

Wind power in electricity markets and the value of forecasting

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10.1 Introduction

Forecasts are to be used as input to decision-making. Today, when it comes to renewable energy generation, such decisions are increasingly made in a liberalized electricity market environment, where future power generation has to be offered through contracts and auction mechanisms, hence based on forecasts. Taking the example of the European Nordic region, in 2014 around 84% of all electricity exchanges were made through Nord Pool. These markets were mainly designed considering the needs of conventional plants which, due to a limited flexibility, have to settle in advance on their production schedule.

Since renewable energy sources are to eventually participate in market mechanisms under the same rules than that for conventional generators, mismatches between contracted generation and actual deliveries may induce financial penalties. Indeed, the energy production from wind and solar power plants can be predicted with a limited accuracy that degrades with further lead times. This, in addition to uncertainties in market prices, yield uncertain market returns. However, even under such high levels of uncertainty, renewable energy producers may make optimal use of all information available, either in a deterministic or probabilistic form. It is our aim here to describe these processes of decision-making in electricity markets based on forecast information, also allowing to assess the value of various types of forecasts perceived by market participants.

Such problem of participation of renewable energy producers in electricity markets has received increasing attention over the last decade, mainly considering the case of wind power generation. One of the very first work in that field was that of Bathurst et al. (2002), which showed that, even using naive models for wind power forecasts, the expected profit can be increased by means of risk analysis. Consequently, many alternative and complementary proposals were made for obtaining optimal participation strategies and to assess the value of probabilistic forecast information in electricity markets. For instance, the aim of Bremnes (2004) was to show that one could find an optimal quantile forecasts that would be the best forecasts to use for market participation. This naturally justified the idea of further developing probabilistic forecasting methods, so as to be able to obtain such optimal quantiles. Focusing on expected revenue maximization and risk management, Pinson et al. (2007) gave analytical expressions for the optimal amount of contracted energy based on probabilistic forecasts, by

giving focus on the utility function of market participants. As a bridge between forecast quality and value in electricity markets, [Bitar et al. \(2012\)](#) presented an extensive analysis on this topic, highlighting the link between the expected profit and quality measure for the input wind power forecasts.

Our aim here is to describe the framework of market participation for renewable energy producers and to show how forecasts information directly translates to market value for these participants. As a basis for the development and discussion, we will consider the case of wind power producers, though similar developments could be made for solar power. This chapter is structured as follows. [Section 10.2](#) introduced the reader to the basic concepts of competitive electricity markets, focusing on their structure and underlying timeline. These concepts are translated into equations in [Section 10.3](#), by formulating the market revenue of wind power producers. [Section 10.4](#) presents different offering strategies that a market participant may apply, depending on the available information of future wind power production. Then, in [Section 10.5](#), the different trading strategies are tested and compared in a case study. Finally, conclusions are drawn in [Section 10.6](#).

10.2 Electricity market context

Over the last decades, power systems moved from a centralized organization to new frameworks that aim to enhance competition. Initially, state-owned and vertically integrated companies were in charge of the management of the whole power system, from generation to retail. Then, aiming in privatizing the electricity supply sector and attracting new investors, deregulation processes have occurred worldwide. The key feature of the processes was the separation between activities of generation, transmission, distribution and retail, while banning the vertical integration among different sectors. Competition has been promoted mainly in generation and retail, while the transmission sector is still a natural monopoly, because of the prohibitive investment cost of transmission lines. The essential role of operation and management of the transmission grid is carry out by noncommercial entities, called transmission system operator (TSO) in Europe and independent system operator (ISO) in the United States.

The aim of this section is to introduce the reader to the concepts of electricity markets and their timeline. It is structured as follows. [Section 10.2.1](#) presents the general structure of electricity markets, distinguishing between futures markets and electricity pools. [Sections 10.2.2](#), [10.2.3](#) and [10.2.4](#) present the main trading stages of an electricity pool, i.e., day-ahead, intraday, and balancing markets, respectively.

10.2.1 Overview of various markets and their timeline

In electricity markets, two different trading floors are typically available, depending on the proximity of the trading. Medium/long-term markets (i.e., futures markets) allow trading on long-term horizons. The market participants can trade both physical and financial products, those by mean of forward contracts and options. A forward contract is signed between a seller who undertakes to produce a certain amount of energy

and a buyer who consumes that energy. Forward contracts are usually standard products, e.g., base load contracts include all the hours of the contracted time span, whereas peak load contracts only hours with high demand, typically from 8 a.m. to 7 p.m. of working days. Forward contracts can be associated with options. An option allows the buyer to decide after the agreement whether to benefit or not of the forward contract.

Differently, short-term markets (i.e., electricity pools) allow the trading of electricity on a daily and hourly horizon. Generally, they include several trading floors, i.e., day-ahead, intraday adjustment, and balancing market. Power producers can participate both in futures market and electricity pools. Usually a part of the capacity of thermal plants is contracted in medium/long contracts, since these ensure fixed revenues for the producers, avoiding the uncertainties of short-term trading. Remaining capacity is usually contracted in electricity pools. Contrariwise, renewable energy plants, e.g., wind farms and solar plants, have a stochastic nature and can be predicted with a limited accuracy. Therefore, they are not suitable for long-term contracts, as it is hard to guarantee a certain level of production, long time before the real-time operation. In this section we will focus on the participation of stochastic producers in electricity pools, neglecting futures market.

10.2.2 Day-ahead market mechanism

The day-ahead market hosts transactions for selling and buying electric energy 1 day prior to delivery day. Buyers and sellers submit their offers to a market operator, which acts as central counterpart. A market offer includes a quantity of energy and the price at which the market participant is willing to contract this amount of energy. In case of sell/buy offers/bids, the price denotes the minimum/maximum price at which the seller/buyer is willing to provide/consume electricity. All sell offers are ranked in price-increasing order, to build a cumulative selling curve. The cumulative buying curve is carried out similarly, by ordering buy bids in price-decreasing order. The intersection between the two curves identifies the market-clearing price and volume. All the offers on the left of the clearing volume are accepted, while all the offers on the right are rejected. Accepted offers/bids are, generally, remunerated at the clearing price, disregarding the offer/bid price.

