# 31761 - Renewables in Electricity Markets

## Exercise session 6: Verification of renewable energy forecasts - [SOLUTION]

The aim of this exercise session is go through the basics of renewable energy analytics, and to more particularly focus on the verification of forecasts in their various forms. Principles of forecast verification are generic, hence the various approaches discussed can also be used for verifying forecasts for other market quantities (e.g., load and prices). This exercise session relies on Modules 7 and 8 mainly.

### Problem 1: Know your forecasts!

Consider the participation of a 12MW solar power plant in the electricity market. A forecast provider was contracted to provide necessary forecasts for designing and applying various market participation strategies.

1.1 For an optimal-quantile type participation strategy, the nominal level is defined as $\boldsymbol{\alpha} = \mathbf{0.4}$. On a given day before gate closure (say, at noon), the solar power plant operator is provided with the following quantile forecasts for the first 4 market time units of the folowing day:

$$\underbrace{\hat{q}^{(0.4)}_{t+14|t} = 5.3\text{MW},}_{00:00 \to 00:01} \quad \underbrace{\hat{q}^{(0.4)}_{t+15|t} = 9\text{MW},}_{00:01 \to 00:02} \quad \underbrace{\hat{q}^{(0.4)}_{t+16|t} = 11\text{MW},}_{00:02 \to 00:03} \quad \underbrace{\hat{q}^{(0.4)}_{t+17|t} = 10\text{MW}}_{00:03 \to 00:04}$$

What is the predicted probability that solar power generation will be above 9MW for the market time unit between 1am and 2am of the following day?

The predicted probability that solar power generation will be above 9MW between 1am and 2am the following day is:

$$1 - \alpha = 1 - 0.4 = 0.6 \quad \rightarrow \quad 60\%$$

1.2 Since the nominal level of the optimal quantile to offer in the market may change depending on estimated penalties for surplus and deficit, the forecast provider generates a number of quantile forecasts with different nominal levels:

$$\hat{q}^{(0.2)}_{t+14|t} = 5.3\text{MW}, \quad \hat{q}^{(0.3)}_{t+14|t} = 6.9\text{MW}, \quad \hat{q}^{(0.4)}_{t+14|t} = 9\text{MW},$$
$$\hat{q}^{(0.5)}_{t+14|t} = 9.7\text{MW} \quad \hat{q}^{(0.6)}_{t+14|t} = 10.5\text{MW} \quad \hat{q}^{(0.7)}_{t+14|t} = 10.9\text{MW}$$

Deduce the value of the quantile forecasts to offer in the cases where the optimal nominal levels would be set to $\boldsymbol{\alpha} = \mathbf{0.37}$ and $\boldsymbol{\alpha} = \mathbf{0.54}$

Assuming that the relationship is linear between two given quantile forecasts level:

$\alpha = 0.37$

$$\left.\begin{array}{l} \hat{q}^{(0.3)}_{t+14|t} = 6.9\text{MW} \\ \hat{q}^{(0.4)}_{t+14|t} = 9\text{MW} \end{array}\right\} \rightarrow \qquad \hat{q}^{(0.37)}_{t+14|t} = 8.37\text{MW}$$

$\alpha = 0.54$

$$\left.\begin{aligned} \hat{q}^{(0.5)}_{t+14|t} &= 9.7\text{MW} \\ \hat{q}^{(0.6)}_{t+14|t} &= 10.5\text{MW} \end{aligned}\right\} \rightarrow \qquad \hat{q}^{(0.54)}_{t+14|t} = 10.2\text{MW}$$

1.3 Based on the quantile forecasts given in 1.2, what is the central prediction interval with nominal coverage of 20%? and of 40%?

Based on the quantile forecasts given in 1.2, the central prediction interval with nominal coverage of 20% and 40% are given as follows:

Nominal coverage rate of 20%

$$\begin{aligned} 1 - \beta &= 0.2 \rightarrow \beta = 0.8 \\ \underline{\alpha} &= \beta/2 = 0.4 \\ \bar{\alpha} &= 1 - \underline{\alpha} = 0.6 \end{aligned}$$

Nominal coverage rate of 40%

$$\begin{aligned} 1 - \beta &= 0.4 \rightarrow \beta = 0.6 \\ \underline{\alpha} &= \beta/2 = 0.3 \\ \bar{\alpha} &= 1 - \underline{\alpha} = 0.7 \end{aligned}$$

1.4 More "traditional" point forecasts are also provided to the solar power plant operator. For this same set-up, the forecast is $\hat{y}_{t+14|t} = 9.2$MW. Is this supposed to tell you anything in terms of probabilities? How to interpret that forecast?

