta_2^0 - \delta_1^0)}{0.13} \leq 100, \tag{3.11e}$$

$$P_1 + R_1^{\mathrm{U}} \leq 50, \tag{3.11f}$$

$$P_1 - R_1^{\mathrm{D}} \geq 0, \tag{3.11g}$$

$$P_2 + R_2^{\mathrm{U}} \leq 110, \tag{3.11h}$$

$$P_2 - R_2^{\mathrm{D}} \geq 0, \tag{3.11i}$$

$$P_3 + R_3^{\mathrm{U}} \leq 100, \tag{3.11j}$$

$$P_3 - R_3^{\mathrm{D}} \geq 0, \tag{3.11k}$$

$$r_{1h}^{\mathrm{U}} + r_{2h}^{\mathrm{U}} - r_{1h}^{\mathrm{D}} - r_{2h}^{\mathrm{D}} + L_{1h}^{\mathrm{shed}} + 50 - W^{\mathrm{S}} - W_h^{\mathrm{spill}} = \frac{(\delta_{1h} - \delta_1^0 + \delta_2^0 - \delta_{2h})}{0.13}, \tag{3.11l}$$

$$r_{1l}^{\mathrm{U}} + r_{2l}^{\mathrm{U}} - r_{1l}^{\mathrm{D}} - r_{2l}^{\mathrm{D}} + L_{1l}^{\mathrm{shed}} + 10 - W^{\mathrm{S}} - W_l^{\mathrm{spill}} = \frac{(\delta_{1l} - \delta_1^0 + \delta_2^0 - \delta_{2l})}{0.13}, \tag{3.11m}$$

$$r_{3h}^{\mathrm{U}} - r_{3h}^{\mathrm{D}} + L_{2h}^{\mathrm{shed}} = \frac{(\delta_{2h} - \delta_2^0 + \delta_1^0 - \delta_{1h})}{0.13}, \tag{3.11n}$$

$$r_{3l}^{\mathrm{U}} - r_{3l}^{\mathrm{D}} + L_{2l}^{\mathrm{shed}} = \frac{(\delta_{2l} - \delta_2^0 + \delta_1^0 - \delta_{1l})}{0.13}, \tag{3.11o}$$

$$\frac{(\delta_{1h} - \delta_{2h})}{0.13} \leq 100, \quad \frac{(\delta_{2h} - \delta_{1h})}{0.13} \leq 100, \tag{3.11p}$$

$$\frac{(\delta_{1l} - \delta_{2l})}{0.13} \leq 100, \quad \frac{(\delta_{2l} - \delta_{1l})}{0.13} \leq 100, \tag{3.11q}$$

$$\delta_1^0 = 0, \quad \delta_{1l} = 0, \quad \delta_{1h} = 0, \tag{3.11r}$$

$$r_{1h}^{\mathrm{U}} \leq R_1^{\mathrm{U}}, \;\; r_{1h}^{\mathrm{D}} \leq R_1^{\mathrm{D}}, \;\; r_{2h}^{\mathrm{U}} \leq R_2^{\mathrm{U}}, \;\; r_{2h}^{\mathrm{D}} \leq R_2^{\mathrm{D}}, \;\; r_{3h}^{\mathrm{U}} \leq R_3^{\mathrm{U}}, \;\; r_{3h}^{\mathrm{D}} \leq R_3^{\mathrm{D}}, \tag{3.11s}$$

$$r_{1l}^{\mathrm{U}} \leq R_1^{\mathrm{U}}, \;\; r_{1l}^{\mathrm{D}} \leq R_1^{\mathrm{D}}, \;\; r_{2l}^{\mathrm{U}} \leq R_2^{\mathrm{U}}, \;\; r_{2l}^{\mathrm{D}} \leq R_2^{\mathrm{D}}, \;\; r_{3l}^{\mathrm{U}} \leq R_3^{\mathrm{U}}, \;\; r_{3l}^{\mathrm{D}} \leq R_3^{\mathrm{D}}, \tag{3.11t}$$

$$W_h^{\mathrm{spill}} \leq 50, \quad W_l^{\mathrm{spill}} \leq 10, \tag{3.11u}$$

$$L_{1h}^{\mathrm{shed}} \leq 40, \quad L_{1l}^{\mathrm{shed}} \leq 40, \tag{3.11v}$$

$$L_{2h}^{\mathrm{shed}} \leq 100, \quad L_{2l}^{\mathrm{shed}} \leq 100, \tag{3.11w}$$

$$R_1^{\mathrm{U}}, R_2^{\mathrm{U}}, R_3^{\mathrm{U}}, R_1^{\mathrm{D}}, R_2^{\mathrm{D}}, R_3^{\mathrm{D}}, P_1, \; P_2, \; P_3, \; W^{\mathrm{S}} \geq 0, \tag{3.11x}$$

$$r_{1h}^{\mathrm{U}}, \; r_{2h}^{\mathrm{U}}, \; r_{3h}^{\mathrm{U}}, \; r_{1h}^{\mathrm{D}}, \; r_{2h}^{\mathrm{D}}, \; r_{3h}^{\mathrm{D}}, \; L_{1h}^{\mathrm{shed}}, \; L_{2h}^{\mathrm{shed}}, \; W_h^{\mathrm{spill}} \geq 0, \tag{3.11y}$$

$$r_{1l}^{\mathrm{U}}, \; r_{2l}^{\mathrm{U}}, \; r_{3l}^{\mathrm{U}}, \; r_{1l}^{\mathrm{D}}, \; r_{2l}^{\mathrm{D}}, \; r_{3l}^{\mathrm{D}}, \; L_{1l}^{\mathrm{shed}}, \; L_{2l}^{\mathrm{shed}}, \; W_l^{\mathrm{spill}} \geq 0. \tag{3.11z}$$

We can use stochastic programming model (3.11) to assess the impact of the uncertain wind power production on the expected value of the total system operation cost. Table 3.2 includes a breakdown of this cost into energy production and reserve capacity costs. For ease of comparison, Fig. 3.2 provides a graphical illustration of this cost breakdown, which is calculated for four different cases, namely:

Case a) The future wind power production is perfectly known, and it coincides with its expected value, given by $0.6 \times 50\mathrm{MW} + 0.4 \times 10\mathrm{MW} = 34\mathrm{MW}$.

Case b) The wind farm is removed from the system in Fig. 3.1. Therefore, loads are exclusively supplied by the thermal units.

Case c) Wind power production is uncertain in keeping with the two-scenario representation indicated in the beginning of this illustrative example. Results in this case are directly obtained by solving optimization problem (3.11).

Case d) Wind power production is uncertain as in case (c), and the capacity of the single transmission line in the system is reduced from 100 to 40 MW.

By comparing case (a) or (c) with case (b), it becomes clear that wind generation leads to a significant reduction in the costs of energy production, since a substantial portion of the electricity demand, which otherwise would be covered by thermal generation, is satisfied by renewable and free energy instead. However, the expected cost in case (a) is significantly smaller than in case (c). This difference is largely driven by the cost of the reserve capacity required to cope with wind production uncertainty. Case (d) highlights the key role played by the network in the integration of stochastic production into power systems. If the capacity of the single line in the system is not high enough to make the most of the wind energy produced at bus 1 in all scenarios, a part of the wind power production is likely to be wasted and the two-bus power system will not fully benefit from the cost-free generation of the wind farm.

We conclude this section by generalizing the two-stage stochastic programming model introduced in the previous illustrative example. For this purpose, we define the following sets and indices:

**Table 3.2** Breakdown of the expected cost in dollars for four different cases. **a** No wind generation uncertainty. **b** No wind generation. **c** Uncertain wind generation. **d** Network congestion

| Case | a | b | c | d |
|---|---|---|---|---|
| Energy | 2180 | 3200 | 2260 | 2880 |
| Reserve | 0 | 0 | 360 | 180 |
| Total | 2180 | 3200 | 2620 | 3060 |

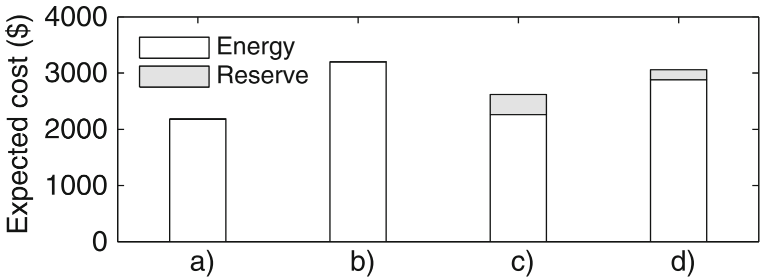



**Fig. 3.2** Illustration of the expected cost breakdown. **a** No wind generation uncertainty. **b** No wind generation. **c** Uncertain wind generation. **d** Network congestion

- $I$ Set of conventional production units.
- $J$ Set of loads.
- $Q$ Set of stochastic production units.
- $N$ Set of buses.
- $\Lambda$ Set of transmission lines.
- $\Omega$ Set of scenarios.
- $\Phi_n^I$ Set of conventional units located at bus $n$.
- $\Phi_n^J$ Set of loads located at bus $n$.
- $\Phi_n^Q$ Set of stochastic production units located at bus $n$.
- $e(\ell)$ Receiving-end bus of line $\ell$.
- $o(\ell)$ Sending-end bus of line $\ell$.

In addition, we build the general formulation on the following assumptions:

- A1 The day-ahead market is cleared using a single-period network-constrained auction. Therefore, inter-temporal constraints, such as ramping limits, are not included in the problem formulation. Hourly periods are considered, and thus, power and energy magnitudes, i.e., MW and MWh, are treated equivalently.
- A2 A DC model is used to account for the transmission network; see [10].
- A3 Electricity consumption is inelastic, with a large value of lost load. Thus, the maximization of the social welfare boils down to the minimization of the operating costs.
- A4 Supply cost functions are linear.
- A5 The uncertainty affecting the market-clearing process is assumed to be solely induced by stochastic producers.

A6 The uncertainty associated with the stochastic producers can be efficiently modeled through a finite set of outcomes or scenarios $\{(W_{q\omega}, \pi_\omega), \omega = 1, \ldots,$ and $card(\Omega)\}$, where $\{\pi_\omega, \forall \omega \in \Omega\}$ are their associated probabilities of occurrence and $card(\cdot)$ is a function that gives the cardinality of a set.

A7 Conventional units are considered to be fully dispatchable from zero to their maximum capacities.

Assumptions A1–A5 are mere simplifications for the purpose of rendering the subsequent model formulation easier to follow. Indeed, the day-ahead market model used in the following two-stage stochastic programming formulation can be extended to a multi-period setup that includes ramping constraints, a piecewise linear approximation of the supply cost functions, elastic demand, and other sources of uncertainty such as equipment failures and/or demand uncertainty. We refer the interested reader to Chap. 5 for further details on these extensions. Assumption A6 is typical in stochastic programming and is needed to cast the stochastic market-clearing formulation in a form manageable by optimization solvers, while exploiting the scenario generation tools presented in Chap. 2. Finally, assumption A7 will be actually needed in the following section, where we will use the two-stage stochastic programming model here introduced to price electricity in spot markets under uncertainty. This assumption allows us to sidestep the problem of pricing in markets with non-convexities, which is out of the scope of this book. For further information on this specific problem, the interested reader is referred to [13], [15] and references therein.