The day-ahead market gate closure occurs the day before the delivery day, usually at 12 a.m. The day-ahead market includes 24 separate auctions, one per each hour of the day. After the gate closure the market operator clears the market and informs each seller/buyer of their production/consumption schedule.

10.2.3 Intraday adjustment and continuous trading

The intraday market is the market for sale/purchase energy during the day of delivery. It opens after the day-ahead market gate closure and closes from hours to minutes prior to energy delivery. Intraday market can be a useful tool for market participants to adjust their positions. Conventional producers may access the market to fix an infeasible schedule, since intertemporal constraints (e.g., ramping constraints) cannot, usually, be directly included in the market offers. On the other hand, stochastic producers can use this additional trading floor to modify their market position as their forecasts

may be more accurate, closer to real-time operation. Trading in the intraday market is, generally, continuous. The negotiation mechanism is based on automatic matching of demand bids and supply offers, which allows a continuous submission of new offers/bids during the whole session. Similarly to the day-ahead market, the intraday market is managed by the market operator.

10.2.4 Balancing market mechanism

The balancing market is the last stage for trading electric energy. It plays an essential role, as production and consumption levels must match during the operation of electric power systems. This is a key feature, given that, at the moment, storage of large quantities of electric energy is not economically convenient.

Balancing markets are generally single-period markets, i.e., a separate session for each trading period. They allow the possibility to trade, in addition to electric energy *ancillary services* (e.g., voltage control) needed to maintain the stability of the electric system.

Conventional producers, usually, participate at the balancing market for providing regulating power, both in upward (i.e., increasing production) and downward (i.e., decreasing production) directions. Differently, stochastic producers, access the balancing stage to settle deviations from contracted production. These deviations are priced differently, depending on the pricing imbalance system of the market. We can distinguish between single-price imbalance system and two-price imbalance system.

In a single-price imbalance system the deviations are settled at the market price, disregarding the sign of producer imbalances (i.e., excess or lack of production). As a general rule, the balancing price is higher/lower than the day-ahead market price if the system is in up/downregulation, i.e., when a lack/excess of power production occurs. This price settlement leads to arbitrage opportunities for power producers. For instance, when the producer and the system imbalance are of opposite sign (i.e., when the power producer deviation helps to reduce the whole system imbalance), the producer receives a bonus for its deviation. Conversely, when the two imbalances occur in the same direction, the producer is penalized. Fig. 10.1a shows the arbitrage opportunity as function of the producer imbalance and the system status (up- or downregulation).

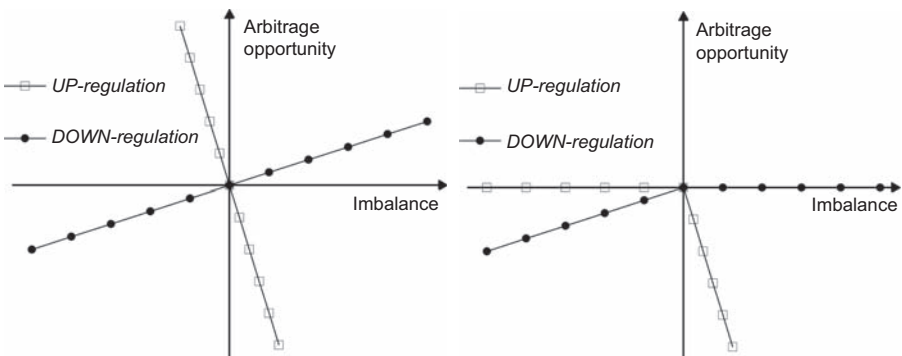


Figure 10.1 Arbitrage opportunity for different imbalance settlement schemes.

In a dual-price imbalance system, deviations from the production schedule are traded at different prices, conditional upon the imbalance sign. When the deviation of the producer and system imbalance occur in opposite directions (i.e., the producer helps in reducing the system imbalance), its deviation is traded at the day-ahead market price, avoiding possible bonuses. Conversely, when the two imbalances occur in the same direction, the deviation of the producer is priced at the balancing market price (i.e., usually penalized). Fig. 10.1b shows the absence of arbitrage opportunity in a two-price imbalance system.

10.3 From market revenue to forecast value

In this section we will translate into equations the qualitative concepts of electricity pools introduced in Section 10.2. This section is structured as follows. Section 10.3.1 presents the assumptions that define the framework of the trading problem. The formulation of market revenues are developed in Section 10.3.2, while Section 10.3.3 presents a performance parameter that helps in comparing different trading strategies.

10.3.1 Assumptions

Let us introduce some assumptions that allow us to simplify the structure of the market and define the framework of the analysis.

- A1 The wind power producer trades only in the day-ahead market and in the balancing market, while the intraday trading is neglected.
- A2 The producer is risk neutral, i.e., he aims to maximize his profit, disregarding possible huge losses.
- A3 The producer is price taker, i.e., he cannot influence market equilibrium with his behavior.
- A4 The producer offer its energy generation at zero marginal cost.
- A5 The producer is provided with the cumulative distribution function $F_E(\cdot)$ of future wind power production E . This writes

$$F_E(e) = \mathbb{P}(E \leq e) = \int_0^e f_E(x) dx, \quad (10.1)$$

where $f_E(\cdot)$ is the probability density function

$$f_E(x) = \frac{\mathbb{P}(x < E < x + dx)}{dx}. \quad (10.2)$$

Assumptions A1, A3, and A5 are necessary for obtaining an analytical solution of the wind power producer profit maximization problem (Morales et al., 2013) and have been used in several studies (Bremnes, 2004; Pinson et al., 2007; Zugno et al., 2013a). As alternative, the problem can be formulated as a stochastic optimization problem (Morales et al., 2010; Rahimiyan et al., 2011), which can be easily extended by including the intraday trading. On the other hand, assumptions A2 and A4 are not

strictly required for the analytical formulation of the problem, but they help the reader in focusing on the fundamental concepts. However, risk aversion can be added subsequently, as [Zugno et al. \(2013a\)](#) show in their work, by anchoring the market quantity to the expected value of wind power production, either in the decision space or the probability space. When the problem is formulated using stochastic optimization, risk aversion can be included by adding the Conditional Value At Risk in the objective function ([Morales et al., 2010](#); [Rahimiyan et al., 2011](#)).