When the renewable energy forecast issued at time $t$ for $t + k$ is single-valued, it is referred to as a point prediction and denoted by: $\hat{y}_{t+k|t}$ . The fact that this forecast is single-valued makes that point forecasts issued in a deterministic or stochastic process framework look similar. However, they are not in essence. In a deterministic framework, the forecaster is somewhat sure that the prediction ought to realize there is no uncertainty involved. In a stochastic process framework, instead, $\hat{y}_{t+k|t}$ is an estimate only, hence acknowledging the presence of uncertainty.

→ For more details: *Integrating renewables in electricity market* - Chapter 2 - Section 3.2 Point Forecasts, p20

**Problem 2: Verification of point forecasts of wind power generation**

You operate a 15MW wind farm and have a contract with 2 forecast providers, *"Guess-it-all"* and *"Just-doing-my-best"*. Considering a given lead time $k$ (say, 12 hour ahead), you receive a number of forecasts day after day (in MW), which you would like to evaluate:

$$\text{Guess-it-all:} \quad \{2,\ 3.5,\ 4.2,\ 5.6,\ 7.4,\ 5.6,\ 6.4,\ 5.3,\ 6.7,\ 8.6,\ 9.3,\ 4.7\}$$
$$\text{Just-doing-my-best:} \quad \{2.6,\ 3.2,\ 3.6,\ 5.9,\ 6.9,\ 5.7,\ 6.3,\ 5.7,\ 6.1,\ 8.7,\ 9.8,\ 8.3\}$$

The corresponding observations are obtained a posteriori:

$$\{1.8,\ 3.9,\ 4,\ 5.1,\ 7.2,\ 6.1,\ 6.7,\ 5.9,\ 6.6,\ 8.3,\ 10.5,\ 6.2\}$$

2.1 Normalize the forecasts and measurements by the wind farm nominal capacity

→ **Check linked Excel file**

2.2 Calculate forecast errors and normalized forecast errors

→ **Check linked Excel file**

2.3 Evaluate these point forecasts with the common scores that are bias, MAE, and RMSE, calculated both in MW and in their normalized version, i.e., as percentage of installed capacity

$$\text{bias}(k) = \frac{1}{T}\sum_{t=1}^{T} \varepsilon_{t+k|t}$$

$$\text{MAE}(k) = \frac{1}{T}\sum_{t=1}^{T} |\varepsilon_{t+k|t}|$$

$$\text{RMSE}(k) = \sqrt{\frac{1}{T}\sum_{t=1}^{T} (\varepsilon_{t+k|t})^2}$$

→ **Check linked Excel for detailed bias, MAE and RMSE calculation**

2.4 Interpret the value of bias, MAE and RMSE obtained. For instance, do you expect the MAE value to be less than the RMSE one, and why? Can the MAE value relate to an average imbalance? and what about the RMSE one?

The *Bias* being the mean of all errors over the period evaluated, it gives only an information about a systematic error occurring. However an error in one direction (overshooting) can be compensated by an error in the other direction (undershooting) and it will not be possible to identify it only using the *Bias*.

That naturally brings the *Mean Absolute Error (MAE)* which considered the absolute forecast errors instead. That would give an additional information comparing to the *bias*, since a negative (undershooting) and positive (overshooting) error would be identified. One could argue that the *MAE* value is usually greater than the *bias* since errors cannot compensate each other when considering absolute values. The *MAE* is based on a linear loss function

To finish with the *Root Mean Square Error (RMSE)* is the square root of the sum of the absolute forecast errors. Unlike the *MAE* it is defined by a quadratic loss function. Therefore it is expected the *RMSE* to be greater than the *MAE* most of the time.

