



# What we'll cover

- Explain the basic assumptions of game theory
- Describe utility functions and find the Nash equilibrium of simple games
- Explain how electricity markets can be gamed
- Model the outcome and impact of gaming in simple market setups
- Explain how gaming is essential in a pay-as-bid market

Point of note: Discussion will be confined to deterministic production (but can be generalized).

But first ...

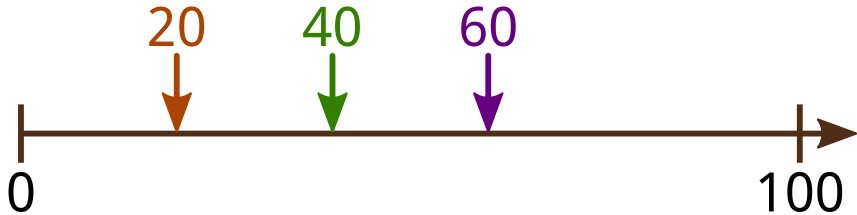


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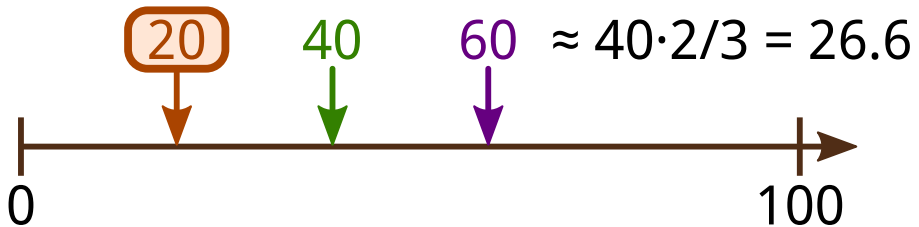
The person who picks the number closest to 2/3rds of the average of all numbers picked will win a prize.

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The person who picks the number closest to  $2/3$ rds of the average of all numbers picked will win a prize.

Type in your guess at:

[bit.ly/elmacourse1](http://bit.ly/elmacourse1)

(Please set your browser to accept cookies.)

# What's in a game?

(Of the game-theoretic kind)

- Multiple players

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- Multiple players
- Each player chooses a strategy
- Each player's utility depends on the chosen strategy of all players
- Each player attempts to choose a strategy which maximizes their utility

## Games are everywhere

Game	Players	Strategy	Utility
Guess 2/3rd	All of you	Chosen number	Win prize

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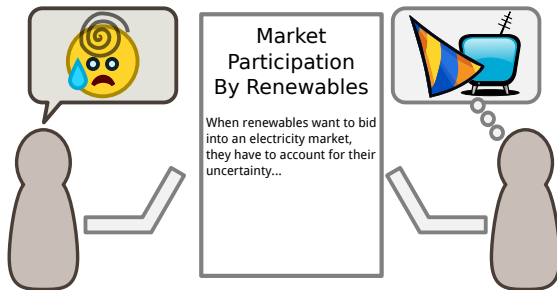
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Which games do you meet in everyday life?

## A classic game

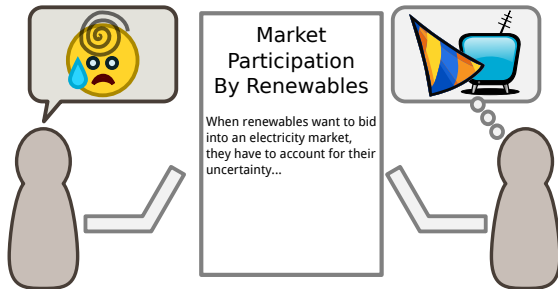


You and your partner are working on the report.

You can choose to either stay up late and work hard, or slack off and hope your partner does most of the work.

Q: Should you work hard?

## A classic game



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Q: Should you work hard?

- If you both work hard, you get a 12.
- If one of you slacks off, you get a 7.
- If both of you slack off, you get a 2.

# What ends up happening?

A combination of strategies is called a Nash Equilibrium if no player wants to change just their own strategy.



