31761 - Renewables in Electricity Markets

Exercise session 1: Day-ahead electricity markets [SOLUTION]

The aim of this exercise session is to appraise and better understand the basic structure of electricity markets, and most particularly its day-ahead mechanism. The session relies on Lectures 1 and 2 available at "Lecture notes for 31761 - Renewables in Electricity Markets".

Problem 1: General description of a day-ahead market and the example of the Nord Pool

This Problem is based on the Nord Pool website, and more particularly its sets of webpages titled "How does it work?". Some hints and answers are also in the lecture slides, and on the wikipedia page for the Nord Pool

1.1 What is the common name of the day-ahead market in Nord Pool?

The common name of the day-ahead market in Nord Pool is Elspot.

1.2 How many participants are they in the day-ahead market? And how many power producers in the whole area covered by the Nord Pool? Can you explain why all power producers do not trade through the Nord Pool, and how they then proceed?

The power market has many actors involved, such as system operators, producers, distributors, traders, brokers, clearing companies, financial analysts etc. There are approximately 14 million end-users, more than 370 companies responsible for power production, around 500 distribution companies and around 370 companies supplying the Nordic and Baltic end-users of power.
All power producers do not trade through the Nord Pool, as financial market or direct contract between producers and big consumers also exist. These products are traded through Nasdaq Commodities and are used for price hedging and risk management.

1.3 What is the overall volume (on average) of energy generated over the Nordic and Baltic countries?

The annual average generation in the Nordic and Baltic countries is around 420 TWh in total.

1.4 What was the overall amount of energy exchanged through the Nord Pool (Nordic and Baltic) day-ahead market in 2016? What about the UK?

The overall amount of energy exchanged through the Nord Pool (Nordic and Baltic) day-ahead market in 2016 was of 391 TWh.
The overall amount of energy exchanged through the Nord Pool (UK) day-ahead market in 2016 was of 109 TWh.

1.5 What are the various types of power production technologies in the Nordic and Baltic countries? Rank them in terms of marginal production costs (in increasing order). What are the most important ones?

In Norway almost all power is generated by hydro power while Sweden and Finland have a mixture of hydro, nuclear and thermal power (steam driven). Denmark uses predominantly thermal power, but wind power is becoming increasingly important. In Estonia and Lithuania there is mostly thermal driven power. In dry years, Nordic countries become more dependent on the import of power from other countries: Russia, Estonia, Netherlands, Poland and Germany.
In terms of marginal production cost, the ranking in an increasing order is the following: renewables(wind and solar), hydro power, combined heat and power, nuclear, condensing coal, condensing oil and gas turbines.

Each of the power production technologies serves a role in the power supply for the Nordic countries. Renewables help towards reducing carbon emissions and achieving sustainability. Hydro power is also a non polluting technology and serves as an energy reservoir for the Nordic countries. Moreover, nuclear, CHP and coal power production units serve as base and intermediate load units ensuring the security of supply. Finally, oil and gas power production units help with their ramping capabilities when needed or when there is a lack of supply from hydro power.

1.6 Who are the various participants in the day-ahead market? What is their role?

A **producers** is responsible for power production.
A **distributor** ensures that power reaches the end-user. Power is transmitted from the power plant through the central grid and the transmission net to the end-user.
A **supplier** buys power either directly from a producer, or through Nord Pool Spot. In general, a supplier then resells it to small and medium-sized companies and households.
A **trader** represents the entity which owns the power while the trading process is taking place. For example, the trader may buy power from a producer and sell it to a retailer, or the trader may choose to buy power from one retailer and sell it to another retailer. There are many routes from the producer to the end-user.

1.7 Who owns Nord Pool? (see webpage "About us")

Nord Pool Spot AS is owned by the Nordic transmission system operators Statnett SF (28.2%), Svenska Kraftnt (28.2%), Fingrid Oyj (18.8%), Energinet.dk (18.8%) and the Baltic transmission system operators Elering (2%), Litgrid (2%)and Augstsprieguma tikls (AST 2%).

1.8 What is the market time unit (i.e, a few minutes, a whole day, or...?), and what are the bidding areas?

The market time unit is based on an hourly resolution. The bidding areas are the following: Norway is divided in 5 bidding areas, Denmark in two bidding areas (DK east and DK West), Finland, Estonia, Lithuania, Latvia and Sweden that is divided in 4 bidding areas. There is a total of 15 bidding areas.