*<u>N.B:</u> For an easier comparison of these indicator it is recommended to normalize them by the nominal capacity of the renewable energy site considered.*

→ For more details: *Integrating renewables in electricity market* - Chapter 2 - Section 4.2 Point Forecasts, p33

**Problem 3: The case of probabilistic forecasts of wind power generation**

You also get probabilistic forecasts from the same 2 forecast providers ( *"Guess-it-all"* and *"Just-doing-my-best"*) for your 15MW wind farm. At a given time $t$ and for a given time $t+k$, you receive the following quantile forecasts $\hat{q}^{(\alpha)}_{t+k|t}$, $\alpha = 0.1, 0.25, 0.5, 0.75, 0.9$, from both forecast providers:

| Nom. level | Quant. forecasts (MW, Guess-it-all) | Quant. forecasts (MW, Guess-it-all) |
|---|---|---|
| 0.1 | 4.5 | 1.5 |
| 0.25 | 7.5 | 3.75 |
| 0.5 | 11 | 7.5 |
| 0.75 | 13 | 11.25 |
| 0.9 | 14.5 | 13.5 |

The power measurement obtained a posteriori is $y_{t+k} = 12.6$MW.

3.1 Draw the cumulative distribution functions (cdf) for the 2 probabilistic forecasts from Guess-it-all and Just-doing-my-best. How would you also draw the cdf for the actual observation?

$\rightarrow$ **Check linked Excel file**

The *CDF* for the actual observation could be drawn using the *Heaviside step function.* More details on: *Integrating renewables in electricity market* - Chapter 2 - Section 4.3.2 Skill of Probabilistic Forecasts, p38

3.2 Recall the definition of the indicator variable (permitting to assess the reliability of quantile forecasts) and calculate it for $\hat{q}^{(\alpha)}_{t+k|t}$, $\alpha = 0.1, 0.25, 0.5, 0.75, 0.9$ from both forecast providers. Is that enough to assess the reliability of these forecasts?

The indicator variable $\xi^{\alpha}_{t,k}$ for a given quantile forecast $\hat{q}^{(\alpha)}_{t+k|t}$ and corresponding realization $y_{t+k}$ is defined as:

$$\xi^{\alpha}_{t,k} = 1 y_{t+k} < \hat{q}^{(\alpha)}_{t+k|t} = \begin{cases} 1, & \text{if} \quad y_{t+k} < \hat{q}^{(\alpha)}_{t+k|t} \\ 0, & \text{otherwise} \end{cases}$$

i.e., as a binary variable indicating if the quantile forecasts actually cover, or not, the renewable power measurements which is enough to assess the reliability of these forecasts.

3.3 Recall the definition of the Continuous Rank Probability Score (CRPS), and its interpretation based on the cdfs for the forecasts and observation.

The CRPS for predictive densities $\hat{F}_{t+k|t}$ and corresponding measurement $y_{t+k}$, is calculated as:

$$\text{CRPS}(k) = \frac{1}{T} \sum_{t=1}^{T} \int_x \left( \hat{F}_{t+k|t}(x) - H(x - y_{t+k}) \right)^2 dx$$

over an evaluation set of length *T. (x)* is the Heaviside step function, taking the value 1 for $x \geq y$ and 0 otherwise.

The *CRPS* evaluates the area between the predictive cdf and the observation (which would have been the perfect forecast). It is a proper skill score: it is minimal when the true distribution of events is used as predictive density. It is a negatively oriented score (the lower the better) and has the same unit than the variable of interest, while taking a minimum of 0. Note that it can be directly compared to the *MAE* criterion used for point forecasts, since the *CRPS* is its generalization in a probabilistic forecasting framework.

$\rightarrow$ More details on: *Integrating renewables in electricity market* - Chapter 2, Section 4.3.2 Skill of Probabilistic Forecasts, p38

3.4 Then visually in this case, which of the two forecasters would get the lowest CRPS value?

According to the graphs (**See linked Excel file**) *"Guess-it-all"* would get the best CRPS value since it has the smallest area under the curve.

3.5 Would you be able to calculate the CRPS value by hand, or to program a function to do so in your favourite programming language?

Calculating the CRPS by hand would require to integrate by hand ... Otherwise most of the programming languages (Python, Matlab, R) have build in CRPS functions. Otherwise a simple integral function implemented by hand would be suitable.