The two-stage stochastic programming model that results from the assumptions above is formulated as follows:

$$\begin{aligned}
\underset{\Xi}{\text{Min.}} \quad & \sum_{i\in I}\left(C_i P_i + C_i^{\text{RU}} R_i^{\text{U}} + C_i^{\text{RD}} R_i^{\text{D}}\right) \\
& + \sum_{q\in Q} C_q W_q^{\text{S}} + \sum_{\omega\in\Omega} \pi_\omega \left[\sum_{i\in I}\left(C_i^{\text{U}} r_{i\omega}^{\text{U}} - C_i^{\text{D}} r_{i\omega}^{\text{D}}\right)\right. \\
& \left. + \sum_{q\in Q} C_q \left(W_{q\omega} - W_q^{\text{S}} - W_{q\omega}^{\text{spill}}\right) + \sum_{j\in J} V_j^{\text{LOL}} L_{j\omega}^{\text{shed}}\right]
\end{aligned} \tag{3.12a}$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{i\in\Phi_n^I} P_i + \sum_{q\in\Phi_n^Q} W_q^{\text{S}} - \sum_{j\in\Phi_n^J} L_j - \sum_{\ell\in\Lambda|o(\ell)=n} b_\ell\left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0\right) \\
& + \sum_{\ell\in\Lambda|e(\ell)=n} b_\ell\left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0\right) = 0 : \lambda_n^{\text{D}}, \quad \forall n \in N,
\end{aligned} \tag{3.12b}$$

$$\begin{aligned}
& \sum_{i\in\Phi_n^I}\left(r_{i\omega}^{\text{U}} - r_{i\omega}^{\text{D}}\right) + \sum_{j\in\Phi_n^J} L_{j\omega}^{\text{shed}} + \sum_{q\in\Phi_n^Q}\left(W_{q\omega} - W_q^{\text{S}} - W_{q\omega}^{\text{spill}}\right) \\
& + \sum_{\ell\in\Lambda|o(\ell)=n} b_\ell\left(\delta_{o(\ell)}^0 - \delta_{o(\ell)\omega} - \delta_{e(\ell)}^0 + \delta_{e(\ell)\omega}\right)
\end{aligned}$$

$$- \sum_{\ell \in \Lambda | e(\ell)=n} b_\ell \left(\delta^0_{o(\ell)} - \delta_{o(\ell)\omega} - \delta^0_{e(\ell)} + \delta_{e(\ell)\omega}\right) = 0 \colon \gamma_{n\omega}, \quad \forall n \in N, \ \forall \omega \in \Omega, \tag{3.12c}$$

$$b_\ell \left(\delta^0_{o(\ell)} - \delta^0_{e(\ell)}\right) \le C^{\max}_\ell, \quad \forall \ell \in \Lambda, \tag{3.12d}$$

$$b_\ell \left(\delta^0_{e(\ell)} - \delta^0_{o(\ell)}\right) \le C^{\max}_\ell, \quad \forall \ell \in \Lambda, \tag{3.12e}$$

$$b_\ell \left(\delta_{o(\ell)\omega} - \delta_{e(\ell)\omega}\right) \le C^{\max}_\ell, \quad \forall \ell \in \Lambda, \ \forall \omega \in \Omega, \tag{3.12f}$$

$$b_\ell \left(\delta_{e(\ell)\omega} - \delta_{o(\ell)\omega}\right) \le C^{\max}_\ell, \quad \forall \ell \in \Lambda, \ \forall \omega \in \Omega, \tag{3.12g}$$

$$\delta^0_1 = 0, \tag{3.12h}$$

$$\delta_{1\omega} = 0, \quad \forall \omega \in \Omega, \tag{3.12i}$$

$$W^{\mathrm{S}}_q \le W^{\max}_q, \quad \forall q \in Q, \tag{3.12j}$$

$$P_i + R^{\mathrm{U}}_i \le P^{\max}_i, \quad \forall i \in I, \tag{3.12k}$$

$$P_i - R^{\mathrm{D}}_i \ge 0, \quad \forall i \in I, \tag{3.12l}$$

$$R^{\mathrm{U}}_i \le R^{\mathrm{U,max}}_i, \quad \forall i \in I, \tag{3.12m}$$

$$R^{\mathrm{D}}_i \le R^{\mathrm{D,max}}_i, \quad \forall i \in I, \tag{3.12n}$$

$$r^{\mathrm{U}}_{i\omega} \le R^{\mathrm{U}}_i, \quad \forall i \in I, \ \forall \omega \in \Omega, \tag{3.12o}$$

$$r^{\mathrm{D}}_{i\omega} \le R^{\mathrm{D}}_i, \quad \forall i \in I, \ \forall \omega \in \Omega, \tag{3.12p}$$

$$L^{\mathrm{shed}}_{j\omega} \le L_j, \quad \forall j \in J, \ \forall \omega \in \Omega, \tag{3.12q}$$

$$W^{\mathrm{spill}}_{q\omega} \le W_{q\omega}, \quad \forall q \in Q, \ \forall \omega \in \Omega, \tag{3.12r}$$

$$P_i, R^{\mathrm{U}}_i, R^{\mathrm{D}}_i \ge 0, \ \forall i \in I; \quad r^{\mathrm{U}}_{i\omega}, r^{\mathrm{D}}_{i\omega} \ge 0, \ \forall i \in I, \ \forall \omega \in \Omega; \ W^{\mathrm{S}}_q \ge 0, \ \forall q \in Q,$$
$$W^{\mathrm{spill}}_{q\omega} \ge 0, \ \forall q \in Q, \ \forall \omega \in \Omega; \ L^{\mathrm{shed}}_{j\omega} \ge 0, \ \forall j \in J, \ \forall \omega \in \Omega, \tag{3.12s}$$

where $\Xi = \{P_i, R^{\mathrm{U}}_i, R^{\mathrm{D}}_i, r^{\mathrm{U}}_{i\omega}, r^{\mathrm{D}}_{i\omega}, W^{\mathrm{S}}_q, \ W^{\mathrm{spill}}_{q\omega}, \ L^{\mathrm{shed}}_{j\omega}, \delta^0_n, \delta_{n\omega}, \forall i \in I, \forall q \in Q, \forall j \in J, \forall n \in N, \forall \omega \in \Omega\}$ is the set of decision variables.

The objective function (3.12a) to be minimized is the sum of the day-ahead energy-reserve dispatch cost and the expected balancing costs. We distinguish here between the *upward* and *downward* reserve capacity costs, represented by $C^{\mathrm{RU}}$ and $C^{\mathrm{RD}}$, respectively. We also make a distinction between the cost of the energy sold by conventional producers in the day-ahead market, denoted as $C$, and the costs of increasing/reducing generation by these producers at balancing time, represented by $C^{\mathrm{U}}$ and $C^{\mathrm{D}}$ in that order. Note that we have included the energy costs incurred by stochastic producers, calculated as

$$\sum_{q \in Q} C_q W^{\mathrm{S}}_q + \sum_{\omega \in \Omega} \pi_\omega \sum_{q \in Q} C_q \left(W_{q\omega} - W^{\mathrm{S}}_q - W^{\mathrm{spill}}_{q\omega}\right)$$

$$= \sum_{\omega \in \Omega} \pi_\omega \sum_{q \in Q} C_q \left( W_{q\omega} - W_{q\omega}^{\text{spill}} \right).$$

Furthermore, since the term $\sum_{\omega \in \Omega} \pi_\omega \sum_{q \in Q} C_q W_{q\omega}$ is constant, it can be removed from the objective function. Constraints (3.12b) and (3.12c) are power balance equations, with $b_\ell$ being the susceptance of line $\ell$. In particular, constraints (3.12b) enforce the power balance on the day-ahead energy dispatch, while constraints (3.12c) do so on the energy redispatch resulting from real-time balancing. The group of equations (3.12d)–(3.12g) enforces the transmission capacity limits. Equations (3.12h) and (3.12i) set, without loss of generality, bus 1 as the reference node. Constraints (3.12j) limit the power dispatched for each stochastic producer $q$ to its capacity, $W_q^{\max}$. Equations (3.12k) and (3.12l) model the physical coupling between energy and reserve capacity. Constraints (3.12m) and (3.12n) restrict the amount of upward and downward reserve capacity sold by each conventional producer $i$ to its reserve offer limits, $R^{\text{U,max}}$ and $R^{\text{D,max}}$, respectively. The amount of additional energy that each conventional producer $i$ produces for balancing in scenario $\omega$, i.e., $r_{i\omega}^{\text{U}}$, is obtained from its upward reserve capacity $R_i^{\text{U}}$. This is stated by Eq. (3.12o). Analogously, the amount of energy reduction that each conventional producer $i$ implements for balancing in each scenario $\omega$, i.e., $r_{i\omega}^{\text{D}}$, is obtained from its downward reserve capacity $R_i^{\text{D}}$. This is enforced through Eq. (3.12p). As already mentioned in Example 3.3, constraints (3.12q) and (3.12r) are commonsense bounds according to which the amount of load that is involuntarily shed and the amount of stochastic production that is curtailed are smaller than or equal to the actual demand value and the actual stochastic production, respectively. The set of constraints (3.12s) constitutes non-negative variable declarations.

Lastly, we point out that the family of dual variables $\{\lambda_n^{\text{D}}, \forall n \in N\}$ and $\{\gamma_{n\omega}, \forall n \in N, \forall \omega \in \Omega\}$ associated with the power balance equations (3.12b) and (3.12c), respectively, are explicitly indicated in optimization problem (3.12) after these equations, separated by a colon, because these dual variables will play a fundamental role in pricing electricity in spot markets that are cleared using a two-stage stochastic programming approach. This is indeed the subject matter of the following section.

## 3.3 Pricing Energy in the Day-Ahead Market Under Uncertainty

In the presence of a high penetration of stochastic production in electricity markets, balancing costs may become strongly dependent on the day-ahead dispatch. If this is such that insufficient flexible and competitive generation capacity is left to the balancing market, managing uncertainties during the real-time operation of the power system may become problematic and costly. The rationale behind the use of the two-stage stochastic programming approach (3.12) is to explicitly account for the potential impact of the day-ahead dispatch on the balancing operation with the aim of improving the overall system performance. The strong coupling between the day-ahead and balancing market stages does not only manifest itself through the system operation cost, but also through its dual counterpart: the *electricity price*.

In this section, we present a settlement scheme that supports the day-ahead dispatch given by the stochastic programming model (3.12) in an economic sense. In other words, we define a set of prices that make market participants satisfied with the day-ahead dispatch outcomes resulting from such a model.

### *3.3.1 Towards an Energy-Only Electricity Pricing*

Reserve capacity is purchased by the system operator prior to balancing time to guarantee that enough flexible generation will be available to deal with system uncertainties in real time. In essence, the need for reserve capacity is actually a need for balancing energy.

In those markets where energy and reserve capacity are traded as different commodities, the day-ahead energy dispatch is altered by the provision of reserve capacity. This alteration is financially supported by reserve capacity payments (see, for instance, [16; 17]). The dual variable associated with the constraint enforcing the procurement of demand for reserve may serve as the reserve capacity price, as we saw in Example 3.1. The demand for reserve is estimated by the system operator based on the need for energy at the balancing stage.

The stochastic programming model (3.12) does not include, however, a reserve requirement constraint. In this formulation, the day-ahead energy dispatch is determined by explicitly modeling the balancing operation as the second-stage or recourse problem. For this purpose, optimization problem (3.12) exploits the information submitted by market participants about their flexibility and willingness to supply balancing energy. Reserve requirements can be computed *ex post* as a byproduct of model (3.12), inasmuch as the provision of reserve boils down to pre-positioning the system in a way that balancing energy can be traded as anticipated. Therefore, in an electricity market cleared using the two-stage stochastic programming approach (3.12), reserve capacity does not need to be a commodity anymore.

Optimization problem (3.12) does include, on the other hand, two different sets of power balance equations, namely (3.12b) and (3.12c). The former are enforced in the day-ahead market, while the latter are imposed on the energy deployed at the balancing stage. The dual variables associated with these two group of constraints, $\{\lambda_n^{\mathrm{D}}, \forall n \in N\}$ and $\{\gamma_{n\omega}, \forall n \in N, \forall \omega \in \Omega\}$, respectively, are particularly meaningful from an economic point of view. Specifically,

- $\lambda_n^{\mathrm{D}}$ accounts for the impact on the expected system operation costs of a marginal increase in the *forecast* load at bus $n$. Therefore, to supply this foreseen marginal increase in load, inflexible units can be used with advance planning;
- $\gamma_{n\omega}$ accounts for the impact on the expected system operation costs of a marginal *uncertain* increase in the load at bus $n$ under scenario $\omega$. This marginal increase

in load, for being uncertain, cannot be supplied by inflexible units, as they cannot provide balancing energy. Besides, this impact is weighted by the probability of occurrence $\pi_\omega$ of scenario $\omega$.

Based on the economic interpretation of these dual variables, we build the following settlement scheme:

1. Each conventional producer $i$ located at bus $n$ is paid for its day-ahead energy dispatch $P_i$ at a price $\lambda_n^{\mathrm{D}}$.
2. Each consumer $j$ located at bus $n$ is charged for its scheduled energy consumption $L_j$ at a price $\lambda_n^{\mathrm{D}}$.
3. Each stochastic producer $q$ located at bus $n$ is paid for its day-ahead dispatch $W_q^{\mathrm{S}}$ at a price $\lambda_n^{\mathrm{D}}$.
4. Each conventional producer $i$ located at bus $n$ is paid for the additional energy $r_{i\omega}^{\mathrm{U}}$ required for balancing in scenario $\omega$ at a price $\frac{\gamma_{n\omega}}{\pi_\omega}$.
5. Each conventional producer $i$ located at bus $n$ is charged for the energy reduction $r_{i\omega}^{\mathrm{D}}$ required for balancing in scenario $\omega$ at a price $\frac{\gamma_{n\omega}}{\pi_\omega}$.
6. Each stochastic producer $q$ located at bus $n$ with production surplus in scenario $\omega$ is paid for its excess of generation $W_{q\omega} - W_q^{\mathrm{S}} - W_{q\omega}^{\mathrm{spill}}$ at a price $\frac{\gamma_{n\omega}}{\pi_\omega}$.
7. Each stochastic producer $q$ located at bus $n$ with generation shortage in scenario $\omega$ is charged for its production deficit $W_q^{\mathrm{S}} + W_{q\omega}^{\mathrm{spill}} - W_{q\omega}$ at a price $\frac{\gamma_{n\omega}}{\pi_\omega}$.
8. Each consumer $j$ located at node $n$ suffering from a load curtailment $L_{j\omega}^{\mathrm{shed}}$ in scenario $\omega$ is compensated for this curtailment at a price $\frac{\gamma_{n\omega}}{\pi_\omega}$.