1.9 What are the assumptions for the definition of bidding areas?

The different bidding areas help indicate constraints in the transmission systems, and ensure that regional market conditions are reflected in the price. Due to bottlenecks in the transmission system, the bidding areas may get different prices called area prices. When there are constraints in transmission capacity between two bidding areas, the power will always go from the low-price area to the high-price area. This principle is right for society: the commodity ought to move towards the high-price where the demand for power is the highest.
This system also secures that no market members are assigned privileges on any bottleneck, which is an important feature of a liberal market. Nord Pool Spot calculates a price for each bidding area for each hour of the following day.

Problem 2: Consideration and ordering of supply offers in a pool

For this and some of the following Problems, we set up our own day-ahead electricity market, with characteristics similar to that of the Nord Pool and analysed in Problem 1. An example of such a day-ahead electricity market was given in Lecture 2.

The market has 5 players on the supply side. For a given time unit (say, between 2pm and 3pm the following day), the market operator has received a set of single-hourly supply offers (i.e., blocks of energy for a single market time unit) from these 5 participants. These offers are defined as:

Supplier name	Supplier id.	Quantity [MWh]	Price [€/MWh]
FlexiGas	G_1	15	75
Nuke22	G_2	100	15
ShinyPower	G_3	32	0
RoskildeCHP	G_4	25	42
BlueWater	G_5	70	10

2.1 What do we call the "supply curve" in the day-ahead market?

The supply curve is also called the bidding curve or the aggregated supply curve. It gathers all the offers on the supply side.

2.2 How is it defined based on a set of offers?

It is the curve that shows all the supply offers aggregated for each delivery hour. The orders are ranked based on an ascending order of price together with the associated amount of energy that will be produced.

2.3 Draw (paper, or plot based on Matlab/R/Excel/etc.) the supply curve for this market time unit.

$\rightarrow$ See Figure 1

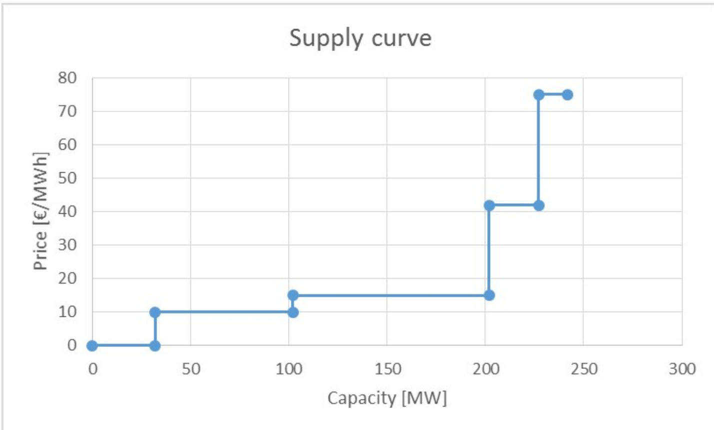

Figure 1: Supply curve

2.4 What is the total amount of energy offered through the market?

The total amount of energy offered is $15 + 100 + 32 + 25 + 70 = 242$MWh.

2.5 By the way... Can they be other types of offers than single-hourly offers?

There also time dependent bids, for instance a producer can offer a bid that is valid for several hours (e.g., regular block orders).

Problem 3: Consideration and ordering of demand offers in a pool

We continue here based on the previous Problem and our day-ahead electricity market setup.

The market has 7 players on the demand side. For the same time unit as in Problem 2 (say, between 2pm and 3pm the following day) the market operator has received a set of single-hourly consumption offers defined as:

Demand name	Demand id.	Quantity [MWh]	Price [€/MWh]
WeLovePower	D_1	35	65
CleanCharge	D_2	23	78
JyskeEl	D_3	12	10
ElRetail	D_4	38	46
QualiWatt	D_5	43	63
IntelliWatt	D_6	16	32
El-Forbundet	D_7	57	50

3.1 What do we call the "demand curve" in the day-ahead market?

The demand curve is based on the same principle than the supply curve besides that it is taken from the "buying side" perspective. Meaning that consumers "bid" their willingness to pay (buy) for the energy. These offers are ranked from the most expensive to the cheapest.