10.3.2 Formulation of market revenues

Let λ and q denote prices and energy quantities, respectively. Then, let D and B be the subscripts denoting the day-ahead and the balancing market, respectively. Moreover, let us denote with t the time of placing offers (i.e., day-ahead market closure) and with k the time delay between t and the real-time operation. In each trading period k the wind power producer can submit an offer in the day-ahead market, specifying the amount of energy q^D he or she is willing to contract. The market revenue ρ_k of a market participant is computed as

$$\rho_k = \rho_k^D + \rho_k^B = \lambda_k^D q_k^D + \lambda_k^B q_k^B. \quad (10.3)$$

The quantities contracted in the two market stages, i.e., q_k^D and q_k^B , are linked by

$$q_k^B = E_k - q_k^D, \quad (10.4)$$

where E_k is the wind power production measured during the hourly interval k . [Eq. \(10.4\)](#) shows that the quantity contracted at the balancing stage, i.e., q_k^B , is not a decision variable. Indeed, E_k is not under control of the power producer and q_k^D is fixed at balancing stage. By rearranging [Eq. \(10.3\)](#),

$$\rho_k = \lambda_k^D q_k^D + \lambda_k^B (E_k - q_k^D) = \lambda_k^D E_k - (\lambda_k^B - \lambda_k^D)(q_k^D - E_k) = \lambda_k^D E_k - L_k. \quad (10.5)$$

The first term of [Eq. \(10.5\)](#), i.e., $\lambda_k^D E_k$, is the product between the day-ahead market price and the effective wind power production during interval k . It represents the profit that the producer may have in case of perfect information, i.e., if he could know at t the wind production at $t + k$. Differently, the term L_k represents the penalties for imbalance creation, and it is always positive (in a dual-price settlement scheme). Therefore, $\lambda_k^D E_k$ is the maximum profit that can be reached by the wind power producer. Note that all these considerations are true only for a dual-price settlement scheme (L_k can be either positive or negative in a single-pricing scheme).

10.3.3 Linkage to forecast value

Let us introduce a performance parameter that represents a coherent measure to access the effectiveness of a market offering strategy. As shown in [Section 10.3.2](#), the profit

of the power producer can be seen as the sum between the profit in case of perfect information ($\lambda_k^D E_k$), which represents the upper limit and the imbalance penalty term (L_k). The first term is the maximum profit that could be reached by the power producer in each trading period, and it can be used as reference. The performance ratio γ_{t+k} of an offering strategy during interval k , is defined as

$$\gamma_{t+k} = \frac{\rho_k}{\lambda_k^D E_k} = 1 - \frac{L_k}{\lambda_k^D E_k}. \quad (10.6)$$

However, the result of specific strategy in a single time interval may not be the statistically significant. Therefore, the performance ratio γ is usually evaluated over N days, i.e.,

$$\gamma = \sum_{t=1}^N \sum_{k=13}^{36} \gamma_{t+k}. \quad (10.7)$$

The upper limit of γ is 1, which is obtained when the power producer is never penalized for its imbalances, during the N days of reference. Note that γ is not bounded inferiorly. As for [Section 10.3.2](#), all these considerations are true only for a dual-price settlement scheme.

10.4 Formulation of offering strategies

In this section we will develop different trading strategies of a wind power producer who aims to maximize his or her market revenue. The section is developed as follows. [Section 10.4.1](#) presents some basic offering strategies that can be used as a benchmark. Then, [Sections 10.4.2 and 10.4.3](#) present optimal offering strategies for the single-price and the dual-price imbalance settlement, respectively.

10.4.1 Benchmark offering strategies

At the time of placing bids, the wind power producer does not know exactly the future amount of wind power production. However, he can be provided with forecasts, either deterministic or probabilistic. Depending on the information that the producer may have available at t , we can identify three different categories of offering strategy. In category 1 we include the strategies that a power producer may develop when no forecasts of E_k are available. Then, in category 2 we consider the offering strategies based on point forecasts of E_k . Finally, in category 3, we include offering strategies based on probabilistic forecasts of E_k . An example of probabilistic forecasts and point forecasts is shown in [Fig. 10.2](#).

Strategy of category 1 should not require forecasts of E_k . However, the power producer can use past observations to develop a naive forecast model. We consider two different strategies in this category. The first, called Strategy 1A, proposes to offer at day-ahead stage the average value of wind power production over the previous

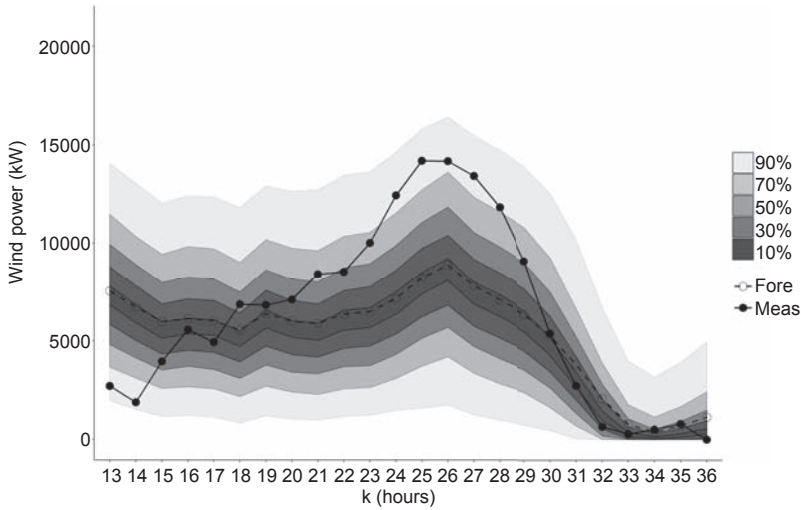


Figure 10.2 Probabilistic and point forecasts for June 7, 2014.