If we now define

| | |
|---|---|
| $s(k)$ | Index of the bus where market participant $k$ is located; |
| $E_k$ | Energy sold, if positive, or energy purchased, if negative, by market participant $k$ in the day-ahead market; |
| $\Delta E_{k\omega}$ | Additional energy sold, if positive, or repurchased, if negative, by market participant $k$ for balancing in scenario $\omega$, |

the previous settlement scheme can be concisely cast as

$$\lambda_{s(k)}^{\mathrm{D}} E_k + \lambda_{s(k)\omega}^{\mathrm{B}} \Delta E_{k\omega}, \tag{3.13}$$

where $\lambda_{s(k)\omega}^{\mathrm{B}} = \frac{\gamma_{s(k)\omega}}{\pi_\omega}$. In fact, $\lambda_{s(k)\omega}^{\mathrm{B}}$ is a prediction of the balancing market price in scenario $\omega$ and can be alternatively computed by solving the recourse stage of the stochastic programming model (3.12) with the day-ahead dispatch variables fixed to their optimal values, for the specific realization $\omega$ of the uncertain parameters.

Observe that the settlement scheme (3.13) is solely based on energy payments, and consequently, it leads to what we call an *energy-only electricity market*. Under such a market settlement, one may question the actual meaning of the reserve capacity costs $\sum_{i \in I} C_i^{\mathrm{RU}} R_i^{\mathrm{U}} + C_i^{\mathrm{RD}} R_i^{\mathrm{D}}$ included in the objective function (3.12a). In principle, the provision of reserve capacity implies no extra cost to the electricity generation process other than the cost of its actual deployment in the form of balancing energy, which is already counted in (3.12a). Reserve capacity costs may be, nevertheless, justified

**Table 3.3** Two possible sets of market outcomes for the two-bus system in Fig. 3.1. Powers in MW

| | Unit | $P_i$ | $R_i^{\mathrm{U}}$ | $R_i^{\mathrm{D}}$ | $r_{i\omega}^{\mathrm{U}}$ | | $r_{i\omega}^{\mathrm{D}}$ | |
|---|---|---|---|---|---|---|---|---|
| | | | | | High | Low | High | Low |
| (a) Solution A ($W^{\mathrm{S}} = 10$) | 1 | 50 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 2 | 80 | 0 | 40 | 0 | 0 | 40 | 0 |
| | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| (b) Solution B ($W^{\mathrm{S}} = 50$) | 1 | 50 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 2 | 40 | 40 | 0 | 0 | 40 | 0 | 0 |
| | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

for the following reason. The clearing process (3.12) allows for the possibility of withdrawing some flexible capacity from the day-ahead market to have it available at the balancing stage. This action may potentially increase the risk exposure of flexible agents inasmuch as the capacity placed in the day-ahead market brings them *certain* profits, while the capacity committed to beforehand in the balancing market yields *uncertain* returns, the actual value of which depends on the eventual outcome of uncertainties. In this sense, reserve capacity costs provide an extra value to the flexible capacity that is allocated, *in advance*, to the balancing market.

A different way to increase the value of the energy for balancing, more aligned with an energy-only electricity market, is to impose a *price premium* on the electricity traded in the balancing market. In practice, this means that $C_i^{\mathrm{U}} > C_i$ and $C_i^{\mathrm{D}} < C_i$ in the objective function (3.12a) of the two-stage stochastic programming model (3.12). From a mathematical point of view, this price premium, or the aforementioned reserve capacity costs, is required for the market-clearing procedure (3.12) not to have multiple solutions. This is illustrated in the example below.

*Example 3.4 (Multiplicity of Solutions)* Let us consider again the two-bus system described in Example 3.3. The two-stage stochastic programming model (3.12) is now solved by setting the reserve capacity costs to zero, i.e., $C_i^{\mathrm{RU}} = C_i^{\mathrm{RD}} = 0$, for all the three conventional units in the system. That being so, optimization problem (3.12) has infinite solutions. Table 3.3 provides two possible sets of market outcomes leading to the same expected system operation cost, which results in \$2180. There is no need for load curtailment.

Note that, in terms of the expected system operation cost, the following two results are equivalent:

1. To dispatch unit 2 to 80 MW and the wind farm to 10 MW in the day-ahead market, and then redispatch unit 2 to either 40 or 80 MW at the balancing stage, depending on whether the eventual wind power production is 50 (scenario *high*) or 10 MW (scenario *low*), respectively. This is the solution given in Table 3.3(a) (Solution A).
2. To dispatch unit 2 to 40 MW and the wind farm to 50 MW at the scheduling stage, and then redispatch unit 2 to 80 MW in the balancing market if scenario *low* realizes. This is the solution shown in Table 3.3(b) (Solution B).

However, if we allow for reserve capacity costs in the market-clearing problem (3.12), these solutions are not equivalent anymore. In particular, suppose that $C_i^{\mathrm{RU}} = \$2/\mathrm{MW}$ and $C_i^{\mathrm{RD}} = \$1/\mathrm{MW}$ for all the three conventional units in the system. In this case, Solution B is not optimal, as it requires "contracting" more expensive reserve capacity. If we consider a price premium on the balancing energy, say $C_i^{\mathrm{U}} = C_i + 1$ and $C_i^{\mathrm{D}} = C_i - 1$ for all $i$, instead of reserve capacity costs, Solution B becomes optimal. Indeed, this solution requires the same amount of balancing energy as Solution A, but with a lower probability (i.e., the probability of scenario *low*, 0.4).

### *3.3.2 Features of the Settlement Scheme*

The settlement scheme (3.13) exhibits two important features, namely as follows:

1. It is revenue adequate in expectation, i.e., the payments that the system operator must make to and receive from the participants do not cause it to incur a financial deficit. The term *expectation* comes into play here due to the stochastic approach on which the market-clearing tool (3.12) is built. Intuitively speaking, a market settlement is said to be revenue adequate in expectation provided that it does not cause the system operator to run a financial deficit over time if used repeatedly over many trading periods.
2. It guarantees that the expected profit of each producer, either conventional or stochastic, is greater than or equal to its operating costs.

These two properties are proved in an appendix to this chapter on page 92 and illustrated through the example below.

*Example 3.5 (Features of the Settlement Scheme)* Let us turn back to the two-bus system introduced in Example 3.3. This time, though, we assume that, comparatively speaking, unit 1 is cheap, but completely inflexible; unit 2 is relatively expensive, but moderately flexible; and unit 3 is expensive, but very flexible. The new data for the three conventional units are collated in Table 3.4. Reserve capacity costs are not considered, i.e., $C_i^{\mathrm{RU}}$ and $C_i^{\mathrm{RD}}$ are set to zero in market-clearing problem (3.12) for all $i$. Instead, we assume that the market settlement allows for a price premium on the balancing energy. This way, for example, unit 3 is willing to sell balancing energy at a price \$5/MWh higher than in the day-ahead market, i.e., $C_3^{\mathrm{U}} = \$40/\mathrm{MWh}$. Similarly, this unit is willing to purchase balancing energy at a cost \$1/MWh lower than its marginal cost of production, i.e., $C_3^{\mathrm{D}} = \$34/\mathrm{MWh}$.

The electricity market is cleared using the two-stage stochastic programming model (3.12). The results of the clearing process are provided in Table 3.5. The wind farm is dispatched in the day-ahead market to 10 MW, $W^{\mathrm{S}} = 10$ MW. No load-shedding events occur. Electricity prices are the same at the two buses of the system, as the transmission line connecting them does not become congested in any of the two scenarios, *high* and *low*, considered. Given the dispatched quantities in Table 3.5(a)

**Table 3.4** Cost and technical data of conventional units in Example 3.5. Powers in MW and marginal costs in \$/MWh

| | $P_i^{\max}$ | $R_i^{U,\max}$ | $R_i^{D,\max}$ | $C_i$ | $C_i^U$ | $C_i^D$ |
|---|---|---|---|---|---|---|
| Unit 1 | 50 | 0 | 0 | 10 | — | — |
| Unit 2 | 110 | 20 | 30 | 30 | 50 | 20 |
| Unit 3 | 100 | 100 | 100 | 35 | 40 | 34 |

and the day-ahead and balancing prices in Table 3.5(b), the profit made by each market participant can be computed. For instance, the payment to unit 3 in scenario *low* is given by $40 \times 30 = \$1200$. Considering that the energy production cost of this unit is equal to \$35/MWh (recall that we assume that the price premium in the balancing market does not reflect a cost intrinsic to the electricity generation process), the profit it makes in this scenario is $1200 - 40 \times 35 = -200\$/\text{MWh}$. Table 3.6 provides the benefit obtained by each market participant both per scenario and in expectation. Note that, at the day-ahead market stage, the profit made by unit 3 can be seen as a random variable the expected value of which $(173.3 \times 0.6 - 200 \times 0.4)$ is greater than zero. The randomness of this profit stems from the uncertain character of the power produced by the wind farm. The settlement scheme (3.13) guarantees cost recovery for all producers in expectation, but this does not prevent unit 3 from incurring economic losses in scenario *low*. Indeed, unit 3 enters the day-ahead dispatch in a loss-making position (!), as its marginal production cost is equal to \$35/MWh, while the day-ahead market price is just \$30/MWh. This unit is dispatched to 40 MW in the day-ahead market with the aim that the system can benefit from its ability and willingness to decrease its production in the case that scenario *high* eventually realizes. It is actually in this scenario where unit 3 makes enough profit to guarantee the recovery of its production cost in expectation.

**Table 3.5** Market outcomes for Example 3.5. Powers in MW and prices in \$/MWh

(a) Dispatch ($W^S = 10$)

| Unit | $P_i$ | $r_{i\omega}^U$ | | $r_{i\omega}^D$ | |
|---|---|---|---|---|---|
| | | High | Low | High | Low |
| 1 | 50 | 0 | 0 | 0 | 0 |
| 2 | 40 | 0 | 0 | 0 | 0 |
| 3 | 40 | 0 | 0 | 40 | 0 |

(b) Prices

| $\lambda_n^D, \forall n$ | $\lambda_{n\omega}^B, \forall n$ | |
|---|---|---|
| | High | Low |
| 30 | 25.67 | 35.75 |

**Table 3.6** Profit of market participants in Example 3.5. Profit in \$

| | Expected | Per scenario | |
|---|---|---|---|
| | | High | Low |
| Unit 1 | 1000 | 1000 | 1000 |
| Unit 2 | 0 | 0 | 0 |
| Unit 3 | 24 | 173.3 | −200 |
| Load 1 | −1200 | −1200 | −1200 |
| Load 2 | −3000 | −3000 | −3000 |
| Wind farm | 916 | 1326.7 | 300 |

**Table 3.7** Market outcomes for Example 3.5 when line capacity is reduced to 50 MW. Powers in MW and prices in \$/MWh

(a) Dispatch ($W^S = 10$)

| Unit | $P_i$ | $r^U_{i\omega}$ | | $r^D_{i\omega}$ | |
|---|---|---|---|---|---|
| | | High | Low | High | Low |
| 1 | 50 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 |
| 3 | 80 | 0 | 0 | 30 | 0 |

(b) Prices

| | $\lambda^D_n, \forall n$ | $\lambda^B_{n\omega}, \forall n$ | |
|---|---|---|---|
| | | High | Low |
| Bus1 | 14.6 | 0 | 36.5 |
| Bus2 | 35 | 34 | 36.5 |

Revenue adequacy in expectation is also ensured for the system as a whole. To illustrate this, we calculate next the expected payments to conventional producers ($\widehat{\rho}_I$), the expected payment to the wind power producer ($\widehat{\rho}_Q$), and the expected payments from consumers ($\widehat{\rho}_J$), i.e.,

$$\widehat{\rho}_I = \underbrace{50 \times 30}_{Unit\ 1} + \underbrace{40 \times 30}_{Unit\ 2} + \underbrace{40 \times 30 - 0.6 \times 40 \times 25.67}_{Unit\ 3} = \$3284,$$

$$\widehat{\rho}_Q = 10 \times 30 + 0.6 \times (50 - 10) \times 25.67 + 0.4 \times (10 - 10) \times 35.75 = \$916,$$

$$\widehat{\rho}_J = \underbrace{40 \times 30}_{Load\ 1} + \underbrace{100 \times 30}_{Load\ 2} = \$4200.$$

Thus, the system is expected not to incur deficit since \$3284 + \$916 − \$4200 = 0.

To conclude this example, we reduce the capacity of the single line in the system to 50 MW. Dispatched quantities and electricity prices in this new variant are shown in Table 3.7. If the eventual wind power production is *high* (50 MW), the line becomes congested and the balancing price differs between buses, that is, it becomes a *locational marginal price* (LMP). In contrast, no network bottleneck occurs in scenario *low*, and the resulting balancing price is unique accordingly. However, it is worth noting that the day-ahead price is also a locational marginal price even though the optimal day-ahead dispatch $\{P_1^* = 50, P_2^* = 0, P_3^* = 80, W^{S*} = 10\}$ does not cause itself network congestion. This highlights the strong coupling between the day-ahead and the balancing prices, which is captured by the two-stage stochastic programming approach. Intuitively speaking, the day-ahead price *anticipates* probable line bottlenecks during the real-time operation of the power system.