3.2 How is it defined based on a set of offers?

The most "expensive" offers (the players willing to pay the most) are placed at the beginning and all offers are aggregated together as a decreasing demand curve (highest willingness to pay to the lowest willingness to pay for one unit of energy. Demand flexibility is reflected through the demand curve). Players willing to pay the most are the ones that have a prioritized access to the energy that will be available.

3.3 Draw (paper, or plot based on Matlab/R/Excel/etc.) the demand curve for this market time unit.

$\rightarrow$ See Figure 2

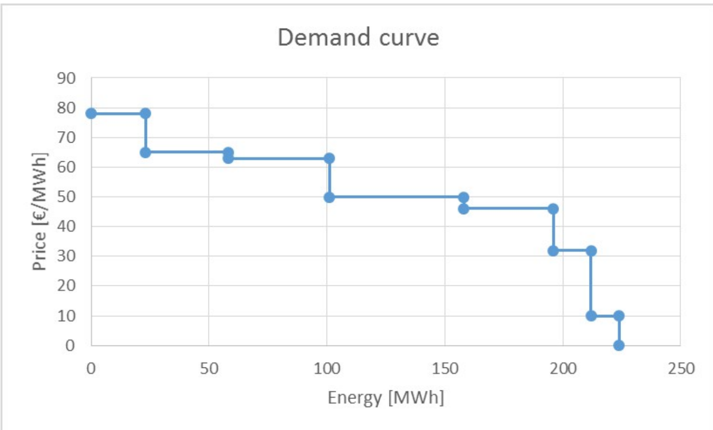

Figure 2: Demand curve

3.4 What is the total amount of energy asked for through the market? Compare it to the total amount of energy supply offers. Is there a problem there?

The total energy asked for through the market is: $35+23+12+38+43+16+57 = 224$ MWh. There does not seem to be a problem since supply is greater than the demand. A market equilibrium can be found which is at the crossing-point between supply and demand curves. There are enough producers willing to produce energy that has been asked for. However both curves have to cross in order to find this market equilibrium.

Problem 4: Equilibrium and market-clearing

We continue here based on Problems 2 and 3, and our day-ahead electricity market setup.

4.1 Have your supply and demand curve on the same drawing/plot.

→ See Figure 3

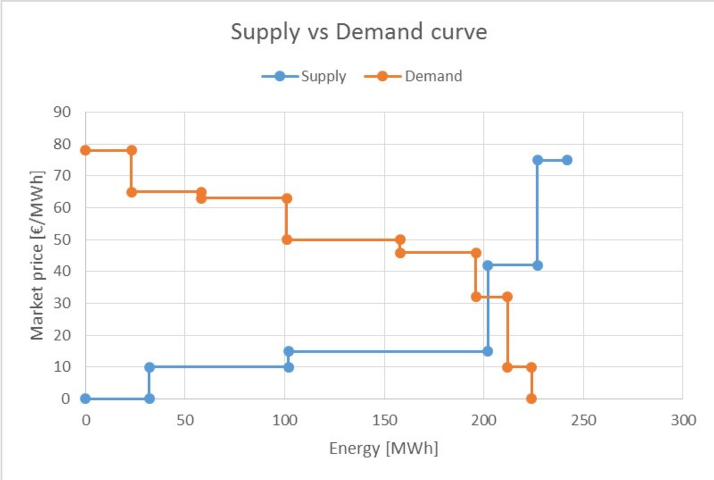

Figure 3: Supply vs Demand curve

4.2 Identify the "equilibrium point". What does it mean in the present case?

The equilibrium point is the crossing point between both curves. In the present case, it means that there are enough producers willing to produce a given amount of energy for which consumers are willing to pay for.

4.3 What is the equilibrium price and quantity?

The equilibrium is at a quantity of **202 MWh** and a price of **32 €/MWh**.

4.4 Who will be effectively supplying power (and how much)? And, who will be effectively consuming (and how much)? Why does that make natural sense?