years. A capacity factor c_f is evaluated, as the ratio between average measured production and total installed capacity (\bar{E}). Then, the value of q_k^D is computed as

$$q_k^D = c_f \bar{E} \tag{10.8}$$

The second, i.e., Strategy 1B, suggests to use wind power production measured at t , as a representative for future values of E_k . This writes

$$q_k^D = E_t \tag{10.9}$$

When more information is available to the power producer, he or she may try to exploit this additional information to increase his expected market profit. In case of Strategy 2, this additional information is provided in the form of deterministic (point) forecasts. Point forecasts represent the expected value of future wind power production, i.e., $\hat{E}_k = \int_0^{\bar{E}} x f_{E_k}(x) dx$. In Strategy 2 the market quantity offer is

$$q_k^D = \hat{E}_k \tag{10.10}$$

Example 4.1. Let us consider an onshore wind farm of 21 MW located in Denmark, having a capacity factor c_f of 24%. Then, let us suppose that at 12 a.m. of June 6, 2014, the producer has to trade in the day-ahead market the wind power production for the following day. Following Strategy 1A would result in offering

$$q_k^D = 5004 \text{ kW}, \quad \forall k \in [13, 36], \tag{10.11}$$

Table 10.1 Point forecasts of wind power production for June 7, 2014

k	13	14	15	16	17	18	19	20
$\hat{E}_{t+k t}$ [kW]	7542	6720	5981	6138	6033	5539	6387	5994
k	21	22	23	24	25	26	27	28
$\hat{E}_{t+k t}$ [kW]	5895	6354	6484	7153	8219	8883	7806	7094
k	29	30	31	32	33	34	35	36
$\hat{E}_{t+k t}$ [kW]	6382	5290	3833	2084	788	386	694	1142

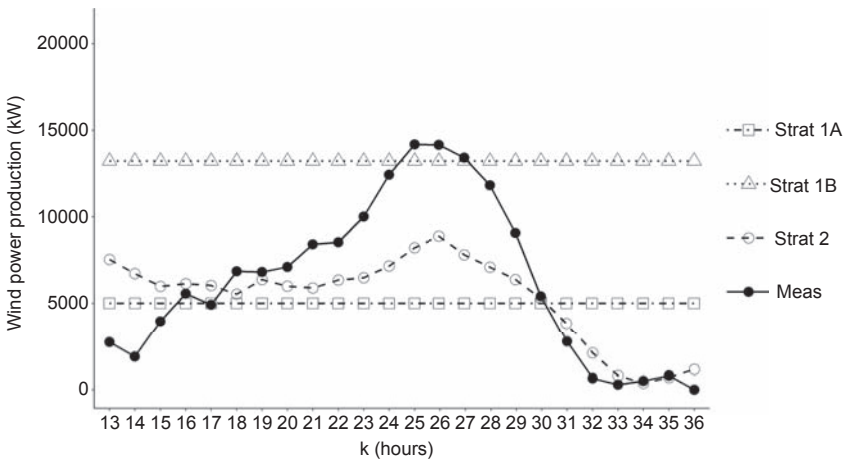


Figure 10.3 Different offering strategies for June 7, 2014.

while under strategy 1B, market offer would be

$$q_k^D = 13222 \text{ kW}, \quad \forall k \in [13, 36] \tag{10.12}$$

Differently, Strategy 2 required the point forecasts of E_k , shown in [Table 10.1](#).

The energy quantities contracted by the three offering strategies are graphically shown in [Fig. 10.3](#). Moreover, [Table 10.2](#) gives the exact value of market offers for the first 8-hourly interval.

10.4.2 Trading strategies in a single-price imbalance system

In a single-price imbalance system, deviations from day-ahead contracted schedule are priced at the balancing market price, disregarding the sign of the imbalance. Let us introduce the *expected monetary value* (EMV), defined in decision theory as the

Table 10.2 Day-ahead market offers under different strategies, expressed in kW

k	13	14	15	16	17	18	19	20
Strategy 1A	5000	5000	5000	5000	5000	5000	5000	5000
Strategy 1B	13,222	13,222	13,222	13,222	13,222	13,222	13,222	13,222
Strategy 2	7542	6720	5981	6138	6033	5539	6387	5994

expected profit due to a specific decision. The EMV for the wind power producer can be obtained by computing the expectation of Eq. (10.5), i.e.,

$$\mathbb{E}[\rho_k] = \lambda_k^D \mathbb{E}[E_k] + (\lambda_k^B - \lambda_k^D) (\mathbb{E}[E_k] - q_k^D). \quad (10.13)$$

The market prices (λ_k^D and λ_k^B) are initially assumed to be known at time t . Then, this strong assumption will be relaxed. When trying to maximize Eq. (10.13), three possible events may occur,

1. If $\lambda_k^B > \lambda_k^D$, the term $(\lambda_k^B - \lambda_k^D)$ of Eq. (10.13) is positive. Therefore, the producer increases his or her profit in expectation when he or she has an excess of production at the balancing stage. This leads the producer to sell nothing at the day-ahead stage and to wait for selling his or her whole production at the balancing one ($q_k^{D*} = 0$);
2. If $\lambda_k^B < \lambda_k^D$, the term $(\lambda_k^B - \lambda_k^D)$ of Eq. (10.13) is negative. In this situation the producer offers all his capacity (\bar{E}) in the day-ahead market ($q_k^{D*} = \bar{E}$). Doing that, he maximizes the volume of the negative imbalance than he can settle at the balancing stage ($q_k^B = E_k - \bar{E}$);
3. If $\lambda_k^B = \lambda_k^D$, the term $(\lambda_k^B - \lambda_k^D)$ of Eq. (10.13) is null and any decision leads to the same expected profit.

All the three events lead to trivial solutions. When market prices are given, the optimal market offer q_k^{D*} is only determined by arbitrage possibility, since $\mathbb{E}[E_k]$ does not influence its optimal level.