The market-clearing procedure (3.12) is designed to produce a day-ahead dispatch $\{P_i^*, \forall i \in I;\ W_q^{S*}, \forall q \in Q\}$ by accounting for its potential impact on the system balancing costs using stochastic programming. The settlement scheme (3.13) underpins this dispatch in a financial sense. In particular, the day-ahead price $\lambda_n^{D*}$ makes all market participants satisfied with their respective day-ahead positions, as long as they seek to maximize their expected profit and are willing to take the risk of incurring losses under certain scenarios.

## 3.4 Clearing the Day-Ahead Market Using Robust Optimization

Robust optimization is an alternative framework to stochastic programming for dealing with optimization problems under uncertainty. This approach aims at determining a solution that is feasible under any realization of the uncertain parameters involved in an optimization problem, and optimal in their worst-case realization. The reader is referred to Appendix D for a brief introduction to the topic. The framework of robust optimization is relevant for the problems considered in this chapter. Indeed, the determination of the optimal dispatch in electricity markets is, as we have seen so far in this chapter, a problem of optimization under uncertainty, where the uncertain parameters include production from renewable sources. Furthermore, *robustness* is a quality that is particularly sought after in these models, as an underestimation of the reserve needs may result in costly load-shedding events. In this section, we shall learn how dispatch problems in electricity markets can be tackled using robust optimization.

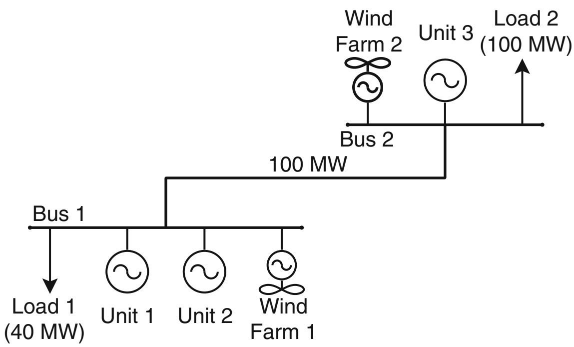
Let us recall the problem of determining the day-ahead energy and reserve dispatch considered in Sect. 3.2. As seen in that section, such a problem is a two-stage optimization problem, where the *here-and-now* decisions comprise day-ahead energy and reserve dispatch, while the redispatch at the balancing stage is a *wait-and-see* decision, which adapts to the realization of the uncertainty. In Sect. 3.2.3, such a problem is cast as a stochastic programming problem that aims at minimizing the total costs of energy dispatch, reserve capacity, and redispatch in expectation over a discrete set of scenarios. An alternative approach to this problem based on robust optimization can be sketched as follows:

*Minimize* Day-ahead dispatch cost + Worst-case balancing cost
subject to

- Day-ahead market constraints:
  - Power balance equations at the day-ahead stage.
  - Reserve capacity determination constraints.
  - Bounds of reserve and energy offers.
- Balancing market constraints:
  - Power balance equations at the balancing stage in the worst-case realization of the uncertain parameters.
  - Network constraints in the worst-case realization of the uncertain parameters.
  - Deployed reserve determination constraints in the worst-case realization of the uncertain parameters.
- Declarations of non-negative variables.

In practice, the mathematical formulation of an adaptive robust optimization problem, i.e., including recourse (*wait-and-see*) decisions, is more complex than the corresponding stochastic programming one. Indeed, the robust formulation

**Fig. 3.3** Modified two-bus system




summarized in the box above involves the determination of the worst-case balancing cost, which confers the problem a *min-max-min* structure. This is illustrated in the following illustrative example.

*Example 3.6 (An Adaptive Robust Optimization Approach to Energy and Reserve Dispatch).* Let us consider a modified version of the two-node system considered in Example 3.3, which is illustrated in Fig. 3.3. Note that the modified two-node system includes two wind farms, one per each node of the network. All the parameters of the system, including cost, production, and transmission limits as well as demand are unchanged with respect to the ones used in Example 3.3.

*Objective Function:* The worst-case system operation cost (WCC) can be expressed as follows:

$$\begin{aligned}\text{WCC} = &\underbrace{\underbrace{10P_1 + 30P_2 + 35P_3}_{\text{Day-ahead energy costs}} + \underbrace{16R_1^{\text{U}} + 15R_1^{\text{D}} + 13R_2^{\text{U}} + 12R_2^{\text{D}} + 10R_3^{\text{U}} + 9R_3^{\text{D}}}_{\text{Reserve capacity costs}}}_{\text{Day-ahead dispatch costs}} \\ &+ \underbrace{\mathcal{Q}(P_1, P_2, P_3, R_1^{\text{U}}, R_2^{\text{U}}, R_3^{\text{U}}, R_1^{\text{D}}, R_2^{\text{D}}, R_3^{\text{D}}, \delta_1^0, \delta_2^0)}_{\text{Worst-case energy redispatch costs}}. \qquad (3.14)\end{aligned}$$

We shall now explicitly write the constraints and the worst-case energy redispatch costs.

*Day-Ahead Market Constraints:* Similarly to Example 3.3, power balance is enforced at both nodes. However, here we assume that the day-ahead dispatch for the wind power producers is equal to their conditional mean forecast, which, for this particular example, is considered to be $\widehat{W}_1 = 15$ MWh and $\widehat{W}_2 = 30$ MWh. This writes as follows:

$$P_1 + P_2 + 15 - 40 = \frac{(\delta_1^0 - \delta_2^0)}{0.13} \qquad \text{(bus 1)},$$

$$P_3 + 30 - 100 = \frac{(\delta_2^0 - \delta_1^0)}{0.13} \qquad \text{(bus 2)},$$

We define bus 1 as the reference node by setting $\delta_1^0$ to 0. Transmission and production capacity are enforced by (3.9) and (3.10), precisely as in the stochastic programming formulation in Example 3.3.

Before enforcing the constraints at the balancing stage, we formulate the worst-case cost of the balancing operation, which was implicitly defined in (3.14) as a function $\mathcal{Q}(\,\cdot\,)$ of the *here-and-now* decisions.

*Worst-Case Balancing Cost:*
For a given set of *here-and-now* decisions, $\Xi^{\mathrm{D}}$, the following *max-min* formulation defines the worst-case balancing cost for the problem at hand:

$$\mathcal{Q}(\,\cdot\,) = \max_{\Delta W_1, \Delta W_2 \in \mathcal{W}} \; \min_{\Xi^{\mathrm{B}} \in \mathcal{B}(\Xi^{\mathrm{D}},\, \Delta W)} \; \Big[10\left(r_1^{\mathrm{U}} - r_1^{\mathrm{D}}\right) + 30\left(r_2^{\mathrm{U}} - r_2^{\mathrm{D}}\right) + 35\left(r_3^{\mathrm{U}} - r_3^{\mathrm{D}}\right) + 200\left(L_1^{\mathrm{shed}} + L_2^{\mathrm{shed}}\right)\Big]. \tag{3.15}$$

The outer maximization problem picks the worst-case realization of the deviations $\Delta W_1$ and $\Delta W_2$ of stochastic production from wind farms 1 and 2, respectively, from their conditional mean forecast. These deviations are to be chosen from within an uncertainty set $\mathcal{W}$, which we shall define later.

Once the worst-case realization of the uncertainty is fixed, the inner minimization problem determines the optimal recourse decision. Notice that the set $\Xi^{\mathrm{B}}$ of recourse decisions includes upward and downward redispatch, $r^{\mathrm{U}}$ and $r^{\mathrm{D}}$, respectively, load-shedding, $L^{\mathrm{shed}}$, wind power spillage, $W^{\mathrm{spill}}$, as well as voltage angles, $\delta$. These decision variables must be optimized within the feasibility set $\mathcal{B}$, which depends on the *here-and-now* decision set $\Xi^{\mathrm{D}}$ and the worst-case realization $\Delta W$ of the uncertainty. In turn, the feasibility set $\mathcal{B}$ is defined by the constraints modeling the balancing operation of the power system (i.e., the recourse problem). These constraints will be introduced later on.

*Definition of the Uncertainty Set ($\mathcal{W}$):* Typically, polyhedral uncertainty sets are chosen in problems of adaptive robust optimization. In this example, we consider symmetrical intervals for the deviation of wind power production from the conditional mean forecast:

$$|\Delta W_1| \leq 10, \tag{3.16a}$$

$$|\Delta W_2| \leq 20. \tag{3.16b}$$

Furthermore, we set a *budget of uncertainty* to limit the overall output deviation for the wind power producers in the network.

$$\frac{|\Delta W_1|}{10} + \frac{|\Delta W_2|}{20} \leq 1.4. \tag{3.17}$$

Basically, such a constraint guarantees that the output for the two wind farms cannot simultaneously be at the lower or upper bound of their respective feasible production intervals resulting from (3.16). Indeed, if the production from one wind farm is at

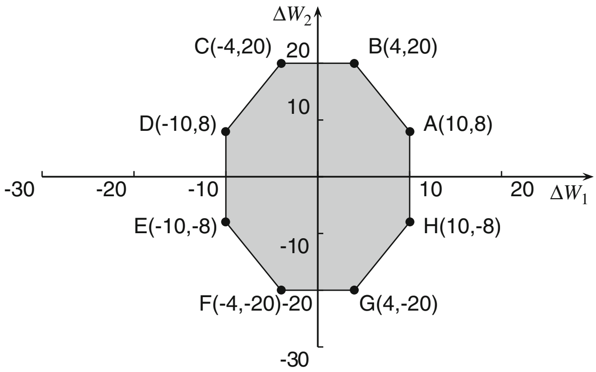



**Fig. 3.4** Uncertainty set for the deviation of wind power production from the conditional mean forecast

the lower bound, then the deviation for the other must be at most equal to 40 % of its maximum value. This reflects nature's behavior.

The constraints (3.16) and (3.17) result in the polyhedral uncertainty set illustrated in Fig. 3.4.

*Balancing Market Constraints:* These constraints define the feasibility set $\mathcal{B}$ in (3.15), which determines the operating region of the power system in real time. Notice that in a robust optimization framework, it is sufficient to enforce one instance of the operation constraints, valid for the worst-case realization of the uncertainty; in this case, the deviation of stochastic power production. In contrast, we had to enforce one set of balancing constraints per scenario in the stochastic programming approach in Example 3.3.

The power balance at each node of the network is enforced by the following constraints:

$$r_1^{\mathrm{U}} + r_2^{\mathrm{U}} - r_1^{\mathrm{D}} - r_2^{\mathrm{D}} + L_1^{\mathrm{shed}} + \Delta W_1 - W_1^{\mathrm{spill}} = \frac{(\delta_1 - \delta_1^0 + \delta_2^0 - \delta_2)}{0.13} \quad \text{(bus 1)},$$

$$r_3^{\mathrm{U}} - r_3^{\mathrm{D}} + L_2^{\mathrm{shed}} + \Delta W_2 - W_2^{\mathrm{spill}} = \frac{(\delta_2 - \delta_2^0 + \delta_1^0 - \delta_1)}{0.13} \quad \text{(bus 2)}.$$

We consider again bus 1 as the reference node in the balancing stage by setting $\delta_1 = 0$.

The following constraints enforce the power transmission capacity between buses 1 and 2 at the balancing stage:

$$\frac{(\delta_1 - \delta_2)}{0.13} \leq 100, \qquad \frac{(\delta_2 - \delta_1)}{0.13} \leq 100.$$

The amount of spilled wind power production must be lower than, or equal to, the actual wind power production value. This is enforced by the following inequalities:

$$W_1^{\mathrm{spill}} \leq 15 + \Delta W_1 \qquad \text{(wind farm 1)},$$

$$W_2^{\mathrm{spill}} \leq 30 + \Delta W_2 \qquad \text{(wind farm 2)}.$$

In a similar fashion, load shedding must be lower than or equal to the actual consumption value:

$$L_1^{\text{shed}} \leq 40 \qquad \text{(load 1)},$$
$$L_2^{\text{shed}} \leq 100 \qquad \text{(load 2)}.$$

The additional energy redispatch is limited by the reserve capacity scheduled in the day-ahead market. This is ensured by the following constraints:

$$r_1^{\text{U}} \leq R_1^{\text{U}}, \quad r_1^{\text{D}} \leq R_1^{\text{D}}, \quad r_2^{\text{U}} \leq R_2^{\text{U}}, \quad r_2^{\text{D}} \leq R_2^{\text{D}}, \quad r_3^{\text{U}} \leq R_3^{\text{U}}, \quad r_3^{\text{D}} \leq R_3^{\text{D}}.$$

*Declarations of Non-Negative Variables:* Finally, we enforce the non-negativity of reserve, production, and consumption quantities in both the day-ahead and balancing stages:

$$R_1^{\text{U}},\ R_2^{\text{U}},\ R_3^{\text{U}},\ R_1^{\text{D}},\ R_2^{\text{D}},\ R_3^{\text{D}},\ P_1,\ P_2,\ P_3 \geq 0 \qquad \text{(day-ahead stage)},$$
$$r_1^{\text{U}},\ r_2^{\text{U}},\ r_3^{\text{U}},\ r_1^{\text{D}},\ r_2^{\text{D}},\ r_3^{\text{D}},\ L_1^{\text{shed}},\ L_2^{\text{shed}},\ W_1^{\text{spill}},\ W_2^{\text{spill}} \geq 0 \qquad \text{(balancing stage)}.$$