Producers G3, G5 and G2 will be supplying power at their maximum offers, 32, 70 and 100 MWh respectively. Regarding the demand side, it is not so obvious at first, since the crossing point does not exactly match a producer's offer. Indeed it crosses the supply curve at a "step" (between two different price offers for a given quantity). Consumers D2, D1, D5, D7, D4 and D6 will be consuming 23, 35, 43, 57, 38, and 6 MWh respectively. As it can be noticed, consumer D6 only partially satisfies his demand.
The producers are willing to supply up to 202 MWh at a price of 15€/MWh. However, if the supply has to be increased by 1 unit (+1 MWh to be supplied), the next power producer has to be activated in order to produce this additional unit. This power producer is willing to produce at a price of 42€/MWh. The last consumer's demand can only be partially satisfied as he gets only 6 MWh of energy, which would both match his willingness to pay as well as the supply offers of the producers. If the last consumer's demand has to be met/satisfied fully, a higher price is required for the last unit to be generated. The last consumer is not willing to pay for this additional 1 MWh and the producer is not willing to produce at a lower price than the one he bid in the market.

4.5 Calculate social welfare.

Producers surplus: Sum over the producers of the difference between market price and bidding price multiplied by the produced quantity.
Total = 4264 €

Consumers surplus: Sum over the consumers of the difference between the market price and bidding price multiplied by the quantity bought.
Total = 5102 €

Social welfare (Sum of the consumers and producers surplus) = 4264 + 5102 = 9366 €

Problem 5: Formulating the market clearing more mathematically

Consider the market setup and list of supply offers of Problem 2, while assuming that the electric power demand to be met is fixed to 180MWh.

5.1 What is the most simple way to find the equilibrium point? Intuitively, what is the clearing price, who will produce and how much?

Draw a vertical straight line at the desired quantity of energy. The clearing price is 15 €/MWh and producers G3, G5 and G2 will be producing 32, 70 and 78 MWh respectively (Figure 4).

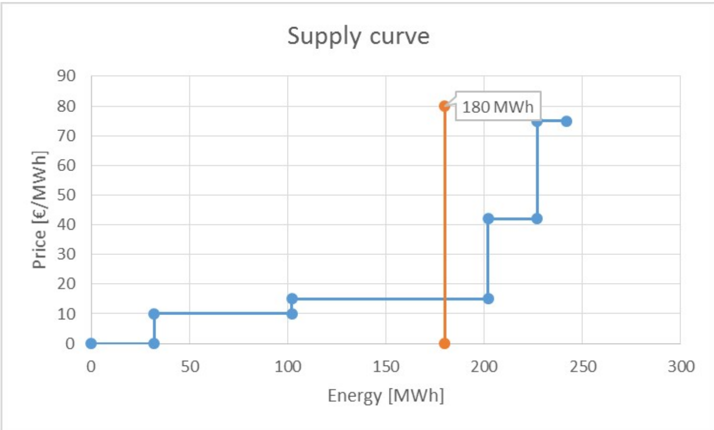

Figure 4: Supply curve with an inelastic demand

5.2 Since demand is fixed, what is the objective of the market clearing with the supply side? Write it as an objective function. Is it a maximization or minimization problem?

As we assume an inelastic demand, the market clearing problem that aims at maximizing social welfare is equivalent to minimizing total production cost. The objective function is:

$\underset{y_j^G}{\text{Minimize}} \ \sum_j \lambda_j^G y_j^G.$

5.3 What is the balance condition for the market (between supply and demand)? Write it as a balance constraint.

The balance condition is that supply should constantly meet demand. The constraint is:

$\sum_j y_j^G = D.$

5.4 Deduce the complete linear program to be used for clearing the market.

$$\begin{aligned}
\underset{y_j^G}{\text{Minimize}} \quad & \sum_j \lambda_j^G y_j^G \\
\text{s.t.} \quad & \sum_j y_j^G = D \\
& 0 \leq y_j^G \leq P_j^{\max}, \ \ \forall j \in J
\end{aligned}$$

As an extension, we now consider the list of demand offers that is given in Problem 3.

5.5 What should be the objective function of the market-clearing (since having to consider both supply and demand sides)? Write it as an objective function. Is it a maximization or minimization problem?