Let us now relax the assumption of known and deterministic prices. Indeed, we consider both the day-ahead and the balancing market prices as random variables with known probability density function. The EMV is obtained by introducing price expectation in Eq. (10.13), i.e.,

$$\mathbb{E}[\rho_k] = \mathbb{E}[\lambda_k^D] \mathbb{E}[E_k] + \mathbb{E}[\lambda_k^B - \lambda_k^D] (\mathbb{E}[E_k] - q_k^D). \quad (10.14)$$

As for deterministic prices, let us distinguish between three possible situations:

1. If $\mathbb{E}[\lambda_k^B - \lambda_k^D] > 0$, optimal bid is $q_k^{D*} = 0$;
2. If $\mathbb{E}[\lambda_k^B - \lambda_k^D] < 0$, optimal bid is $q_k^{D*} = \bar{E}$; and
3. If $\mathbb{E}[\lambda_k^B - \lambda_k^D] = 0$, each bid yields to the same EMV.

Even when the market prices are considered as random variables, the optimal level of q_k^{D*} is not influenced by the forecast of wind power production. Indeed, this settlement scheme pushes the producer to offer, at day-ahead stage, nothing or the whole capacity, depending on the expected value of market prices.

10.4.3 Trading strategies in a two-price imbalance system

In a two-price imbalance settlement scheme, deviations from the contracted generation schedule are priced differently depending on the mutual sign of the producer's imbalance and the system's imbalance. Therefore, we introduce two artificial market prices that allow to represent the imbalance sign of the system. The first, called upregulation price (λ_k^{UP}), is equal to the balancing market price when upregulation energy is required and to the day-ahead market one otherwise. This writes

$$\lambda_k^{UP} = \begin{cases} \lambda_k^B & \text{if } \lambda_k^B \geq \lambda_k^D \\ \lambda_k^D & \text{if } \lambda_k^B < \lambda_k^D \end{cases} \quad (10.15)$$

Conversely, downregulation price (λ_k^{DW}) is equal to the balancing market price when the system needs downregulation energy and to the day-ahead market one otherwise, i.e.,

$$\lambda_k^{DW} = \begin{cases} \lambda_k^D & \text{if } \lambda_k^B \geq \lambda_k^D \\ \lambda_k^B & \text{if } \lambda_k^B < \lambda_k^D \end{cases} \quad (10.16)$$

Then, let us define the differential prices ψ_k^{UP} and ψ_k^{DW} as the difference between the up- and downregulation market prices and the day-ahead market price, respectively. This writes

$$\psi_k^{UP} = \lambda^{UP} - \lambda_k^D \geq 0 \quad (10.17)$$

$$\psi_k^{DW} = \lambda^{DW} - \lambda_k^D \leq 0 \quad (10.18)$$

These differential prices allow to simplify the notation of the imbalance cost λ_k^D of the stochastic producer. Negative imbalances are priced at ψ_k^{UP} , while positive imbalances at ψ_k^{DW} . Therefore, λ_k^D is evaluated as

$$L_k = \begin{cases} \psi_k^{UP} (q_k^D - E_k) & \text{if } q_k^D \geq E_k \\ \psi_k^{DW} (q_k^D - E_k) & \text{if } q_k^D < E_k \end{cases} \quad (10.19)$$

The value of L_k is always positive, indeed.

1. If $q_k^D > E_k$, the terms $(q_k^D - E_k)$ and ψ_k^{DW} are both negative. This yields to a positive value of L_k ;
2. If $q_k^D < E_k$, the terms $(q_k^D - E_k)$ and ψ_k^{UP} are both positive. As before, this yields to a positive value of L_k ;
3. If $q_k^D = E_k$, trivial.

The EMV can be written as the difference of two terms, i.e., $\lambda^D \mathbb{E}[E_k]$ and $\mathbb{E}[L_k]$, which are respectively the expected profit in case of perfect information and the expected opportunity loss called EOL. The EOL is the loss of profit introduced by uncertainties, i.e., all the stochastic processes that the power producer can predict only with a limited accuracy.

As stated before, the term $\lambda^D E_k$ is not under control of the wind power producer. Therefore the problem of maximizing the EMV is equivalent to minimizing the EOL. Let us now write explicitly the expectation of the EOL by moving to the probability space of wind power production. The L_k is a piece-wise function, where E_k is a discontinuity point. Therefore, the integral form of the EOL is the sum of two integrals: the first (upregulation term) is defined for $E_k \in [0, q_k^D]$, while the second (downregulation term) for $E_k \in [q_k^D, \bar{E}]$. Let us, initially, assume known and deterministic market prices. Under such assumption, the EOL is

$$\text{EOL}_k = \int_0^{q_k^D} \psi_k^{UP} (q_k^D - x) f_{E_k}(x) dx + \int_{q_k^D}^{\bar{E}} \psi_k^{DW} (q_k^D - x) f_{E_k}(x) dx, \quad (10.20)$$

where $f_{E_k}(\cdot)$ is the probability density function of the wind power production. The minimum of the EOL can be computed by deriving Eq. (10.20) with respect to q_k^D and by setting it to 0. The solution yields to the following expression for the optimal quantile q_k^{D*}

$$q_k^{D*} = F_{E_k}^{-1} \left(\frac{|\psi_k^{DW}|}{|\psi_k^{DW}| + \psi_k^{UP}} \right), \quad (10.21)$$

where $F_{E_k}^{-1}(\cdot)$ is the inverse of the cumulative density function of wind power production.