*Complete Model Formulation:* By joining the objective function with the constraints defined above, we get the following *min-max-min* problem formulation:

$$\begin{aligned}
\underset{\Xi^{\text{D}}}{\text{Min.}}\ & 10P_1 + 30P_2 + 35P_3 + 16R_1^{\text{U}} + 15R_1^{\text{D}} + 13R_2^{\text{U}} + 12R_2^{\text{D}} + 10R_3^{\text{U}} + 9R_3^{\text{D}} \\
+ & \max_{\Delta W_1, \Delta W_2} \min_{\Xi^{\text{B}}} \Big[10\left(r_1^{\text{U}} - r_1^{\text{D}}\right) + 30\left(r_2^{\text{U}} - r_2^{\text{D}}\right) + 35\left(r_3^{\text{U}} - r_3^{\text{D}}\right) \\
& \qquad + 200\left(L_1^{\text{shed}} + L_2^{\text{shed}}\right)\Big] && \text{(3.18a)} \\
\text{s.t. } & r_1^{\text{U}} + r_2^{\text{U}} - r_1^{\text{D}} - r_2^{\text{D}} + L_1^{\text{shed}} \\
& \qquad + \Delta W_1 - W_1^{\text{spill}} = \frac{(\delta_1 - \delta_1^0 + \delta_2^0 - \delta_2)}{0.13}, && \text{(3.18b)} \\
& r_3^{\text{U}} - r_3^{\text{D}} + L_2^{\text{shed}} + \Delta W_2 - W_2^{\text{spill}} = \frac{(\delta_2 - \delta_2^0 + \delta_1^0 - \delta_1)}{0.13}, && \text{(3.18c)} \\
& -100 \leq \frac{(\delta_1 - \delta_2)}{0.13} \leq 100, && \text{(3.18d)} \\
& \delta_1 = 0, && \text{(3.18e)} \\
& W_1^{\text{spill}} \leq 15 + \Delta W_1, && \text{(3.18f)} \\
& W_2^{\text{spill}} \leq 30 + \Delta W_2, && \text{(3.18g)} \\
& L_1^{\text{shed}} \leq 40, && \text{(3.18h)} \\
& L_2^{\text{shed}} \leq 100, && \text{(3.18i)} \\
& r_1^{\text{U}} \leq R_1^{\text{U}},\ r_2^{\text{U}} \leq R_2^{\text{U}},\ r_3^{\text{U}} \leq R_3^{\text{U}}, && \text{(3.18j)}
\end{aligned}$$

$$r_1^{\mathrm{D}} \le R_1^{\mathrm{D}},\ r_2^{\mathrm{D}} \le R_2^{\mathrm{D}},\ r_3^{\mathrm{D}} \le R_3^{\mathrm{D}}, \tag{3.18k}$$

$$r_1^{\mathrm{U}}, r_2^{\mathrm{U}}, r_3^{\mathrm{U}}, r_1^{\mathrm{D}}, r_2^{\mathrm{D}}, r_3^{\mathrm{D}}, L_1^{\mathrm{shed}}, L_2^{\mathrm{shed}}, W_1^{\mathrm{spill}}, W_2^{\mathrm{spill}} \ge 0, \tag{3.18l}$$

$$\text{s.t. } |\Delta W_1| \le 10, \tag{3.18m}$$

$$|\Delta W_2| \le 20, \tag{3.18n}$$

$$\frac{|\Delta W_1|}{10} + \frac{|\Delta W_2|}{20} \le 1.4, \tag{3.18o}$$

$$\text{s.t. } P_1 + P_2 + 15 - 40 = \frac{(\delta_1^0 - \delta_2^0)}{0.13}, \tag{3.18p}$$

$$P_3 + 30 - 100 = \frac{(\delta_2^0 - \delta_1^0)}{0.13}, \tag{3.18q}$$

$$-100 \le \frac{(\delta_1^0 - \delta_2^0)}{0.13} \le 100, \tag{3.18r}$$

$$\delta_1^0 = 0, \tag{3.18s}$$

$$P_1 + R_1^{\mathrm{U}} \le 50, \tag{3.18t}$$

$$P_1 - R_1^{\mathrm{D}} \ge 0, \tag{3.18u}$$

$$P_2 + R_2^{\mathrm{U}} \le 110, \tag{3.18v}$$

$$P_2 - R_2^{\mathrm{D}} \ge 0, \tag{3.18w}$$

$$P_3 + R_3^{\mathrm{U}} \le 100, \tag{3.18x}$$

$$P_3 - R_3^{\mathrm{D}} \ge 0, \tag{3.18y}$$

$$R_1^{\mathrm{U}}, R_2^{\mathrm{U}}, R_3^{\mathrm{U}}, R_1^{\mathrm{D}}, R_2^{\mathrm{D}}, R_3^{\mathrm{D}}, P_1, P_2, P_3 \ge 0. \tag{3.18z}$$

In the formulation above, $\Xi^{\mathrm{D}}$ indicates the set of day-ahead decision variables, i.e., the energy dispatch, $P_1, P_2, P_3$, the dispatch of upward reserve $R_1^{\mathrm{U}}, R_2^{\mathrm{U}}, R_3^{\mathrm{U}}$, and of downward reserve, $R_1^{\mathrm{D}}, R_2^{\mathrm{D}}, R_3^{\mathrm{D}}$, as well as the voltage angles, $\delta_1^0$ and $\delta_2^0$, at this stage.

Notice that in the model above, we can introduce an auxiliary variable $\beta$ representing the worst-case recourse cost $\mathcal{Q}(\cdot)$, which is the optimal objective function value of the inner *max-min* problem in (3.18a). We could then solve problem (3.18) as a single minimization problem after enforcing the following constraints:

$$\begin{aligned}\beta \ge\, & 10\left[r_1^{\mathrm{U}}(\Delta W_1, \Delta W_2) - r_1^{\mathrm{D}}(\Delta W_1, \Delta W_2)\right] \\ & + 30\left[r_2^{\mathrm{U}}(\Delta W_1, \Delta W_2) - r_2^{\mathrm{D}}(\Delta W_1, \Delta W_2)\right] \\ & + 35\left[r_3^{\mathrm{U}}(\Delta W_1, \Delta W_2) - r_3^{\mathrm{D}}(\Delta W_1, \Delta W_2)\right] \\ & + 200\left[L_1^{\mathrm{shed}}(\Delta W_1, \Delta W_2) + L_2^{\mathrm{shed}}(\Delta W_1, \Delta W_2)\right], \qquad \forall \Delta W_1, \Delta W_2 \in \mathcal{W},\end{aligned} \tag{3.19}$$

where we write the optimal recourse decision, i.e., the redispatch at the balancing stage, as a function of the deviation of wind power production. However, let us recall that the uncertainty set $\mathcal{W}$, defined by (3.16)–(3.17) and illustrated in Fig. 3.4, includes an infinite number of points. Then, there is an instance of each balancing market variable and constraint for each pair $(\Delta W_1, \Delta W_2) \in \mathcal{W}$.

Since the set $\mathcal{W}$ is uncountable, the reformulation described above would result in a problem with an infinite number of constraints and of variables for the balancing stage. In practice, however, we can get around this issue. Indeed, it can be shown that only the vertices $v = A, B, \ldots, H$ of the polyhedral set $\mathcal{W}$ illustrated in Fig. 3.4 can be part of a solution to the inner *max-min* problem in (3.18). The interested reader is referred to the appendix on page 95 at the end of this chapter for a proof of this property. As a result, we can consider only this finite number of vertices $v = A, B, \ldots, H$ of the uncertainty set, and cast problem (3.18) as follows:

$$\text{Min. } 10P_1 + 30P_2 + 35P_3 + 16R_1^{\text{U}} + 15R_1^{\text{D}} + 13R_2^{\text{U}} + 12R_2^{\text{D}} + 10R_3^{\text{U}} + 9R_3^{\text{D}} + \beta \tag{3.20a}$$

s.t. (3.18p)–(3.18z),

$$\beta \geq \Big[10\left(r_{1v}^{\text{U}} - r_{1v}^{\text{D}}\right) + 30\left(r_{2v}^{\text{U}} - r_{2v}^{\text{D}}\right) + 35\left(r_{3v}^{\text{U}} - r_{3v}^{\text{D}}\right) + 200\left(L_{1v}^{\text{shed}} + L_{2v}^{\text{shed}}\right)\Big], \quad v = A, \ldots, H, \tag{3.20b}$$

$$r_{1v}^{\text{U}} + r_{2v}^{\text{U}} - r_{1v}^{\text{D}} - r_{2v}^{\text{D}} + L_{1v}^{\text{shed}} + \Delta W_{1v} - W_{1v}^{\text{spill}} = \frac{(\delta_{1v} - \delta_1^0 + \delta_2^0 - \delta_{2v})}{0.13}, \quad v = A, \ldots, H, \tag{3.20c}$$

$$r_{3v}^{\text{U}} - r_{3v}^{\text{D}} + L_{2v}^{\text{shed}} + \Delta W_{2v} - W_{2v}^{\text{spill}} = \frac{(\delta_{2v} - \delta_2^0 + \delta_1^0 - \delta_{1v})}{0.13}, \quad v = A, \ldots, H, \tag{3.20d}$$

$$\frac{(\delta_{1v} - \delta_{2v})}{0.13} \leq 100, \quad v = A, \ldots, H, \tag{3.20e}$$

$$\frac{(\delta_{2v} - \delta_{1v})}{0.13} \leq 100, \quad v = A, \ldots, H, \tag{3.20f}$$

$$\delta_{1v} = 0, \quad v = A, \ldots, H, \tag{3.20g}$$

$$W_{1v}^{\text{spill}} \leq 15 + \Delta W_{1v}, \quad v = A, \ldots, H, \tag{3.20h}$$

$$W_{2v}^{\text{spill}} \leq 30 + \Delta W_{2v}, \quad v = A, \ldots, H, \tag{3.20i}$$

$$L_{1v}^{\text{shed}} \leq 40, \quad v = A, \ldots, H, \tag{3.20j}$$

$$L_{2v}^{\text{shed}} \leq 100, \quad v = A, \ldots, H, \tag{3.20k}$$

$$r_{1v}^{\text{U}} \leq R_1^{\text{U}},\ r_{2v}^{\text{U}} \leq R_2^{\text{U}},\ r_{3v}^{\text{U}} \leq R_3^{\text{U}}, \quad v = A, \ldots, H, \tag{3.20l}$$

$$r_{1v}^{\text{D}} \leq R_1^{\text{D}},\ r_{2v}^{\text{D}} \leq R_2^{\text{D}},\ r_{3v}^{\text{D}} \leq R_3^{\text{D}}, \quad v = A, \ldots, H, \tag{3.20m}$$

$$r_{1v}^{\text{U}}, r_{2v}^{\text{U}}, r_{3v}^{\text{U}}, r_{1v}^{\text{D}}, r_{2v}^{\text{D}}, r_{3v}^{\text{D}}, L_{1v}^{\text{shed}}, L_{2v}^{\text{shed}}, W_{1v}^{\text{spill}}, W_{2v}^{\text{spill}} \geq 0, \quad v = A, \ldots, H. \tag{3.20n}$$

**Table 3.8** Energy (in MWh) and reserve schedule (in MW) obtained from the robust optimization model. The dispatched wind energy production is 45 MWh

| | $P$ | $R^{\mathrm{U}}$ | $R^{\mathrm{D}}$ |
|---|---|---|---|
| Unit 1 | 50 | 0 | 0 |
| Unit 2 | 45 | 24 | 0 |
| Unit 3 | 0 | 0 | 0 |

Notice that the reformulation above resembles the stochastic programming problem (3.11), although with two fundamental differences. Firstly, instead of employing scenarios $\omega$, we consider the vertices $v$ of the polyhedral feasible set for the deviation of wind power production defined by (3.16)–(3.17). Secondly, as a result of inequalities (3.20b), the objective function (3.20a) aims at minimizing the total worst-case cost of energy dispatch, reserve, and energy redispatch rather than its value in expectation.

Table 3.8 illustrates the day-ahead schedule determined using model (3.20). The following observations should be pointed out.

1. Robust optimization yields a rather conservative schedule in terms of upward reserve. Indeed, the scheduled value for this quantity is sufficient to cover any negative deviation of wind power production in the uncertainty set $\mathcal{W}$. Notice that the largest production deficit, equal to 24 MWh for the aggregation of the two nodes, is attained at vertex F in Fig. 3.4.
2. No downward reserve is scheduled. This stems from the combination of two facts. Firstly, robust optimization focuses on the worst-case realization of the uncertainty. Secondly, there is no cost associated with wind power spillage, while the penalty for load shedding is relatively large. This implies that the worst-case realization of the uncertainty is a negative deviation of wind power production from its mean forecast. Notice that for cases of production deficit, only upward reserve is needed. Therefore, scheduling downward reserve would unnecessarily increase the worst-case system cost.
3. Because of the focus on the worst-case realization of the uncertainty, robust optimization prioritizes the scheduling of reserves with the lowest sum of production and upward reserve cost. In this case, unit 2 is preferred to unit 3, since $C_2 + C_2^{\mathrm{U}} < C_3 + C_3^{\mathrm{U}}$. This may not be the case for the stochastic programming approach. Indeed, in the latter framework, the higher production cost $C_3$ is discounted by the probability of actually deploying reserve. On the contrary, the cost of buying reserve, which is lower for unit 3 than for unit 2, is fixed. As a result, scheduling reserve from unit 3 may be more beneficial if the probability of deploying that reserve is sufficiently low.