John Forbes Nash, Jr.  
(1928–2015)



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		WH	SO
You	WH	<u>12,12</u>	7,7
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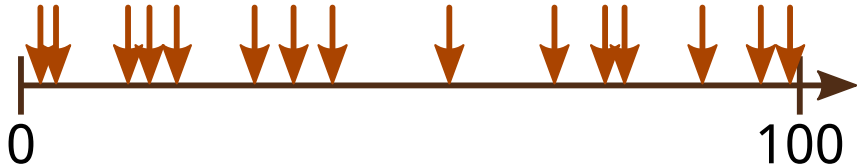


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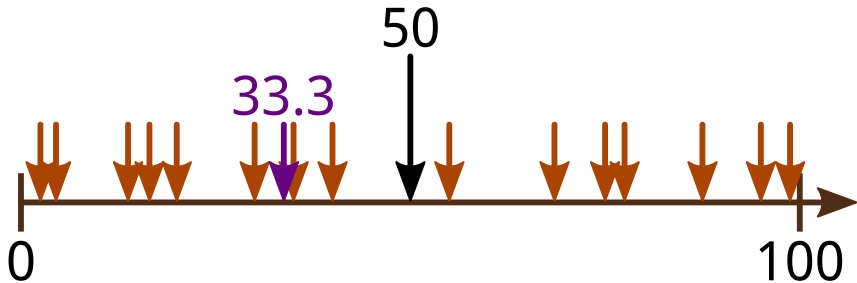
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		Partner	
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You	WH	6,6	1,7
	SO	7,1	<u>2,2</u>

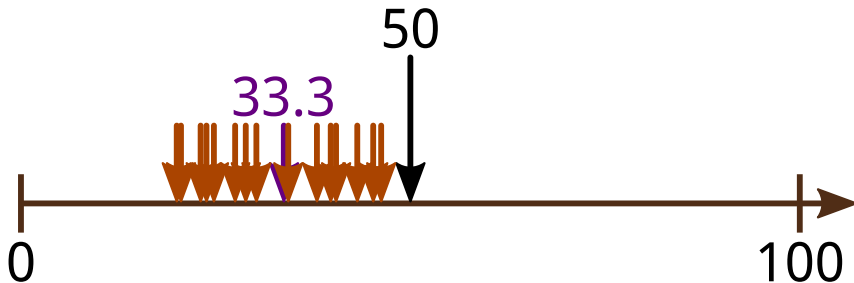
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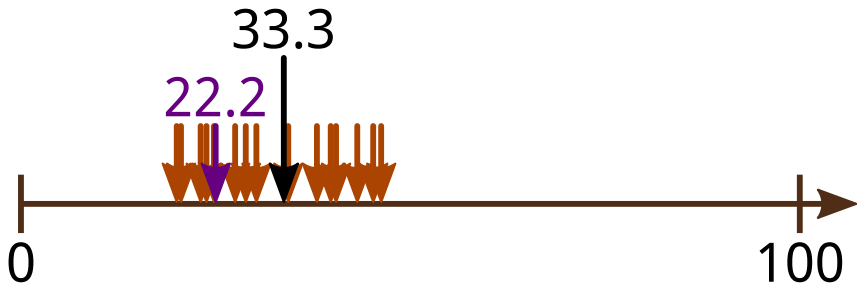
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Nash equilibrium may not be realized in real life!



# BREAK

Next hour: Gaming in electricity markets

# Gaming in electricity markets



## Gaming in electricity markets

Producers offer an amount of energy  $E_i$  and a price  $\pi_i$  they will sell it at.