Let us now relax the assumption of deterministic prices. The extended integral form of the EOL to the probability space of stochastic prices is

$$\begin{aligned} \text{EOL}_k &= \int_0^{q_k^D} \int_0^{\infty} y (q_k^D - x) f_{E_k}(x) f_{\psi_k^{UP}}(y) dx dy \\ &\quad + \int_{q_k^D}^{\bar{E}} \int_{-\infty}^0 y (q_k^D - x) f_{E_k}(x) f_{\psi_k^{DW}}(y) dx dy. \end{aligned} \quad (10.22)$$

Similar to Eq. (10.21), the optimal level of the day-ahead contracted energy is (Bremnes, 2004; Linnet, 2005)

$$q_k^{D*} = F_{E_k}^{-1} \left(\frac{|\widehat{\psi}_k^{DW}|}{|\widehat{\psi}_k^{DW}| + \widehat{\psi}_k^{UP}} \right) = F_{E_k}^{-1}(\alpha_{t+k}^*) \tag{10.23}$$

where

$$\widehat{\psi}_k^{UP} = \int_0^\infty y f_{\psi_k^{UP}}(y) dy \tag{10.24}$$

$$\widehat{\psi}_k^{DW} = \int_{-\infty}^0 y f_{\psi_k^{DW}}(y) dy \tag{10.25}$$

Eq. (10.23) shows that the behavior of a strategic producer in a dual-price settlement is to overestimate future wind power production when $|\widehat{\psi}_k^{DW}| > \widehat{\psi}_k^{UP}$, and underestimate it if $|\widehat{\psi}_k^{DW}| < \widehat{\psi}_k^{UP}$. However, the balancing market prices are hard to forecast with high accuracy, since they generally show a high degree of stochasticity. Then, we want to analyze how the quality of wind power forecasts may affect the optimal value of the EOL. First, we compute the value of the EOL when $q_k^D = q_k^{D*}$ (Bitar et al., 2012)

$$EOL_k^* = -\widehat{\psi}_k^{UP} \int_0^{q_k^{D*}} x f_{E_k}(x) dx + |\widehat{\psi}_k^{DW}| \int_{q_k^{D*}}^{\bar{E}} x f_{E_k}(x) dx \tag{10.26}$$

Then, we apply the change of variable $y = F_{E_k}(x)$, thus leading to

$$EOL_k^* = -\widehat{\psi}_k^{UP} \int_0^{\alpha_{t+k}^*} F_{E_k}^{-1}(y) dy + |\widehat{\psi}_k^{DW}| \int_{\alpha_{t+k}^*}^1 F_{E_k}^{-1}(y) dy \tag{10.27}$$

Fig. 10.4 provides a graphical interpretation of the two integrals of Eqs. (10.27), when for sake of clarity, the wind farm capacity \bar{E} , has been set to 1 MW. $I1$ and $I2$ of Fig. 10.4 are computed as

$$I1 = \int_0^{\alpha_{t+k}^*} F_{E_k}^{-1}(y) dy = \int_0^{q_k^{D*}} x f_{E_k}(x) dx \tag{10.28a}$$

$$I2 = \int_{\alpha_{t+k}^*}^1 F_{E_k}^{-1}(y) dy = \int_{q_k^{D*}}^{\bar{E}} x f_{E_k}(x) dx \tag{10.28b}$$

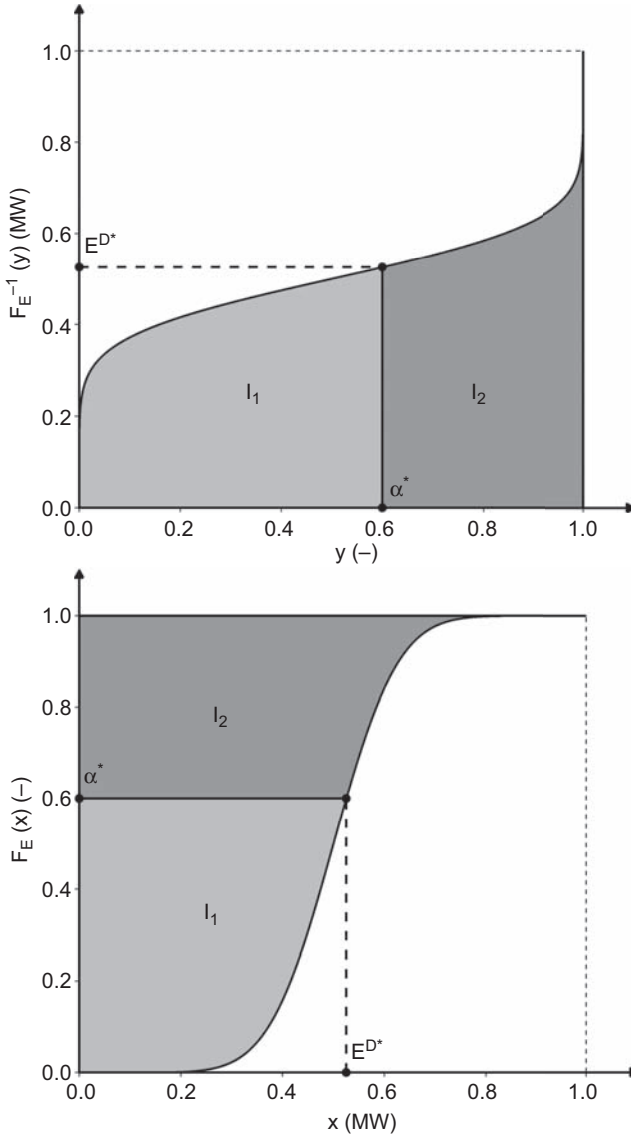


Figure 10.4 Integral form interpretation of $EOL^*_{t+k} (\bar{E} = 1MW)$.

For a complete and wider analysis of this topic, we refer the interested reader to [Bitar et al. \(2012\)](#).