*Example 3.7 (Comparing Robust Optimization and Stochastic Programming).* Let us now compare the results obtained from the robust optimization model presented in the previous example with those of the stochastic programming model (3.11). In the implementation of the latter model, we employ a set of 100 scenarios sampled randomly from a uniform distribution the support of which is the uncertainty set illustrated in Fig. 3.4.

**Table 3.9** Energy (in MWh) and reserve schedule (in MW) obtained from the stochastic programming model. The dispatched wind energy production is 27.72 MWh

| | $P$ | $R^U$ | $R^D$ |
|---|---|---|---|
| Unit 1 | 50 | 0 | 0 |
| Unit 2 | 40.28 | 0 | 0 |
| Unit 3 | 22 | 0 | 22 |

**Table 3.10** Breakdown of system cost with the stochastic programming and the robust optimization approaches. Values in $

| | Stochastic Programming | | Robust Optimization | |
|---|---|---|---|---|
| Energy dispatch | 2478.29 | | 1850 | |
| Reserve dispatch | 197.95 | | 312 | |
| Total day-ahead | 2676.24 | | 2162 | |
| | *Expected* | *Worst-case* | *Expected* | *Worst-case* |
| Energy redispatch | −521.75 | 0 | 144.92 | 720 |
| Load shedding | 26.03 | 1344.53 | 0 | 0 |
| Total balancing | −495.72 | 1344.53 | 144.92 | 720 |
| Total aggregate | 2180.53 | 4020.78 | 2306.92 | 2882 |

Table 3.9 illustrates the day-ahead energy and reserve dispatch obtained for the conventional producers in the stochastic programming approach. Remarkably, reserve in this solution is assigned to unit 3 rather than to unit 2, as occurs in Table 3.8 for the robust optimization approach. This fact should be considered in view of observation 3 in the previous example. Furthermore, it should be noticed that the total day-ahead dispatch for the conventional units is larger in the stochastic programming approach. In turn, this results in a lower dispatch for the wind power producers, which totals 27.72 MWh, i.e., the amount needed to meet the total load (140 MWh). Notice that the dispatch for wind power producers is not constrained to be equal to the conditional mean forecast of production in model (3.11).

A breakdown of the system cost for the two approaches is provided in Table 3.10. The day-ahead cost is higher for the stochastic programming solution than for the robust optimization one, which dispatches more (zero-cost) wind. However, the former solution benefits from the possibility of redispatching unit 3 downward in the balancing stage, resulting in gains in expectation in this stage. The net effect of the combination of these two facts on the total aggregate expected cost is trivial: the total expected cost is lower in the stochastic programming approach than in the robust optimization one.

The situation is quite the opposite when looking at the worst-case realization of the stochastic production within the uncertainty set illustrated in Fig. 3.4. At the vertex F of this polyhedron, the two wind farms combined produce 24 MWh less than their expected output (45 MWh in total). In the stochastic programming solution, no upward reserve is available to cope with a day-ahead dispatch that is short by 6.72 MWh. Therefore, this amount becomes load shedding, which makes the worst-case balancing cost skyrocket to over \$1300. It should also be noticed that smaller load-shedding events take place in some of the scenarios used as input to the stochastic programming problem, which result in a load-shedding cost equal to roughly \$26 in expectation. In comparison, load shedding never takes place with the robust optimization solution.

We now give the general formulation for the market-clearing model for energy and reserve dispatch based on robust optimization. We employ the same notation as in Sect. 3.2.3. Furthermore, the assumptions made at the end of that section still hold with the exception of A6. Contrarily to this assumption, we now model the uncertainty through a polyhedral uncertainty set $\mathcal{W}$, where the stochastic parameters can take values in.

$$\underset{\Xi^{\mathrm{D}}}{\mathrm{Min.}} \sum_{i \in I} \left( C_i P_i + C_i^{\mathrm{RU}} R_i^{\mathrm{U}} + C_i^{\mathrm{RD}} R_i^{\mathrm{D}} \right) + \sum_{q \in Q} C_q \widehat{W}_q$$

$$+ \max_{\Delta W} \min_{\Xi^{\mathrm{B}}} \left[ \sum_{i \in I} \left( C_i^{\mathrm{U}} r_i^{\mathrm{U}} - C_i^{\mathrm{D}} r_i^{\mathrm{D}} \right) + \sum_{q \in Q} C_q \left( \Delta W_q - W_q^{\mathrm{spill}} \right) + \sum_{j \in J} V_j^{\mathrm{LOL}} L_j^{\mathrm{shed}} \right] \tag{3.21a}$$

$$\text{s.t.} \sum_{i \in \Phi_n^I} \left( r_i^{\mathrm{U}} - r_i^{\mathrm{D}} \right) + \sum_{j \in \Phi_n^J} L_j^{\mathrm{shed}} + \sum_{q \in \Phi_n^Q} \left( \Delta W_q - W_q^{\mathrm{spill}} \right)$$

$$+ \sum_{\ell \in \Lambda | o(\ell) = n} b_\ell \left( \delta_{o(\ell)}^0 - \delta_{o(\ell)} - \delta^0{}_{e(\ell)} + \delta_{e(\ell)} \right)$$

$$- \sum_{\ell \in \Lambda | e(\ell) = n} b_\ell \left( \delta_{o(\ell)}^0 - \delta_{o(\ell)} - \delta_{e(\ell)}^0 + \delta_{e(\ell)} \right) = 0, \quad \forall n \in N, \tag{3.21b}$$

$$b_\ell \left( \delta_{o(\ell)} - \delta_{e(\ell)} \right) \le C_\ell^{\max}, \quad \forall \ell \in \Lambda, \tag{3.21c}$$

$$- b_\ell \left( \delta_{o(\ell)} - \delta_{e(\ell)} \right) \le C_\ell^{\max}, \quad \forall \ell \in \Lambda, \tag{3.21d}$$

$$\delta_1 = 0, \tag{3.21e}$$

$$r_i^{\mathrm{U}} \le R_i^{\mathrm{U}}, \quad \forall i \in I, \tag{3.21f}$$

$$r_i^{\mathrm{D}} \le R_i^{\mathrm{D}}, \quad \forall i \in I, \tag{3.21g}$$

$$L_j^{\mathrm{shed}} \le L_j, \quad \forall j \in J, \tag{3.21h}$$

$$W_q^{\mathrm{spill}} \le \widehat{W}_q + \Delta W_q, \quad \forall q \in Q, \tag{3.21i}$$

$$r_i^{\mathrm{U}}, r_i^{\mathrm{D}} \ge 0, \ \forall i \in I; \ W_q^{\mathrm{spill}} \ge 0, \ \forall q \in Q; \ L_j^{\mathrm{shed}} \ge 0, \ \forall j \in J, \tag{3.21j}$$

$$\text{s.t.} \ |\Delta W_q| \le \Delta W_q^{\max}, \quad \forall q \in Q, \tag{3.21k}$$

$$\sum_{q \in Q} \frac{|\Delta W_q|}{\Delta W_q^{\max}} \le \Gamma, \tag{3.21l}$$

$$\text{s.t.} \sum_{i \in \Phi_n^I} P_i + \sum_{q \in \Phi_n^Q} \widehat{W}_q - \sum_{j \in \Phi_n^J} L_j - \sum_{\ell \in \Lambda | o(\ell)=n} b_\ell \left(\delta^0_{o(\ell)} - \delta^0_{e(\ell)}\right)$$

$$+ \sum_{\ell \in \Lambda | e(\ell)=n} b_\ell \left(\delta^0_{o(\ell)} - \delta^0_{e(\ell)}\right) = 0, \quad \forall n \in N, \tag{3.21m}$$

$$b_\ell \left(\delta^0_{o(\ell)} - \delta^0_{e(\ell)}\right) \leq C_\ell^{\max}, \quad \forall \ell \in \Lambda, \tag{3.21n}$$

$$-b_\ell \left(\delta^0_{o(\ell)} - \delta^0_{e(\ell)}\right) \leq C_\ell^{\max}, \quad \forall \ell \in \Lambda, \tag{3.21o}$$

$$\delta^0_1 = 0, \tag{3.21p}$$

$$P_i + R_i^{\mathrm{U}} \leq P_i^{\max}, \quad \forall i \in I, \tag{3.21q}$$

$$P_i - R_i^{\mathrm{D}} \geq 0, \quad \forall i \in I, \tag{3.21r}$$

$$R_i^{\mathrm{U}} \leq R_i^{\mathrm{U,max}}, \quad \forall i \in I, \tag{3.21s}$$

$$R_i^{\mathrm{D}} \leq R_i^{\mathrm{D,max}}, \quad \forall i \in I, \tag{3.21t}$$

$$P_i, R_i^{\mathrm{U}}, R_i^{\mathrm{D}} \geq 0, \ \forall i \in I, \tag{3.21u}$$

where $\widehat{W}_q$ is the conditional mean forecast for the power generated by stochastic producer $q$ and $\Gamma$ is the budget of uncertainty, which limits the overall output deviation for stochastic producers, as in (3.17).

We conclude the section by mentioning that model (3.21) need not be solved by enumeration of all the vertices of the uncertainty set $\mathcal{W}$. Indeed, there exist iterative methods based on Benders decomposition, see [8], where vertices of the feasible polyhedron are generated on demand and a corresponding Benders cut is added at each iteration. In this way, the objective value as a function of the first-stage variables is constructed by sequential approximations, and consequently, more and more accurate estimates of the solution are obtained at each iteration. We refer the interested reader to [2] and [11] for further details on this solution technique.

Finally, note that it is not trivial to derive a pricing scheme for the robust optimization approach. Research is currently underway to develop pricing schemes with desirable short- and long-term properties, such as to convey proper marginal signals and to guarantee investment recovery, respectively.

## 3.5 Summary and Conclusions

This chapter describes market-clearing procedures for the day-ahead market under a large-scale penetration of stochastic renewable production sources.

Firstly, this procedure is formulated as a two-stage stochastic programming problem, which provides production and consumption levels, allocation of reserve capacity, and clearing prices. The clearing algorithm is of particular interest for markets with a significant number of stochastic producers.

To mimic the actual operation of electric energy systems, a two-stage decision framework, as the one proposed in this chapter, is required. However, such two-step decision framework is not that common in the technical literature.

The proposed stochastic programming framework allows anticipating the impact of the realization of uncertain events and, as a result, achieving the best possible pre-positioning of the market against such uncertain events, with the ultimate purpose of minimizing the expected system cost.

Following widely accepted marginal pricing theory, the algorithm proposed results in energy-only prices, which ensure cost recovery, on average, for all operating producers, and revenue adequacy for the system, also on average.

The algorithm proposed is computationally tractable provided that the number of scenarios required to describe the future realization of the uncertainty is small enough.

Finally, an alternative approach based on adaptive robust optimization is introduced. The market-clearing procedure developed in this framework aims at minimizing the total cost of system operation in the worst-case realization of the uncertain parameters, taking into account the operation at the balancing market.

## 3.6 Further Reading

Relevant manuals on electricity markets include [16] and [6]. A standard reference on power system reliability is [3], which elaborates on methods to estimate the amount of reserve required to attain a certain level of reliability. The modeling of reliability metrics within a mixed-integer linear programming formulation can be found in [5; 9]. The concepts of stochastic programming are provided in [4], and its applications to decision making under uncertainty in electricity markets, including the market-clearing problem, are presented in [7]. Settlement schemes based on the simultaneous dispatch of energy and reserve using stochastic optimization are discussed, for example, in [17]. Further details on pricing electricity in energy-only markets cleared using stochastic programming can be found in [12] and [14]. The reader is referred to [1] for a tutorial on robust optimization. Applications of adaptive robust optimization focusing on electricity markets, and in particular on unit commitment, comprise [2] and [11].

## Appendix 1: Settlement Scheme Properties

The properties of the settlement scheme in Sect. 3.3.1 are formally stated below in the form of theorems. For this purpose, we define first the following indices:

$s(i)$ Index of the bus where conventional unit $i$ is located.
$s(j)$ Index of the bus where load $j$ is located.
$s(q)$ Index of the bus where stochastic production unit $q$ is located.

**Theorem 3.1 (Revenue adequacy in expectation).** Consider the market-clearing procedure (3.12), built on a stochastic programming framework, and the resulting sets of dual variables $\{\lambda_n^{\mathrm{D}}, \forall n \in N\}$ and $\{(\lambda_{n\omega}^{\mathrm{B}}, \pi_\omega), \forall n \in N, \forall \omega \in \Omega\}$. The settlement scheme (3.13) is revenue adequate in expectation.