Taking the demand offers given in Problem 3, the demand curve is not a straight line anymore. Therefore the configuration of the market clearing is the following:

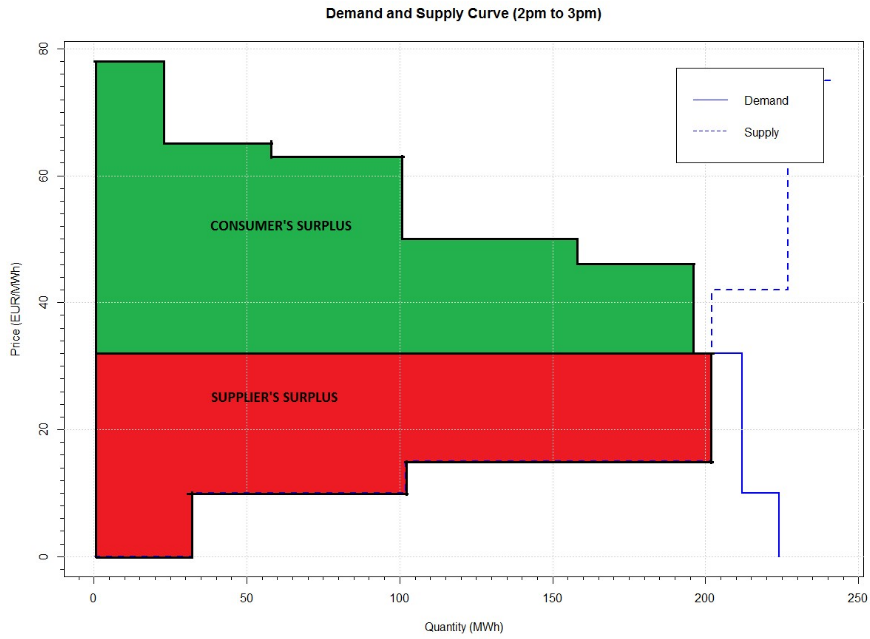

Figure 5: Social welfare maximization with demand/supply offers

As we consider both supply and demand sides, the problem is transformed into a maximization problem (Figure 5). The demand is not a fixed parameter but a set of demand offers that are variables in the optimization problem (notion of "willingness to pay"). The objective function becomes:

$$\underset{y_i^D, y_j^G}{\text{Maximize}} \ \sum_i \lambda_i^D y_i^D - \sum_j \lambda_j^G y_j^G.$$

5.6 What is the balance condition for the market (between supply and demand)? Write it as a balance constraint.

The balance condition is that supply should constantly meet demand. The constraint is $\sum_j y_j^G = \sum_i y_i^D$.

5.7 Deduce the complete linear program to be used for clearing the market.

$$\begin{aligned}
\underset{y_i^D, y_j^G}{\text{Maximize}} \quad & \sum_i \lambda_i^D y_i^D - \sum_j \lambda_j^G y_j^G \\
\text{s.t.} \quad & \sum_j y_j^G = \sum_i y_i^D \\
& 0 \le y_j^G \le P_j^G, \quad \forall j \in J \\
& 0 \le y_i^D \le P_i^D, \quad \forall i \in I
\end{aligned}$$

Feel free to implement those linear programs in R/Matlab/GAMS/etc. in order to verify that you obtain the same solution as in Problem 4. It can only help you for the further work to be done for the first assignment.

Problem 6: Settlement and revenues

The market has been cleared for this time unit (between 2pm and 3pm the following day) based on the list of supply and demand offers given in Problems 2 and 3. It is now time to figure out how much the various participants will be paid, or will have to pay...

6.1 Look through the lecture slides, and define the difference between "pay-as-bid" and "uniform pricing".

Pay as bid means that the producer will get each unit of energy sold, paid at the price he bid in the market. Uniform pricing is the one used for Nord pool. The market is cleared with a common price at which the supply and demand curves are crossing and everyone is selling for that price. Even if their bids were lower than the market price.

6.2 Determine the revenues of various market participants on the supply side under uniform pricing settlement. What if using pay-as-bid instead?

Table 1: Supply side revenues

Supplier name	Supplier id.	Quantity sold [MWh]	Bidding price [€/MWh]	Market price [€/MWh]	Revenue under "Uniform pricing" [€]	Revenue under "pay-as-bid" [€]
ShinyPower	G3	32	0	32	1024	0
BlueWater	G5	70	10	32	2240	700
Nuke22	G2	100	15	32	3200	1500

6.3 Determine the payments for various market participants on the demand side under uniform pricing settlement. What if using pay-as-bid instead?