Example 4.2. Let us consider that the expectations of the differential prices (in \euro/MWh) are:

$$\hat{\psi}^{UP} = 10, \hat{\psi}^{DW} = -10 \tag{10.29}$$

which lead to a nominal level of the optimal quantile of 0.5 ($\alpha^* = 0.5$). Let us now consider 2 different cumulative distribution functions with the same expected value but different variance:

$$F_E^a(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x > 1 \\ x, & \text{otherwise} \end{cases} \quad (10.30)$$

$$F_E^b(x) = \begin{cases} 0, & \text{if } x < 0.25 \\ 1, & \text{if } x > 0.75 \\ \frac{x - 0.25}{0.5}, & \text{otherwise} \end{cases} \quad (10.31)$$

In both cases the optimal quantile is $E^{D^*} = 0.5$ MW. The integrals I_1 and I_2 are straightforward to compute for uniform distributions and led to the following results:

$$I_1^a = 0.125 \text{ MWh} \quad I_1^b = 0.1875 \text{ MWh} \quad (10.32a)$$

$$I_2^a = 0.375 \text{ MWh} \quad I_2^b = 0.3125 \text{ MWh} \quad (10.32b)$$

The reader may refer to [Fig. 10.5](#) for a graphical interpretation of the results. We can now compute the optimal EOL for the two cases:

$$\text{EOL}^{a*} = -\psi^{\text{UP}} I_1^a + |\hat{\psi}^{\text{DW}}| I_2^a = 2.50 \text{ €/MWh} \quad (10.33a)$$

$$\text{EOL}^{b*} = -\psi^{\text{UP}} I_1^b + |\hat{\psi}^{\text{DW}}| I_2^b = 1.25 \text{ €/MWh} \quad (10.33b)$$

10.5 Test case exemplification

A test case based on real data can help in understanding concretely the differences between different offering strategies and the value of wind power forecasts, either deterministic or probabilistic.

10.5.1 Experimental setup

The test case analyses different possible offering strategies of a 21-MW wind farm located in Western Denmark. For this wind farm, both point forecasts and probabilistic forecasts, in form of 19 quantiles and measured values of wind power production, are available for the whole 2014. Market prices of zone *DK1* of Nord Pool Spot are used

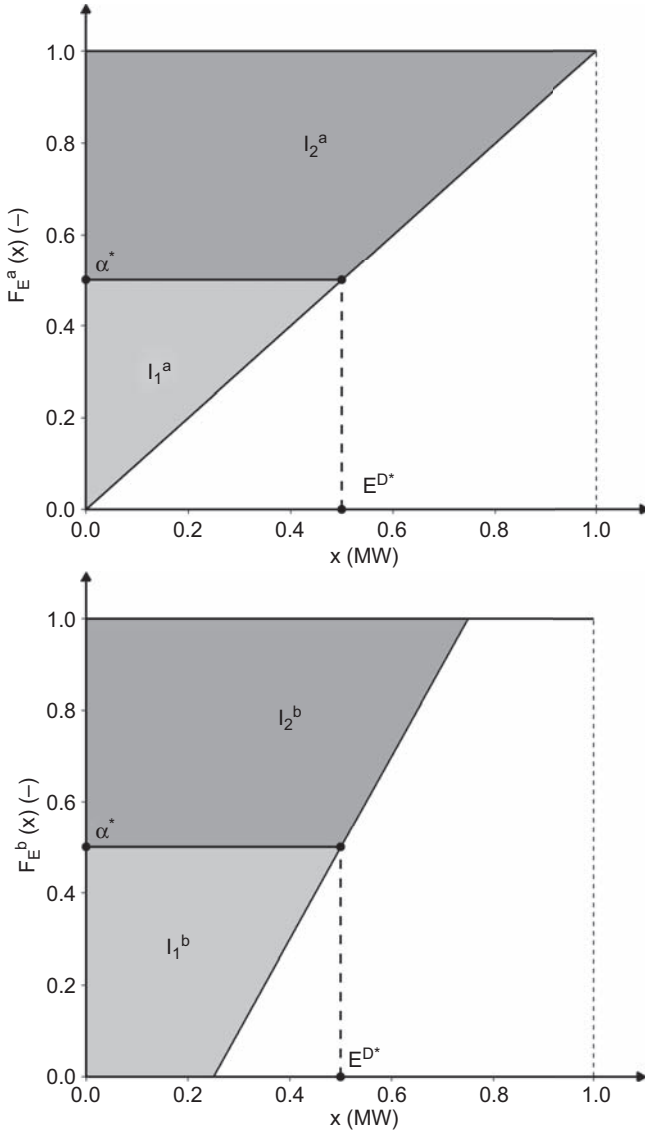


Figure 10.5 Integral form interpretation of Example 4.2.

for both day-ahead and balancing market prices. All the assumptions of [Section 10.3.1](#) are still valid. Five different offering strategies are considered for the test case:

- Strategy 1: naive forecasts
 - Strategy 1A (*capacity factor model*)
 - Strategy 1B (*persistence model*)
- Strategy 2: point forecasts
- Strategy 3: probabilistic forecasts

The producer has to develop a strategy for estimating the nominal level α_{t+k} of the optimal quantile:

$$\alpha_{t+k} = \frac{|\hat{\psi}_k^{DW}|}{|\hat{\psi}_k^{DW}| + \hat{\psi}_k^{UP}} \tag{10.34}$$

The expected price for up(down)-regulation can be evaluated as the product between the expected price, known that the system is in up(down)-regulation and the probability of system to be in up(down)-regulation. In this test case we consider two simple models for the estimation of α_{t+k} :

- Strategy 3A: The producer uses the historical market prices of the previous year to evaluate of the optimal α_{t+k} . Bimonthly averages for each of the 24 trading hours are obtained by analyzing market data of 2013 (*fixed average*).
- Strategy 3B: Optimal α is estimated from the last n_{ts} market prices available at the moment of placing bids, for each specific trading hour. In this test case we chose a time span of 30 days (*moving average*).

Fig. 10.6 shows the different values of α over a year for the sixth trading interval (from 5 to 6 a.m.) for strategy 3A (solid line) and 3B (dashed line).