*Proof* Mathematically, the settlement scheme (3.13) is revenue adequate in expectation if, at the optimum, it holds

$$\sum_{n\in N}\lambda_n^{\mathrm{D}*}\left(\sum_{i\in\Phi_n^I}P_i^* + \sum_{q\in\Phi_n^Q}W_q^{\mathrm{S}*} - \sum_{j\in\Phi_n^J}L_j\right) + \sum_{\omega\in\Omega}\sum_{n\in N}\pi_\omega\lambda_{n\omega}^{\mathrm{B}*}\Big[\sum_{i\in\Phi_n^I}\left(r_{i\omega}^{\mathrm{U}*} - r_{i\omega}^{\mathrm{D}*}\right)$$
$$- \sum_{q\in\Phi_n^Q}\left(W_q^{\mathrm{S}*} + W_q^{\mathrm{spill}*} - W_{q\omega}\right) + \sum_{j\in\Phi_n^J}L_{j\omega}^{\mathrm{shed}*}\Big] \le 0, \tag{3.22}$$

where $\{\lambda_{n\omega}^{\mathrm{B}*} = \gamma_{n\omega}^*/\pi_\omega, \forall n \in N, \forall \omega \in \Omega\}$ are the probability-removed balancing prices and superscript "$*$" denotes optimal values.

Using the power balance equations (3.12b) and (3.12c), expression (3.22) can be equivalently rewritten as follows:

$$\sum_{n\in N}\lambda_n^{\mathrm{D}*}\left[\sum_{\ell\in\Lambda|o(\ell)=n}b_\ell\left(\delta_{o(\ell)}^{0*} - \delta_{e(\ell)}^{0*}\right) - \sum_{\ell\in\Lambda|e(\ell)=n}b_\ell\left(\delta_{o(\ell)}^{0*} - \delta_{e(\ell)}^{0*}\right)\right]$$
$$- \sum_{\omega\in\Omega}\sum_{n\in N}\pi_\omega\lambda_{n\omega}^{\mathrm{B}*}\Big[\sum_{\ell\in\Lambda|o(\ell)=n}b_\ell\left(\delta_{o(\ell)}^{0*} - \delta_{o(\ell)\omega}^{*} - \delta_{e(\ell)}^{0*} + \delta_{e(\ell)\omega}^{*}\right)$$
$$- \sum_{\ell\in\Lambda|e(\ell)=n}b_\ell\left(\delta_{o(\ell)}^{0*} - \delta_{o(\ell)\omega}^{*} - \delta_{e(\ell)}^{0*} + \delta_{e(\ell)\omega}^{*}\right)\Big] \le 0. \tag{3.23}$$

Let us consider the following partial Lagrangian function of problem (3.12):

$$\mathcal{L} = \sum_{i\in I}\left(C_iP_i + C_i^{\mathrm{RU}}R_i^{\mathrm{U}} + C_i^{\mathrm{RD}}R_i^{\mathrm{D}}\right) + \sum_{q\in Q}C_qW_q^{\mathrm{S}} + \sum_{\omega\in\Omega}\pi_\omega\Big[\sum_{i\in I}\left(C_i^{\mathrm{U}}r_{i\omega}^{\mathrm{U}} - C_i^{\mathrm{D}}r_{i\omega}^{\mathrm{D}}\right)$$
$$+ \sum_{q\in Q}C_q\left(W_{q\omega} - W_q^{\mathrm{S}} - W_{q\omega}^{\mathrm{spill}}\right) + \sum_{j\in J}V_j^{\mathrm{LOL}}L_{j\omega}^{\mathrm{shed}}\Big] - \sum_{n\in N}\lambda_n^{\mathrm{D}}\Big[\sum_{i\in\Phi_n^I}P_i + \sum_{q\in\Phi_n^Q}W_q^{\mathrm{S}}$$
$$- \sum_{j\in\Phi_n^J}L_j - \sum_{\ell\in\Lambda|o(\ell)=n}b_\ell\left(\delta_{o(\ell)}^{0} - \delta_{e(\ell)}^{0}\right) + \sum_{\ell\in\Lambda|e(\ell)=n}b_\ell\left(\delta_{o(\ell)}^{0} - \delta_{e(\ell)}^{0}\right)\Big]$$
$$- \sum_{\omega\in\Omega}\sum_{n\in N}\gamma_{n\omega}\Big[\sum_{i\in\Phi_n^I}\left(r_{i\omega}^{\mathrm{U}} - r_{i\omega}^{\mathrm{D}}\right) + \sum_{j\in\Phi_n^J}L_{j\omega}^{\mathrm{shed}} + \sum_{q\in\Phi_n^Q}\left(W_{q\omega} - W_q^{\mathrm{S}} - W_{q\omega}^{\mathrm{spill}}\right)$$
$$+ \sum_{\ell\in\Lambda|o(\ell)=n}b_\ell\left(\delta_{o(\ell)}^{0} - \delta_{o(\ell)\omega} - \delta_{e(\ell)}^{0} + \delta_{e(\ell)\omega}\right)$$
$$- \sum_{\ell\in\Lambda|e(\ell)=n}b_\ell\left(\delta_{o(\ell)}^{0} - \delta_{o(\ell)\omega} - \delta_{e(\ell)}^{0} + \delta_{e(\ell)\omega}\right)\Big]. \tag{3.24}$$

Since problem (3.12) is linear and thus convex, $\mathcal{L}$ is minimized subject to the rest of constraints, i.e., constraints (3.12d)–(3.12s), at the optimum. Note that by moving the power balance equations (3.12b) and (3.12c) to the objective function (3.12a) to form the partial Lagrangian function $\mathcal{L}$, the resulting optimization problem {minimize (3.24), subject to (3.12d)–(3.12s)} can be decomposed into appropriate minimization subproblems for any given set of Lagrange multipliers $\{\lambda_n^{\mathrm{D}}, \forall n \in N; \gamma_{n\omega}, \forall n \in N, \forall \omega \in \Omega\}$. In particular, the summation of the following terms, extracted from (3.24),

$$\sum_{n\in N} \lambda_n^{\mathrm{D}} \left[ \sum_{\ell\in\Lambda|o(\ell)=n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0\right) - \sum_{\ell\in\Lambda|e(\ell)=n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{e(\ell)}^0\right) \right]$$
$$- \sum_{\omega\in\Omega} \sum_{n\in N} \gamma_{n\omega} \left[ \sum_{\ell\in\Lambda|o(\ell)=n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{o(\ell)\omega} - \delta_{e(\ell)}^0 + \delta_{e(\ell)\omega}\right) \right.$$
$$\left. - \sum_{\ell\in\Lambda|e(\ell)=n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{o(\ell)\omega} - \delta_{e(\ell)}^0 + \delta_{e(\ell)\omega}\right) \right] \tag{3.25}$$

is minimized subject to constraints (3.12d)–(3.12i) at the optimum.

A solution such that $\delta_n^0 = 0, \forall n \in \Omega$; $\delta_{n\omega} = 0, \forall n \in N, \forall \omega \in \Omega$, is feasible for the minimization subproblem {minimize (3.25), subject to (3.12d)–(3.12i)}, as long as the capacity of transmission lines is non-negative, i.e., $C_\ell^{\max} \geq 0$, $\forall \ell \in \Lambda$. This solution allows us to set the upper bound of expression (3.25) to zero. Therefore, inequality (3.23) holds, and this concludes the proof.

**Theorem 3.2 (Cost recovery in expectation).** Consider the market-clearing procedure (3.12), built on a stochastic programming framework, and the resulting sets of dual variables $\left\{\lambda_n^{\mathrm{D}}, \forall n \in N\right\}$ and $\left\{\left(\lambda_{n\omega}^{\mathrm{B}}, \pi_\omega\right), \forall n \in N, \forall \omega \in \Omega\right\}$. The settlement scheme (3.13) guarantees cost recovery for all market participants in expectation.

*Proof* The settlement scheme (3.13) ensures that both conventional and stochastic producers recover their energy production costs in expectation. Mathematically, this is expressed as follows:

$$C_i P_i^* + C_i^{\mathrm{RU}} R_i^{\mathrm{U}*} + C_i^{\mathrm{RD}} R_i^{\mathrm{D}*} + \sum_{\omega\in\Omega} \pi_\omega \left(C_i^{\mathrm{U}} r_{i\omega}^{\mathrm{U}*} - C_i^{\mathrm{D}} r_{i\omega}^{\mathrm{D}*}\right) - \lambda_{s(i)}^{\mathrm{D}*} P_i^*$$
$$- \sum_{\omega\in\Omega} \pi_\omega \lambda_{s(i)\omega}^{\mathrm{B}*} \left(r_{i\omega}^{\mathrm{U}*} - r_{i\omega}^{\mathrm{D}*}\right) \leq 0, \quad \forall i \in I; \tag{3.26}$$

and

$$\sum_{\omega\in\Omega} \pi_\omega C_q \left(W_{q\omega} - W_{q\omega}^{\mathrm{spill}*}\right) - \lambda_{s(q)}^{\mathrm{D}*} W_q^{\mathrm{S}*}$$
$$- \sum_{\omega\in\Omega} \pi_\omega \lambda_{s(q)\omega}^{\mathrm{B}*} \left(W_{q\omega} - W_q^{\mathrm{S}*} - W_{q\omega}^{\mathrm{spill}*}\right) \leq 0, \forall q \in Q, \tag{3.27}$$

where superscript "$*$" denotes optimal values and $\lambda_{n\omega}^{\mathrm{B}*} = \frac{\gamma_{n\omega}^*}{\pi_\omega}, \forall n \in N, \forall \omega \in \Omega$.

Let us consider again the partial Lagrangian function (3.24). At the optimum, this function is minimized subject to constraints (3.12d)–(3.12s). As stated in the proof for revenue adequacy in expectation, the optimization problem {minimize (3.24), subject to (3.12d)–(3.12s) can be decomposed into appropriate minimization subproblems for any given set of shadow prices $\{\lambda_n^{\mathrm{D}}, \forall n \in N; \gamma_{n\omega}, \forall n \in N, \forall \omega \in \Omega\}$. Specifically, the series of terms extracted from (3.24)

$$C_i P_i + C_i^{\mathrm{RU}} R_i^{\mathrm{U}} + C_i^{\mathrm{RD}} R_i^{\mathrm{D}} + \sum_{\omega \in \Omega} \pi_\omega \left( C_i^{\mathrm{U}} r_{i\omega}^{\mathrm{U}} - C_i^{\mathrm{D}} r_{i\omega}^{\mathrm{D}} \right) - \lambda_{s(i)}^{\mathrm{D}} P_i$$
$$- \sum_{\omega \in \Omega} \gamma_{s(i)\omega} \left( r_{i\omega}^{\mathrm{U}} - r_{i\omega}^{\mathrm{D}} \right), \tag{3.28}$$

and

$$\sum_{\omega \in \Omega} \pi_\omega C_q \left( W_{q\omega} - W_{q\omega}^{\mathrm{spill}} \right) - \lambda_{s(q)}^{\mathrm{D}} W_q^{\mathrm{S}} - \sum_{\omega \in \Omega} \gamma_{s(q)\omega} \left( W_{q\omega} - W_q^{\mathrm{S}} - W_{q\omega}^{\mathrm{spill}} \right), \tag{3.29}$$

are minimized, for all $i \in I$ and for all $q \in Q$, subject to the set of constraints {(3.12k)–(3.12p),(3.12s)} and {(3.12j),(3.12r),(3.12s)}, respectively.

The collection of decision variables such that $P_i = R_i^{\mathrm{U}} = R_i^{\mathrm{D}} = 0, \forall i \in I$ (here, we appeal to assumption A7, according to which conventional producers are fully dispatchable) and $r_{i\omega}^{\mathrm{U}} = r_{i\omega}^{\mathrm{D}} = 0, \forall i \in I, \forall \omega \in \Omega$, constitutes a feasible solution to the minimization subproblem made up of the objective function (3.28) and the group of constraints (3.12k)–(3.12p), and (3.12s). Likewise, the set of decision variables such that $W_q^{\mathrm{S}} = 0$, $\forall q \in Q$, and $W_{q\omega}^{\mathrm{spill}} = W_{q\omega}$, $\forall q \in Q$, $\forall \omega \in \Omega$, is a feasible solution to the minimization subproblem composed of the objective function (3.29) and constraints (3.12j),(3.12r), and(3.12s). This pair of solutions sets the upper bound of expressions (3.28) and (3.29) to zero. Consequently, inequalities (3.26) and (3.27) hold, which concludes the proof.

It is important to underline that the decomposition-based reasoning employed to prove Theorem 3.2 cannot be used, however, to prove cost recovery per scenario due to the day-ahead dispatch variables $P_i$ and $W_q^{\mathrm{S}}$, which link all the scenarios together. This is so because the settlement scheme (3.13) allows power producers to *incur economic losses in some scenarios* as long as they recover their production costs in expectation, i.e., in the long run and under similar conditions.

## Appendix 2: Worst-Case Realization of Uncertain Production in Robust Optimization

In this appendix, we prove that the worst-case uncertainty realization for the adaptive robust optimization problem (3.21) occurs at an extreme point of the polyhedral uncertainty set $\mathcal{W}$ for the deviation of wind power production from its conditional mean forecast.