Table 2: Demand side payments

Demand name	Demand id.	Quantity purchased [MWh]	Bidding price [€/MWh]	Market price [€/MWh]	Payment under "Uniform pricing" [€]	Payment under "pay-as-bid" [€]
CleanCharge	D2	23	78	32	736	1794
WeLovePower	D1	35	65	32	1120	2275
QualiWatt	D5	43	63	32	1376	2709
El-Forbundet	D7	57	50	32	1824	2850
ElRetail	D4	38	46	32	1216	1748
IntelliWatt	D6	6	32	32	192	192

Problem 7: Day-ahead market with 2 zones

Let us now complexify a bit the market set-up and make it more realistic. Our market is now split into two zones (West and East). The various suppliers and demands are associated to these zones as follows:

Supplier name	Zone	Supplier id.	Quantity [MWh]	Price [€/MWh]
FlexiGas	East	G_1	15	75
Nuke22	West	G_2	100	15
ShinyPower	East	G_3	32	0
RoskildeCHP	East	G_4	25	42
BlueWater	West	G_5	70	10

Demand name	Zone	Demand id.	Quantity [MWh]	Price [€/MWh]
WeLovePower	East	D_1	35	65
CleanCharge	East	D_2	23	78
JyskeEl	East	D_3	12	10
ElRetail	East	D_4	38	46
QualiWatt	West	D_5	43	63
IntelliWatt	East	D_6	16	32
El-Forbundet	West	D_7	57	50

The available transmission capacity between these 2 zones is of 30MW. In the following we will assess how this may affect the previous market clearing and revenues that were obtained when not having such transmission constraints.

7.1 Make a schematic representation of the system layout (i.e., the two zones with its players, as well as the transmission constraints between these two).

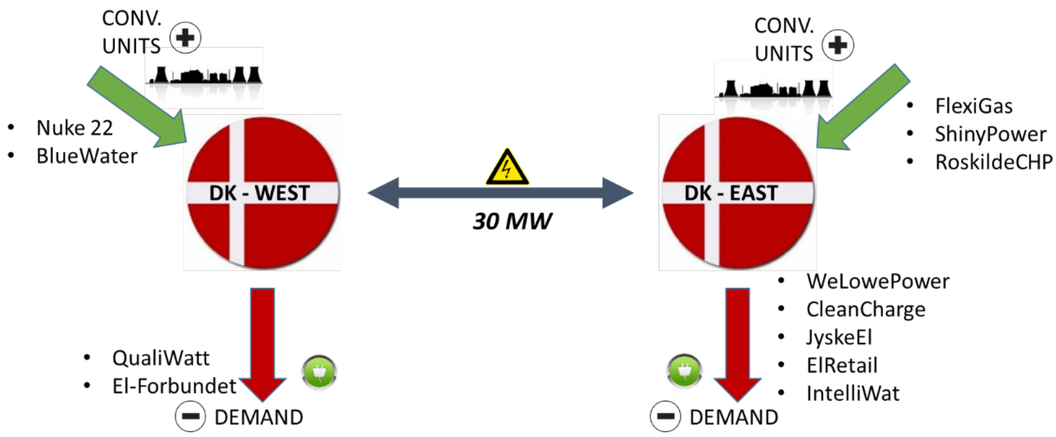

Figure 6: Schematic representation of the system layout

7.2 Assess whether the previous market clearing (from Problem 5) is feasible or not.

Based on previous clearing and resulting schedules for the various suppliers and demands, one can map them on both West and East side:

- Supply on the West side: G5 with 70MWh and G2 with 100MWh
- Demand on the West side: D5 with 43MWh and D7 with 57MWh
- Supply on the East side: G3 with 32MWh
- Demand on the East side: D2 with 23MWh, D1 with 35MWh, D4 with 38MWh and D6 with 6MWh

This translates to a total supply of 170MWh on the West side, for a total demand of 100MWh. On the East side, the total supply is of 32MWh, and the demand is of 102MWh. Consequently, there would be a need to transmit 70MWh from the West to the East. THis is definitely not feasible in view of the capacity of 30MW of the West-East transmission link.