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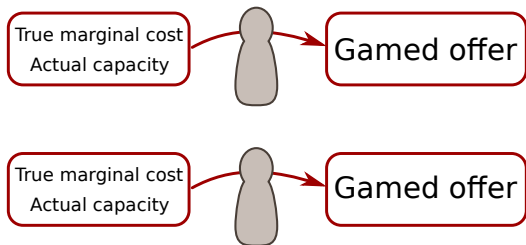
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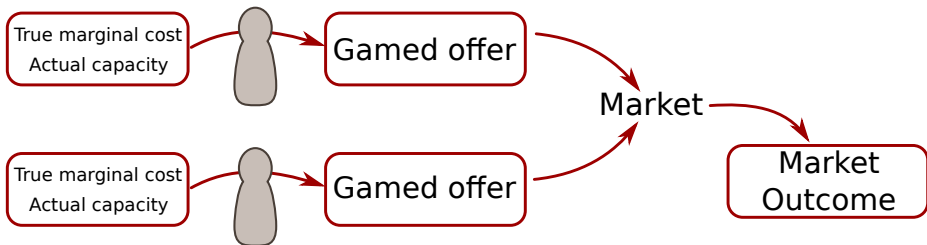
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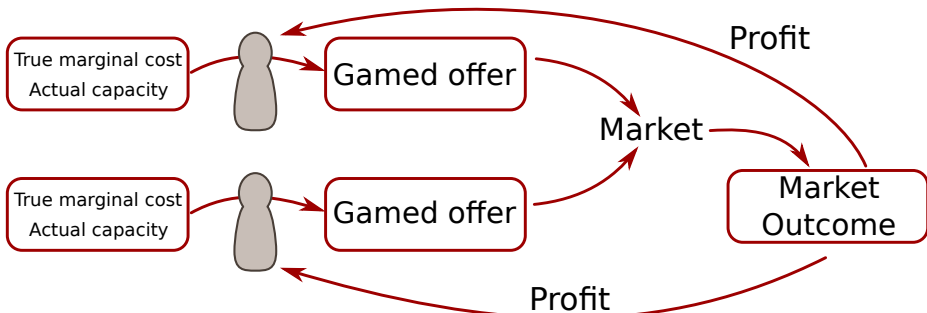
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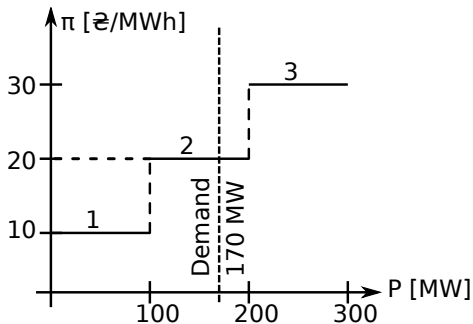
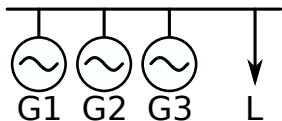
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Market players game using their price and production.





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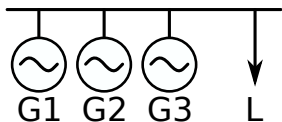
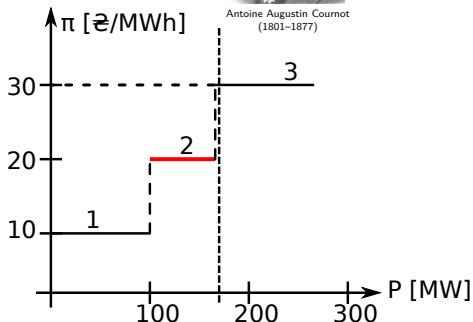
Cournot strategy: Withhold production

→ More expensive marginal producer

→ ??? → Profit!



Antoine Augustin Cournot  
(1801-1877)



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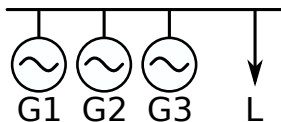
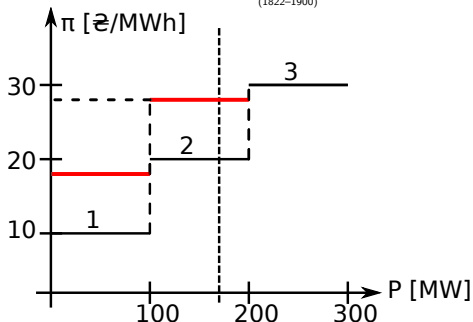
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Bertrand strategy: Increase bid price above marginal cost



Joseph Louis François Bertrand  
(1822–1900)