For each strategy we evaluated the total revenue over a year, called ρ . The total revenue, following the approach of Section 10.3.2, can be split into two terms: the revenue in case of perfect information, called ρ_{PI} , and the imbalance penalties term, ρ_L . The difference between ρ_{PI} , which is common among each trading strategy, and ρ_L gives the total revenue. The values of ρ_{PI} and ρ_L for the whole year ($t \in [1,365]$ and $k \in [13,36]$) can be computed as follow:

$$\rho_{PI} = \sum_{t=1}^{365} \sum_{k=13}^{36} \lambda^D E_k, \tag{10.35}$$

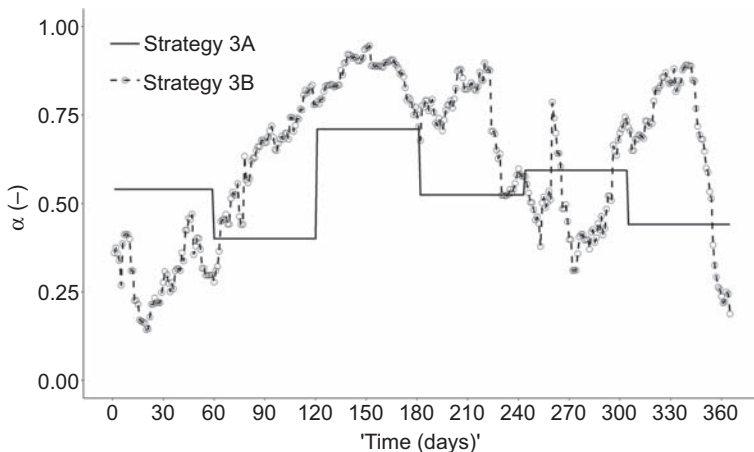


Figure 10.6 Coefficient α estimated by mean of strategies 3A and 3B, sixth trading hour.

$$\rho_L = \sum_{t=1}^{365} \sum_{k=13}^{36} L_{t+k} \quad (10.36)$$

10.5.2 Trading results and value of various forecasts

The cumulative values of the imbalance penalties term ρ_L for the different strategies are displayed in Fig. 10.7.

The results of each trading strategy are shown in Fig. 10.8 and Table 10.3.

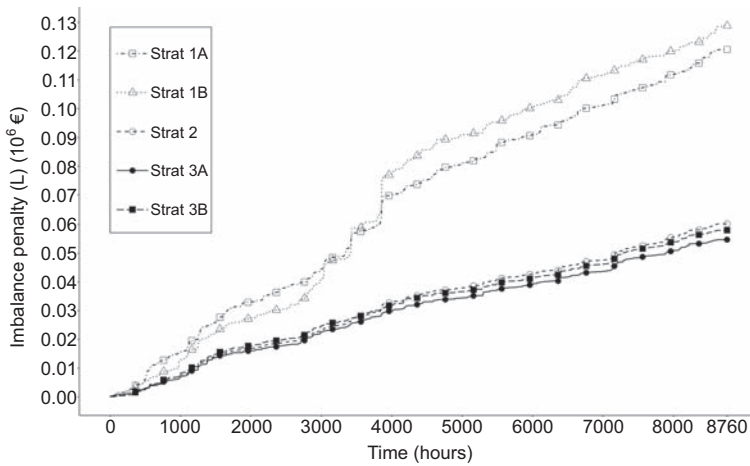


Figure 10.7 Cumulative imbalance penalties for different strategies.

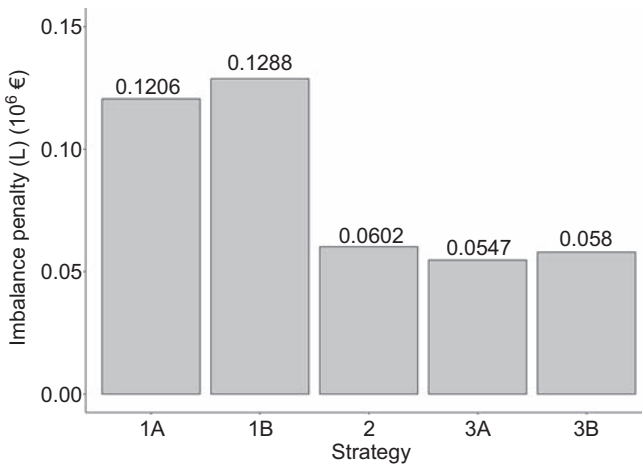


Figure 10.8 Bar plot of imbalance penalties for different strategies.

Table 10.3 Revenues and performance ratio of each trading strategy

Strategy	ρ_{PI}	ρ_L	ρ	γ
	[10 ⁶ €]	[10 ⁶ €]	[10 ⁶ €]	[%]
1A	1.256	0.121	1.135	90.40
1B	1.256	0.129	1.127	89.74
2	1.256	0.060	1.196	95.20
3A	1.256	0.055	1.201	95.64
3B	1.256	0.058	1.198	95.38

Analyzing the results, the improvement that forecasts give in terms of reducing the imbalance penalties term is clear. We notice that the term ρ_L is more than the double in case of naive forecast (Strategies 1A and 1B) compare to point forecast (Strategy 2). The penalty term is further decreased when probabilistic forecasts are available. If we take producer 2 as reference, ρ_L is reduced of around 4% for strategy 3B and 9% for strategy 3A. The difference, mainly if we analyze the performance ratio, may not appear so significant to the reader; nevertheless, for this test case we used very simple methods for the estimation of optimal α . More advances models can help in increasing the performance. Furthermore, in [Zugno et al. \(2013a\)](#) the authors show how the effectiveness of this strategy can be improved by modeling the risk aversion of the wind power producer.

10.6 Overall conclusions and perspectives

In this chapter we presented the trading problem of a wind power producer. Here we consider that the stochastic producer has to access the electricity market under the same rules of conventional generators, thus becoming responsible of its deviations. We have shown how the wind power producer can exploit information from wind and market price forecasts, to maximize his or her expected profit. The stochastic producer offers in the day-ahead market (which ensures more high and stable prices), considering his forecasts on balancing market prices. We have shown how he can exploit all the information he has available and how it can affect his or her profit. We developed our analysis under assumptions that may not be always acceptable, e.g., assumption A3 (price taker). We refer the interested reader to [Zugno et al. \(2013b\)](#) and [Baringo and Conejo \(2013\)](#), where a stochastic mathematical program with equilibrium constraints is used to model the price-maker behavior, both at day-ahead stage and balancing stage ([Zugno et al., 2013b](#)). Furthermore, this chapter considers the typical structure of European electricity markets. Our assumption of uniform prices is not more true if considering US electricity markets. The reader is referred to [Botterud et al. \(2012\)](#) for a formulation of the trading problem of wind production in locational marginal price markets.

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