7.3 Obtain the supply and demand curves for both zones.

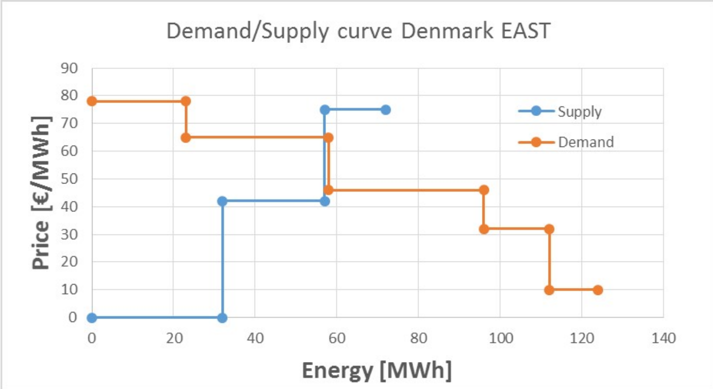

Figure 7: Demand/supply curve for DK-EAST

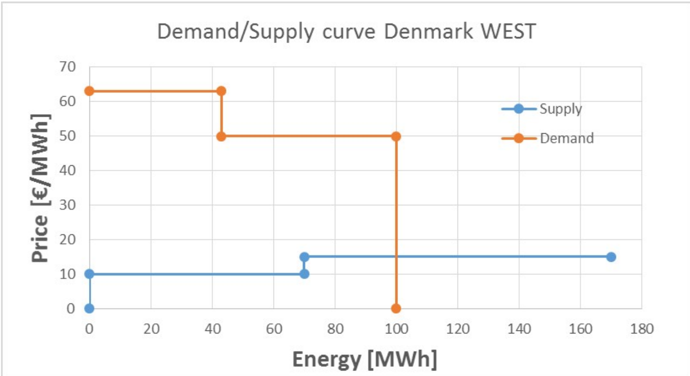

Figure 8: Demand/supply curve for DK-WEST

7.4 Add extra virtual offers representing transfer of power from one zone to the next. From which and to which zone should the power flow?

From Figure 7 and Figure 8 we can notice that DK-EAST is the high-price area and DK-WEST the low-price area. It is know that if transmission between areas is possible, energy should flow from the low-price area to the high-price area (DK-WEST $\xrightarrow[\text{Power flow}]{}$ DK-EAST). Therefore the transmission is added on the supply curve for the high-price area (Figure 9) and to the demand curve on the low-price area (Figure 10).

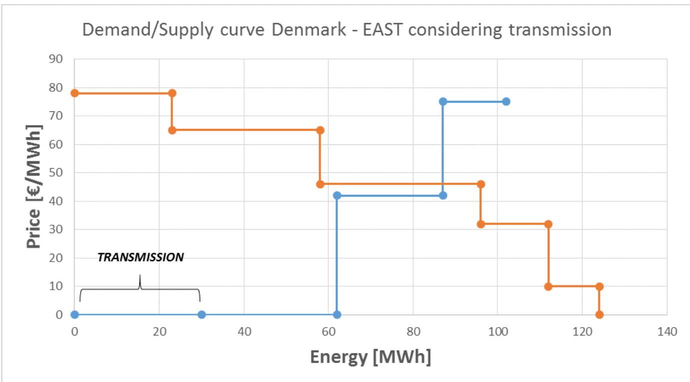

Figure 9: Demand/supply curve for DK-EAST with transmission line

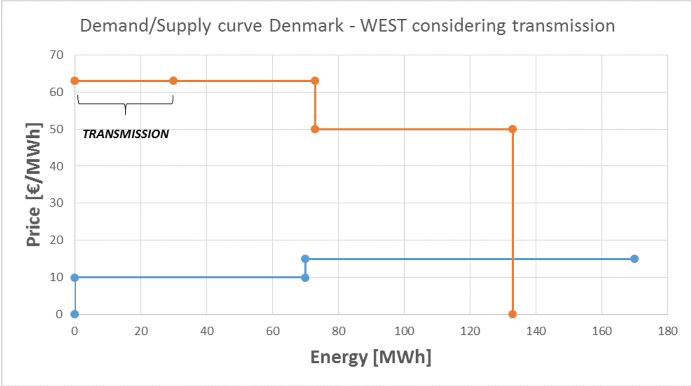

Figure 10: Demand/supply curve for DK-WEST with transmission line

7.5 Determine equilibrium price in both zones. Deduce revenues and payments.

Equilibrium price in DK-EAST (with transmission): 46€/MWh (Figure 9)
Equilibrium price in DK-WEST (with transmission): 15€/MWh (Figure 10)