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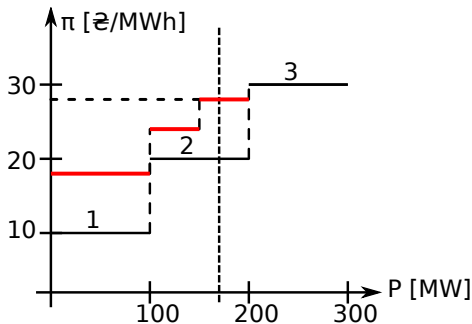
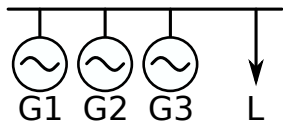
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Hybrid strategies: Both, split bid, ...



## Try your hand at gaming a market

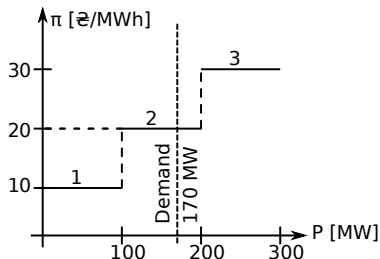
### In groups of two:

One is Generator 1, one is Generator 2.

Secretly write down an offer.

(Offer = Price and quantity)

Reveal at same time, record price and profits.



	Marg. Cost	Capacity
Gen. 1	10	100
Gen. 2	20	100
Gen. 3	30	100

Assume Generator 3 offers full production at marginal cost.

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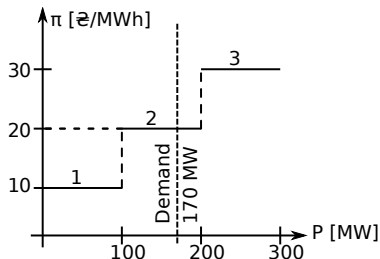
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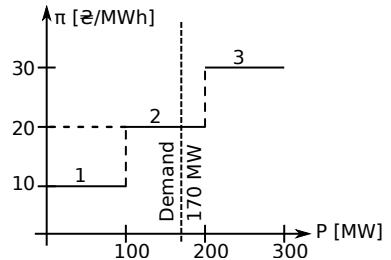
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- What makes gaming hard here?
- Did you ever earn 0 profit?
- Did plant 3 ever earn a profit?

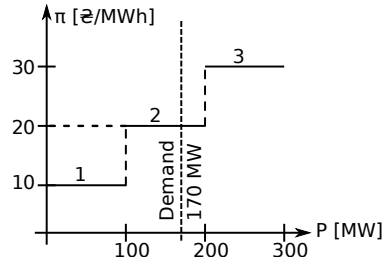
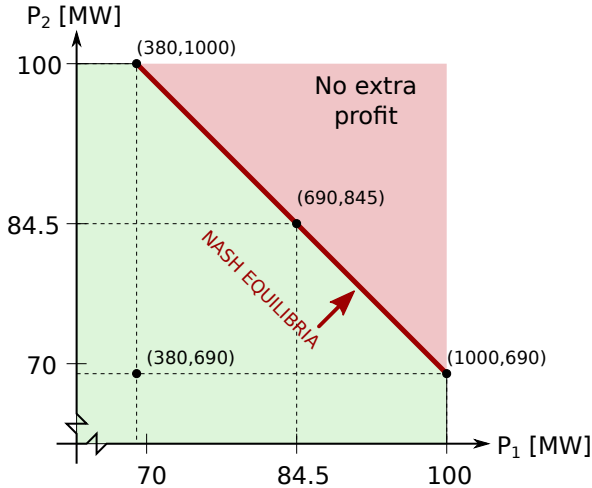
## Example: Cournot strategy only

We can draw the profit earned by gaming of producers 1 and 2 as a function of their offer.



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When a plant influences the market price by changing its bid, it is called a price maker.

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Open questions:

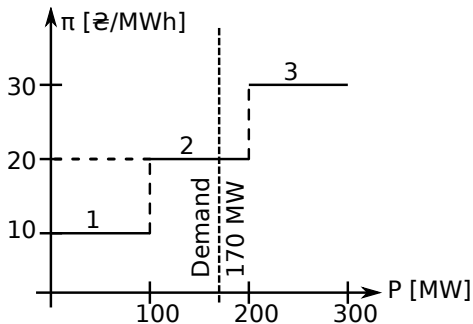
- How can we detect use of market power?
- Can we quantify its impact? Who loses?
- How do we design markets that minimize use of market power?