Let us consider the inner *max-min* problem in (3.21):

$$\underset{\Delta W}{\text{Max}} \ \underset{\Xi^{\text{B}}}{\min} \left[ \sum_{i\in I} \left(C_i^{\text{U}} r_i^{\text{U}} - C_i^{\text{D}} r_i^{\text{D}}\right) + \sum_{q\in Q} C_q \left(\Delta W_q - W_q^{\text{spill}}\right) + \sum_{j\in J} V_j^{\text{LOL}} L_j^{\text{shed}} \right] \tag{3.30a}$$

$$\begin{aligned} \text{s.t.} \ & \sum_{i\in\Phi_n^I} \left(r_i^{\text{U}} - r_i^{\text{D}}\right) + \sum_{j\in\Phi_n^J} L_j^{\text{shed}} + \sum_{q\in\Phi_n^Q} \left(\Delta W_q - W_q^{\text{spill}}\right) \\ & + \sum_{\ell\in\Lambda|o(\ell)=n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{o(\ell)} - \delta_{e(\ell)}^0 + \delta_{e(\ell)}\right) \\ & - \sum_{\ell\in\Lambda|e(\ell)=n} b_\ell \left(\delta_{o(\ell)}^0 - \delta_{o(\ell)} - \delta_{e(\ell)}^0 + \delta_{e(\ell)}\right) = 0 \quad : \lambda_n\,, \quad \forall n \in N, \end{aligned} \tag{3.30b}$$

$$b_\ell \left(\delta_{o(\ell)} - \delta_{e(\ell)}\right) \le C_\ell^{\max} \quad : \sigma_\ell^{\text{U}}, \quad \forall \ell \in \Lambda, \tag{3.30c}$$

$$- b_\ell \left(\delta_{o(\ell)} - \delta_{e(\ell)}\right) \le C_\ell^{\max} \quad : \sigma_\ell^{\text{D}}, \quad \forall \ell \in \Lambda, \tag{3.30d}$$

$$\delta_1 = 0 \quad : \nu, \tag{3.30e}$$

$$r_i^{\text{U}} \le R_i^{\text{U}} \quad : \mu_i^{\text{U}}, \quad \forall i \in I, \tag{3.30f}$$

$$r_i^{\text{D}} \le R_i^{\text{D}} \quad : \mu_i^{\text{D}}, \quad \forall i \in I, \tag{3.30g}$$

$$L_j^{\text{shed}} \le L_j \quad : \epsilon_j^{\text{shed}}, \quad \forall j \in J, \tag{3.30h}$$

$$W_q^{\text{spill}} \le \widehat{W}_q + \Delta W_q \quad : \epsilon_q^{\text{spill}} \quad \forall q \in Q, \tag{3.30i}$$

$$r_i^{\text{U}}, r_i^{\text{D}} \ge 0, \ \forall i \in I; \ W_q^{\text{spill}} \ge 0, \ \forall q \in Q; \ L_j^{\text{shed}} \ge 0, \ \forall j \in J, \tag{3.30j}$$

$$\text{s.t.} \ |\Delta W_q| \le \Delta W_q^{\max}, \quad \forall q \in Q, \tag{3.30k}$$

$$\sum_{q\in Q} \frac{|\Delta W_q|}{\Delta W_q^{\max}} \le \Gamma. \tag{3.30l}$$

Notice that we indicated the dual variables for the inner minimization problem on the right-hand side of the corresponding constraints, preceded by a colon.

First of all, we can reformulate inequalities (3.30k) and (3.30l) as follows:

$$- \Delta W_q^{\max} \le \Delta W_q \le \Delta W_q^{\max}, \qquad \forall q \in Q, \tag{3.31a}$$

$$\Delta W_q = \Delta W_q^+ - \Delta W_q^-, \qquad \forall q \in Q, \tag{3.31b}$$

$$\sum_{q\in Q} \frac{\Delta W_q^+ + \Delta W_q^-}{\Delta W_q^{\max}} \le \Gamma, \tag{3.31c}$$

$$\Delta W_q^+, \Delta W_q^- \ge 0, \qquad \forall q \in Q. \tag{3.31d}$$

It is worth to point out that reformulation (3.31) is linear, on the contrary of (3.30k)–(3.30l).

Denoting the set of dual variables of the inner problem (3.30a)–(3.30j) with $\varXi'$, we can replace the inner minimization problem with its dual (see Appendix B of the book). This renders the following problem:

$$\begin{aligned}
\operatorname*{Max}_{\varDelta W} \max_{\varXi'} & \sum_{n\in N}\Bigg[\sum_{\ell\in\varLambda|o(\ell)=n} b_\ell\left(-\delta^0_{o(\ell)}+\delta^0_{e(\ell)}\right)-\sum_{\ell\in\varLambda|e(\ell)=n} b_\ell\left(-\delta^0_{o(\ell)}+\delta^0_{e(\ell)}\right) \\
& -\sum_{q\in\varPhi^Q_n}\varDelta W_q\Bigg]\lambda_n+\sum_{\ell\in\varLambda}C^{\max}_\ell\left(\sigma^{\mathrm{U}}_\ell+\sigma^{\mathrm{D}}_\ell\right)+\sum_{i\in I}\left(R^{\mathrm{U}}_i\mu^{\mathrm{U}}_i+R^{\mathrm{D}}_i\mu^{\mathrm{D}}_i\right) \\
& +\sum_{j\in J}L_j\epsilon^{\mathrm{shed}}_j+\sum_{q\in Q}\left[\left(\widehat{W}_q+\varDelta W_q\right)\epsilon^{\mathrm{spill}}_q+C_q\varDelta W_q\right] \qquad (3.32a)
\end{aligned}$$

$$\text{s.t. } \lambda_{s(i)}+\mu^{\mathrm{U}}_i\le C^{\mathrm{U}}_i, \quad \forall i\in I, \qquad (3.32b)$$

$$-\lambda_{s(i)}+\mu^{\mathrm{D}}_i\le -C^{\mathrm{D}}_i, \quad \forall i\in I, \qquad (3.32c)$$

$$\lambda_{s(j)}+\epsilon^{\mathrm{shed}}_j\le V^{\mathrm{LOL}}_j, \quad \forall j\in J, \qquad (3.32d)$$

$$-\lambda_{s(q)}+\epsilon^{\mathrm{spill}}_q\le -C_q, \quad \forall q\in Q, \qquad (3.32e)$$

$$\begin{aligned}
& \nu-\left(\sum_{\ell\in\varLambda|o(\ell)=1} b_\ell+\sum_{\ell\in\varLambda|e(\ell)=1} b_\ell\right)\lambda_1+\sum_{\ell\in\varLambda|e(\ell)=1} b_\ell\lambda_{o(\ell)} \\
& +\sum_{\ell\in\varLambda|o(\ell)=1} b_\ell\lambda_{e(\ell)}+\sum_{\ell\in\varLambda|o(\ell)=1} b_\ell\left(\sigma^{\mathrm{U}}_\ell-\sigma^{\mathrm{D}}_\ell\right) \\
& -\sum_{\ell\in\varLambda|e(\ell)=1} b_\ell\left(\sigma^{\mathrm{U}}_\ell-\sigma^{\mathrm{D}}_\ell\right)=0, \qquad (3.32f)
\end{aligned}$$

$$\begin{aligned}
& -\left(\sum_{\ell\in\varLambda|o(\ell)=n} b_\ell+\sum_{\ell\in\varLambda|e(\ell)=n} b_\ell\right)\lambda_n+\sum_{\ell\in\varLambda|e(\ell)=n} b_\ell\lambda_{o(\ell)} \\
& +\sum_{\ell\in\varLambda|o(\ell)=n} b_\ell\lambda_{e(\ell)}+\sum_{\ell\in\varLambda|o(\ell)=n} b_\ell\left(\sigma^{\mathrm{U}}_\ell-\sigma^{\mathrm{D}}_\ell\right) \\
& -\sum_{\ell\in\varLambda|e(\ell)=n} b_\ell\left(\sigma^{\mathrm{U}}_\ell-\sigma^{\mathrm{D}}_\ell\right)=0, \quad \forall n\in N\setminus\{1\}, \qquad (3.32g)
\end{aligned}$$

$$\sigma^{\mathrm{U}}_\ell,\sigma^{\mathrm{D}}_\ell\le 0,\ \forall \ell\in\varLambda;\ \mu^{\mathrm{U}}_i,\mu^{\mathrm{D}}_i\le 0,\ \forall i\in I; \qquad (3.32h)$$

$$\epsilon^{\mathrm{shed}}_j\le 0,\ \forall j\in J;\ \epsilon^{\mathrm{spill}}_q\le 0,\ \forall q\in Q, \qquad (3.32i)$$

$$\text{s.t. } -\varDelta W^{\max}_q\le \varDelta W_q\le \varDelta W^{\max}_q, \quad \forall q\in Q, \qquad (3.32j)$$

$$\varDelta W_q=\varDelta W^+_q-\varDelta W^-_q, \quad \forall q\in Q, \qquad (3.32k)$$

$$\sum_{q\in Q}\frac{\varDelta W^+_q+\varDelta W^-_q}{\varDelta W^{\max}_q}\le \varGamma, \qquad (3.32l)$$

$$\varDelta W^+_q,\varDelta W^-_q\ge 0, \quad \forall q\in Q. \qquad (3.32m)$$

The following observations on model (3.32) are in order.

1. The two *max* operators can be merged. Therefore, (3.32) is a single maximization problem.
2. Constraints (3.32b)–(3.32m) are linear.
3. Objective function (3.32a) is bilinear, owing to the cross products between variables $\Delta W_q$ and $\lambda_n$ as well as $\epsilon_q^{\text{spill}}$.

In view of the observations above, if we fix the variables in $\Xi'$ at their optimal value, model (3.32) boils down to a linear programming problem in the decision variables $\Delta W_q$, constrained by (3.32j)–(3.32m). For any linear program, at least one of the solutions (if it exists) is a vertex of the feasible set. Since the feasible set $\mathcal{W}$ is compact, at least an optimal solution of the bilinear program (3.32) is a vertex of $\mathcal{W}$.

## Exercises

**3.1** Reformulate the auction in Example 3.3 to include two time periods. Enforce ramping limits on the thermal generation units and analyze numerically the impact of such limits on market outcomes. Hint: the reader is advised to consult Sect. 5.3.3.

**3.2** Consider multiple Gaussian distributed wind power production scenarios in the problem of Example 3.3. Analyze numerically the impact of increasing the number of scenarios on market outcomes. Compare these outcomes with those obtained considering solely the average value scenario.

**3.3** Consider just two extreme scenarios (very-high wind production and no wind production), and analyze the outcomes of the auction in Example 3.3. Compare these outcomes with the outcomes obtained considering solely the average value scenario. What happens as scenarios become increasingly extreme?

**3.4** Analyze the market-clearing algorithm in Example 3.3 in a case in which only wind producers are available. Study the behavior of prices, both day-ahead prices and balancing prices.

**3.5** Consider the market-clearing algorithm in Example 3.3, but involving thermal plants with significantly high start-up costs. What happens with the clearing prices (both day-ahead and balancing) in such situation? Hint: you can get inspiration on how to model the start-up cost of a thermal power plant from Sect. 8.2.1 in the book.

**3.6** Consider the market-clearing algorithm in Example 3.3, but involving thermal plants with minimum power outputs. What happens with the clearing prices (both day-ahead and balancing) in such situation? Hint: you can get inspiration from Sect. 5.3.2 for the modeling of capacity limits.

**3.7** Consider the auction in Example 3.3, and solve it for a wide range of values of lost load. Study how market outcomes change as a result of an increasingly high unserved-energy value.

**3.8** Consider wind production offers at non-zero price in the auction of Example 3.3. Analyze numerically how market outcomes change as wind offering prices increase.

**3.9** Reformulate the auction in Example 3.3 to include two time periods involving highly different load levels, and a pumped storage plant. Is the availability of such pumped storage plant beneficial? Analyze how the impact of the pumped storage plant on market outcomes changes as the efficiency of the pumping-turbine cycle increases. Hint: Sect. 5.5 provides insight into how to model a pumped-storage power plant.

**3.10** Reformulate the auction in Example 3.3 to include two time periods involving highly different load levels, and a pumped storage plant. Consider that the transmission line has such a low capacity that often leads to transmission bottleneck. Analyze the ability of the pumped storage plant to alleviate the detrimental effect of transmission bottlenecks.

**3.11** Solve the robust optimization problem (3.20) considered in Example 3.6 for different values of the *budget of uncertainty* in (3.17). Start by enumerating the vertices of the polyhedral uncertainty set. What is the effect of increasing the uncertainty budget on the amount of dispatched reserve?

**3.12** Include constraints of the following type

$$|\Delta W_1 - \Delta W_2| \leq \rho$$

in the definition of the uncertainty set for the dispatch model based on robust optimization presented in Example 3.6. Determine the uncertainty set and enumerate its vertices, then solve the dispatch problem.

**3.13** Reformulate the robust optimization model (3.20) to include two time periods and a pumped storage plant. Consider the two-node system of Example 3.3, which includes only one wind power plant. Introduce intervals for the deviation of wind power production during each time period, and a *budget of uncertainty* to limit the total deviation of energy production over the two periods, similarly to (3.16) and (3.17). Analyze the effect on the robust dispatch of a storage facility with limited capacity.

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