Equilibrium price in DK-EAST (without transmission): 65€/MWh (Figure 7)
Equilibrium price in DK-WEST (without transmission): 15€/MWh (Figure 8)

Table 3: Producer's revenues with transmission

Supplier name	Zone	Supplier id.	Quantity sold [MWh]	Market price [€/MWh]	Revenue [€]
FlexiGas	East	G_1	0	46	0
ShinyPower	East	G_3	32	46	1472
RoskildeCHP	East	G_4	25	46	1150
Nuke22	West	G_2	60	15	900
BlueWater	West	G_5	70	15	1050

Table 4: Consumer's payments with transmission

Demand name	Zone	Demand id.	Quantity purchased [MWh]	Market price [€/MWh]	Payment [€]
WeLowePower	East	D_1	35	46	1610
CleanCharge	East	D_2	23	46	1058
JyskeEl	East	D_3	0	46	0
ElRetail	East	D_4	29	46	1334
IntelliWatt	East	D_6	0	46	0
QualiWatt	West	D_5	43	15	645
El-Forbundet	West	D_7	57	15	855

7.6 Compare with the case where there was not transmission constraint.

With no transmission the price in DK-WEST remains unchanged. However, in DK-EAST the price is increased to 65 €/MWh

Table 5: Power producer's revenues without transmission

Supplier name	Zone	Supplier id.	Quantity sold [MWh]	Market price [€/MWh]	Revenue [€]
FlexiGas	East	G_1	0	65	0
ShinyPower	East	G_3	32	65	2080
RoskildeCHP	East	G_4	25	65	1625
Nuke22	West	G_2	30	15	450
BlueWater	West	G_5	70	15	1050

Table 6: Consumer's payments without transmission

Demand name	Zone	Demand id.	Quantity purchased [MWh]	Market price [€/MWh]	Payment [€]
WeLowePower	East	D_1	34	65	2210
CleanCharge	East	D_2	23	65	1495
JyskeEl	East	D_3	0	65	0
ElRetail	East	D_4	0	65	0
IntelliWatt	East	D_6	0	65	0
QualiWatt	West	D_5	43	15	645
El-Forbundet	West	D_7	57	15	855

7.7 What would be the minimum transmission capacity needed here for the price to be the same in the 2 zones?

In this case, the minimum capacity should be 70 MW to have the same price in both zones. This is since it represents the power deficit on the East side and power surplus on the West side. With a 70MW transmission, one would not need to split the market and would then end up with a single price for all.

Problem 8: Extract and analyse data for a day-ahead market

Besides some of the basic modelling and market concepts dealt with through the previous problems, a key aspect of working with electricity markets (including the day-ahead stage) is to develop an ability to find and analyse relevant data. In the present problem, emphasis is then placed on extracting data from the Nord Pool website in order to appraise what is going on there.

8.1 Pay a visit to the market data page of the Nord Pool website and have a look at prices in tables in chart for the last cleared day. How similar are prices for the 2 market areas of Denmark? What are the daily variations, and can you explain them?

DK1 (West) tend to have slightly lower prices at some hours of the day compared to DK2 (East). This is due to the high share of wind power installed in that region. Prices are subject to unpredictable generation from production units such as wind turbines.

8.2 One may also download more extensive datasets from the historical market data webpage of the Nord Pool website. There you may for instance get some of the data for 2018 so far:

- Hourly consumption data used at the time of clearing the market,
- Hourly wind power forecasts used at the time of clearing the market,
- hourly market prices as the result of the market-clearing process.

Download these data and choose your favorite data analysis environment (R/Matlab/Excel/etc,).

8.3 Find a typical day with high wind power production in DK1, and look at the corresponding prices. Do the same with a typical day with very low wind power production. Is there something to learn here?

Prices are lower when there is a high wind production compared to a day with almost no wind blowing. This is because wind is integrated on the bottom left of the supply curve with a low bidding price (typically equal to 0). This results in dragging the equilibrium point to a lower point with a reduced market price on the supply/demand curve.

8.4 What is the average day-ahead, also called spot prices, for DK1 (Western Denmark) as a function of the time of the day? Its maximum and minimum? Are they defined limits for these minimum and maximum values (i.e., as set by the market rules)?

8.5 What is the average consumption for DK1 and DK2 (Eastern Denmark) as a function of the time of the day?