## Why not pay-as-bid?

Thus far, markets have had uniform pricing.

Merit order, last bid sets price for everyone.

But this is not the only type of pricing!

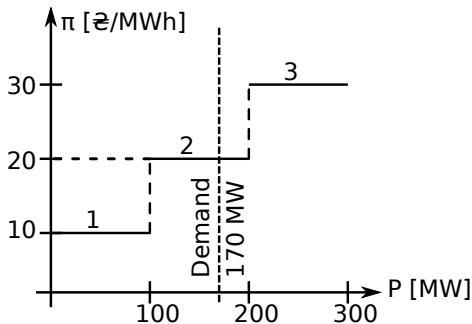


## Why not pay-as-bid?

In a pay-as-bid market you are paid at your bid price.

In order to make any profit at all, you have to game.

$$U_i = (\pi_i - MC_i) * E_i^*$$

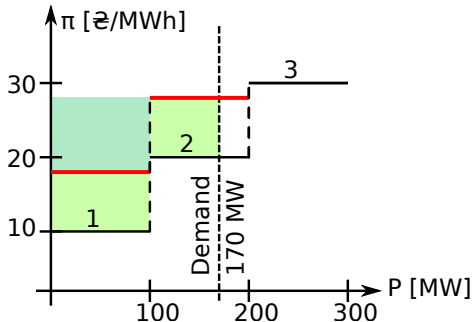


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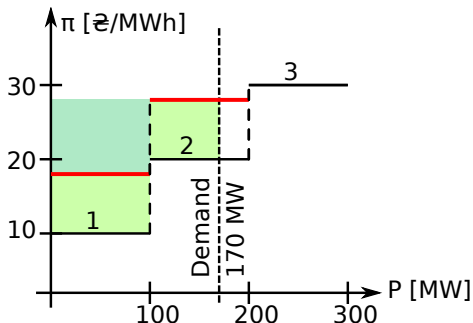
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Let's try it out!

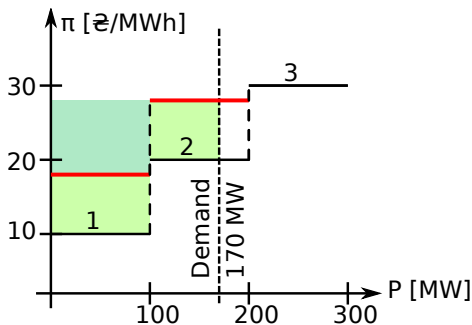
Go to [bit.ly/elmacourse2](http://bit.ly/elmacourse2) and get bidding!



## Pay-as-bid in practice

A player's maximum profit is obtained by correctly guessing the marginal price.

Huge loss if they guess slightly too high.



Can lead to market power for big producers with a diverse portfolio.  
But costs to consumers are more stable!

## Summary

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- The equilibrium strategy is not necessarily the one that will emerge in practice
- Market players game using price and quantity
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What about:

- The transmission grid?
- Ramping?
- Storage?
- Uncertainty?
- Reserves?
- Incomplete information?
- Irrational actors?

Other courses at CEE, Compute,  
Management

## Further resources

Some good search terms Public Good Game, Cournot vs. Bertrand competition, Braess' Paradox, Mixed Nash Equilibrium, Market Power

<https://www.youtube.com/watch?v=nM3rTU927io> Yale lecture series; Thorough, practical introduction to game theory. Many good examples!

[https://www.youtube.com/watch?v=TM\\_QFmQU\\_VA](https://www.youtube.com/watch?v=TM_QFmQU_VA) Stanford lecture series: Designing policies knowing that gaming will occur. More advanced and in depth.

<http://www.iro.umontreal.ca/~marcotte/ARTIPS/AOR2007.pdf> For the OR-interested; using bilevel programs to model competition.